AAE 538: Air-Breathing Propulsion

Lecture 9: Analysis of Ideal Engine Cycles

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Introduction

- Now we will investigate both ideal and real engine cycles of air-breathing engines.
  - As the name suggests, ideal engines are 'perfect world' representations of real engines. The properties of an ideal engine include:
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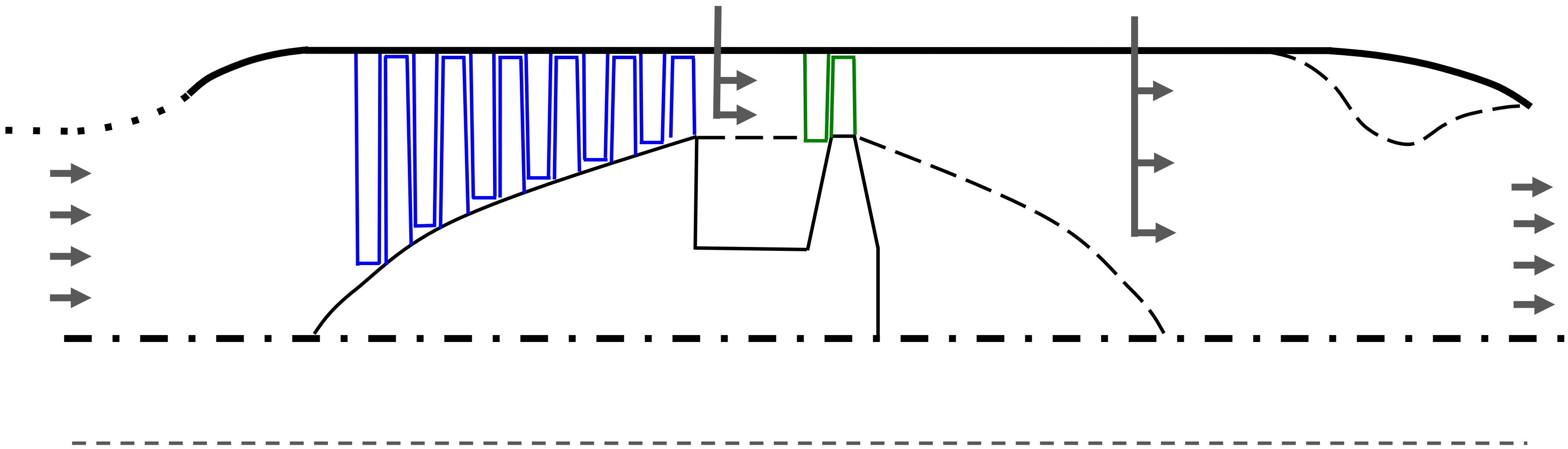
    -
    -
    -
  - As we know very well, these assumptions are quite restrictive, but they permit us to rapidly (and cheaply) analyze the effects of design limitations (such as the turbine inlet temperature), flight conditions, and design parameters (like the compressor pressure ratio) on the tradeoffs in real engine design.
  - To study the performance of real engines, we will introduce ____, quantified relative to this ideal condition. By handling dissipative phenomena in this manner, we have a powerful tool to accurately predict real engine performance. Of course, this requires us to actually ____ these components efficiencies.

