AAE 538: Air-Breathing Propulsion

Lecture 24: Inlet Starting

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Introduction

- Inlet starting is one of the classical topics in supersonic propulsion that is even more critical in hypersonics.
  - The questions to be answered are:
    - How can the flow within the compression system be arranged to handle the change from subsonic to supersonic flight conditions?
    - How does one achieve an efficient compression process over the entire range of flight Mach numbers?
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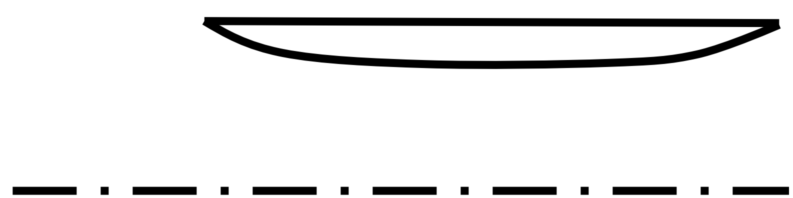
Introduction

- Consider this example of an internal compression inlet:

At this flow condition, the isentropic flow tables (for $\gamma = 1.4$) yield

Neglecting losses in the oblique shock system, ______________________, which gives __________________. The terminal normal shock will be positioned at $M_2 = 1.3$, where our normal shock relations give

While this is a stable operating point, the stability is tenuous. If the terminal shock moves to the throat, it can be ______________. This process is referred to as unstart.



Introduction

- The critical flight Mach number to place the normal shock at the throat can be determined by setting
  - Under this condition, $M_{th} = 1$ and $A_1/A^* = 2.56$. This condition corresponds to a in-flow Mach number, $M_1 = 2.47$.
  -
- To re-start the inlet, you have two options:
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  -
- The key to understanding this process is the recognition and treatment of a sequence of states.
  - We will develop our treatment with the following assumptions:
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    -
    -
  - We will also assume steady, one-dimensional, frictionless, isentropic flow
    -
    - Allows us to return to the compressible flow functions!

Condition 1

Completely Subsonic Flow-Field

- We start by observing from the energy equation that the stagnation temperature is uniform throughout the entire flow-field for all Mach numbers.
- We also note that, at low speeds where the flow is everywhere subsonic, no shocks are present and the stagnation pressure is constant throughout.
- Here, the continuity equation becomes:

Condition 1

Completely Subsonic Flow-Field

where $$D(M) = \frac{M}{\left(1 - \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

- Taking the ratio of specific heats, the gas constant (the molecular weight), and the stagnation conditions as constant, the continuity equation simplifies to:

$$A_0 D_0 =$$

- Given that the flow is entirely subsonic, we know that the pressure at the exit plane must equal

Condition 1

Completely Subsonic Flow-Field

- Therefore, the continuity equation implies that, for a completely subsonic flow field, the capture area is equal to the exit area.
  - This condition holds as long as the the Mach number at the throat remains subsonic.

Condition 2a

Subsonic Freestream with Choked Throat and Subsonic Expansion

- As the free-stream Mach number if increased, the Mach number at the throat continues to increase until it becomes choked.
- At the free-stream Mach number at which the throat Mach number first reaches the choked condition, the flow in the divergent section ______________________________ to the exit plane. At this one condition:
    -
    -
    -

Condition 2a

Subsonic Freestream with Choked Throat and Subsonic Expansion

- At this single-point, we have

  and we have two equal relations for the capture area:

Condition 2

Subsonic Free Stream with a Choked Throat and Shock in the Nozzle

- As the free-stream Mach number increases above this choking Mach number, the throat _________________________.
- A shock-wave appears in the divergent section of the nozzle so that the stagnation pressure at the exit plane is less than the free stream stagnation pressure.
    -
- The capture area is given by the relation:

Condition 2

Subsonic Free Stream with a Choked Throat and Shock in the Nozzle

- As the free-stream Mach number continues to increase within this subsonic, choked throat regime, the value of D_0 continues to increase and the capture area continues to decrease until the freestream Mach number reaches the sonic value.
    -

- As the Mach number of the flying nozzle increases from zero to one:
    - The capture area starts out equal to the exit area
    - Once the nozzle chokes, the capture area starts to decrease until it reaches the throat area.

$$A_0 = A_2 \frac{D_2}{D_0} = A_2$$

Condition 2

Subsonic Free Stream with a Choked Throat and Shock in the Nozzle

- Depending on the ratio of the inlet area to the exit area:
    - The free-stream Mach number may reach the sonic condition before the shock reaches the exit plane.
    -

Condition 3

Supersonic Free-Stream with a Choked Throat

- For free-stream Mach numbers greater than unity, a shock wave forms in front of the inlet.
- We start by considering a free-stream Mach number that is just slightly larger than unity so the flow-field is similar to the conditions for a Mach number slightly less than unity.
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- The supersonic flow cannot deviate from its free-stream condition (no change in the streamline direction)

Condition 3

Supersonic Free-Stream with a Choked Throat

- Upstream of the shock, the two streamlines that define the capture area will be straight and parallel from upstream infinity until the shock.

