Motion Planning

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- All sources of material must be cited. The University Academic Code of Conduct will be strictly enforced.

THIS IS A GROUP ASSIGNMENT: submit one from each team.

Goals and Learning outcomes
The goal of this lab is to provide you with tools that will help you in a head-to-head race on a race track. After finishing this lab, your car should be able to do something like https://www.youtube.com/watch?v=llHCRqwIllM. The following fundamentals should be understood by the students upon completion of this lab:

- Motion Planning basic concepts.

  - Configuration space vs. Workspace: you should understand the difference between configuration space and workspace, and the advantages and disadvantages of planning in each of them.
  - Free space vs. Obstacle space: you should understand the difference between free space and obstacle space.
  - Occupancy grids and Costmaps: you should understand what occupancy grids and costmaps are, how to use them, and how to create them.

- Motion Planning algorithms. You should understand the basic ideas of these following planning algorithms, their advantages and disadvantages, and when to use them.

  - Grid-based search: Dijkstra's, A*, and their variants
  - Sampling based algorithms: RRT and its variants

1 Part A: Written assignment

Turn **Part A** in at the end of lab 1 of motion planning.

0	1	2	3	4	5	6	GOAL
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39
START	41	42	43	44	45	46	47

Figure 1: Grid World

1.1 Grid world planning with Dijkstra's

Using figure 1, write out steps for Dijkstra's algorithm (**8-connected**, assume uniform cost for each action). At each step, list the grid cells in the open set with their running cost and the grid cells in the visited set. Write the final path found as a list of grid cell ids.

Dijkstra's Path Planning
For each step, the state of every node is shown in the form {visited(v) or unvisited(u); running cost} next to the node number. Since uniform cost is used for any move, we take c to be the cost of a single move to any of the 8-connected neighbors of a node.
Key: inf - infinity, u - unvisited, v - visited
Step 1:Start from start node, assign cost to 32, 33, 41

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; inf}	25 {u; inf}	26 {u; inf}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {u; c}	33 {u; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; c}	42 {u; inf}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 2: Visit node 32, assign cost to 24, 25

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; 2c}	25 {u; 2c}	26 {u; inf}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {u; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; c}	42 {u; inf}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 3: Visit node 33, assign cost to 26, 42

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; 2c}	25 {u; 2c}	26 {u; 2c}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; c}	42 {u; 2c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 4: Visit node 41, no changes to any cost

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; 2c}	25 {u; 2c}	26 {u; 2c}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {u; 2c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 5:Visit node 24, no changes to any costs

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {u; 2c}	26 {u; 2c}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {u; 2c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 6: Visit node 25, no changes to any cost

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {u; 2c}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {u; 2c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 7: Visit node 42, assign cost to 43

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {u; 2c}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {u; 3c}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 8: Visit node 26, assign cost to 27

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {u; 3c}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {u; 3c}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 9: Visit node 43, assign cost to 44

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {u; 3c}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {u; 4c}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 10: Visit node 27, assign cost to 20, 28

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; 4c}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {u; 4c}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {u; 4c}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 11: Visit node 44, assign cost to 45

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; 4c}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {u; 4c}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {u; 5c}	46 {u; inf}	47 {u; inf}

Step 12: Visit node 20, assign cost to 12, 21, 29

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 5c}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {u; 5c}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {u; 4c}	29 {u; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {u; 5c}	46 {u; inf}	47 {u; inf}

Step 13: Visit node 28, no changes to any costs

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 5c}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {u; 5c}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {u; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {u; 5c}	46 {u; inf}	47 {u; inf}

Step 14: Visit node 45, assign cost to 38, 46

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 5c}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {u; 5c}	22 {u; inf}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {u; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; 6c}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {u; 6c}	47 {u; inf}

Step 15: Visit node 21, assign cost to 14, 22

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 5c}	13 {u; inf}	14 {u; 6c}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {u; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; 6c}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {u; 6c}	47 {u; inf}

Step 16: Visit node 12, assign cost to 3, 4, 5

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; 6c}	4 {u; 6c}	5 {u; 6c}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {u; 6c}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {u; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; 6c}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {u; 6c}	47 {u; inf}

