AAE 538: Air-Breathing Propulsion

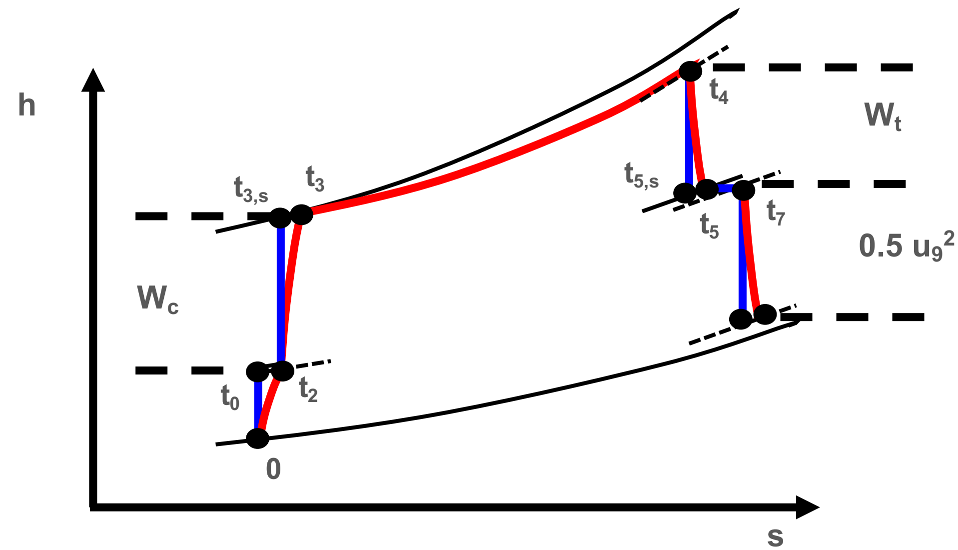
Lecture 20: Component Matching

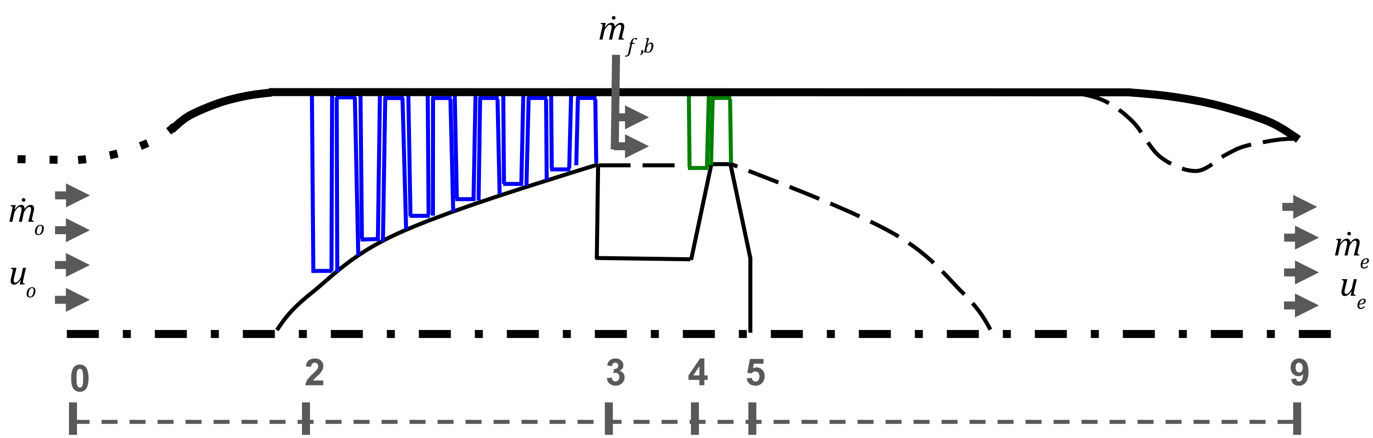
Prof. Carson D. Slabaugh

Purdue University
School of Aeronautics and Astronautics
Maurice J. Zucrow Laboratories

Introduction to Component Matching

- Consider the simple non-afterburning single-spool turbo jet engine
  - Assuming viable operation of the engine (stable compression and combustion), there are obvious constraints relating the gas generator components to each other.
    -
    -
    -
  - The basic problem at hand is to relate the thermophysical states of the turbomachinery components based on their 'pumping' characteristics;

