AAE 538: Air-Breathing Propulsion

Lecture 11: Analysis of Ideal Engine Cycles (continued)

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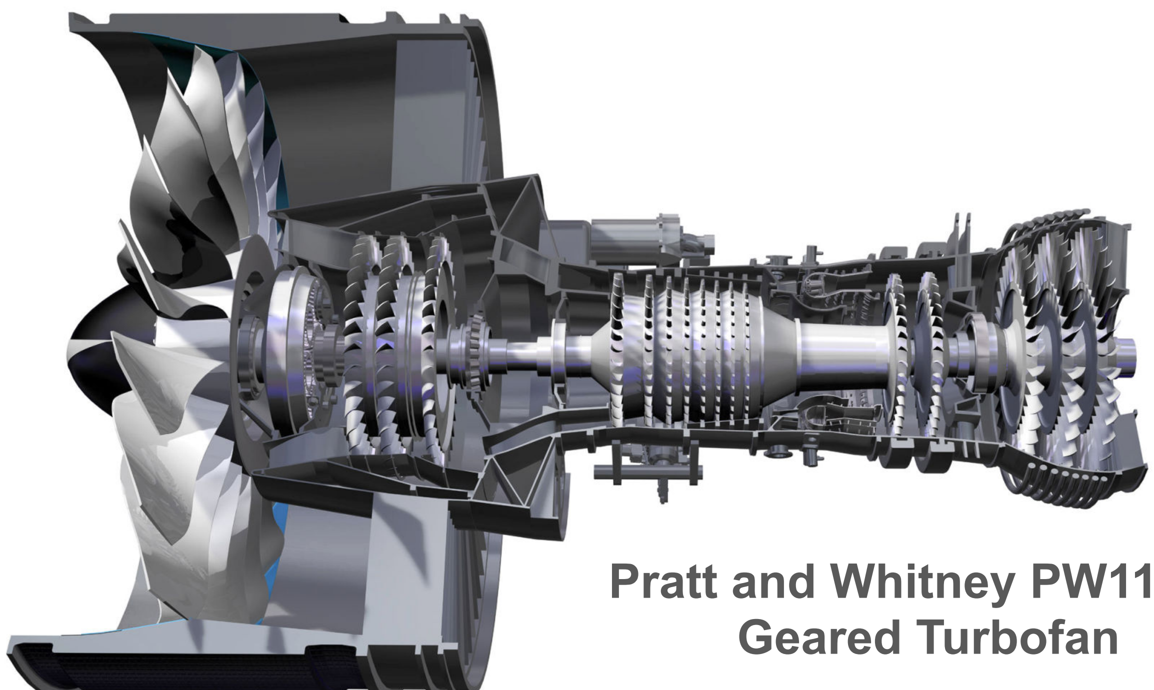
Introduction

The Ideal Turbofan Cycle

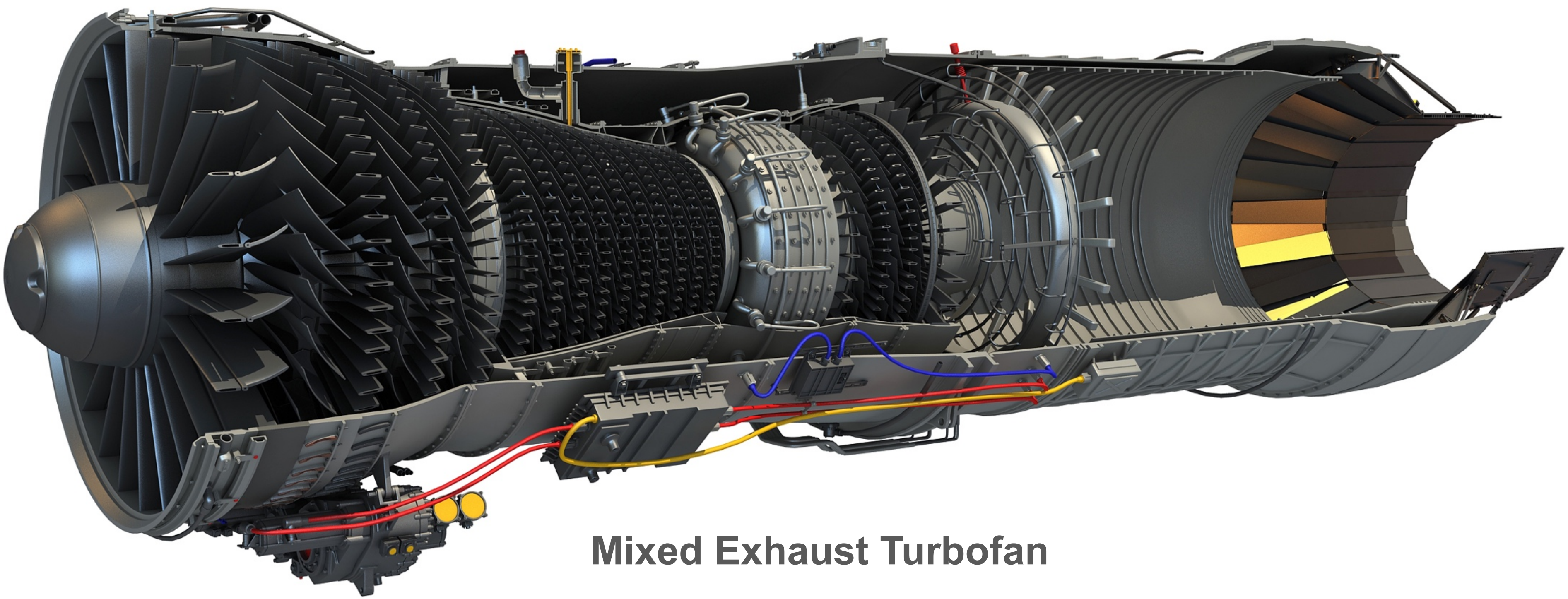
- There are two types of turbofan engines in operation today
  - The first type contains a ______________________________ for air passing through the fan.
    - The length of the fan duct varies in many designs from a short 'half-length' cowl to a full-length duct with the same axial exit station at the core engine.
    - This is the most common integration for a turbofan, particularly in commercial aircraft