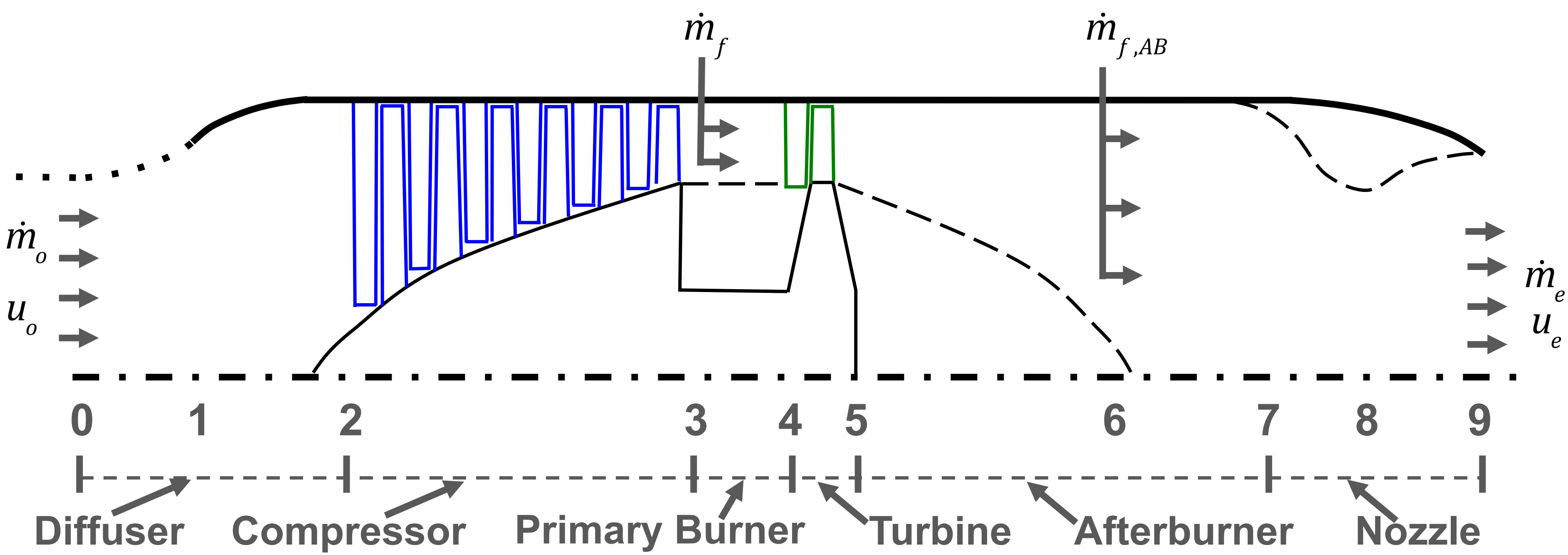
The Ideal Turbojet Cycle

- Stations throughout turbojet engine for the proceeding analysis based on SAE Aerospace Recommended Practice (ARP 755A)
  - 0: Free-stream (far upstream)
  - 1: Diffuser Inlet
  - 2: Compressor Inlet
  - 3: Combustor Inlet
  - 4: Turbine Inlet
  - 5: Turbine Exit
  - 6: Afterburner
  - 7: Nozzle Inlet
  - 8: Nozzle Throat
  - 9: Nozzle Exit