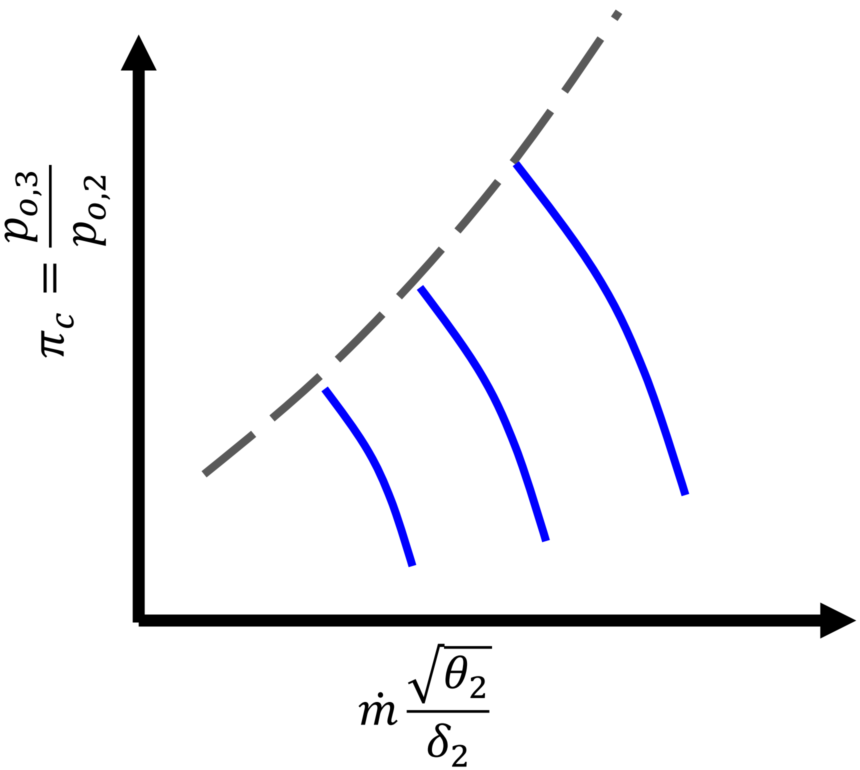


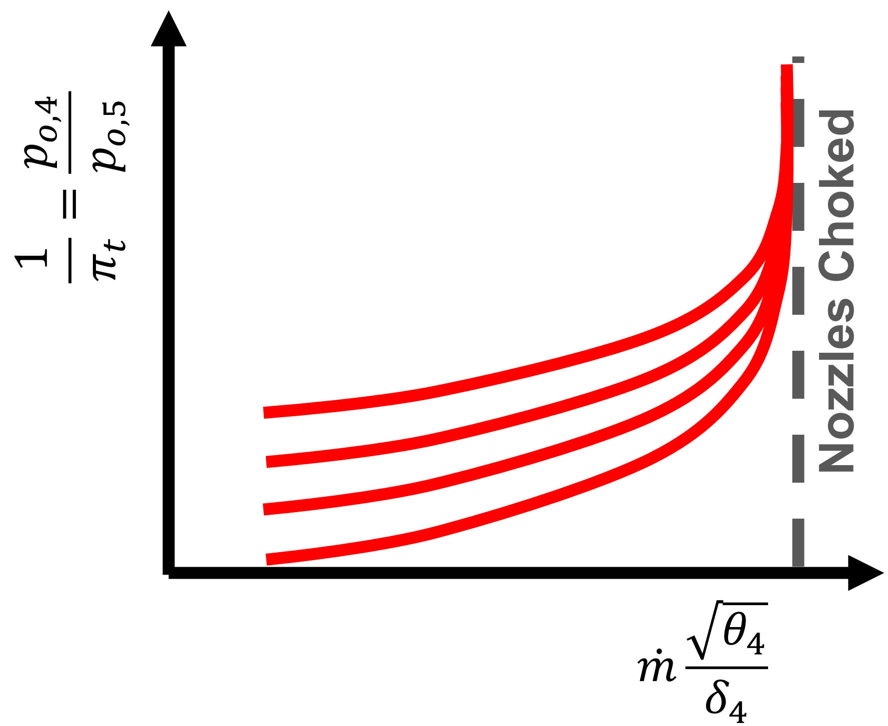


Introduction to Component Matching

- The compressor/turbine maps describe functions in the corrected mass-flow, pressure ratio space.