Pratt and Whitney PW1100G Geared Turbofan

Introduction

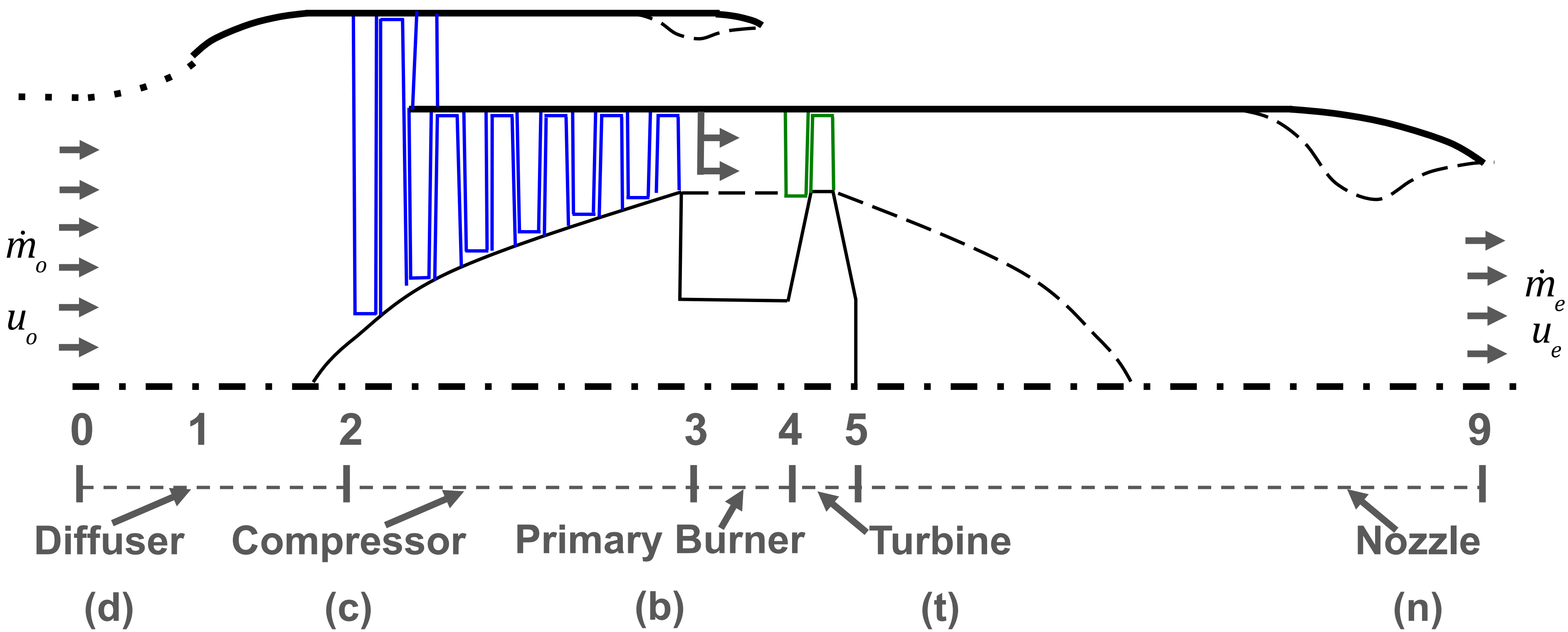
- The second type of turbofan effectively ________________________________ prior to exiting through the nozzle.
  - Advantages of this approach include reduced cowling and lower temperature gases at the exit nozzle
  - Disadvantages include mixing losses (_________________________________) and interdependence between the fan and the core flows (can lead to instabilities).



