AAE 538: Air-Breathing Propulsion

Lecture 5: Fundamentals of Compressible Flow (continued)

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Extensions of 1-D Analysis

Flow with Friction

- We've examined isentropic flows and shocks, which are iso-energetic discontinuities that cause a change in the flow entropy and a loss in total pressure. Now, we extend our analysis to compressible flows with friction.
- ____________________________are a canonical case where the duct area is constant and the flow is adiabatic, but friction forces are present throughout.
  - Clearly many propulsion problems involve duct flow with friction
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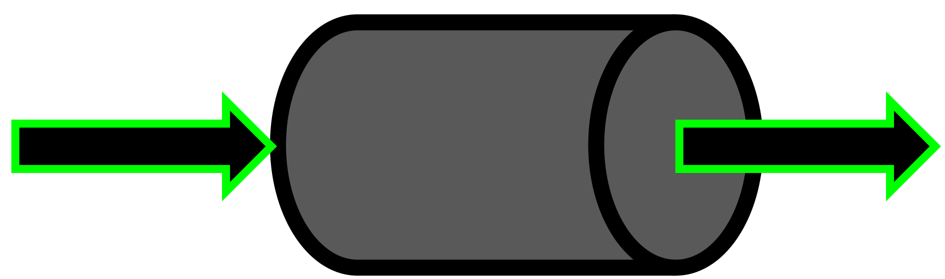
    -
  - For now, we will develop the treatment of friction using these assumptions to highlight the effects on the thermo-physical properties of the flow.
- The addition of friction to the flow makes the process irreversible, and leads to a reduction in the stream thrust in the axial direction.



Fanno Flow

- Beginning our analysis with the force-momentum balance

$$p_{o,1}A_1G_1 + F_x = p_{o,2}A_2G_2$$

- Dividing the conservation of momentum expression by the ______________________,

- From the energy equation, for an Adiabatic flow with no work transfer, we know that the ______________________________.

$$T_{o,1} - \frac{\dot{W}}{\dot{m}c_p} + \frac{\dot{Q}}{\dot{m}c_p} = T_{o,2}$$

- Therefore, assuming constant properties, the momentum equation reduces to:

Fanno Flow

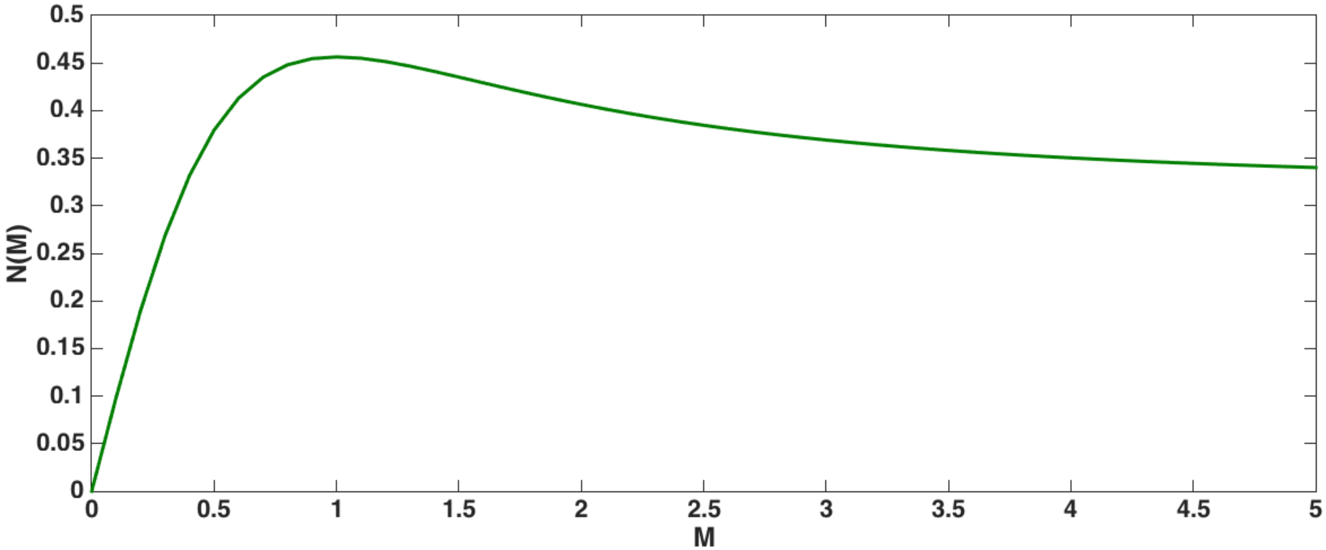
- Pausing briefly to examine the implications of this:

$$\frac{1}{N(M_1)} + \frac{F_x}{p_{o,1} A_1 D_1} = \frac{1}{N(M_2)}$$

- The ratio of the outlet (N_2) to inlet (N_1) thrust flow functions is equal to the ratio of the inlet (F_1) to exit (F_2) stream thrust.

