Thomas Satterly

STAT 51400, Section 9

HW 8, 11/2/2017

1.

Given:

An experiment is conducted to study the influence of operating temperature and three types of face plate glass on the light output of an oscilloscope tube. The data is as follows:

Glass Type	100	125	150
1	58 56.8 57	107, 106.7, 106.5	129.2, 128, 128.6
2	55 53 57.9	107, 103.5, 105	117.8, 116.2, 109.9
3	54.6, 57.5, 59.9	106.5, 107.3, 103.6	101.7, 105.4, 103.9

(Temperature)

Find:

a) Write the statistical model. Use ANOVA to determine if the factorial effects are significant. State the hypotheses and use $\alpha = 5\%$

b) Obtain estimates of the main effects and interactions

c) Use proper plots to check assumptions

d) Generate the interaction plot for the glass type and temperature and interpret their interaction

e) Use the Bonferroni procedure to perform a pairwise comparison for glass type level means. What are the conclusions?

f) Use Tukey's method for pairwise comparison between treatment means

g) Use regression to derive the functional relationship between the light output and temperature for each type of glass

Solution:

a) The statistical model is:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk}$$

μ: grand mean
τ_i: i^{th} effect for factor i
β_j: j^{th} effect for factor j
$(\tau\beta)_{ij}$: Interaction effect of the factor combination ij
ε_{ijk}: Error $\sim N(0, \sigma^2)$

$$\sum_i \tau_i = 0,\ \sum_j \beta_j = 0,\ \sum_i (\tau\beta)_{ij} = 0,\ \sum_j (\tau\beta)_{ij} = 0$$

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There are three hypothesis to test

1: $H_0: \tau_1 = \tau_2 = \dots \tau_3 = 0 \rightarrow$ Glass

2: $H_0: \beta_1 = \beta_2 = \dots \beta_3 = 0 \rightarrow$ Temperature

3: $H_0: (\tau\beta)_{ij} = 0$ for all $i, j \rightarrow$ Glass and Temperature Interaction

See attached for SAS code and output

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	$P_r > F_0$
Glass	310.89	2	155.445	35.81	<0.0001
Temperature	18142.46	2	9071.228	2089.97	<0.0001
Interaction	642.41	4	160.6014	37	<0.0001
Error	78.1267	18	4.3404		
Total	19173.878	26			

1: With $F_0 = 35.81 \Rightarrow (P_r > F_0) < 0.0001 < \alpha$, reject H_0, conclude there is a difference between the glass types used

2: With $F_0 = 2089.97 \Rightarrow (P_r > F) < 0.0001 < \alpha$, reject H_0, conclude there is a difference between the temperatures tested

3: With $F_0 = 37 \Rightarrow (P_r > F) < 0.0001 < \alpha$, reject H_0, conclude at least one $(\tau\beta)_{ij} \neq 0$, there is an interaction effect

```
ods html close;
ods html;

data scope;
    infile 'D:\Grad\+stat514\+HW8\data.dat';
    input glass temperature output;
    if glass=1 then x1=1;
    if glass=1 then x2=0;
    if glass=2 then x1 = 0;
    if glass=2 then x2 = 1;
    if glass=3 then x1=-1;
    if glass=3 then x2=-1;
    t = (temperature-1)*25+100;
    t2 = t*t;
    x1t = x1*t;
    x2t = x2*t;
    x1t2=x1*t2;
    x2t2=x2*t2;
proc print;

proc glm data = scope;
    class glass temperature;
    model output = glass temperature glass*temperature;
    output out=scopeNew r=res p=pred;
    lsmeans glass temperature glass*temperature;
    means glass /bon lines;
    lsmeans glass|temperature/tdiff adjust=tukey;
run;

proc sort; by pred;

symbol1 v=circle;
proc gplot data=scopeNew;
    plot res*pred/frame;
run;

proc univariate data=scopeNew;
    var res; qqplot res / normal (L=1 mu=est sigma=est);
run;

proc means noprint data=scope;
    var output;
    by glass temperature;
    output out=scopemean mean=mn;

symbol1 v=circle i=join;
symbol2 v=square i=join;
symbol3 v=triangle i=join;
proc gplot data=scopemean;
    plot mn*temperature=glass;
run;

proc reg data=scope;
    model output=x1 x2 t x1t x2t t2 x1t2 x2t2;
run;
```

The SAS System

The GLM Procedure

Dependent Variable: output

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	19095.75185	2386.96898	549.95	<.0001
Error	18	78.12667	4.34037		
Corrected Total	26	19173.87852			