Step 17: Visit node 29, no changes to any costs

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; 6c}	4 {u; 6c}	5 {u; 6c}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {u; 6c}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; 6c}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {u; 6c}	47 {u; inf}

Step 18: Visit node 14, assign cost to goal, 5, 6, 15

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; 6c}	4 {u; 6c}	5 {u; 6c}	6 {u; 7c}	goal {u; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {v; 6c}	15 {u; 7c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; 6c}	39 {u; inf}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {u; 6c}	47 {u; inf}

Step 19: Visit node 46, assign cost to 39, 47

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; 6c}	4 {u; 6c}	5 {u; 6c}	6 {u; 7c}	goal {u; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {v; 6c}	15 {u; 7c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; inf}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; 6c}	39 {u; 7c}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {v; 6c}	47 {u; 7c}

Step 20: Visit node 38, assign cost to 31

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; 6c}	4 {u; 6c}	5 {u; 6c}	6 {u; 7c}	goal {u; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {v; 6c}	15 {u; 7c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; 7c}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {v; 6c}	39 {u; 7c}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {v; 6c}	47 {u; 7c}

Step 21: Visit node 3, assign cost to 2

0 {u; inf}	1 {u; inf}	2 {u; 7c}	3 {v; 6c}	4 {u; 6c}	5 {u; 6c}	6 {u; 7c}	goal {u; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {v; 6c}	15 {u; 7c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; 7c}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {v; 6c}	39 {u; 7c}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {v; 6c}	47 {u; 7c}

Step 22: Visit node 4, no changes to any cost

0 {u; inf}	1 {u; inf}	2 {u; 7c}	3 {v; 6c}	4 {v; 6c}	5 {u; 6c}	6 {u; 7c}	goal {u; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {v; 6c}	15 {u; 7c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {u; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; 7c}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {v; 6c}	39 {u; 7c}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {v; 6c}	47 {u; 7c}

Step 23: Visit node 5, no changes to any costs

0 {u; inf}	1 {u; inf}	2 {u; 7c}	3 {v; 6c}	4 {v; 6c}	5 {v; 6c}	6 {u; 7c}	goal {u; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {v; 6c}	15 {u; 7c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {v; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; 7c}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {v; 6c}	39 {u; 7c}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {v; 6c}	47 {u; 7c}

Step 24: Visit goal, terminate algorithm

0 {u; inf}	1 {u; inf}	2 {u; 7c}	3 {v; 6c}	4 {v; 6c}	5 {v; 6c}	6 {u; 7c}	goal {v; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {v; 5c}	13 {u; inf}	14 {v; 6c}	15 {u; 7c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 4c}	21 {v; 5c}	22 {v; 6c}	23 {u; inf}
24 {v; 2c}	25 {v; 2c}	26 {v; 2c}	27 {v; 3c}	28 {v; 4c}	29 {v; 5c}	30 {u; inf}	31 {u; 7c}
32 {v; c}	33 {v; c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {v; 6c}	39 {u; 7c}
start {v; 0}	41 {v; c}	42 {v; 2c}	43 {v; 3c}	44 {v; 4c}	45 {v; 5c}	46 {v; 6c}	47 {u; 7c}

The shortest path to goal from start using Dijkstra's algorithm is through nodes {start, 33, 26, 27, 28, 21, 14, goal} listed here sequentially. Total cost of travel is 7c.

1.2 Grid world planning with A*

Using figure 1, write out steps for A* (**8-connected**, assume uniform cost for each action). At each step, list the grid cells in the open set with their f-value (use Manhattan distance to the goal as the heuristic function) and the grid cells in the visited set. Write the final path found as a list of grid cell ids.

A* Path Planning

For each step, the state of every node is shown in the form {visited(v) or unvisited(u); running cost} next to the node number. Since uniform cost is used for any move, we take c to be the normal cost of a single move to any of the 8-connected neighbors of a node. We also use the unit cost c to calculate Manhattan distance (where total cost of travel to that node is represented as a sum of normal cost and cost of heuristic function).