- In a Fanno Flow,

Fanno Flow

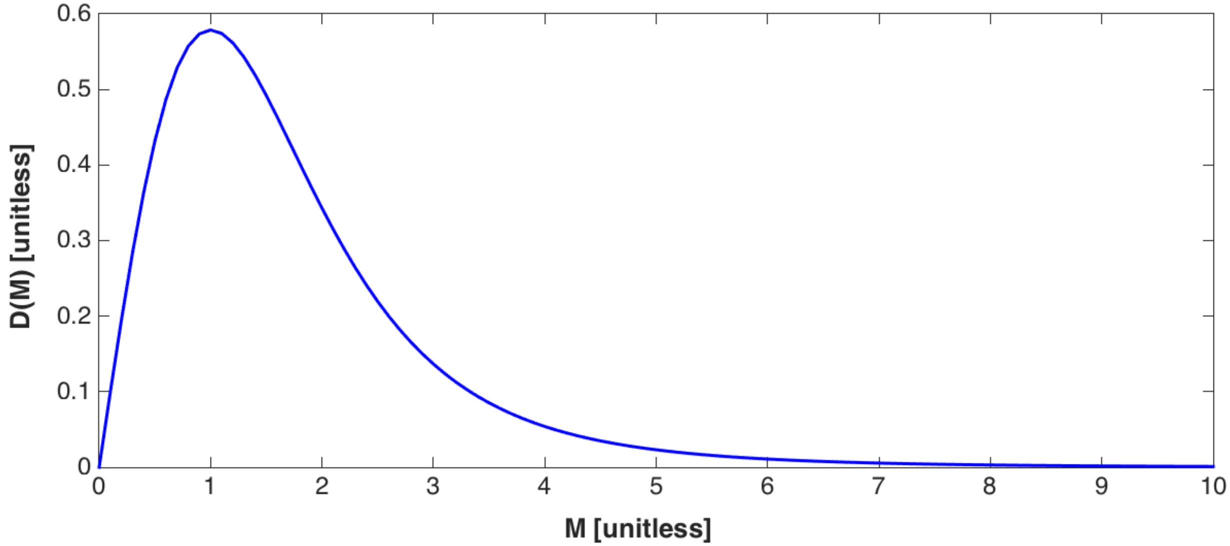


$$N = \frac{M\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}}}{(1+\gamma M^2)}$$

- Thus, friction raises a subsonic Mach number and lowers a supersonic Mach number.
  -

- This process is known as ______________________________.
  - That is, for a given inlet flow condition, there is a maximum stream thrust loss (due to high friction of a long duct) before the flow reach $M = 1$.

Fanno Flow



- The continuity equation for a Fanno flow can be simplified to show:

$$\dot{m} = \left(\frac{p_o A}{\sqrt{T_o}}\right)\sqrt{\frac{\gamma}{R}}\,D$$

- Thus

Fanno Flow

- This can also be shown through the entropy relation
  - Defined with stagnation properties:

