Niklas Vest - A13

Given are the disjoint events A1, A2 and A3 and the (also disjoint) events B1 and B2 about which we know the following:

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P[A1] + P[A2] + P[A3] = 1
P[B1] + P[B2] = 1
```

This means one event of the set { A1, A2, A3 } must occur, which also applies to the set { B1, B2 }. I will take advantage of this information later on.

Furthermore, we know:

```
P[A2] = 0.3
P[B2] = 0.8

P[A2 ⋂ B2] = 0.2
P[A3 ⋂ B2] = 0.2

P[B2 | A1] = 0.8
```

Missing Values

The first task is to derive the values of

- P[A1]
- P[B1]
- P[A1 AND B1]
- P[A1 GIVEN B1]
- P[B1 GIVEN A1]

P[B1]

Since B1 and B2 are complete and B2 is known,

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P[B1] = 1 - P[B2] = 0.2
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(P[A1 ⋂ B2])

Similarly, because A1, A2 and A3 are disjoint and complete, we know that one of the aforementioned events must occur and that the set { A1, A2, A3 } is a partition. With this in mind, we can think of the probability of the concurrent occurrence of B2 and any of the events in said partition as $\sum_{i=1}^{3} P[A_i \cap \text{ B2}]$. Keep in mind that sum means OR and OR means union! This specific consideration of "All A_i AND B2" is the same as B2 due to the completeness of A1, A2 and A3. Hence we can derive P[A1 ⋂ B2]:

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P[A1 ⋂ B2] + P[A2 ⋂ B2] + P[A3 ⋂ B2] = P[B2] = 0.8
P[A1 ⋂ B2] + 0.2 + 0.2 = 0.8
P[A1 ⋂ B2] = 0.4
```

P[B1 | A1]

Given that event A1 occurred, we know that B2 will occur with a probability of 80%, meaning that for the other 20 out of 100 times, B1 will occur since they are complete.

$$P[B1 \mid A1] = 1 - P[B2 \mid A1] = 0.2$$

P[A1]

Using our knowledge of P[B2 | A1] and P[A1 ∩ B2] we can use the formula from definition 7.7

$$P[X \mid Y] = \frac{P[Y \cap X]}{P[Y]}$$

and solve for P[Y] to get P[A1]

$$P[B2 \mid A1] = \frac{P[A1 \cap B2]}{P[A1]}$$

$$0.8 = \frac{0.4}{P[A1]}$$

$$P[A1] = \frac{0.4}{0.8} = 0.5$$

P[A1 ∩ B1]

Since we are now aware of P[A1 ∩ B2] and P[A1] we can use the same formula to get P[A1 ∩ B1]:

$$P[A1 \cap B1] = P[B1\ \text{GIVEN}\ A1] * P[A1]$$
$$P[A1 \cap B1] = 0.2 * 0.5 = 0.1$$

P[A1 ∩ B1]

Lastly, using a consequence of definition 7.7, namely

$$P[X \mid Y] * P[Y] = P[Y \mid X] * P[X]$$

we can calculate the final missing information:

$$P[A1 \mid B1] * P[B1] = P[B1 \mid A1] * P[A1]$$
$$P[A1 \mid B1] * 0.2 = 0.2 * 0.5$$
$$P[A1 \mid B1] = 0.5$$

Dependencies

A1 and B1 are **independent** because

$$P[A1 \cap B1] = P[A1] * P[B1]$$

A1 and B2 are **independent** because

$$P[A1 \cap B2] = P[A1] * P[B2]$$

... I did not expect you to read this far, I did not work out the remaining exercise.. sowwy :(