Key: inf - infinity, u - unvisited, v - visited

Step 1:Start from start node, assign cost to 32, 33, 41

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; inf}	25 {u; inf}	26 {u; inf}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {u; c + 11c}	33 {u; c + 10c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; c + 11c}	42 {u; inf}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 2: Visit 33, assign cost to 24, 25, 26, 42

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; 2c + 10c}	25 {u; 2c + 9c}	26 {u; 2c + 8c}	27 {u; inf}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {u; 12c}	33 {v; 11c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; 12c}	42 {u; 2c + 10c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 3: Visit 26, assign cost to 27

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; inf}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; 12c}	25 {u; 11c}	26 {v; 10c}	27 {u; 3c + 7c}	28 {u; inf}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {u; 12c}	33 {v; 11c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; 12c}	42 {u; 12c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 4: Visit 27, assign cost to 20, 28

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; inf}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {u; 4c + 5c}	21 {u; inf}	22 {u; inf}	23 {u; inf}
24 {u; 12c}	25 {u; 11c}	26 {v; 10c}	27 {v; 10c}	28 {u; 4c + 6c}	29 {u; inf}	30 {u; inf}	31 {u; inf}
32 {u; 12c}	33 {v; 11c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; 12c}	42 {u; 12c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 5: Visit 20, assign cost to 12, 21, 29

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 5c + 4c}	13 {u; inf}	14 {u; inf}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 9c}	21 {u; 5c + 4c}	22 {u; inf}	23 {u; inf}
24 {u; 12c}	25 {u; 11c}	26 {v; 10c}	27 {v; 10c}	28 {u; 10c}	29 {u; 5c + 5c}	30 {u; inf}	31 {u; inf}
32 {u; 12c}	33 {v; 11c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; 12c}	42 {u; 12c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 6: Visit 21, assign cost to 22, 14

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; inf}	6 {u; inf}	goal {u; inf}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 9c}	13 {u; inf}	14 {u; 6c + 2c}	15 {u; inf}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 9c}	21 {v; 9c}	22 {u; 6c + 3c}	23 {u; inf}
24 {u; 12c}	25 {u; 11c}	26 {v; 10c}	27 {v; 10c}	28 {u; 10c}	29 {u; 10c}	30 {u; inf}	31 {u; inf}
32 {u; 12c}	33 {v; 11c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; 12c}	42 {u; 12c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 7: Visit 14, assign cost to 5, 6, goal, 15

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; 7c + 2c}	6 {u; 7c + c}	goal {u; 7c + 0}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 9c}	13 {u; inf}	14 {v; 8c}	15 {u; 7c + c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 9c}	21 {v; 9c}	22 {u; 9c}	23 {u; inf}
24 {u; 12c}	25 {u; 11c}	26 {v; 10c}	27 {v; 10c}	28 {u; 10c}	29 {u; 10c}	30 {u; inf}	31 {u; inf}
32 {u; 12c}	33 {v; 11c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; 12c}	42 {u; 12c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

Step 8: Visit goal, terminate algorithm

0 {u; inf}	1 {u; inf}	2 {u; inf}	3 {u; inf}	4 {u; inf}	5 {u; 9c}	6 {u; 8c}	goal {v; 7c}
8 {u; inf}	9 {u; inf}	10 {u; inf}	11 {u; inf}	12 {u; 9c}	13 {u; inf}	14 {v; 8c}	15 {u; 8c}
16 {u; inf}	17 {u; inf}	18 {u; inf}	19 {u; inf}	20 {v; 9c}	21 {v; 9c}	22 {u; 9c}	23 {u; inf}
24 {u; 12c}	25 {u; 11c}	26 {v; 10c}	27 {v; 10c}	28 {u; 10c}	29 {u; 10c}	30 {u; inf}	31 {u; inf}
32 {u; 12c}	33 {v; 11c}	34 {u; inf}	35 {u; inf}	36 {u; inf}	37 {u; inf}	38 {u; inf}	39 {u; inf}
start {v; 0}	41 {u; 12c}	42 {u; 12c}	43 {u; inf}	44 {u; inf}	45 {u; inf}	46 {u; inf}	47 {u; inf}

The shortest path to goal from start using A* algorithm is through nodes {start, 33, 26, 27, 20, 21, 14, goal} listed here sequentially. Total cost of travel is 7c.