Mixed Exhaust Turbofan

The Ideal Turbofan (Separated Case)

- Stations throughout turbofan engine for the proceeding analysis
  - 0: Free-stream Conditions
  - 1: Diffuser Inlet
  - 2: Compressor Inlet
  - 3: Combustor Inlet
  - 4: Turbine Inlet
  - 5: Turbine Exit
  - 9: Nozzle Exit
  - 3’: Fan Exit
  - 9’: Fan Nozzle Exit



The Ideal Turbofan

- If we let $\dot{m}_F$ and m_c represent the mass flow rates through the fan and core, respectively, we can write the thrust for this device as:

$$T = \dot{m}_c(u_9 - u_0) + \dot{m}_F(u_{9'} - u_0) + (p_9 - p_0)A_9 + (p_{9'} - p_0)A_{9'}$$

  - Now, if we consider the fan and core nozzles to be perfectly expanded, in accordance with our treatment of ideal engine performance, the pressure thrust contributions in this relation vanish.
  - We can now also introduce the ______________________________.

    which represents the ratio of the fan and core mass flows.

    -

The Ideal Turbofan

- Substituting the bypass ratio into the thrust equation and letting the total mass flow rate ($\dot{m}$) represent the sum of the core and fan flows, we can write the specific thrust as:

- So, just as before, we need to compute the exit velocities to determine thrust
  - First we note that, since the core engine operation is still specified in terms of the same parameters as in the turbojet analysis, we can use our previous results to determine the ____________________________________.

$$\frac{u_9}{u_0} = \frac{M_9\sqrt{T_9}}{M_0\sqrt{T_0}} = \sqrt{\frac{\tau_b(\tau_r\tau_c\tau_t - 1)}{\tau_r - 1}}$$

The Ideal Turbofan

- To calculate the fan duct exit velocity, we proceed as before to express this quantity in terms of the engine operating characteristics.

- Note that the stagnation to static temperature ratio at the fan exit can be written

where

$\tau_f =$

is the stagnation temperature ratio across the fan

The Ideal Turbofan

- We can also write the stagnation-static pressure ratio at the exit

$$\frac{p_{o,9'}}{p_{9'}} = \left(1 + \frac{\gamma - 1}{2} M_{9'}^2\right)^{\frac{\gamma}{\gamma - 1}} =$$

Where we have used the fact that $p_0 = p_{9'}$.

- If we 'solve' these two equations for the function of Mach number and equate the results, we can write

and since the recovery and fan compression processes are isentropic, the result is simply stated

The Ideal Turbofan

- Therefore, the fan exit velocity ratio is equal to the Mach number ratio, which can be obtained by dividing through our stagnation-static temperature ratio, such that

$$\frac{M_{9'}}{M_0} = \frac{1}{M_0}\sqrt{\frac{2}{\gamma - 1}(\tau_r \tau_f - 1)} =$$

- We now have all of the information we need to calculate thrust, except for the turbine temperature ratio, which appears in the equation for the core engine exit velocity
    - Because the turbine must drive the compressor and the fan, we have to write a new __________________________________ .

The Ideal Turbofan

- Nondimensionalizing and solving for τ_t

  Which demonstrates that the bypass ratio increases demand on the turbine (τ_t decreases)

- With the solution for the turbine temperature ratio, we have enough information to calculate the exit velocities for both streams as well as the engine thrust.

- To evaluate the specific fuel consumption of the ideal turbofan, we need to calculate the fuel/air ratio in the core of the engine with respect to engine parameters.

