1) The goal is to minimize the weight of the beam while staying within constraints:

Minimize : $f(x) = m = \rho A L = \rho \pi t(2R-t)L$, $x = \begin{Bmatrix} R \\ t \end{Bmatrix}$

Constraints:

1. $0.02 \leq R \leq 0.2$ (m)
2. $0.001 \leq t \leq 0.01$ (m)
3. $R/t \leq 18$
4. $\delta \leq 0.001L \Rightarrow \frac{5wL^4}{384EI} = \frac{5 \cdot \frac{mg}{L} \cdot L^4}{384 E \pi R^3 t} = \frac{5 \cdot \frac{\rho \pi t(2R-t)Lg}{L} \cdot L^4}{384 E \pi R^3 t}$

$$= \frac{5}{384} \cdot \frac{\rho(2R-t)gL^3}{ER^3} \leq 0.001$$

5. $\sigma \leq \sigma_{all} \Rightarrow \frac{P}{\pi t(2R-t)} \leq \sigma_{all}$

Translating constraint equations:

$g_1(x) = 1 - \frac{R}{0.02} \leq 0$

$g_2(x) = \frac{R}{0.2} - 1 \leq 0$

$g_3(x) = 1 - \frac{t}{0.001} \leq 0$

$g_4(x) = \frac{t}{0.01} - 1 \leq 0$

$g_5(x) = \frac{R}{18t} - 1 \leq 0$ or: $g_5(x) = R - 18t \leq 0$

$g_6(x) = \frac{5}{0.384} \cdot \frac{\rho(2R-t)gL^3}{ER^3} - 1 \leq 0$ or: $g_6(x) = 5\rho(2R-t)gL^3 - 0.384ER^3 \leq 0$

$g_7(x) = \frac{P}{\pi t(2R-t)\sigma_{all}} - 1 \leq 0$ or: $g_7(x) = 1 - \frac{\pi t(2R-t)\sigma_{all}}{P} \leq 0$

Minimize : $f(x) = \rho \pi t(2R-t)L$, $x = \begin{Bmatrix} R \\ L \end{Bmatrix}$

$L = 3.5$ m
$\sigma_{all} = 405 \cdot 10^6$ Pa
$\rho = 7{,}850$ kg/m^3
$P = 55 \cdot 10^3$ N
$E = 250 \cdot 10^9$ Pa

Alternate equations for g_5, g_6, and g_7 are presented in case design variables in the denominator give ill-conditioned effects. In this case, constraints will also be scaled to better the problem conditioning.

Moving forward, the alternate expressions without design variables in the denominator will be used

AAE550 10/29/17 HW#2 Thomas Slattery

The analytic gradients are found to ease computation:

$$\left.\begin{aligned} \frac{df}{dR} &= 2\rho\pi t L \\ \frac{df}{dt} &= -\rho\pi t L + \rho\pi L(2R-t) = 2\rho\pi L(R-t) \end{aligned}\right\} \nabla f$$

$$\left.\begin{aligned} \frac{dg_1}{dR} &= -\frac{1}{0.02} \\ \frac{dg_1}{dt} &= 0 \end{aligned}\right\} \nabla g_1$$

$$\left.\begin{aligned} \frac{dg_2}{dR} &= \frac{1}{0.2} \\ \frac{dg_2}{dt} &= 0 \end{aligned}\right\} \nabla g_2$$

$$\left.\begin{aligned} \frac{dg_3}{dR} &= 0 \\ \frac{dg_3}{dt} &= -\frac{1}{0.001} \end{aligned}\right\} \nabla g_3$$

$$\left.\begin{aligned} \frac{dg_4}{dR} &= 0 \\ \frac{dg_4}{dt} &= \frac{1}{0.01} \end{aligned}\right\} \nabla g_4$$

$$\left.\begin{aligned} \frac{dg_5}{dR} &= 1 \\ \frac{dg_5}{dt} &= -18 \end{aligned}\right\} \nabla g_5$$

$$\left.\begin{aligned} \frac{dg_6}{dR} &= 10\rho g L^3 - 1.152 E R^2 \\ \frac{dg_6}{dt} &= -5\rho g L^3 \end{aligned}\right\} \nabla g_6$$

$$\left.\begin{aligned} \frac{dg_7}{dR} &= -2\pi t \sigma_{all}/P \\ \frac{dg_7}{dt} &= \frac{\left(-\pi t \sigma_{all}(-1) - \pi\sigma_{all}(2R-t)\right)}{P} = \frac{-2\pi\sigma_{all}(t-R)}{P} \end{aligned}\right\} \nabla g_7$$