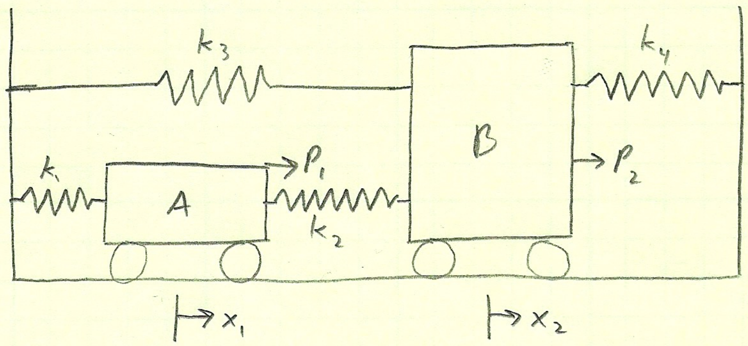
Part I

Given:



The potential energy of the system is:

$$f(x) = \frac{1}{2} x^T K x - x^T P$$

Where x is the displacement vector $x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$ and is the design variable vector.

K is the global stiffness vector is given by:

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 + k_4 \end{bmatrix}$$

Minimizing $f(x)$ will provide displacements x under applied load P

$$P = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

The following is known:

$k_1 = 3000$ N/m, $k_2 = 1000$ N/m, $k_3 = 2500$ N/m, $k_4 = 1500$ N/m
$P_1 = 500$ N, $P_2 = 1000$ N

Find:

1) Develop analytic expressions for the gradient vector components and Hessian matrix.

2-8) See attached for problem statements and solutions.

Solution:

1) $f(x) = \frac{1}{2} x^T K x - x^T P$

$$= \frac{1}{2}[x_1, x_2]\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3+k_4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - [x_1, x_2]\begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$= \frac{1}{2}\left[x_1(k_1+k_2) - x_2k_2,\ -x_1k_2 + x_2(k_2+k_3+k_4)\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - x_1P_1 - x_2P_2$$

$$= \frac{1}{2}\left(x_1^2(k_1+k_2) - x_1x_2k_2 - x_1x_2k_2 + x_2^2(k_2+k_3+k_4)\right) - x_1P_1 - x_2P_2$$

$$f(x) = \frac{1}{2}x_1^2(k_1+k_2) + \frac{1}{2}x_2^2(k_2+k_3+k_4) - x_1x_2k_2 - x_1P_1 - x_2P_2$$

$$\frac{df}{dx_1} = x_1(k_1+k_2) - x_2k_2 - P_1$$

$$\frac{df}{dx_2} = x_2(k_2+k_3+k_4) - x_1k_2 - P_2$$

$$\boxed{\nabla f = \begin{Bmatrix} x_1(k_1+k_2) - x_2k_2 - P_1 \\ x_2(k_2+k_3+k_4) - x_1k_2 - P_2 \end{Bmatrix} = x^T K - P}$$

$$\frac{d^2f}{dx_1 dx_2} = -k_2$$

$$\frac{d^2f}{dx_1^2} = k_1 + k_2$$

$$\frac{d^2f}{dx_2^2} = k_2 + k_3 + k_4$$

$$\boxed{H = \begin{Bmatrix} k_1+k_2, & -k_2 \\ -k_2, & k_2+k_3+k_4 \end{Bmatrix} = K}$$