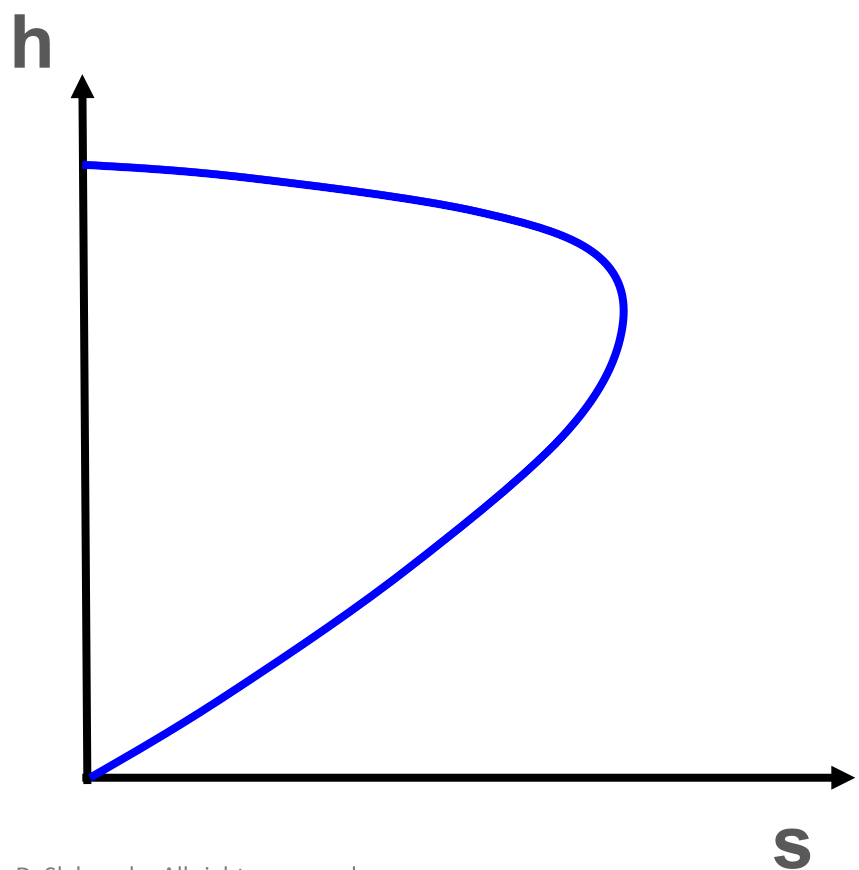
$$s_2 - s_1 = c_p \ln\left(\frac{T_{o,2}}{T_{o,1}}\right) - R \ln\left(\frac{p_{o,2}}{p_{o,1}}\right)$$

-
  -

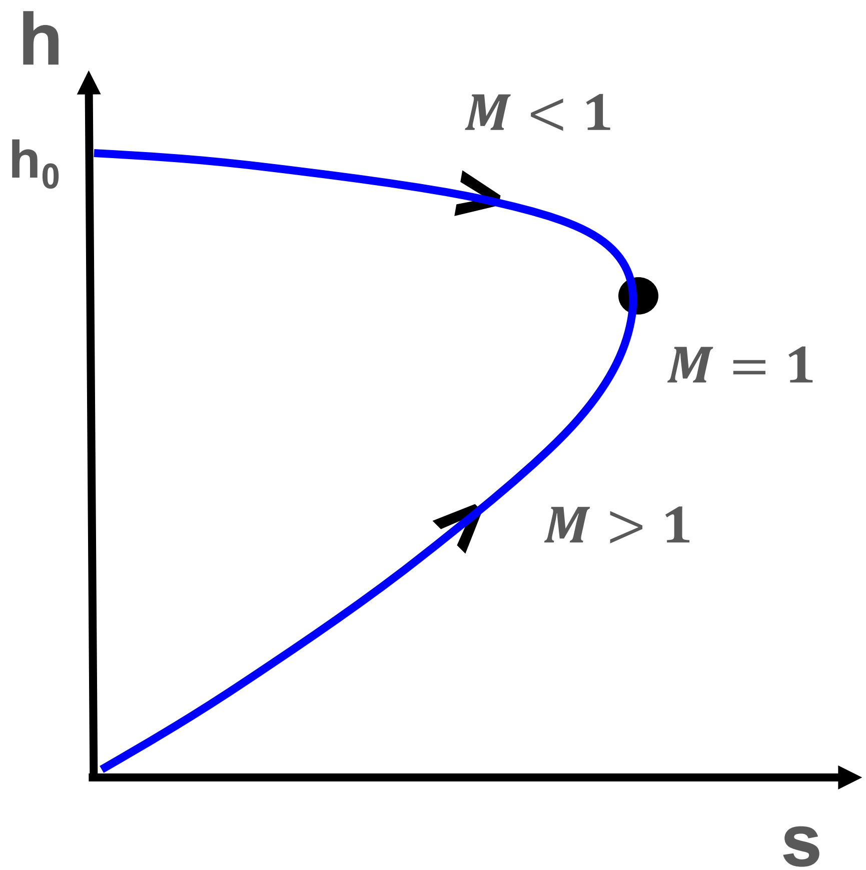
- Static pressure change depends on the starting Mach number of the flow
  - Subsonic:
  - Supersonic:

Property Changes in Fanno Flow

- By definition, Fanno flow is an iso-energetic with a constant area.
    -
    -
- Velocity change within the duct as a result friction is caused by a change in entropy
    - Friction can only act to _______________ entropy
    - Friction can only to push the flow toward choking.
    - Friction, alone cannot act to transition a flow between the sonic regimes
-



