

A26 Vest

$$f_x(x) := \begin{cases} 1 - \frac{x}{2} & \text{für } 0 \leq x \leq 2 \\ 0 & \text{sonst} \end{cases}$$

$$f_x(1) = \frac{1}{2} \quad (P(1) = 0! \text{ thx @ Konsti})$$

$$P(X \geq 1) = \int_1^2 1 - \frac{x}{2} dx = 1 - \frac{1}{2} \int_1^2 x dx = 1 - \frac{1}{2} \cdot \frac{2^2 - 1^2}{2} = \frac{1}{4}$$

$$P(0,5 \leq X \leq 1,5) = \int_{0,5}^{1,5} 1 - \frac{x}{2} dx = 1 - \frac{1}{2} \int_{0,5}^{1,5} x dx = 1 - \frac{1}{2} \cdot \frac{1,5^2 - 0,5^2}{2} = \frac{1}{2}$$

$$\begin{aligned} E(X) &= \int_0^2 x \cdot \left(1 - \frac{x}{2}\right) dx = \int_0^2 x - \frac{x^2}{2} dx = \int_0^2 x - \frac{1}{2} \int_0^2 x^2 \\ &= \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^3}{3} \Big|_0^2 = 2 - \frac{1}{2} \cdot \frac{8}{3} = 2 - \frac{8}{6} = \frac{2}{3} \end{aligned}$$

$$\text{Var}(X) = E(X^2) - E(X)^2$$

$$\begin{aligned} E(X^2) &= \int_0^2 x^2 \cdot \left(1 - \frac{x}{2}\right) dx = \int_0^2 x^2 dx - \frac{1}{2} \int_0^2 x^3 dx = \frac{8}{3} - \frac{1}{2} \cdot \frac{2^4 - 0^4}{4} \\ &= \frac{8}{3} - \frac{1}{2} \cdot 4 = \frac{8}{3} - \frac{6}{3} = \frac{2}{3} \end{aligned}$$

$$\text{Var}(X) \Rightarrow \frac{6}{9} - \frac{4}{9} = \frac{2}{9}$$