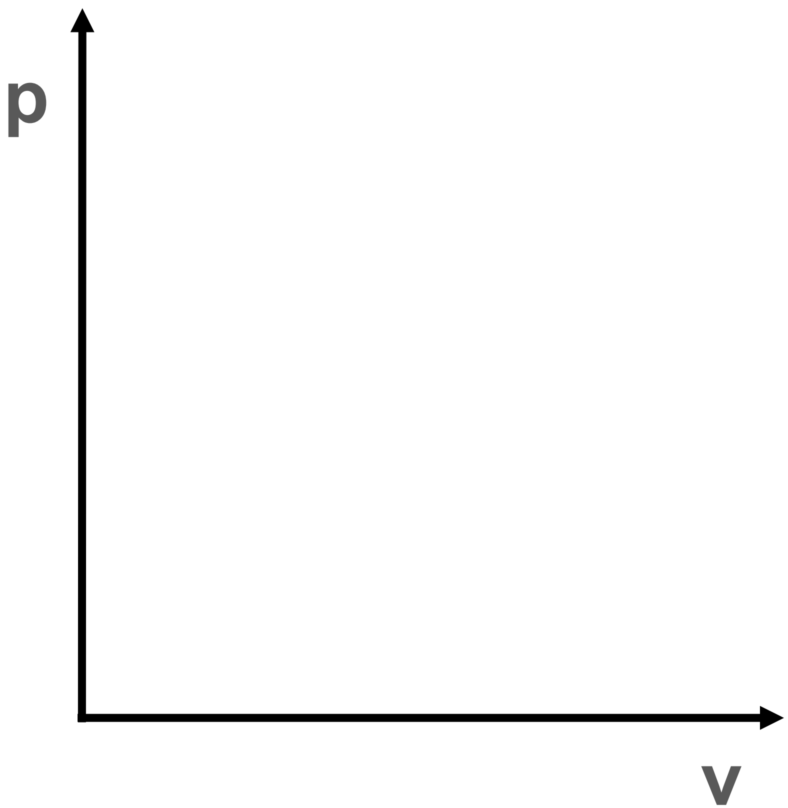
The Ideal Turbojet Cycle

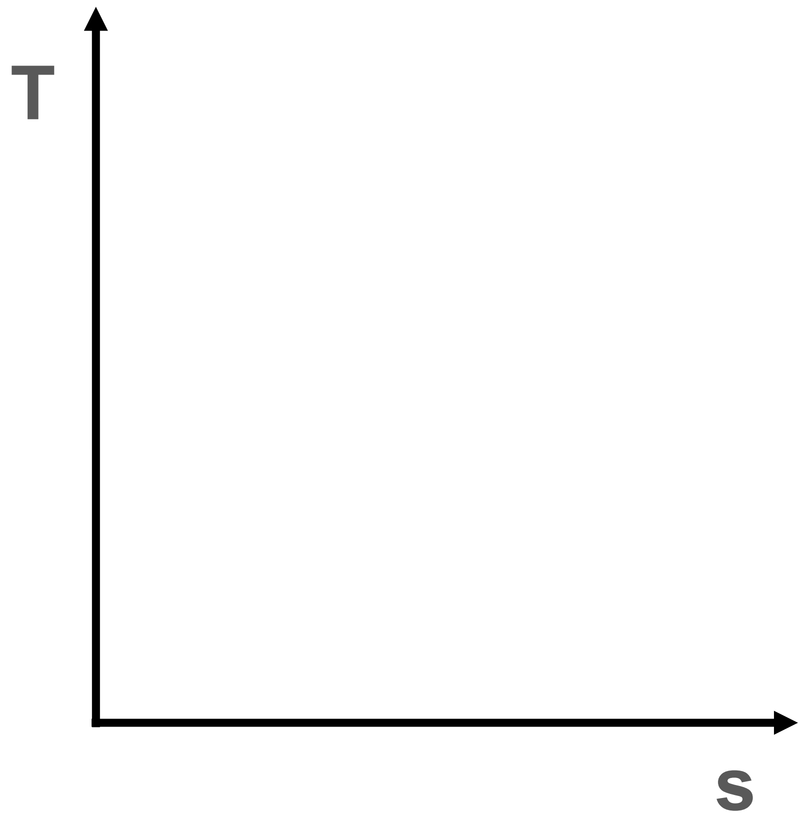
- To establish our ideal engine cycle analysis (reference case), we represent the processes inside the engine as follows:
  - Isentropic Compression
  - Constant Pressure Combustion
  - Isentropic Expansion
  - Constant Pressure Heat Rejection
- Note the similarity with our Air-Standard Brayton cycle, but with additional system processes that we can now analyze within that same framework.



The Ideal Turbojet Cycle

- Beginning with the non-afterburning case:
  - 0-2 and 2-3: Isentropic Compression
  - 3-4: Constant Pressure Combustion
  - 4-9: Isentropic Expansion
  - 9-0: Constant Pressure Heat Rejection





The Ideal Turbojet Cycle

- Defining some nomenclature, before we get started:
  - It is useful for us to define changes throughout the engine in terms of ratios
    - Just like with our canonical compressible flow analyses (nozzle flows, Fanno flows, and Rayleigh flows), we can work through the engines station-by-station, process-by-process, then compute the overall change by combining these ratios.
    - Defining the change in ___________ conditions through a given process in the engine:
  - e.g. relating states 2 and 3 across the compressor (c)

The Ideal Turbojet Cycle

- These characters also represent the 'recovery' temperature and pressure ratios; ____ noted with a subscript r and are functions of M_0, only.

$$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2 =$$

$$\pi_r = \left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{\frac{\gamma}{\gamma - 1}} =$$

  - Thus the freestream stagnation conditions can be represented as:

and

The Ideal Turbojet Cycle

- Beginning a step-wise analysis through the engine
  - Diffuser
    - Consistent with the assumptions for an ideal engine, the ideal diffuser would decelerate and compress free-stream gases in an isentropic manner.
    - Recall for an adiabatic process with no shaft work, ______________________________ ______________________________ .

    So that for constant specific heats, we obtain

  Furthermore, if the process is also reversible (and, hence, isentropic), the pressure and temperature are related through our isentropic relations

The Ideal Turbojet Cycle

- Compressor
  - In the ideal cycle the compressor is modeled as an adiabatic, reversible (hence, isentropic) compression device. Under these assumptions the stagnation pressure ratio and stagnation temperature ratio are related through isentropic relations.