Property Changes in Fanno Flow



- The total (integral) amount of friction acting on the flow between 1 and 2 increases with ______________.
- If L is large enough,
- If L is increased ______________ for a given inlet stagnation enthalpy, then it is ____________________________.
  How does it respond?
  -
  -

Property Changes in Fanno Flow

- Since friction loss is a path-dependent process, we need to integrate the differential forms of the conservation equations to quantify property changes:
  - For a constant area channel, we find:

$$\frac{dM^2}{M^2} = \frac{\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{(1-M^2)}\frac{f\,dx}{D}$$

$$\frac{dp}{p} = \frac{-\gamma M^2(1+(\gamma-1)M^2)}{2(1-M^2)}\frac{f\,dx}{D}$$

$$\frac{dT}{T} = \frac{dh}{h} = \frac{-\gamma(\gamma-1)M^4}{2(1-M^2)}\frac{f\,dx}{D}$$

$$\frac{d\rho}{\rho} = -\frac{du}{u} = \frac{-\gamma M^2}{2(1-M^2)}\frac{f\,dx}{D}$$

$$\frac{ds}{R} = -\frac{dp_o}{p_o} = -\frac{\gamma M^2}{2}\frac{f\,dx}{D}$$

Property Changes in Fanno Flow

- Integrating between states 1 and 2, we can relate the conditions between the initial and final state:

$$\frac{T_2}{T_1} = \left(\frac{1 + \frac{(\gamma - 1)}{2} M_1^2}{1 + \frac{(\gamma - 1)}{2} M_2^2} \right)$$

$$\frac{p_2}{p_1} = \frac{M_1}{M_2} \sqrt{\frac{T_2}{T_1}}$$

$$\frac{p_{o,2}}{p_{o,1}} = \frac{M_1}{M_2} \left(\frac{T_2}{T_1} \right)^{\frac{\gamma+1}{2(1-\gamma)}}$$

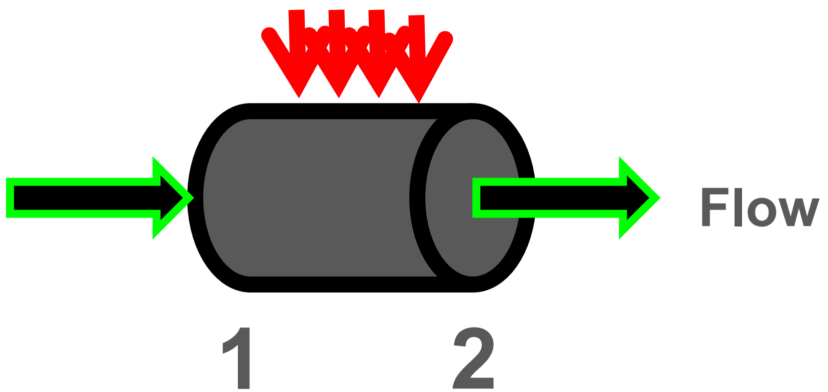
$$\frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} = \frac{M_2}{M_1} \sqrt{\frac{T_2}{T_1}}$$

Property	$M < 1$	$M > 1$
Entropy, s		
Stagnation temperature, T_o		
Stagnation pressure, p_o		
Mach number, Ma		
Static Enthalpy, h		
Static Temperature, T		
Static Pressure, p		
Density, ρ		
Velocity, u		

Extensions of 1-D Analysis

Flow with Heat Transfer

- Another special case of non-isentropic flow that we need to examine is one where heat is added (or subtracted).
  - Specific applications in air-breathing engines:
    -
    -
    -
  - Other (more far-reaching) applications
    -
    -
- ____________________ corresponds to compressible flow in a frictionless, constant-area duct with heat transfer.
  -
  -



