Part II

Design a minimum-mass flag pole of fixed-height H. The pole is made of a uniform hollow circular tubing with inner diameter $d_o$ and outer diameter $d_i$. The pole must not fail under high winds. The pole is treated as a cantilever beam that is subjected to wind load w (kN/m). In addition, the wind induces a concentrated load P at the top of the pole.

The design variable have the bounds:

$5 \le d_o \le 50$ cm
$4 \le d_i \le 45$ cm
The tube thickness must be between 0.5 and 2 cm
$\tau < \tau_a$
$\sigma < \sigma_a$
Total deflection is less than 10 cm
Ratio of mean diameter to thickness must not exceed 60

Find:

1. Formula the optimal design problem

   a) Write an objective function, $f(x)$, in terms of $d_o$ and $d_i$. This objective function should minimize the mass of the flag pole. Include bounds in the formulation.

   b) Write the inequality constraint functions, $g_i(x)$, in terms of $d_o$ and $d_i$. Convert to the form $g_i(x) \le 0$. Note if any design variables are in the denominator.

   c) Write any side constraints or bounds on the design variables. Convert into additional $g_i(x) \le 0$.

2. See attached for problem statement and solution

3. Compare the total number of unconstrained minimization and iterations needed for each method. Additionally, compare each solution. Which method was the easiest to implement and use? Can any conclusions be made about the different penalty methods for this problem?

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Solution:

1.

a)

the fundamental structure equations are:

$A = \frac{\pi}{4}(d_o^2 - d_i^2)$ — Cross section area

$I = \frac{\pi}{64}(d_o^4 - d_i^4)$ — Moment of inertia

$M = PH + 0.5wH^2$, kN-m — Moment at base

$\sigma = \frac{M}{2I} d_o$, kPa — Bending stress

$S = P + wH$, kN — Shear at base

$\tau = \frac{S}{12I}(d_o^2 + d_o d_i + d_i^2)$, kPa — Shear stress

$\delta = \frac{PH^3}{3EI} + \frac{wH^4}{8EI}$ — Deflection at top

Total mass of the pole is as follows:

$m = \rho A H$, where $\rho$ is the mass density of the pole in $kg/m^3$

Let $x = \begin{Bmatrix} d_o \\ d_i \end{Bmatrix}$, with $d_o, d_i$ in meters

$$\boxed{\begin{array}{l} f(x) = \rho \frac{\pi}{4}(d_o^2 - d_i^2)H \\ \text{For simplicity:} \\ f(x) \geq 0 \\ \text{Other constraints will} \\ \text{handle } d_o \text{ and } d_i \end{array}}$$

b) $d_o \geq 0.05 \Rightarrow g_1(x) = 0.05 - d_o \leq 0$

$d_o \leq 0.5 \Rightarrow g_2(x) = d_o - 0.5 \leq 0$

$d_i \geq 0.04 \Rightarrow g_3(x) = 0.04 - d_i \leq 0$

$d_i \leq 0.45 \Rightarrow g_4(x) = d_i - 0.45 \leq 0$

$(d_i + d_o)/(2(d_o - d_i)) \leq 60 \Rightarrow g_5(x) = \frac{d_i + d_o}{2(d_o - d_i)} - 60 \leq 0$

$d_o - d_i \geq 0.005 \Rightarrow g_6(x) = 0.005 - d_o + d_i \leq 0$

$d_o - d_i \leq 0.02 \Rightarrow g_7(x) = d_o - d_i - 0.02 \leq 0$

$$\tau \le \tau_a \Rightarrow \frac{S}{12I}(d_o^2 + d_o d_i + d_i^2) < \tau_a \Rightarrow \frac{16(P+wH)}{3\pi(d_o^4 - d_i^4)}(d_o^2 + d_o d_i + d_i^2) \le \tau_a$$

$$\Rightarrow g_8(x) = \frac{16(P+wH)}{3\pi(d_o^4 - d_i^4)}(d_o^2 + d_o d_i + d_i^2) - \tau_a \le 0$$

$$\sigma \le \sigma_a \Rightarrow \frac{M}{2I} d_o \le \sigma_a \Rightarrow \frac{32(PH + 0.5wH^2)}{\pi(d_o^4 - d_i^4)} d_o \le \sigma_a$$

$$\Rightarrow g_9(x) = \frac{32(PH + 0.5wH^2)}{\pi(d_o^4 - d_i^4)} d_o - \sigma_a \le 0$$

$$\delta \le \delta_a \Rightarrow \frac{PH^3}{3EI} + \frac{wH^4}{8EI} \le \delta_a \Rightarrow \frac{64}{\pi E(d_o^4 - d_i^4)}\left(\frac{PH^3}{3} + \frac{wH^4}{8}\right) \le \delta_a$$

$$\Rightarrow g_{10}(x) = \frac{64}{\pi E(d_o^4 - d_i^4)}\left(\frac{PH^3}{3} + \frac{wH^4}{8}\right) - \delta_a \le 0$$

All constraint equations can then be transformed to be on the order of 1.

$$g_1(x) = 1 - \frac{d_o}{0.05} \le 0$$
$$g_2(x) = \frac{d_o}{0.5} - 1 \le 0$$
$$g_3(x) = 1 - \frac{d_i}{0.04} \le 0$$
$$g_4(x) = \frac{d_i}{0.45} - 1 \le 0$$
$$g_5(x) = \frac{d_i + d_o}{120(d_o - d_i)} - 1 \le 0$$
$$g_6(x) = 1 - [(d_o + d_i)/0.005] \le 0$$
$$g_7(x) = [(d_o - d_i)/0.02] - 1 \le 0$$

$$g_8 = \frac{16(P+wH)}{3\pi \tau_a (d_o^4 - d_i^4)}(d_o^2 + d_o d_i + d_i^2) - 1 \le 0$$ *Design variables in denominator

$$g_9 = \frac{32(PH + 0.5wH^2)}{\pi \sigma_a (d_o^4 - d_i^4)} d_o - 1 \le 0$$ *Design variables in denominator

$$g_{10} = \frac{64}{\pi E \delta_a (d_o^4 - d_i^4)}\left(\frac{PH^3}{3} + \frac{wH^4}{8}\right) - 1 \le 0$$ *Design variables in denominator

c) No additional constraints were found to be necessary to solve this problem!