The Ideal Turbofan

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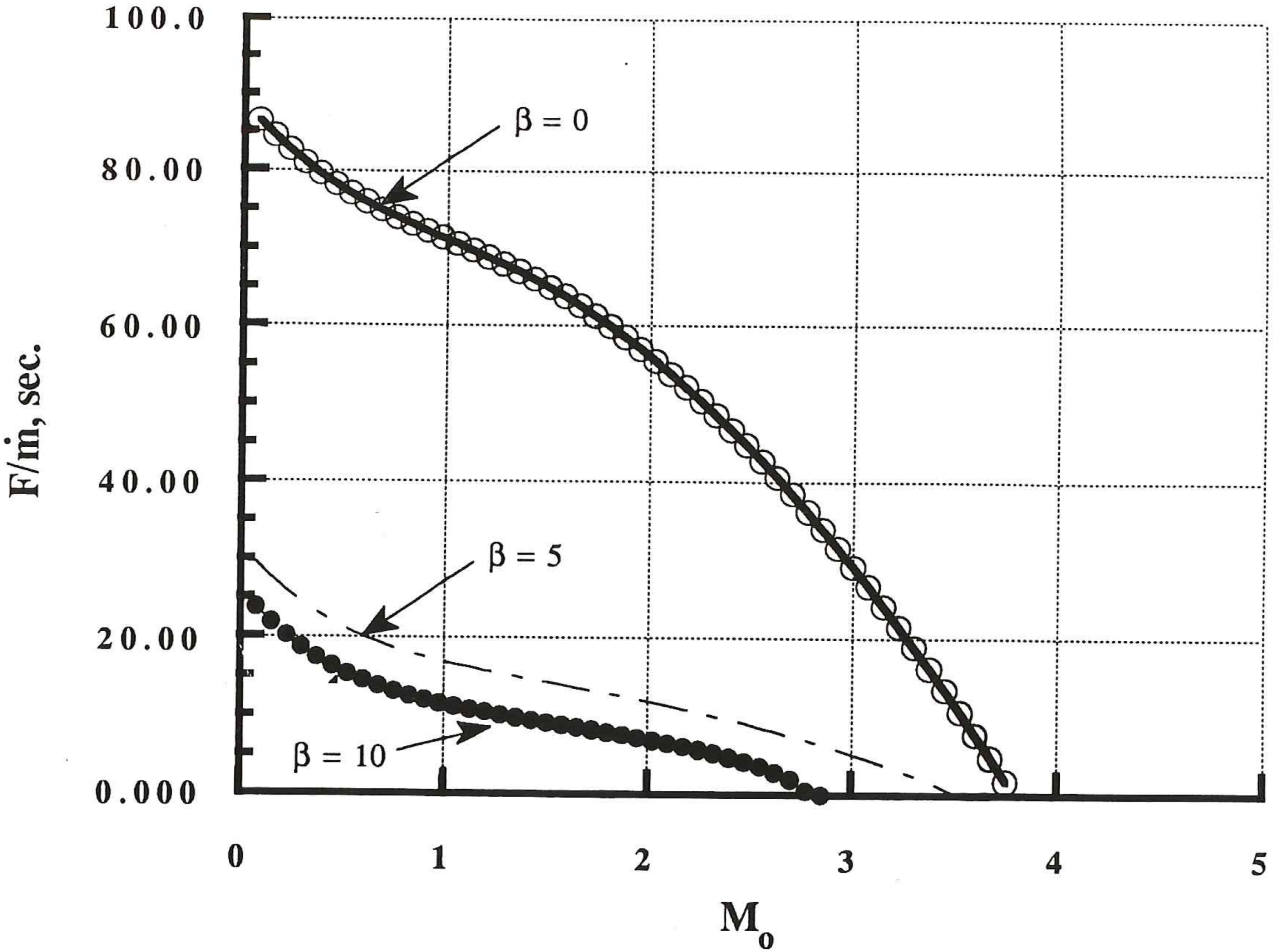
- If we now view f as the ratio of the fuel flow to the core engine air mass flow, we obtain the same result as before.