- At the shock wave, we can compute conditions behind the shock by the applying continuity, momentum, and energy equations.
  - Recall that, because the shock is very thin, the area is the same on both side of the shock so that there are no lateral areas on which an external force can act.

- The flow passing through the shock undergoes a ______________________ in Mach number that is given by the combination of continuity, momentum, and energy:
  - Denoting the downstream conditions of the unattached shock by the subscript ‘0d’, we can write:

From conservation of energy:

Condition 3

Supersonic Free-Stream with a Choked Throat

From conservation of momentum:

There is no external force acting on the shock, so we have:

From conservation of mass:

$$\dot{m} = \sqrt{\frac{\gamma}{R}}\frac{p_{o,0}A_0D_0}{\sqrt{T_{o,0}}} = \sqrt{\frac{\gamma}{R}}\frac{p_{o,0d}A_0D_{0d}}{\sqrt{T_{o,0d}}}$$

Condition 3

Supersonic Free-Stream with a Choked Throat

- We find that the conditions across the shock can be reduced to:

- From this relation, we can determine that the Mach number behind the shock: M_{0d}

Condition 3

Supersonic Free-Stream with a Choked Throat

- The stagnation pressure behind the shock can then be obtained from either continuity or momentum:

- Since the flow between the shock and the throat is isentropic, we can again use the conservation of mass to relate the conditions behind the shock to the conditions at the throat.
  - Taking into account the constant stagnation temperature and the constant stagnation pressure, the continuity equation gives:

Condition 3

Supersonic Free-Stream with a Choked Throat

- As the free-stream Mach number is increased,

  - D_{0d} ________________ and, thus, the capture area ______________________.

- Updating our summary of the capture area as a function of the free-stream Mach number:

  - The capture area starts out equal to the exit area
  - Once the nozzle chokes, the capture area starts to decrease until it reaches a minimum at the throat area.
  -

- As the capture area increases, the amount of spillage

  - As the capture area approaches the inlet area, the spillage decreases ________________ and the shock approaches the inlet face.
  - At this limiting condition, the streamlines remain un-deviated all the way to the inlet and the capture area becomes equal to the inlet and the capture area becomes equal ______________________.

Condition 3

Supersonic Free-Stream with a Choked Throat

- At this condition, we have:

- Here, the capture area becomes:

- The free-stream Mach number at which the shock sits on the lip is referred to as the ______________________________.
    - Supersonic speeds below the critical Mach number are designated as sub-critical speeds.
    - Speeds above the critical Mach number are called super-critical speeds.

Condition 4

Unchoked Supersonic Flow with a Swallowed Shock

- In the sub-critical Mach number number regime, the shock wave in front of the inlet moves ever closer to the inlet as the free-stream Mach number is increased.
- At the critical Mach number, the shock sits on the nozzle lip
  - At this condition, we can also compute a second solution in which the shock has jumped from the inlet lip to a location downstream of the throat.

Condition 4

Unchoked Supersonic Flow with a Swallowed Shock

- When the shock lies inside the inlet:
  - The flow is supersonic and isentropic up to the throat
  - The throat is unchoked, with a supersonic Mach number
  - We say that the shock has been swallowed.

- The second solution is also readily found from the conservation equations. In this case:
  - The flow is supersonic and isentropic up to the throat
  - The inlet Mach number is equal to the free-stream Mach number
  - The capture area is equal to the inlet area

Condition 4

Unchoked Supersonic Flow with a Swallowed Shock

- Summarizing this whole progression:
  - At low speeds, the capture area starts out equal to the exit area and remains constant with free-stream Mach number until the throat chokes.
  - Once the throat chokes, the capture area starts to decrease until it reaches a minimum at the throat area for a free-stream Mach number of unity.
  - For a free-stream Mach number greater than unity, the capture area increases until it equals the inlet area at the critical Mach number.
  - At Mach number greater than the critical Mach number, the capture area remains equal to the inlet area

Condition 4

Unchoked Supersonic Flow with a Swallowed Shock

- For the supercritical Mach number case, the application of conservation of mass and energy gives us the following condition at the throat:

  since

  This serves as an equation for computing the Mach number at the throat.
  - Note, that in solving for M_2 , one must take the supersonic solution.