**Trigonometry Approximations**

**(approximations.c)**

**Technical Note**

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## Introduction

### Summary

This Application Note documents the functions in file approximations.c. These functions implement highly accurate approximations to inverse trigonometry functions to reduce the computational overhead of using the standard C floating point library functions on integer microcontrollers. The result is in degrees rather than radians saving an additional multiplication to convert from radians to degrees.

The benchmarks in the following table were measured on the Freescale FRDM-KL25Z board which uses a 32 bit ARM M0+ integer core running at 48MHz (giving 48 million clock ticks per second).

|  |  |
| --- | --- |
| **C99 library function** | **approximations.c** |
| float asinf(float x)  4000-6000 clock ticks | float fasin\_deg(float x)  3000-4000 clock ticks |
| float acosf(float x)  4000-6000 clock ticks | float facos\_deg(float x)  3000-4000 clock ticks |
| float atanf(float x)  3800-4800 clock ticks | float fatan\_deg(float x)  1900-3500 clock ticks |

### Functions

|  |
| --- |
| float fasin\_deg(float x);  Inverse sine function (deg) range -90 deg to 90 deg |
| float facos\_deg(float x);  Inverse cosine function (deg) range 0 deg to 180 deg |
| float fatan\_deg(float x);  Inverse arctangent (deg) range -90 deg to 90 deg |
| float fatan2\_deg(float y, float x);  Inverse arctangent (deg) range -180 deg to 180 deg |
| float fatan\_15deg(float x);  Inverse arctangent for range -15 deg to +15 deg only |

## Mathematics

### Approximation to Inverse Sine (-90 deg to +90 deg)

Function fasin\_deg computes the inverse sine of as the inverse tangent of the new argument . The overhead of the square root and division is still less than the overhead of the standard C inverse sine function.

Putting into the definition of the tangent gives:

Eq 2.1.1

### Approximation to Inverse Cosine (0 deg to +180 deg)

Function facos\_deg allows the inverse cosine of to be determined as the inverse tangent of the new argument . Since the inverse tangent returns an angle in the range -90o to +90o, 180o is added if the argument is negative to give the inverse cosine in the range 0o to 180o.

Putting into the definition of the tangent gives.

Eq 2.2.1

### Approximation to Inverse Tangent (-90 deg to +90 deg)

The general inverse tangent for angles in the range -90o to +90o is computed in function fatan\_deg which successively maps its argument to the inverse tangent of an angle in the range 0o to +15o.

Negative arguments are mapped to positive arguments using the identity:

Eq 2.3.1

An argument greater than 1 (implying an angle above +45o) is mapped to argument less than 1 (implying an angle below +45o) using the identity:

Eq 2.3.2

The new argument is then compared with . If the angle is above 15o (in the range 15o to 45o) then it is mapped to the range -15o to 15o using the identity:

Eq 2.3.3

Substituting gives:

Eq 2.3.4

With the substitution :

Eq 2.3.5

Once the inverse tangent is computed in the range -15o to 15o, the manipulations applied in equations 2.3.1 to 2.3.5 are then reversed to give an angle in the range -90o to 90o.

### Pade[3, 3] Approximation to Inverse Tangent (-15 deg to +15 deg)

The Pade[3,3] rational approximation to the inverse tangent expanded about is:

Eq 2.5.1

Eq 2.5.2

Equation 2.5.2 is anti-symmetric as required. It is used in function fatan\_15deg with slightly modified Pade parameters selected to minimize the maximum error in the range -15o to +15o.