- Burner
  - The ideal combustor is modeled such that there is no stagnation pressure drop across this component; i.e. __________. Recall from Rayleigh flow that:

So that π_b is unity only when ___________. This corresponds to the case where there is no heat addition at all. However, if M_3 is small, changes in Mach number across the burner are also quite small. Furthermore, the combustor does not necessarily act as a Rayleigh flow (with fixed inlet Mach number). Nevertheless, regardless of BC, we typically burn at low M and in this case we can write:

The Ideal Turbojet Cycle

- Turbine
  - In the ideal cycle the expansion through the turbine is modeled as an adiabatic, reversible (hence, isentropic) process. Under these assumptions the stagnation pressure ratio and stagnation temperature ratio are related through isentropic relations.

- Nozzle
  - As in the diffuser, the ideal nozzle flow is modeled as an isentropic process with no flow work so that the stagnation temperature and stagnation pressure are constant:

Isentropic Rel. of Stagnation Prop.

- Pausing for a moment here, recall that for an isentropic process:

$$\Delta s = 0 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$$

From which we can derive the isentropic relations for a ____________________ .

$$\frac{\mathrm{p}_2}{\mathrm{p}_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

and the stagnation-to-static temperature and pressure ratios:

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

- The expressions relate the static conditions through some process, but in this analysis, we are focused on the stagnation quantities. The stagnation quantities give us more useful information based on our analysis of the governing equations
  -

Isentropic Rel. of Stagnation Prop.

- So, as we are defining these ratios through various processes in the engine in terms of stagnation conditions, we better make sure it's true…
  - Taking the turbine as an example, here's the proof:

Isentropic Rel. of Stagnation Prop.

The Ideal Turbojet Cycle

- Using the component information, we can relate the nozzle exit properties to the free-stream conditions
  - Consider the stagnation to static temperature ratio at the nozzle exit

    Since the nozzle flow is isentropic, we can also relate this temperature ratio to the pressure ratio

    Equating these two expression gives

The Ideal Turbojet Cycle

- Now, let's relate the exit Mach number to the inlet condition
  - We just wrote that

$$\frac{T_{o,9}}{T_9} = \tau_r \tau_c \tau_t$$

From the definition of the ______________________________, we can write

Combining these equations, we develop a relation between the inlet and exit Mach numbers

The Ideal Turbojet Cycle

- With both Mach number and temperature change across the engine, we can now compute the change in _______________

- We want to know cast thrust in terms of our dimensionless parameters (setting up for real engine analysis)
  - Recalling from previous notes

$$T = \dot{m}(u_9 - u_0) = \dot{m}u_0\left(\frac{u_9}{u_0} - 1\right)$$

The Ideal Turbojet Cycle

- Substitution gives

- The specific thrust (or specific impulse) can then be obtained by dividing the thrust equation by the mass flow ($\dot{m}$) or the weight flow ($\dot{m}g$) for English units

The Ideal Turbojet Cycle

- We can gain further information about the turbine stagnation pressure and temperature by considering the ________________
  - In a turbojet, the power output from the turbine must equal the power input to the compressor; i.e. all of the the work provided by the turbine goes into driving the compressor of the ideal turbojet. We represent this as:

    Assuming constant specific heats (as we do throughout our ideal engine), we can divide through by $T_{o,3}$