Rayleigh Flow

- Beginning our analysis with the energy equation, we see

$$T_{o,1} - \frac{\dot{W}}{\dot{m}c_p} + \frac{\dot{Q}}{\dot{m}c_p} = T_{o,2}$$

- The force-momentum balance, in this case is simplified

$$p_{o,1}A_1G_1 + F_x = p_{o,2}A_2G_2$$

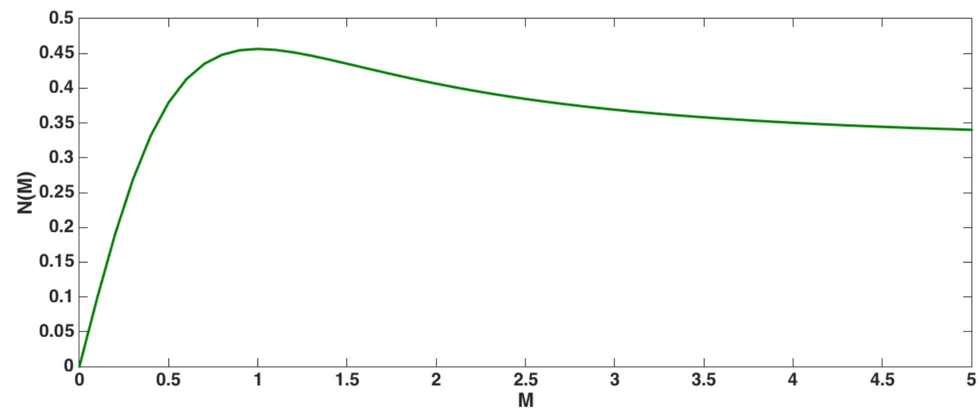
- Dividing the conservation of momentum expression by the continuity equation,
  -
  -

Rayleigh Flow

- In a Rayleigh flow,

  where

  - When heat is transferred _______ the flow, the Mach number is driven toward



- This process is known as

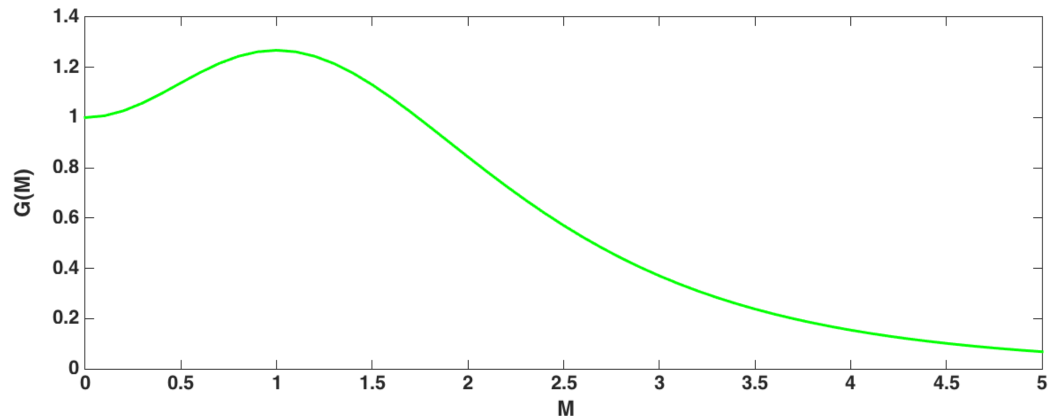
  - That is, heat addition to the fluid acts to ________________ the Mach number of a _________________ flow and _______________the Mach number of a _____________ flow towards $M = 1$ (choking).

- While heat addition acts to raise the total temperature of the flow, it also lowers the __________________ . To show this, we again consider the momentum equation for a flow without friction:

$$p_{o,1}A_1G_1 = p_{o,2}A_2G_2$$

Rayleigh Flow

- Examining the behavior of the $G(M)$ thrust flow function:
  - The maximum at $M = 1$ requires that, for a uniquely subsonic or supersonic flow,
  - Therefore,