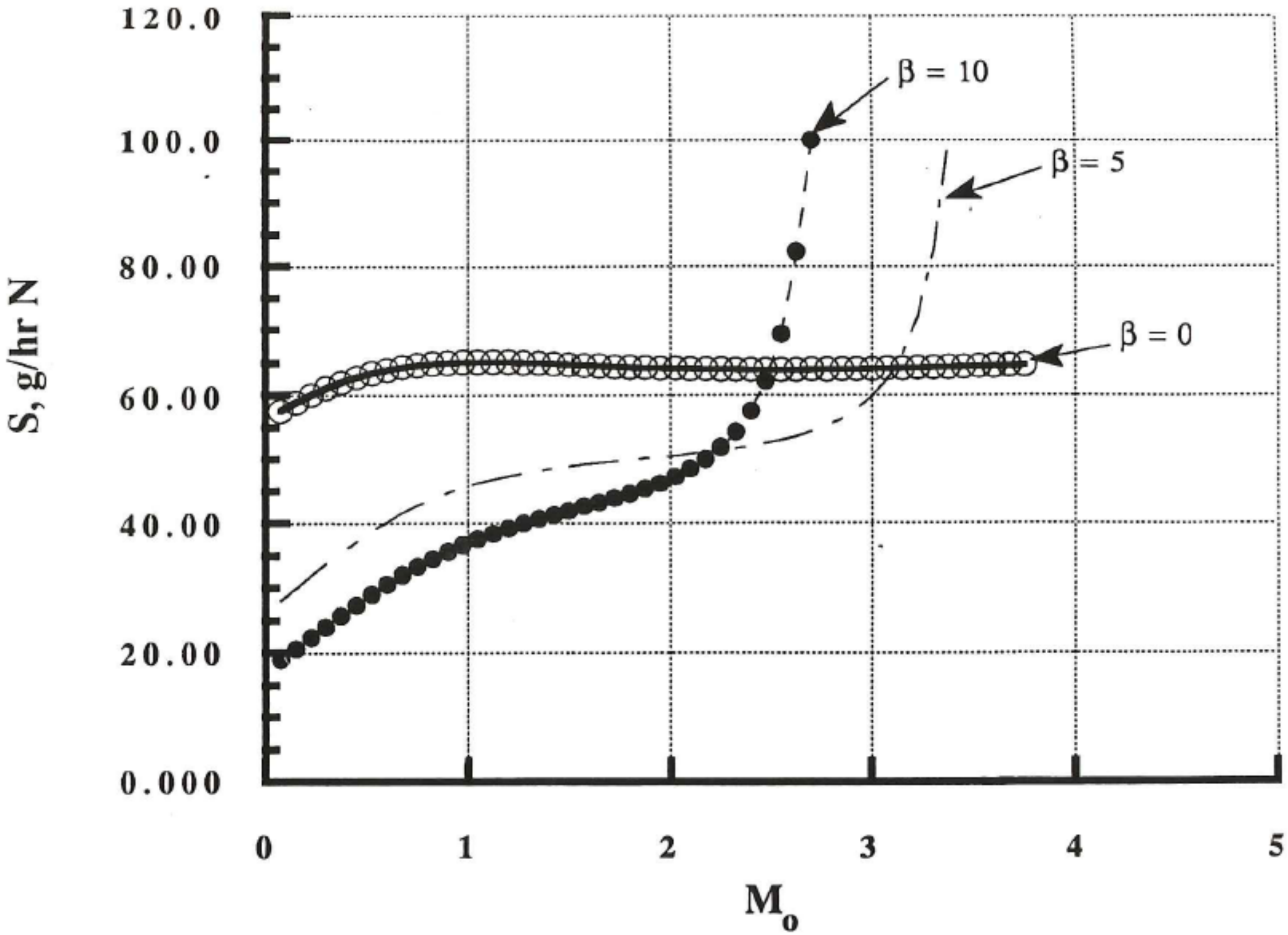
$$f = \frac{\dot{m}_f}{\dot{m}_c} = \frac{c_p T_0}{h}(\tau_\lambda - \tau_r \tau_c)$$

- And, as before, the specific fuel consumption can then be written

The Ideal Turbofan



The Ideal Turbofan



Bypass Ratio Optimization

- To minimize the specific fuel consumption, an optimum bypass ratio exists for given flight conditions and engine parameters.

$$S = \frac{f}{(1+\beta)(T/\dot{m})}$$

We want to minimize S with respect to β , but

$$(1+\beta)\left(\frac{T}{\dot{m}}\right) = u_0\left[\left(\left(\frac{u_9}{u_0}\right) - 1\right) + \beta\left(\frac{u_{9'}}{u_0} - 1\right)\right]$$

so

Bypass Ratio Optimization

Where we have noted that f, u_0 and $u_{9'}$ are all independent of β

Since ______________________________, we have to know

Bypass Ratio Optimization

Now, we can use the chain rule to calculate the variation of the core exit velocity with bypass ratio:

Differentiating our equation for (u_9/u_0) with respect to τ_t,

$$\frac{\partial u_9}{\partial \tau_t}\frac{1}{u_0} = \frac{\partial}{\partial \tau_t}\sqrt{\frac{\tau_\lambda \tau_t - \tau_b}{\tau_r - 1}} = \frac{1}{2}\left[\frac{\tau_\lambda \tau_t - \tau_b}{\tau_r - 1}\right]^{-\frac{1}{2}}\frac{\tau_\lambda}{\tau_r - 1} = \frac{1}{2}\frac{u_0}{u_9}\left(\frac{\tau_\lambda}{\tau_r - 1}\right)$$

and

$$\frac{\partial \tau_t}{\partial \beta} = \frac{\partial}{\partial \beta}\left[1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1) + \beta\left(\tau_f - 1\right)\right] = -\frac{\tau_r}{\tau_\lambda}\left(\tau_f - 1\right)$$