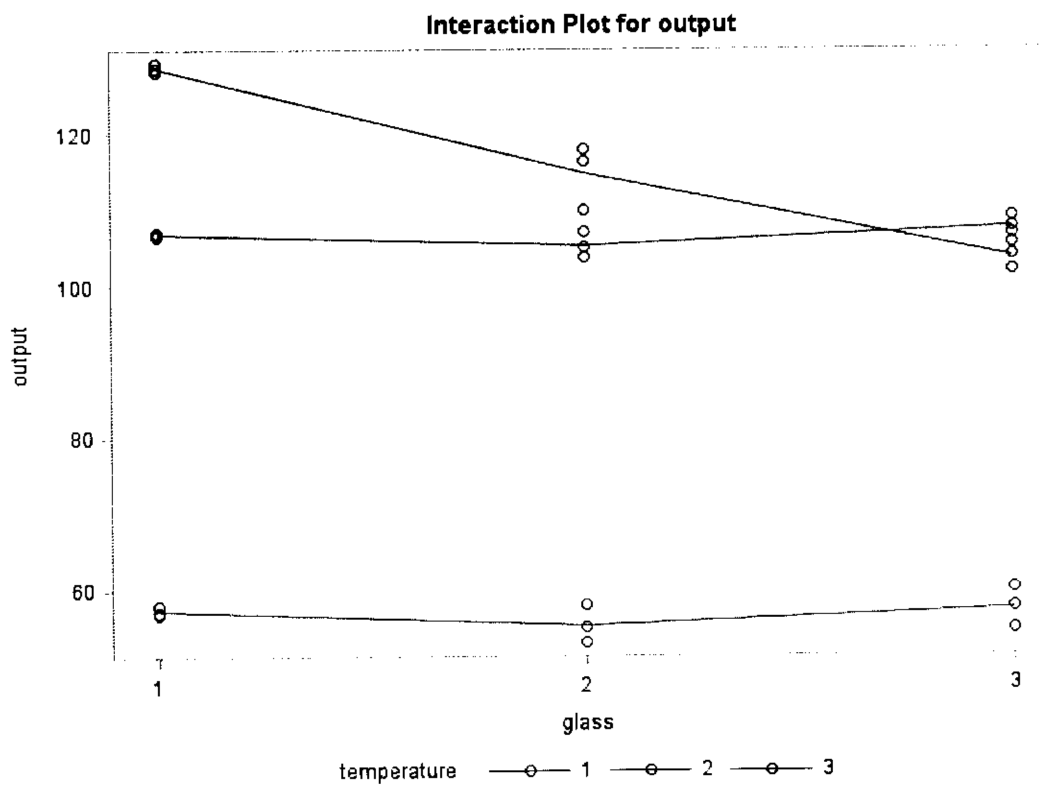
Part (b)

R-Square	Coeff Var	Root MSE	output Mean
0.995925	2.242400	2.083356	92.90741

Part (a)

Source	DF	Type I SS	Mean Square	F Value	Pr > F
glass	2	310.88963	155.44481	35.81	<.0001
temperature	2	18142.45630	9071.22815	2089.97	<.0001
glass*temperature	4	642.40593	160.60148	37.00	<.0001

Source	DF	Type III SS	Mean Square	F Value	Pr > F
glass	2	310.88963	155.44481	35.81	<.0001
temperature	2	18142.45630	9071.22815	2089.97	<.0001
glass*temperature	4	642.40593	160.60148	37.00	<.0001



b) Glass effect estimate : $\hat{\tau}_i$

$$\hat{\tau}_1 = \bar{y}_{1..} - \bar{y}_{...} = 97.5\overline{33} - 92.90741 = 4.6259$$
$$\hat{\tau}_2 = \bar{y}_{2..} - \bar{y}_{...} = 91.7 - 92.90741 = -1.20741$$
$$\hat{\tau}_3 = \bar{y}_{3..} - \bar{y}_{...} = 89.4\overline{88} - 92.90741 = -3.41852$$

$\Rightarrow$ $\boxed{\hat{\tau}_1 = 4.6259,\ \hat{\tau}_2 = -1.2074,\ \hat{\tau}_3 = -3.4185}$

Temperature effect estimate : $\hat{\beta}_j$

$$\hat{\beta}_1 = \bar{y}_{.1.} - \bar{y}_{...} = 56.6\overline{33} - 92.90741 = -36.2741$$
$$\hat{\beta}_2 = \bar{y}_{.2.} - \bar{y}_{...} = 106.4\overline{55} - 92.90741 = 13.5481$$
$$\hat{\beta}_3 = \bar{y}_{.3.} - \bar{y}_{...} = 115.6\overline{33} - 92.90741 = 22.7259$$

$\boxed{\hat{\beta}_1 = -36.2741,\ \hat{\beta}_2 = 13.5481,\ \hat{\beta}_3 = 22.7259}$

Interaction Effect:

$$\widehat{(\tau\beta)}_{ij} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}$$

$$(\tau\beta)_{11} = 57.2\overline{66} - 97.5\overline{33} - 56.6\overline{33} + 92.90741 = -3.9926$$
$$(\tau\beta)_{12} = 106.7\overline{33} - 97.5\overline{33} - 106.4\overline{55} + 92.90741 = -4.3481$$
$$(\tau\beta)_{13} = 128.6 - 97.5\overline{33} - 115.6\overline{33} + 92.90741 = 8.3407$$
$$(\tau\beta)_{21} = 55.3 - 91.7 - 56.6\overline{33} + 92.90741 = -0.1259$$
$$(\tau\beta)_{22} = 105.1\overline{66} - 91.7 - 106.4\overline{55} + 92.90741 = -0.0815$$
$$(\tau\beta)_{23} = 114.6\overline{33} - 91.7 - 115.6\overline{33} + 92.90741 = 0.2074$$
$$(\tau\beta)_{31} = 57.\overline{33} - 89.4\overline{88} - 56.6\overline{33} + 92.90741 = 4.1185$$
$$(\tau\beta)_{32} = 107.4\overline{66} - 89.4\overline{88} - 106.4\overline{55} + 92.90741 = 4.4296$$
$$(\tau\beta)_{33} = 103.\overline{66} - 89.4\overline{88} - 115.6\overline{33} + 92.90741 = -8.5481$$