    Solving for τ_t, we find

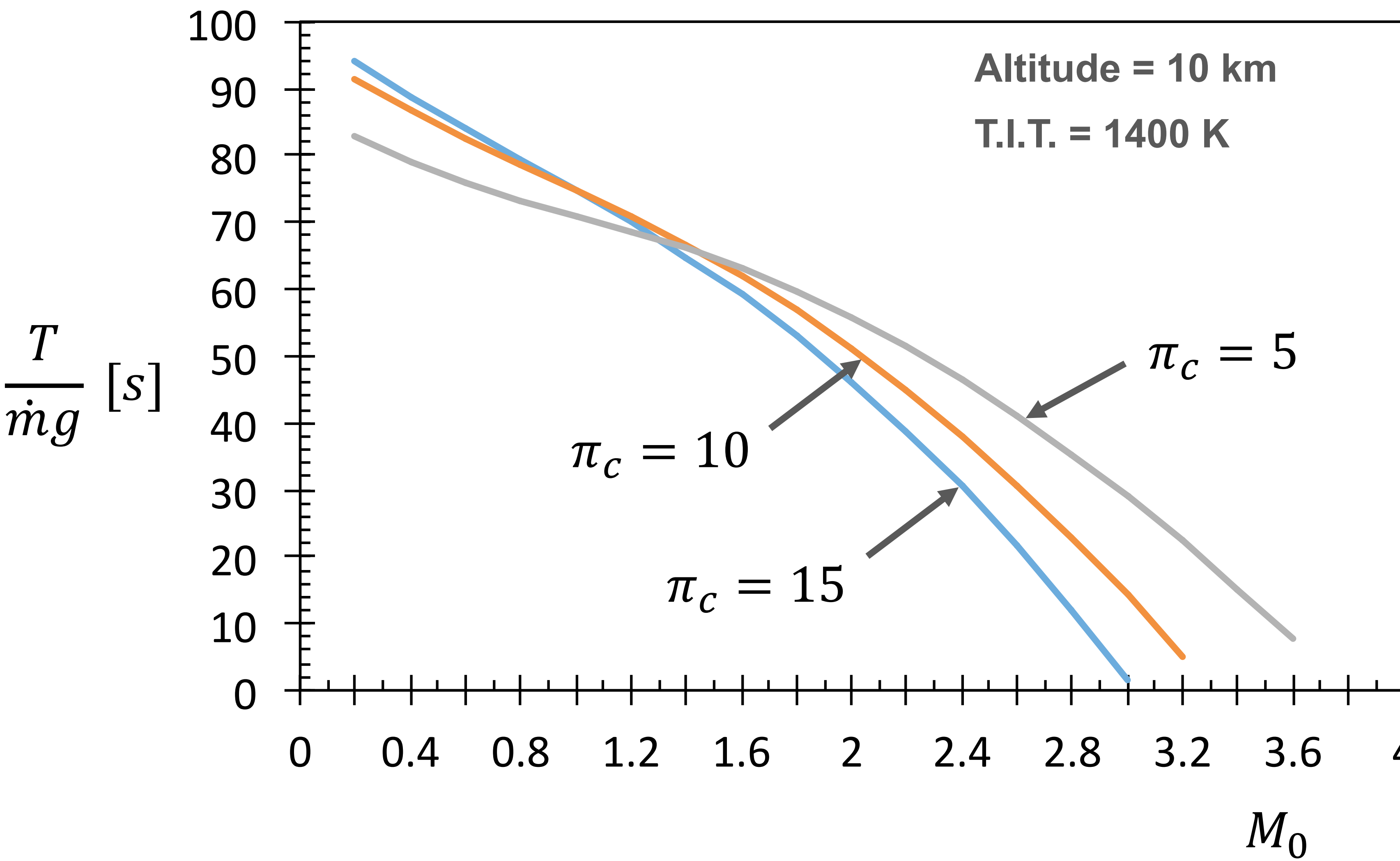
The Ideal Turbojet Cycle

Substituting the result for τ_t into the equation for specific thrust

$$\frac{T}{\dot{m}g} = \frac{a_0 M_0}{g}\left(\sqrt{\frac{\tau_r}{\tau_r - 1}(\tau_b - 1)(\tau_c - 1) + \tau_b} - 1\right)$$

Where we have now related the aircraft operating conditions (M_0, τ_r, and a_0) and engine design characteristics ($\tau_b$and τ_c) to the specific thrust of the engine.