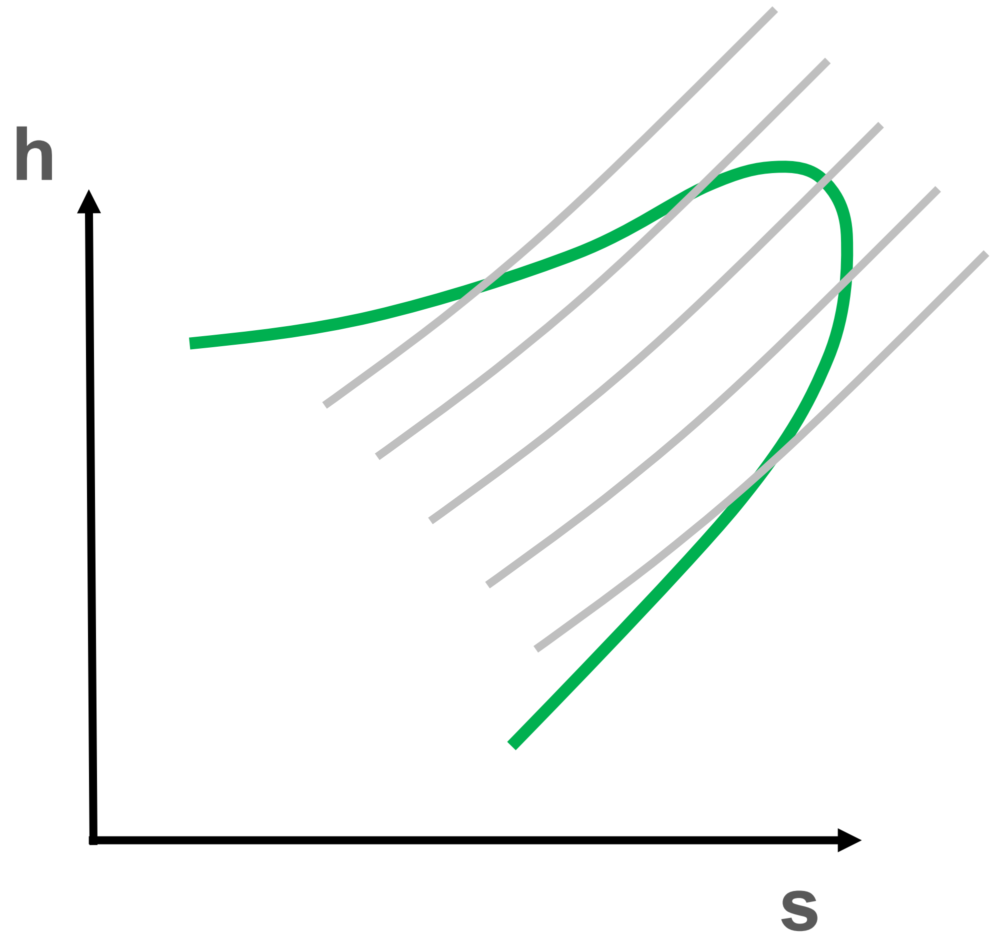
- We can again correlate this to entropy change through our understanding of the stagnation pressure as our 'entropy' variable.

$$s_2 - s_1 = c_p \ln\left(\frac{T_{o,2}}{T_{o,1}}\right) - R \ln\left(\frac{p_{o,2}}{p_{o,1}}\right)$$

Property Changes in Rayleigh Flow

- The relationship between the flow (static) enthalpy and entropy can be shown by the Rayleigh line, where:

$$\frac{ds}{c_p} = \frac{M^2 - 1}{\gamma M^2 - 1}\frac{dh}{h}$$



- Note that:

Property Changes in Rayleigh Flow

- Heat addition corresponds to an ______________ in the entropy while cooling causes a ____________in entropy
  -

- For the case of heat subtraction ($dT_o < 0$), changes in Mach number must drive the flow ________________________; deceleration of a subsonic flow and acceleration of a supersonic flow.
- Note that, under these conditions, we have the ability to decelerate a supersonic flow to a subsonic flow:

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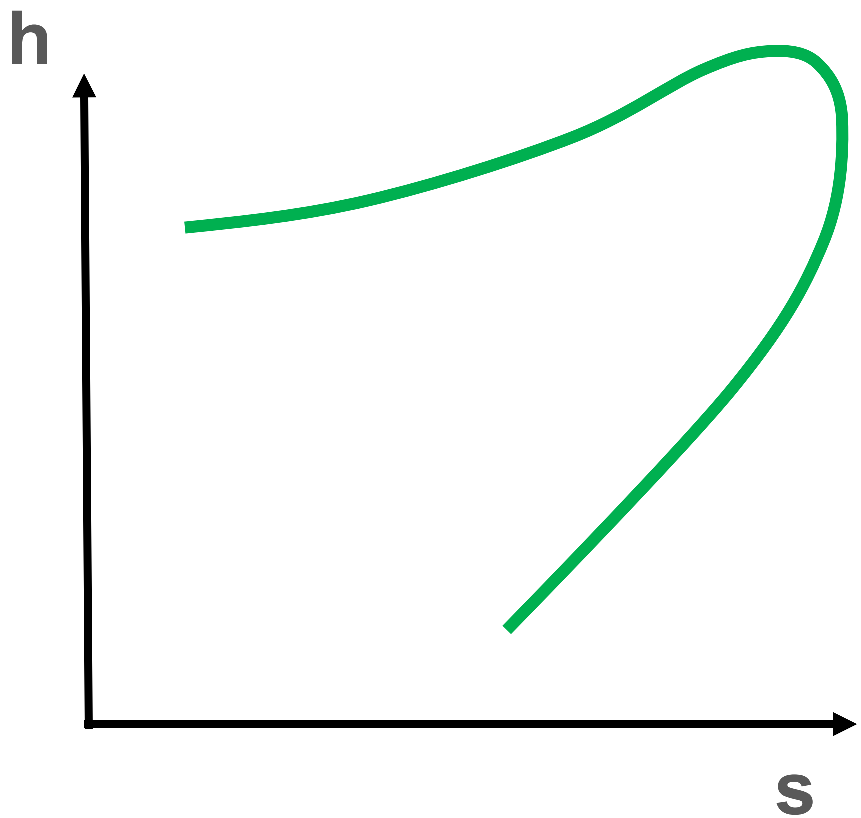