$$\boxed{\begin{aligned}
(\tau\beta)_{11} &= -3.9926\\
(\tau\beta)_{12} &= -4.3481\\
(\tau\beta)_{13} &= 8.3407\\
(\tau\beta)_{21} &= -0.1259\\
(\tau\beta)_{22} &= -0.0815\\
(\tau\beta)_{23} &= 0.2074\\
(\tau\beta)_{31} &= 4.1185\\
(\tau\beta)_{32} &= 4.4296\\
(\tau\beta)_{33} &= -8.5481
\end{aligned}}$$

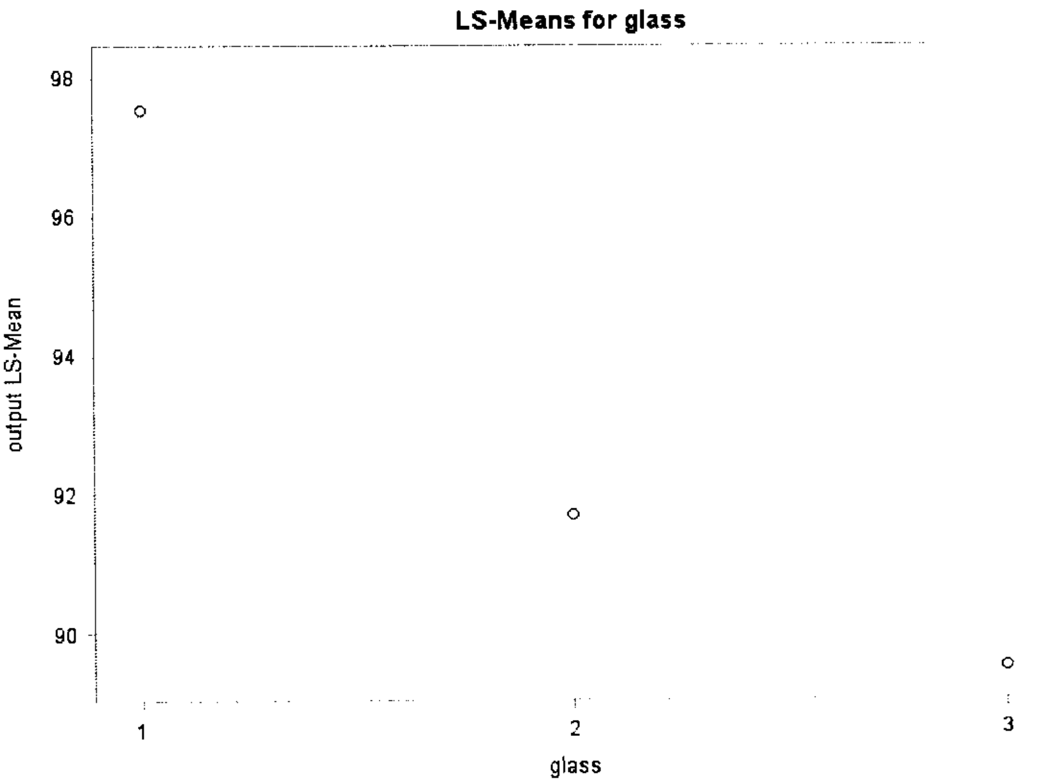
See attached for SAS code and outputs

The SAS System

The GLM Procedure
Least Squares Means

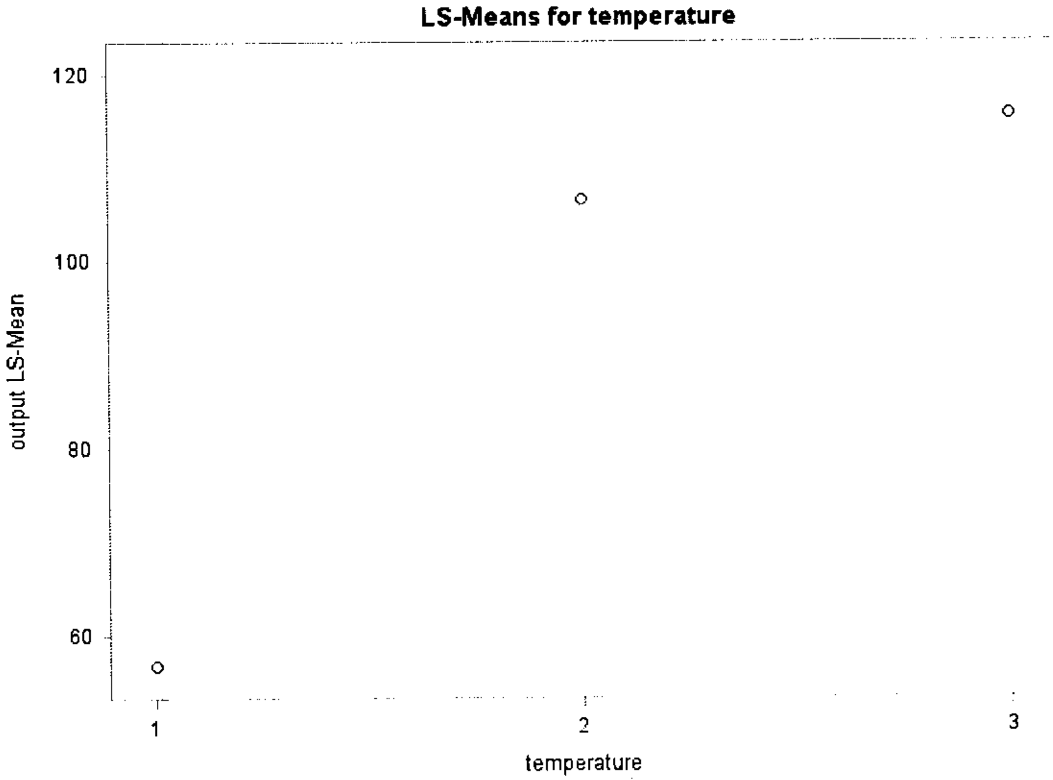
glass	output LSMEAN
1	97.5333333
2	91.7000000
3	89.4888889

Part (b)



temperature	output LSMEAN
1	56.633333
2	106.455556
3	115.633333

Part (b)



glass	temperature	output LSMEAN
1	1	57.266667
1	2	106.733333
1	3	128.600000
2	1	55.300000
2	2	105.166667
2	3	114.633333
3	1	57.333333
3	2	107.466667
3	3	103.666667

– Part (b)

c) See attached for SAS code and plots

Constant Variance: Based on the residual plot, there is no apparent violation of the constant variance assumption. However, there looks to be a possible outlier and, at the far right, a slight decrease in variance. A formal test is needed to determine if these are significant