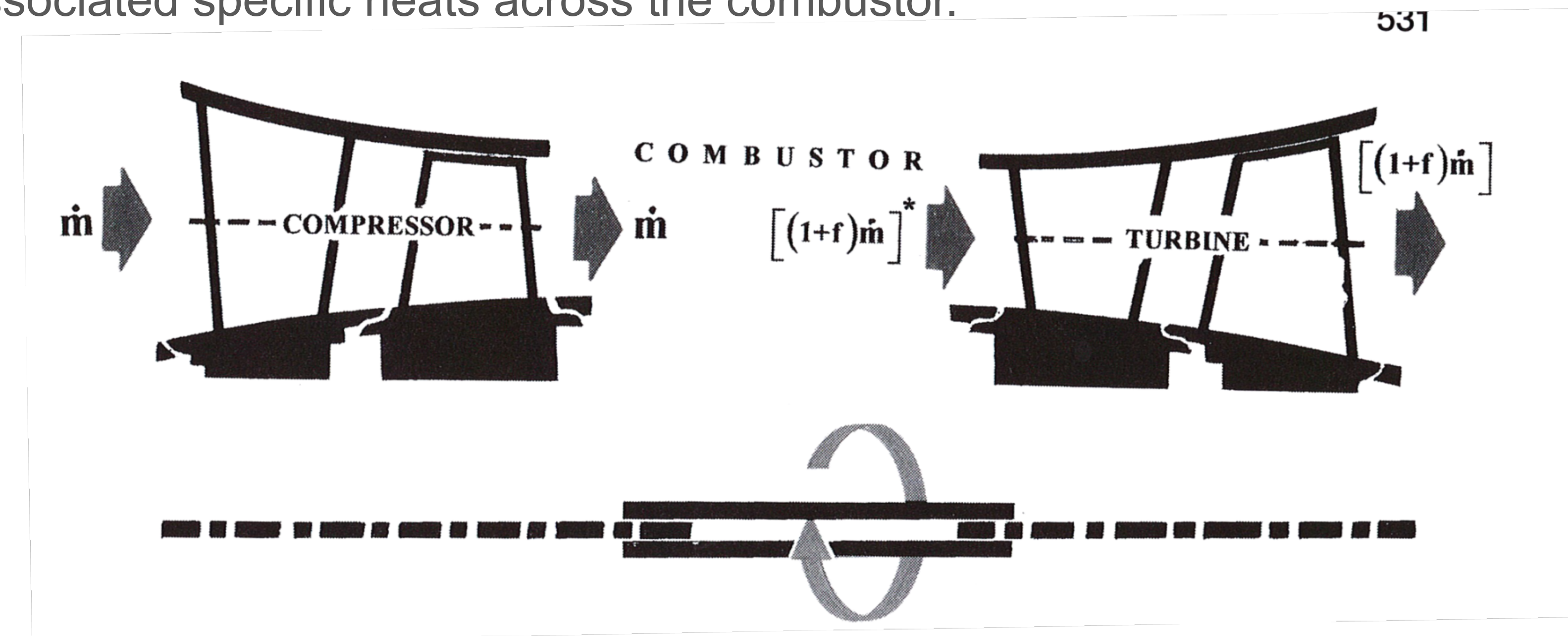
- As we discussed, the charts are generated from ____________________________, so they represent actual performance with losses (from shocks, friction, heat loss, etc.)





Introduction to Component Matching

- For improved accuracy in this analysis, we will consider a __________________________ in the gas composition and associated specific heats across the combustor.