Bypass Ratio Optimization

So that,

$$\frac{1}{u_0}\frac{\partial u_9}{\partial \beta} = -\frac{1}{2}\left(\frac{u_0}{u_9}\right)\left(\frac{\tau_r(\tau_f - 1)}{\tau_r - 1}\right) = -\frac{1}{2}\left(\frac{u_0}{u_9}\right)\left(\frac{\tau_r\tau_f - 1 + 1 - \tau_r}{\tau_r - 1}\right)$$

$$=$$

$$=$$

Plugging in from our original differentiation and setting equal to zero:

$\Rightarrow$

Bypass Ratio Optimization

Rearranging, we find the expression for the optimal β:

$$\frac{1}{2}\left(\left(\frac{u_{9'}}{u_0}\right) + 1\right) = \left(\frac{u_9}{u_0}\right)$$

Substituting

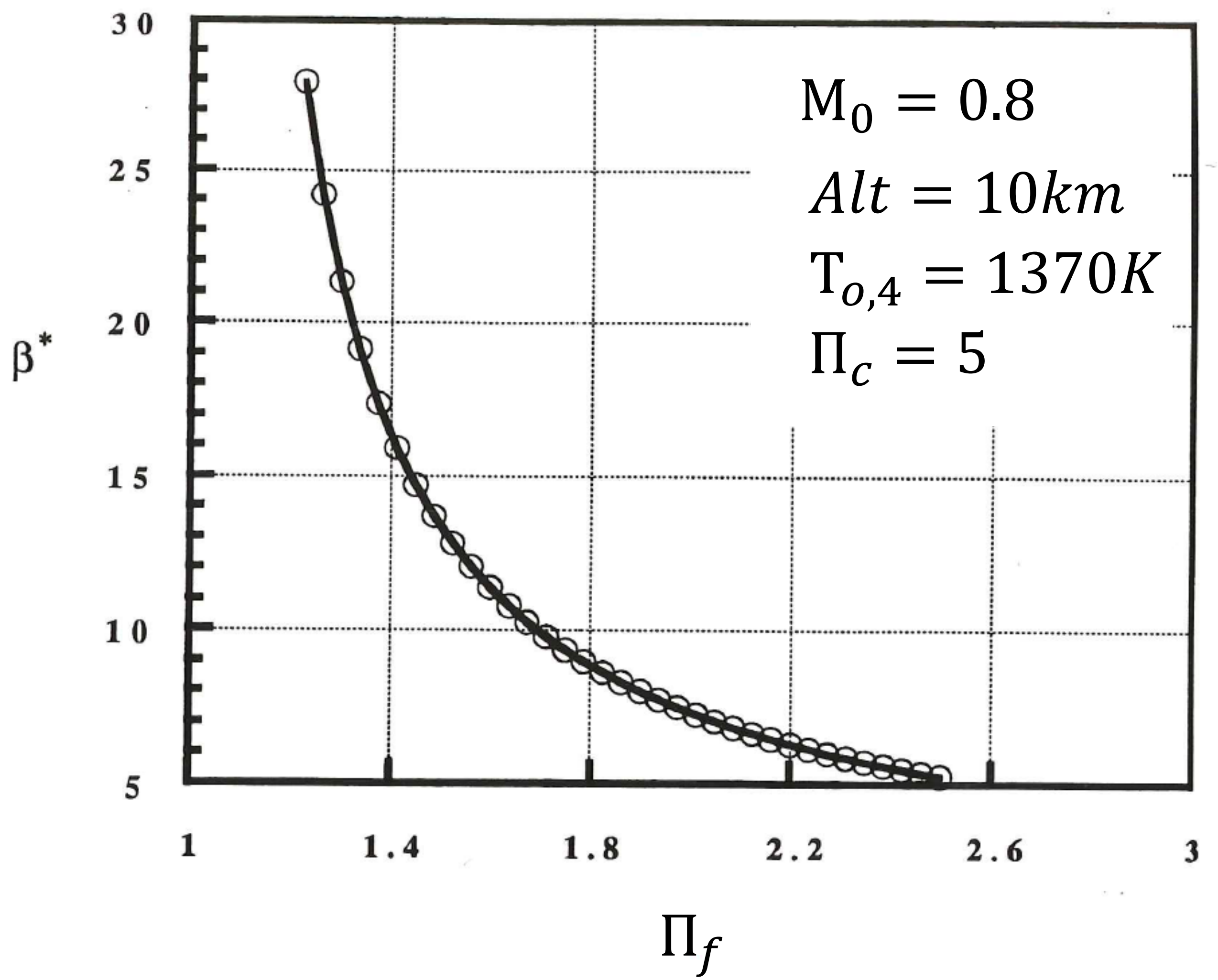
$$\frac{1}{2}\left(\sqrt{\frac{\tau_r\tau_f - 1}{\tau_r - 1}} + 1\right) = \sqrt{\frac{\tau_\lambda\tau_t^* - \tau_\lambda/(\tau_r\tau_c)}{\tau_r - 1}}$$