Normal Distribution of Error: The Q-Q plot shows that the residuals do not greatly deviate from the normal curve, but there does appear to be skew in the data, as shown by a curved pattern in the residuals. A formal test is needed to determine if this is significant

d) Interaction: Between glass 1 and 2, there appears to be no interaction with temperature. However, glass 3 no longer appears to follow the parallel lines between glass 1 and 2, indicating that there is an antagonistic interaction effect with glass 3 and temperature.

e) See attached for SAS code and output.

Based on the Bonferonni pairwise comparison, there is no significant difference between glass 2 and 3. However, glass 1 is significantly different from both glass 2 and 3.

f) See attached for SAS code and output

Most pairs of glass temperature comparisons are significantly different, with only 9 of the 36 combinations being of no significant difference.

g) See attached for SAS code and output as well as Matlab code and output.

The functions for each glass type are as follows, with no transformation required for input temperature "t"

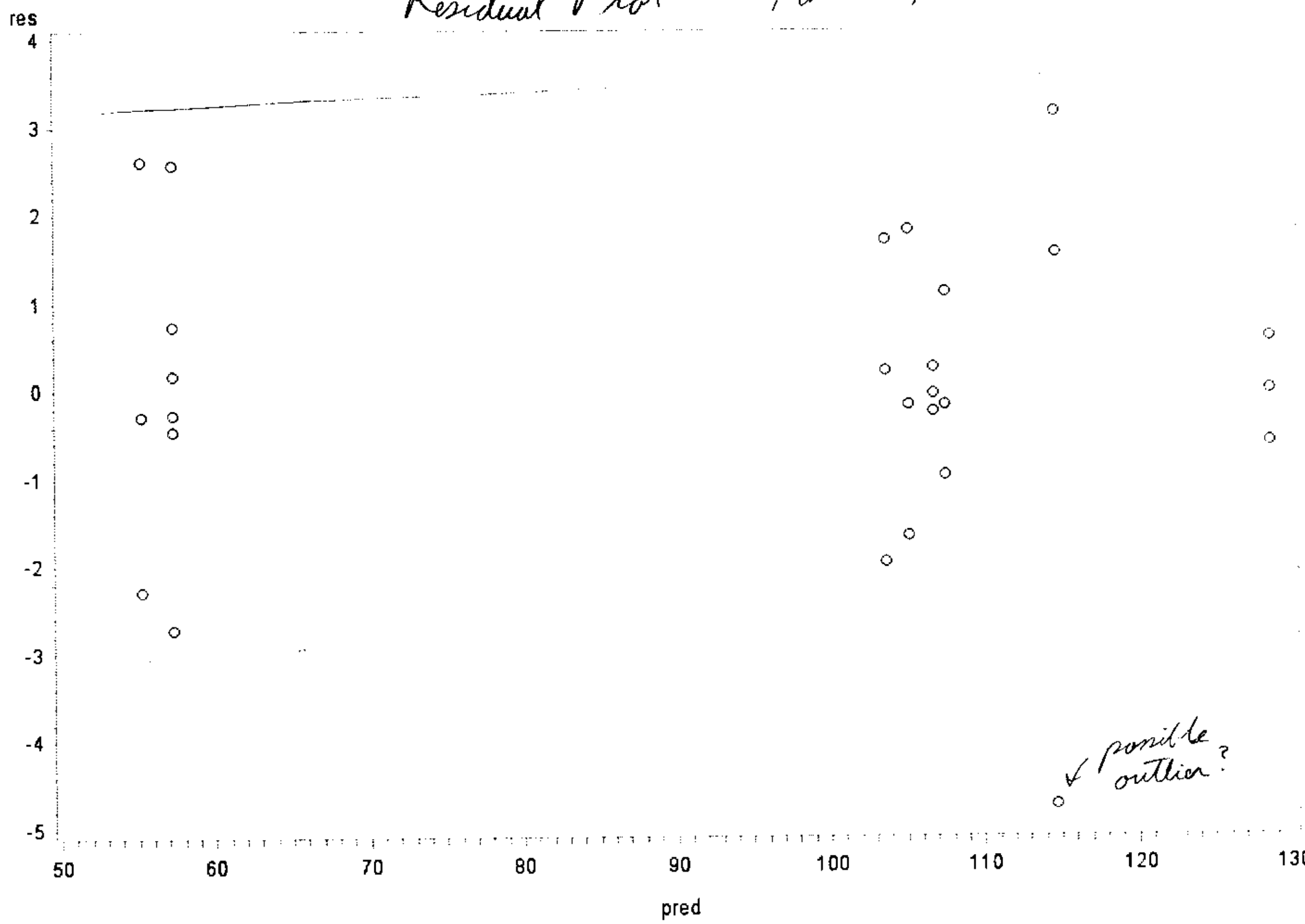
Glass 1: $f(t) = -416.6 + 6.9467t - 0.0221t^2$