- Recognizing this, the expression for the power balance can be re-written as:

Introduction to Component Matching

Where η_m is a new mechanical efficiency term to account for losses in bearings, etc. and

$$c_{p,c} = \left(\frac{\gamma_c}{\gamma_c - 1}\right) R_{air} \qquad c_{p,t} = \left(\frac{\gamma_t}{\gamma_t - 1}\right) R_{mix}$$

- In the absence of afterburning, it's usually a safe assumption to say $R_{mix} \approx R_{air}$, but in any case, it's pretty simple to compute the equilibrium composition with a tool like CEA.
- Rewriting in terms of our non-dimensional property ratios

and combining with the shaft speed correspondence and mass conservation:

Introduction to Component Matching

- As a quick reminder, the expressions for the corrected speeds and corrected mass flow rates are as follows:

  Where the non-dimensional variables θ and δ are defined with respect to the sea-level static conditions

Turbine-Compressor Matching

Problem Category 1

Consider the situation where the state C, on the compressor map, is defined.

- Given
  - Ambient conditions (p_0 and T_0)
  - Flight Mach number (M_0)
  - Machine speed (Ω)
  - Core mass flow rate ($\dot{m}_C$)
  - Fuel-air ratio (f)
  - System Mechanical Efficiency (η_m)
  - Turbine inlet total temperature ($T_{0,4}$)

Find the corresponding state T on the turbine map.

Turbine-Compressor Matching

- Step 1:
  - Unless explicitly provided, read/extract the corrected mass-flow rate ($\dot{m}_c$), the corrected speed (Ω_c), and the compressor stagnation pressure ratio (π_c) at state C on the compressor map (contour function).
- Step 2:
  - Knowing the ambient (inlet) conditions (p_0 and T_0) and the flight Mach number (M_0), calculate the compressor-inlet total properties, as follows:
- Step 3:
  - Calculate the compressor total temperature ratio

Turbine-Compressor Matching

- Step 4:
  - Calculate the specific heat ratio between the hot and cold gases; assuming a ______________________________ occurs through the combustor and calorically-perfect gas behavior continues everywhere else.

    Because the combustion products are still predominantly diatomic nitrogen, regardless of flame condition, the specific heat ratios of the reactants and products are typically considered to be roughly 1.4 and 1.33, respectively.

- Step 5:
  - Calculate the stagnation temperature ratio (τ_t) by substitution into:

$$\tau_t = 1 - \left[\frac{1}{\eta_m(1+f)}\right]\left(\frac{c_{p,c}}{c_{p,t}}\right)\left(\frac{T_{o,2}}{T_{o,4}}\right)(\tau_c - 1)$$

Turbine-Compressor Matching

- Step 6:
  - Calculate the turbine corrected speed ($\Omega_{c,4}$) as follows:
- Step 7:
  - At this point, we need to determine the thermophysical state at point ‘T’ on the turbine map via iteration.
    - With the turbine corrected speed known, pick an arbitrary point T^k along that constant speed line on the turbine map.
    - Compute/extract the corresponding magnitudes of $(\pi_t)_{T^k}$, $\left(\dot{m}_{c,4}\right)_{T^k}$, and $(\eta_t)_{T^k}$
    - Compute the corresponding stagnation temperature ratio

Turbine-Compressor Matching

- With state C fully-defined by the compressor map, you also now compute the total temperature ratio for the turbine using the expression from the energy balance

$$\tau_t = 1 - \left[\frac{1}{\eta_m(1+f)}\right]\left(\frac{c_{p,c}}{c_{p,t}}\right)\left(\frac{T_{o,2}}{T_{o,4}}\right)(\tau_c - 1)$$

This becomes your basis of error checking.

- Compare the magnitudes of the total temperature ratios computed from the energy balance and from your guessed condition on the turbine map. Iterate on a series of points constrained to the same corrected speed line. Continue iteration until the two magnitudes of τ_t converge to an acceptable level of error. AT that point, you have determined the thermo-physical state 'T' corresponding to the state C on the compressor map.