The Ideal Turbojet Cycle



The Ideal Turbojet Cycle

- We previously defined the Specific Fuel Consumption as

$$S = \frac{\dot{m}_f}{T} =$$

  - Substituting the specific thrust into this relation

The Ideal Turbojet Cycle

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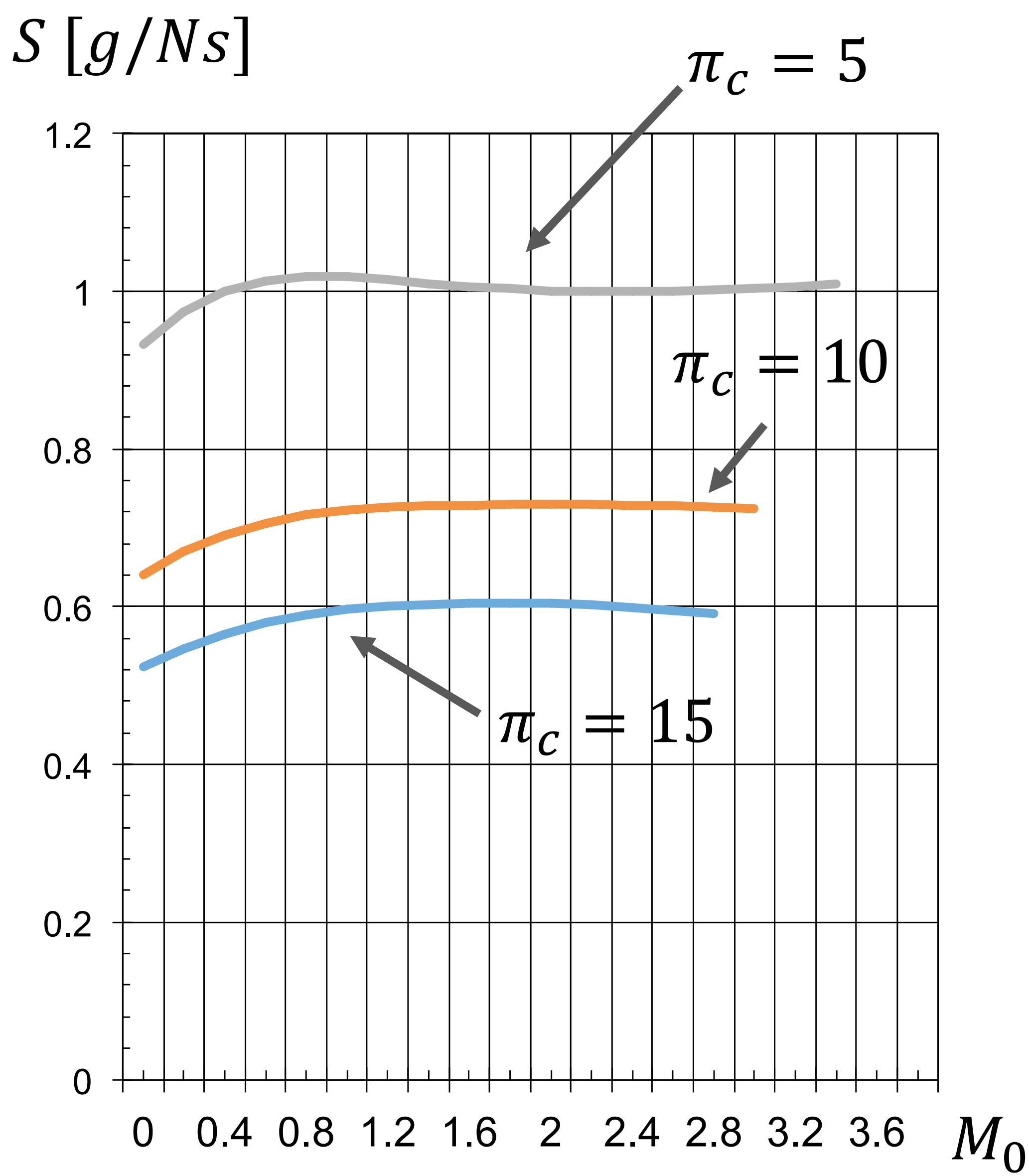
- We can express the fuel/air ratio in terms of the heating value of the fuel by writing the energy equation across the combustor for f << 1.

$$\dot{m}c_p\left(T_{o,4} - T_{o,3}\right) = \dot{m}_f h \qquad \textbf{or} \qquad c_p T_{o,3}(\tau_b - 1) = f h$$

- If $f \ll 1$, $\mathrm{h} = \mathrm{q} = \Delta H_B$. Therefore, by solving for f and substituting for $T_{o,3}$, we can show that

- Substituting back into the equation for S, we can now represent the specific fuel consumption in terms of the performance of each stage, throughout the engine.

The Ideal Turbojet Cycle



Altitude = 10 km

T.I.T. = 1400 K

H = 18000 cal/g

The Maximum Thrust Turbojet

- We can represent the turbine inlet temperature by a group of temperature ratios

$$\frac{T_{o,4}}{T_0} = \tau_r \tau_c \tau_b$$

We will define this parameter as:

Evidently, the minimum possible turbine inlet temperature is realized when no combustion takes place

Consequently, for a given, maximum allowable T.I.T., τ_b is limited by ______________________ ____. We just discussed this at length in accordance with the SFC vs M_0 and F/m vs. M_0 plots – the flight conditions and pressure ratio are very important to thrust and fuel consumption performance of the engine.