Glass 2: $f(t) = -550.0333 + 9.2667t - 0.0323t^2$

Glass 3: $f(t) = -680.6667 + 11.7133t - 0.0432t^2$

The attached plots show the regression overlaying the data

Residual Plot Part (c)

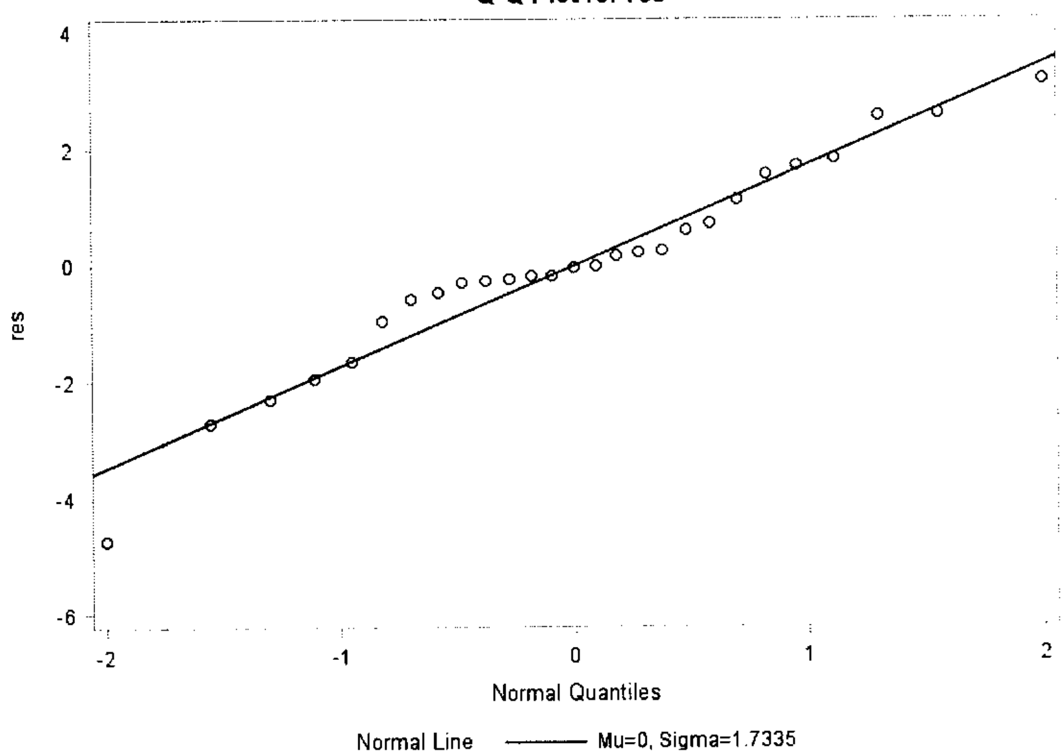


The SAS System

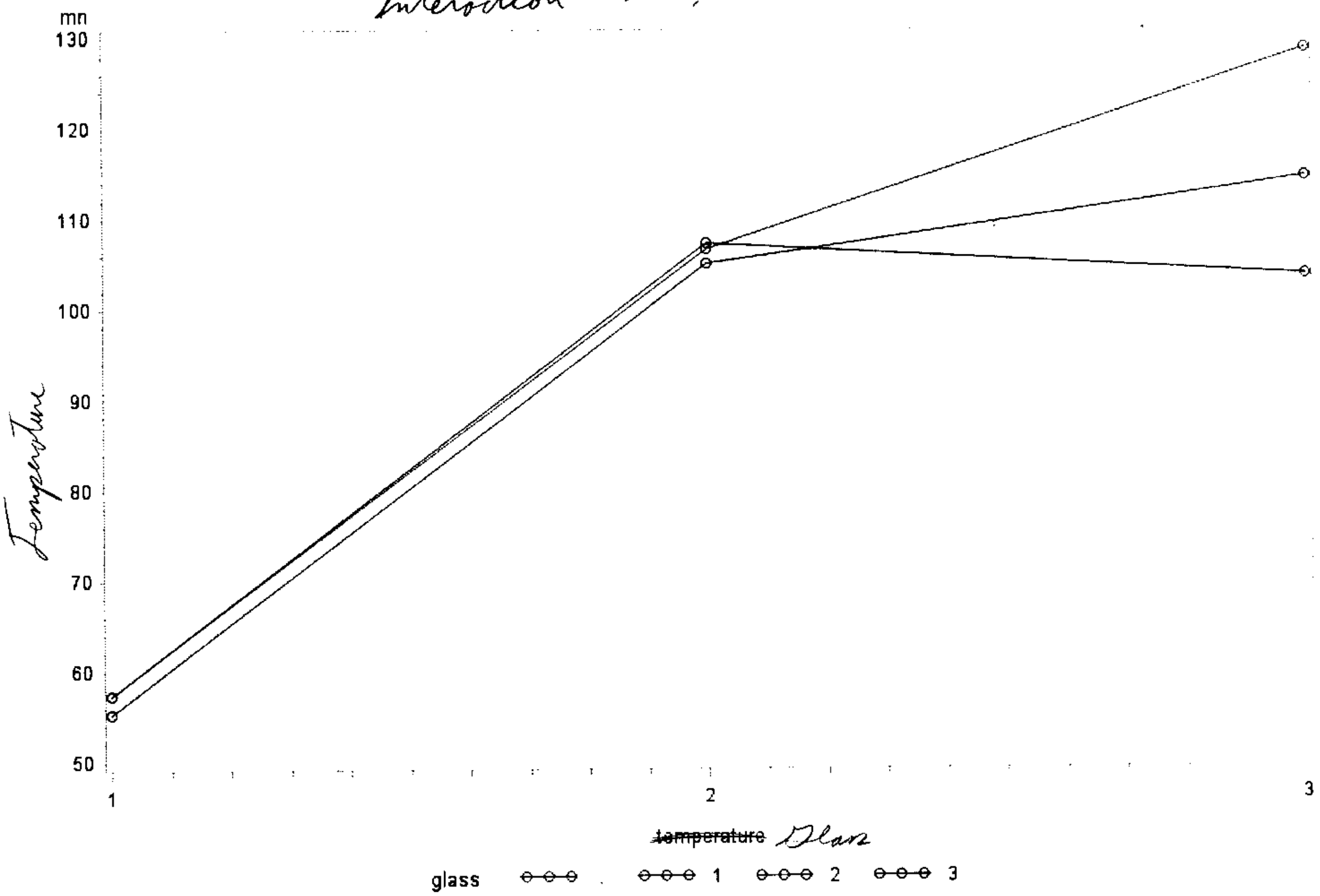
The UNIVARIATE Procedure

Part (c)

Q-Q Plot for res



Interaction Plot, Part (d)



The SAS System

Part (e)

The GLM Procedure

Bonferroni (Dunn) t Tests for output

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	18
Error Mean Square	4.34037
Critical Value of t	2.63914
Minimum Significant Difference	2.5919

Means with the same letter are not significantly different.

Bon Grouping	Mean	N	glass
A	97.5333	9	1
B	91.7000	9	2
B			
B	89.4889	9	3

The SAS System

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

glass	temperature	output LSMEAN	LSMEAN Number
1	1	57.266667	1
1	2	106.733333	2
1	3	128.600000	3
2	1	55.300000	4
2	2	105.166667	5
2	3	114.633333	6
3	1	57.333333	7
3	2	107.466667	8
3	3	103.666667	9

Least Squares Means for Effect glass*temperature
t for H0: LSMean(i)=LSMean(j) / Pr > |t|
Dependent Variable: output

i/j	1	2	3	4	5	6	7	8	9