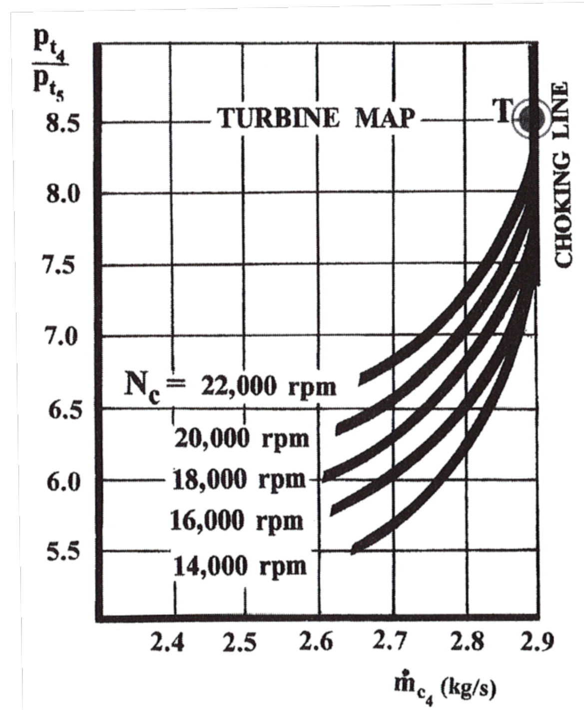
Turbine-Compressor Matching

Turbine-Compressor Matching

Problem Category 2

Another category of turbine-compressor matching is one where the turbine-operation point 'T' is known to be on the choking line. Again, the difficulty, here, arises from the fact that the turbine corrected speed cannot be found, since the corrected speed lines become indistinguishable.

The requirement, here, is to locate the operating point 'C' on the compressor map that corresponds to the point 'T' on the turbine map.



- Step 1:
  - Calculate $T_{o,2}$ in the same manner as in Category 1. The compressor corrected speed ($\Omega_{c,2}$) can then be calculated, as follows:

Turbine-Compressor Matching

This gives us one locus of the compressor point of operation (‘C’), which is simply the corrected speed line on the compressor map.

- Step 2:
  - From continuity, we can show:

    Where the turbine inlet temperature is considered a given design value. Rearranging, we find:

Turbine-Compressor Matching

where:

- The $\dot{m}_{c,4}$ term is a known variable, for it is the choking magnitude of the corrected mass flow on the turbine map.
- The term $\frac{1}{(1+f)}$ is a function of the air-fuel ratio, which is given/known
- The $\frac{p_{o,4}}{p_{o,3}}$ term is the combustion total pressure ratio, which either measured or assumed to be unity
- The $\frac{T_{o,2}}{T_{o,4}}$ term is known, as well. $T_{o,4}$ is always a known design limitation and the compressor inlet total temperature is computed from the given flight and ambient conditions.

○

Which represents another straight line on the compressor map; our second locus for operating state 'C' on the compressor map.