Rearranging, we can now show that

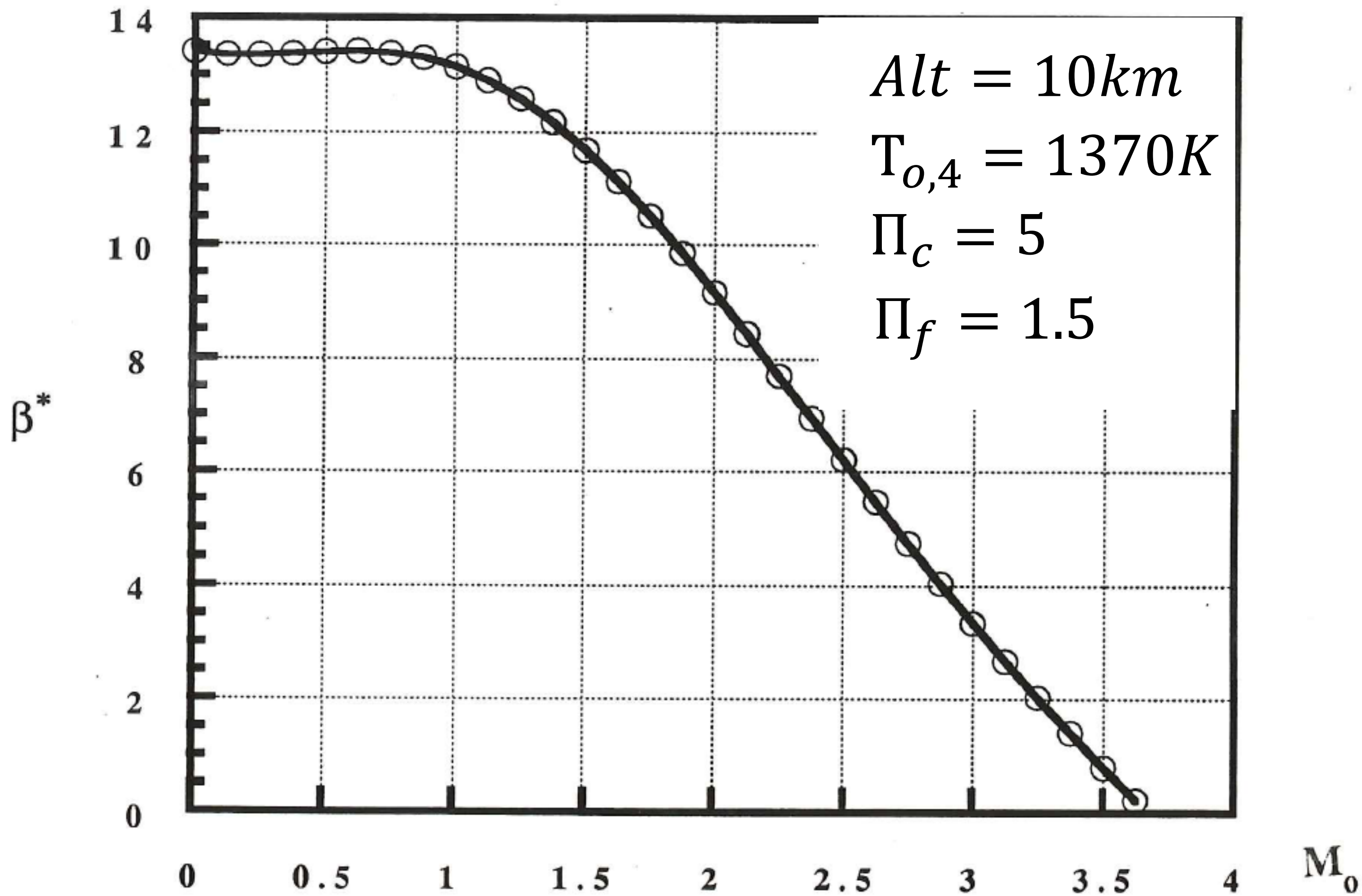
Bypass Ratio Optimization

Now having an expression for the optimal value of the turbine temperature ratio, τ_t^*, we can use our expression for τ_t as a $fn(\beta)$ to give us β^*, where