1		-29.08	-41.9348	1.156147	-28.159	-33.7242	-0.03919	-29.5111	-27.2772
		<.0001	<.0001	0.9561	<.0001	<.0001	1.0000	<.0001	<.0001
2	29.08003		-12.8548	30.23618	0.920998	-4.64418	29.04084	-0.43111	1.802805
	<.0001		<.0001	<.0001	0.9886	0.0049	<.0001	0.9999	0.6802
3	41.93482	12.85479		43.09096	13.77578	8.210602	41.89563	12.42368	14.65759
	<.0001	<.0001		<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
4	-1.15615	-30.2362	-43.091		-29.3152	-34.8804	-1.19534	-30.6673	-28.4334
	0.9561	<.0001	<.0001		<.0001	<.0001	0.9474	<.0001	<.0001
5	28.15903	-0.921	-13.7758	29.31518		-5.56518	28.11984	-1.3521	0.881807
	<.0001	0.9886	<.0001	<.0001		0.0007	<.0001	0.9015	0.9913
6	33.72422	4.644183	-8.2106	34.88036	5.565181		33.68502	4.213077	6.446988
	<.0001	0.0049	<.0001	<.0001	0.0007		<.0001	0.0120	0.0001
7	0.039191	-29.0408	-41.8956	1.195338	-28.1198	-33.685		-29.4719	-27.238
	1.0000	<.0001	<.0001	0.9474	<.0001	<.0001		<.0001	<.0001
8	29.51114	0.431106	-12.4237	30.66728	1.352104	-4.21308	29.47195		2.233911
	<.0001	0.9999	<.0001	<.0001	0.9015	0.0120	<.0001		0.4261
9	27.27723	-1.80281	-14.6576	28.43337	-0.88181	-6.44699	27.23804	-2.23391	
	<.0001	0.6802	<.0001	<.0001	0.9913	0.0001	<.0001	0.4261	