Property Changes in Rayleigh Flow

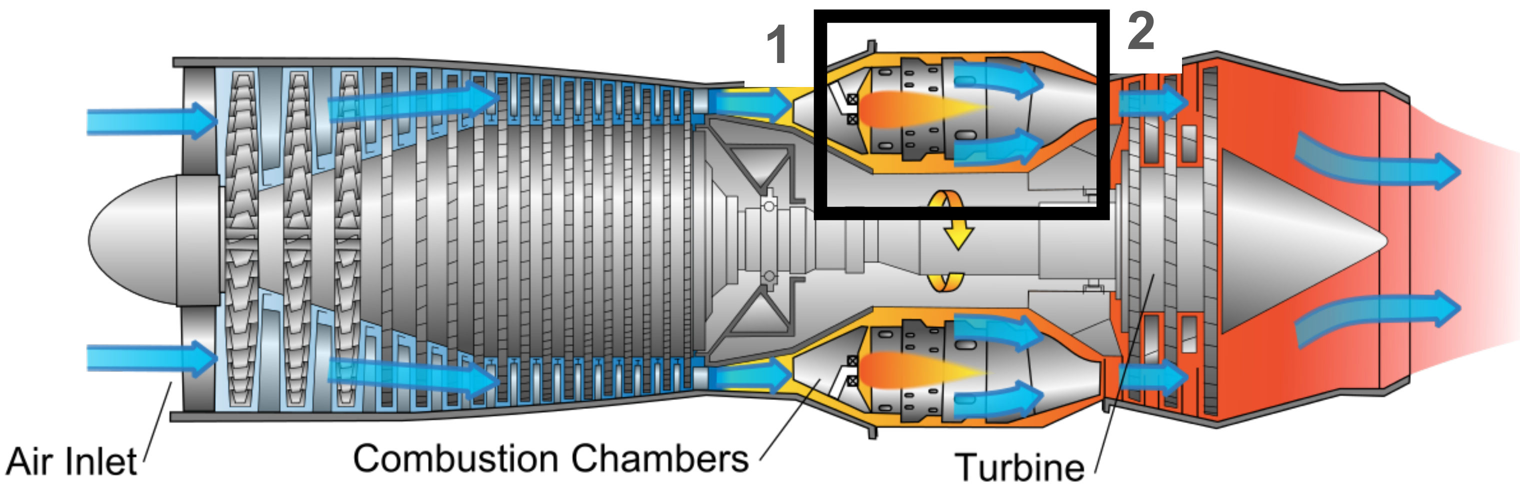
Property	Heating		Cooling	
	$M < 1$	$M > 1$	$M < 1$	$M > 1$
Entropy, s				
Stagnation temperature, T_o				
Static Temperature, T				
Mach number, Ma				
Stagnation Pressure, p_o				
Static Pressure, p				
Velocity, u				
Density, ρ				

- Heat addition ($\frac{T_{o,2}}{T_{o,1}} > 1$) always decreases P_o.
  - The opposite is also true.
- The sensitivity of P_o to changes in T_o is the lowest when M is low.

Example Analysis – Rayleigh Flow

Heat Addition in a Constant Area Duct

In a section of a duct between points 1 and 2, heat is added in a volumetric sense; i.e. not through the walls and into the boundary layer, but uniformly across the complete passage in such a manner that the temperature field remains constant across the duct at every axial location. Given the pressure, temperature, and velocity at location 1, compute the pressure, temperature, and velocity at location 2.