Using the definition of τ_λ, the specific thrust can be re-written as

The Maximum Thrust Turbojet

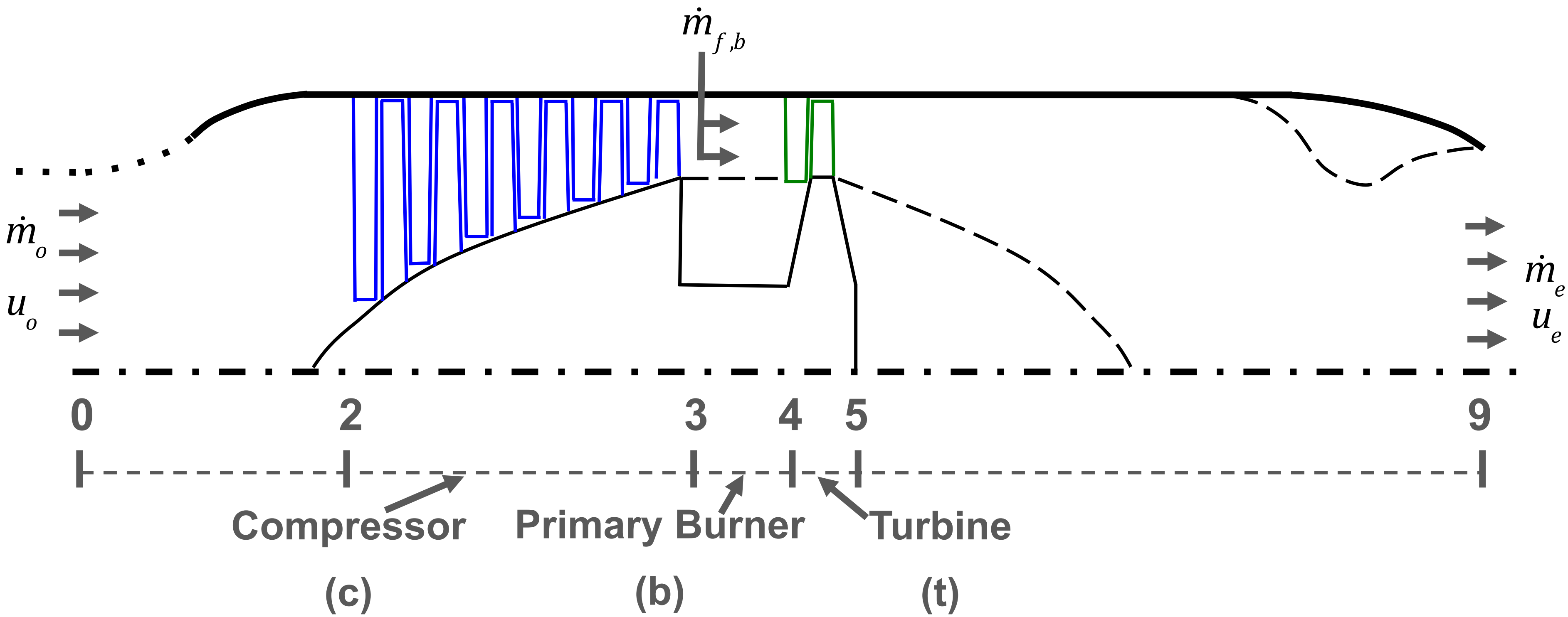
We note from this equation that, for a fixed $\tau_\lambda \tau_r$, there must be an optimal value of τ_c which maximizes (or minimizes) thrust.

Differentiation of the specific thrust relation with respect to τ_c and setting the result to zero gives:

Substitution back into the specific thrust equation gives the maximum specific thrust:

Ideal Turbojet Example

Compute the Specific Thrust and the Specific Fuel Consumption of an ideal turbojet engine operating with the following parameters:



Ideal Turbojet Example

Find: $T/\dot{m}\ ,\ S$

Ideal Turbojet Example

Ideal Turbojet Example

Ideal Turbojet Example

Ideal Turbojet Example