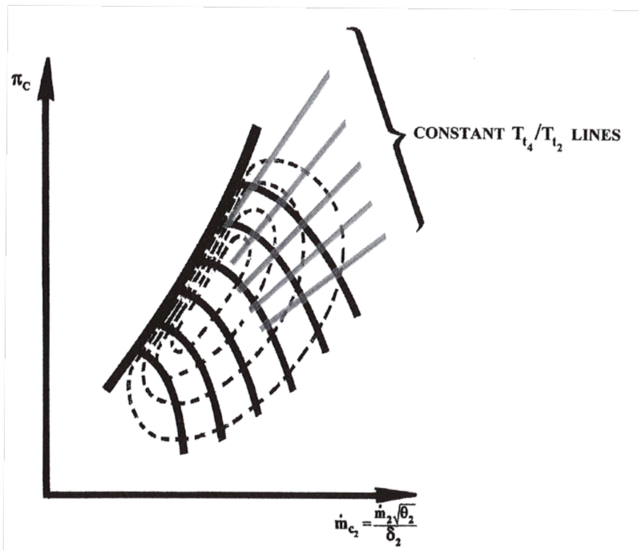
Turbine-Compressor Matching

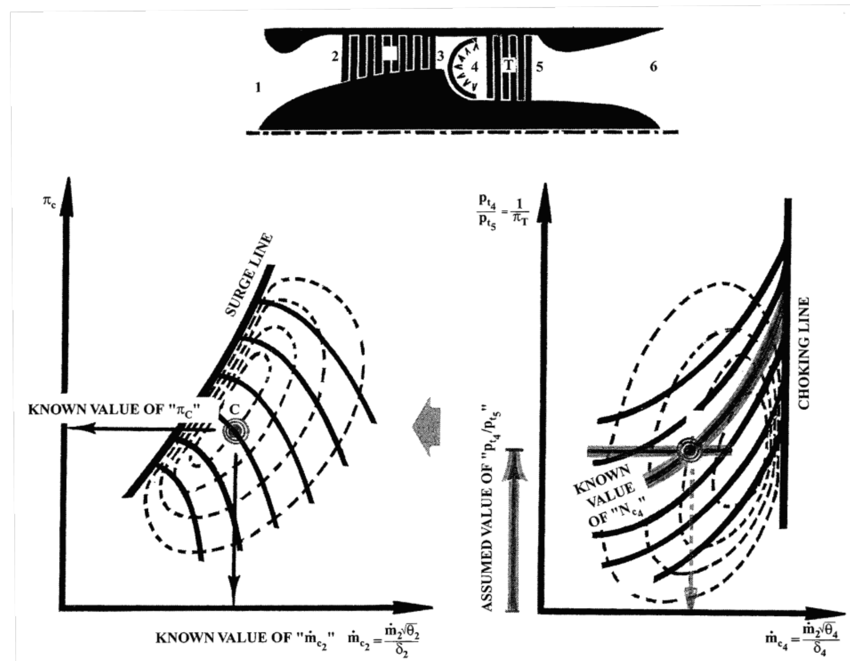
The intersection of this line with the corrected speed line gives the required thermophysical states at operating point 'C'.

Gas Generator Operating Lines

Constant $T_{o,4}/T_{o,2}$ Isocontours

The importance of these lines stems from the fact that the temperature ratio represents the single most persistent linkage in the turbine-compressor matching process.





Gas Generator Operating Lines

- Outer Loop:
  - Step 1 – Select a point 'C' on the compressor map and compute/extract the variables
  - Step 2 – Assume a reasonable magnitude of $T_{o,4}/T_{o,2}$ and compute the corrected speed, $\Omega_{c,4}$, where

    And $\Omega_{c,2}$ is known from step 1.
- Inner Loop:
  - Step 1 – Assume the turbine stagnation pressure ratio: $\pi_t = \frac{p_{o,5}}{p_{o,4}}$
  - Step 2 – Locate the operation point 'T' on the turbine map, based on the corrected speed $\Omega_{c,4}$ and the assumed π_t. Compute/Extract the turbine corrected mass flow rate $\dot{m}_{c,4}$ and the turbine efficiency η_t.

Gas Generator Operating Lines

- Step 3 – Compute the corresponding compressor corrected flow rate where, again, continuity gives

  Where all of the terms on the RHS are either known or assumed.

- Step 4 – Now, with a **new** compressor corrected mass flow rate, we compare this value to the one computed from step 1 of the outer loop. If the assumed and computed values don't match, iterate on the ______________________________ (within the inner loop) until a suitable level of error is achieved.

- Step 5 – Exit the inner loop.

Gas Generator Operating Lines

- (Back into the) Outer Loop:
  - Step 3 – We now calculate the value of $T_{o,4}/T_{o,2}$ for the (inner-loop converged) conditions and compare it with our originally assumed value from step 2 of the outer loop. From our ______________________________, we know

$$\eta_m \eta_t (1+f) c_{p,t} T_{o,4} \left[1 - \left(\frac{p_{o,5}}{p_{o,4}} \right)^{\frac{\gamma_t - 1}{\gamma_t}} \right] = \frac{c_{p,c} T_{o,2}}{\eta_c} \left[\left(\frac{p_{o,3}}{p_{o,2}} \right)^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right]$$

Where basic manipulations give us:

With all other terms known, this equation give us

Gas Generator Operating Lines

- Step 4 – Compare the computed value of $T_{o,4}/T_{o,2}$ to assumed value and iterate (using your favorite method) until a negligible level of error is achieved. This converged answer is the total temperature ratio that corresponds to that operating point (on the compressor map). Repeat as necessary to populate this information throughout the compressor map

- Step 5 – Repeat as necessary to populate this information throughout the compressor map. The result will be a scattered series of data points from which one can interpolate to a desired operating point, or fit a function.