Example Analysis – Rayleigh Flow

As always, we start with the three basic relations for 1-D Compressible Flow

$$\dot{m} = \left(\frac{p_{o,1}A_1}{\sqrt{T_{o,1}}}\right)\sqrt{\frac{\gamma}{R}}D_1 = \left(\frac{p_{o,2}A_2}{\sqrt{T_{o,2}}}\right)\sqrt{\frac{\gamma}{R}}D_2$$

$$p_{o,1}A_1G_1 + F_x = p_{o,2}A_2G_2$$

$$T_{o,1} - \frac{\dot{W}}{\dot{m}c_p} + \frac{\dot{Q}}{\dot{m}c_p} = T_{o,2}$$

Making the following assumptions/approximations:

Example Analysis – Rayleigh Flow

We reduce these expressions to the form:

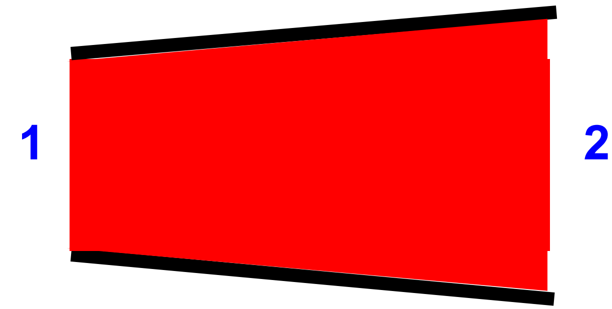
Example Analysis – Rayleigh Flow

Example Analysis – Rayleigh Flow

Example Analysis – CP Combustor

Constant Pressure Heat Addition

Compressible flow textbooks will often directly state that heat addition will cause Mach numbers of subsonic flows to increase and will cause a decrease in Mach number for supersonic flows. The fact is, that for typical conditions encountered in propulsive engines, the exact opposite can also be true. The difference is a matter of boundary conditions.



Another simple, one-dimensional problem concerns flow through a duct with heat addition occurring at constant pressure; i.e. the static pressure between locations 1 and 2 remains constant.

Again, we start from the conservation equations and simplify through the following assumptions/approximations:

Example Analysis – CP Combustor

Mass:

$$\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

or, in terms of stagnation quantities

$$\dot{m} = \left(\frac{p_{o,1} A_1}{\sqrt{T_{o,1}}}\right) \sqrt{\frac{\gamma}{R}} D_1 = \left(\frac{p_{o,2} A_2}{\sqrt{T_{o,2}}}\right) \sqrt{\frac{\gamma}{R}} D_2$$

Momentum:

$$\rho_1 u_1^2 A_1 + p_1 A_1 + F_x = \rho_2 u_2^2 A_2 + p_2 A_2$$

or, in terms of stagnation quantities

$$p_{o,1} A_1 G_1 + F_x = p_{o,2} A_2 G_2$$

Energy:

$$T_{o,1} - \frac{\dot{W}}{\dot{m} c_p} + \frac{\dot{Q}}{\dot{m} c_p} = T_{o,2}$$

Example Analysis – CP Combustor

Example Analysis – CP Combustor

Example Analysis – CP Combustor

Example Analysis – CP Combustor

Example Analysis – CP Combustor

Example Analysis – CP Combustor

Example Analysis – CP Combustor

Solution for $\frac{T_{o,2}}{T_{o,1}} = 2.0$

M_1	M_2	A_2/A_1	T_2/T_1	u_2/u_1	ρ_2/ρ_1	p_2/p_1	$p_{o,2}/p_{o,1}$
0.100	0.071	2.002	2.002	1.000	0.500	1.000	0.997
0.200	0.141	2.008	2.008	1.000	0.498	1.000	0.986
0.400	0.281	2.032	2.032	1.000	0.492	1.000	0.946
0.600	0.417	2.072	2.072	1.000	0.483	1.000	0.884
0.800	0.548	2.128	2.128	1.000	0.470	1.000	0.805
1.000	0.674	2.200	2.200	1.000	0.455	1.000	0.716
1.200	0.793	2.288	2.288	1.000	0.437	1.000	0.624
1.400	0.905	2.392	2.392	1.000	0.418	1.000	0.534
1.600	1.010	2.512	2.512	1.000	0.398	1.000	0.450
1.800	1.106	2.648	2.648	1.000	0.378	1.000	0.374
2.000	1.195	2.800	2.800	1.000	0.357	1.000	0.308
2.500	1.387	3.250	3.250	1.000	0.308	1.000	0.183
3.000	1.539	3.800	3.800	1.000	0.263	1.000	0.106
4.000	1.754	5.200	5.200	1.000	0.192	1.000	0.035