Part (f)

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: output

Number of Observations Read	28
Number of Observations Used	27
Number of Observations with Missing Values	1

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	19096	2386.96898	549.95	<.0001
Error	18	78.12667	4.34037		
Corrected Total	26	19174			

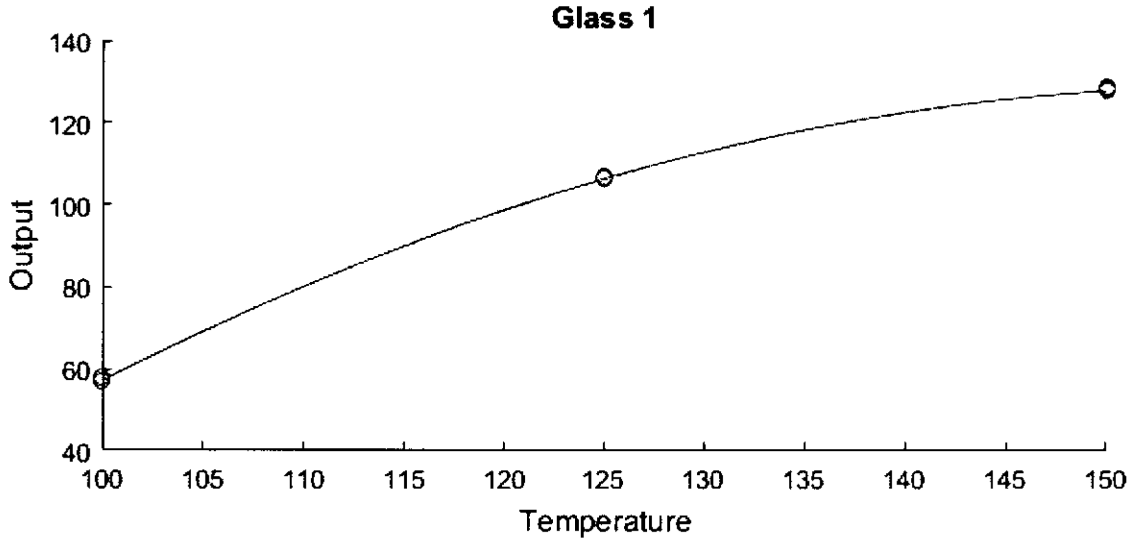
Root MSE	2.08336	**R-Square**	0.9959
Dependent Mean	92.90741	**Adj R-Sq**	0.9941
Coeff Var	2.24240		

Parameter Estimates

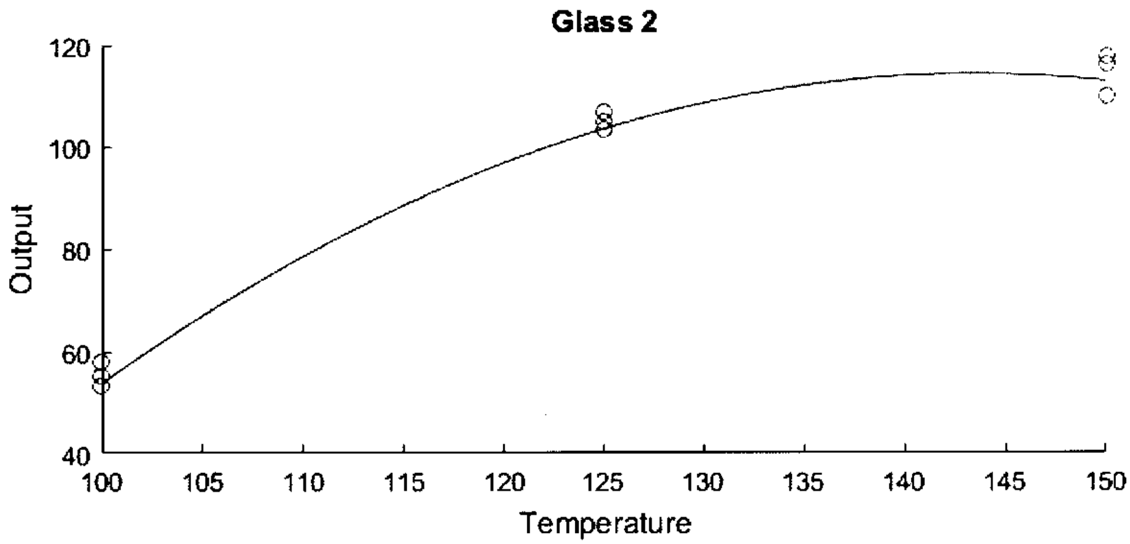
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	-549.10000	20.84513	-26.34	<.0001
x1	1	132.50000	29.47946	4.49	0.0003
x2	1	0.93333	29.47946	0.03	0.9751
t	1	9.30889	0.34078	27.32	<.0001
x1t	1	-2.36222	0.48193	-4.90	0.0001
x2t	1	-0.04222	0.48193	-0.09	0.9312
t2	1	-0.03252	0.00136	-23.89	<.0001
x1t2	1	0.01044	0.00192	5.42	<.0001
x2t2	1	0.00019556	0.00192	0.10	0.9202