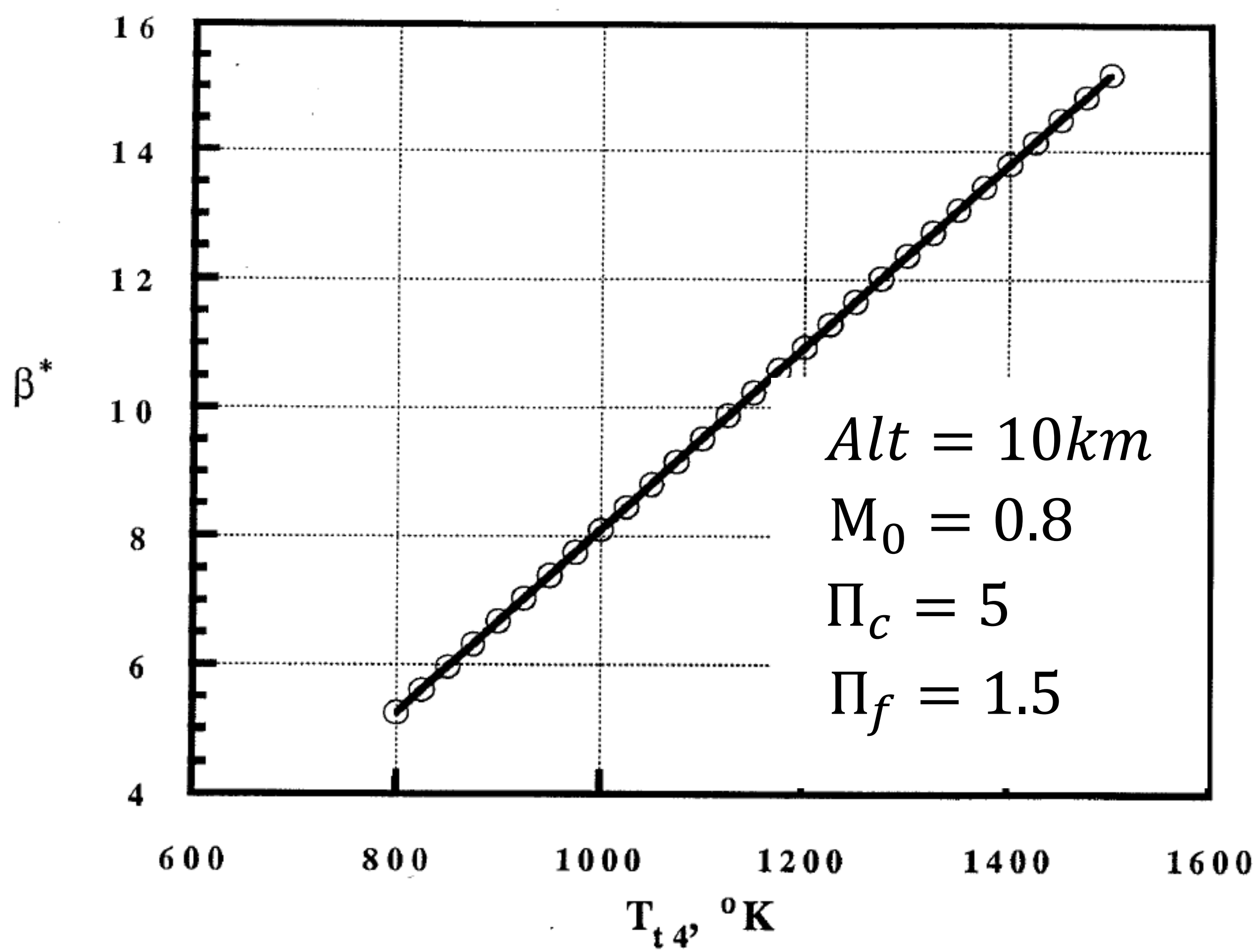
Bypass Ratio Optimization



Bypass Ratio Optimization



Bypass Ratio Optimization



Example

Comparison of Ideal Turbojet and Turbofan Engines

Given

$altitude = 35000\ ft$

$T_{o,4} = 394\ R$

$p_0 = 3.5\ psi$

$a_0 = 973\ ft/s$

$\rho_0 = 8.9\ e10^{-4}\ slug/ft^3$

$M_0 = 0.8 \rightarrow u_0 = 778\ ft/s$

$\Delta H_B = 18000\ BTU/lb_m$

$T_{o,4} = 2000F = 2460R$

$Desired\ Thrust\ Level \rightarrow 20000\ lb_f$

$$\frac{A_1}{A_0} = 1.5$$

Engine Design Parameters

Turbojet

$\pi_c = 19.5$

Turbofan

$\pi_c = 19.5, \pi_f = 1.3, \beta = 6$

Find:

Example

Example

Example

Example

Example