Part (g)

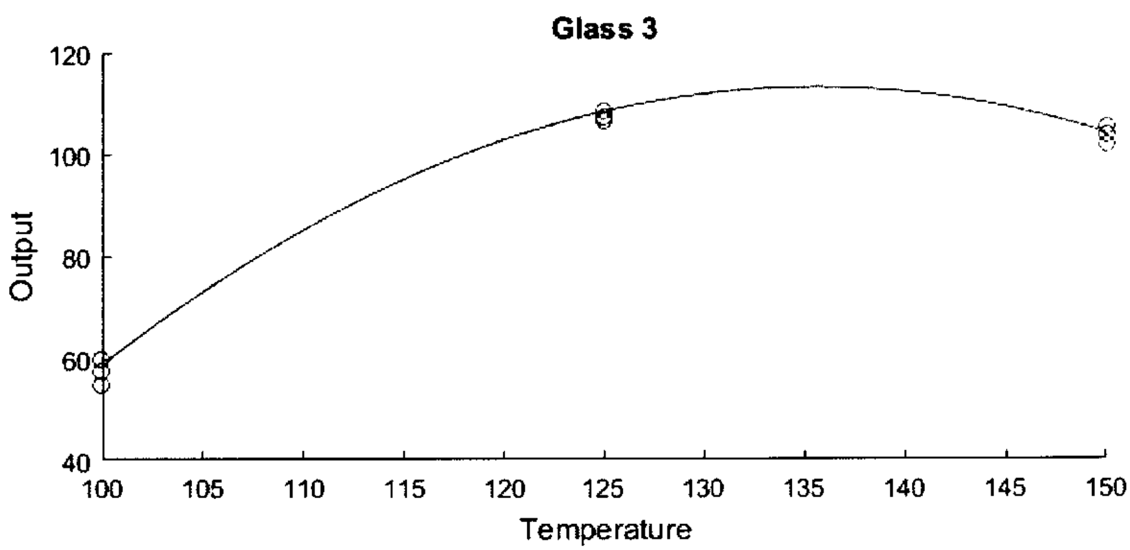
Part (g)



Glass 1
Output
Temperature
140
120
100
80
60
40
100
105
110
115
120
125
130
135
140
145
150



Glass 2
Output
Temperature
120
100
80
60
40
100
105
110
115
120
125
130
135
140
145
150



Glass 3
Output
Temperature
120
100
80
60
40
100
105
110
115
120
125
130
135
140
145
150

```
% Thomas Satterly
% STAT 514, HW 8

% Full regression from SAS
% f = @(x1, x2, t) -549.1 + 132.5 * x1 - 0.93333 * x2 + 9.30889 * t +...
%     -2.36222 * x1 * t - 0.0422 * x2 * t +...
%     -0.03252 * t^2 + 0.01044 * x1 * t^2 + 0.00019556 * x2 * t^2;

% % Regression split up between glass types
f1 = @(t) -416.6 + 6.9467 * t - 0.0221 * t^2;
f2 = @(t) -550.0333 + 9.2667 * t - 0.0323 * t^2;
f3 = @(t) -680.6667 + 11.7133 * t - 0.0432 * t^2;

% Temperature space
temps = linspace(100, 150, 50);

% Raw data
t = [100 100 100 125 125 125 150 150 150];
g1 = [58 56.8 57 107 106.7 106.5 129.2 128 128.6];
g2 = [55 53 57.9 107 103.5 105 117.8 116.2 109.9];
g3 = [54.6 57.5 59.9 106.5 107.3 108.6 101.7 105.4 103.9];

for i = 1:numel(temps)
    y11(i) = f1(temps(i));
    y22(i) = f2(temps(i));
    y33(i) = f3(temps(i));
end

% Plot
figure;
subplot(3, 1, 1);
hold on;
plot(temps, y11);
plot(t, g1, 'o');
title('Glass 1');
xlabel('Temperature');
ylabel('Output');

subplot(3, 1, 2);
hold on;
plot(temps, y22);
plot(t, g2, 'o');
title('Glass 2');
xlabel('Temperature');
ylabel('Output');

subplot(3, 1, 3);
hold on;
plot(temps, y33);
plot(t, g3, 'o');
title('Glass 3');
```

```
xlabel('Temperature');
ylabel('Output');
```