#### ESE-680 - Autonomous Racing

# Motion Planning

(Due: 03/30/2020)

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

• All sources of material must be cited. The University Academic Code of Conduct will be strictly enforced.

#### THIS IS A GROUP ASSIGNMENT: submit one from each team.

#### Goals and Learning outcomes

The goal of this lab is to provide you with tools that will help you in a head-to-head race on a race track. After finishing this lab, your car should be able to do something like <a href="https://www.youtube.com/watch?v=llHCRqwIllM">https://www.youtube.com/watch?v=llHCRqwIllM</a>. The following fundamentals should be understood by the students upon completion of this lab:

- Motion Planning basic concepts.
  - Configuration space vs. Workspace: you should understand the difference between configuration space and workspace, and the advantages and disadvantages of planning in each of them.
  - Free space vs. Obstacle space: you should understand the difference between free space and obstacle space.
  - Occupancy grids and Costmaps: you should understand what occupancy grids and costmaps are, how
    to use them, and how to create them.
- Motion Planning algorithms. You should understand the basic ideas of these following planning algorithms, their advantages and disadvantages, and when to use them.
  - Grid-based search: Dijkstra's, A\*, and their variants
  - Sampling based algorithms: RRT and its variants

## 1 Part A: Written assignment

Turn **Part A** in at the end of lab 1 of motion planning.

0	1	2	3	4	5	6	GOAL
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39
START	41	42	43	44	45	46	47

Figure 1: Grid World

## 1.1 Grid world planning with Dijkstra's

Using figure 1, write out steps for Dijkstra's algorithm (8-connected, assume uniform cost for each action). At each step, list the grid cells in the open set with their running cost and the grid cells in the visited set. Write the final path found as a list of grid cell ids.

## Dijkstra's Path Planning

For each step, the state of every node is shown in the form {visited(v) or unvisited(u); running cost} next to the node number. Since uniform cost is used for any move, we take c to be the cost of a single move to any of the 8-connected neighbors of a node.

Key: inf - infinity, u - unvisited, v - visited

Step 1:Start from start node, assign cost to 32, 33, 41

```
0 {u: inf}
                1 {u; inf}
                               2 {u; inf}
                                              3 {u; inf}
                                                            4 {u; inf}
                                                                           5 {u; inf}
                                                                                          6 {u; inf}
                                                                                                        goal {u; inf}
 8 {u; inf}
                9 {u; inf}
                                             11 {u; inf}
                                                            12 {u; inf}
                                                                           13 {u; inf}
                                                                                                         15 {u; inf}
                              10 {u; inf}
                                                                                          14 {u; inf}
16 {u; inf}
               17 {u; inf}
                              18 {u; inf}
                                             19 {u; inf}
                                                            20 {u; inf}
                                                                          21 {u; inf}
                                                                                          22 {u; inf}
                                                                                                         23 {u; inf}
24 {u; inf}
               25 {u; inf}
                              26 {u; inf}
                                             27 {u; inf}
                                                            28 {u; inf}
                                                                           29 {u; inf}
                                                                                          30 {u; inf}
                                                                                                         31 {u; inf}
 32 {u; c}
                33 {u; c}
                              34 {u; inf}
                                             35 {u; inf}
                                                            36 {u; inf}
                                                                          37 {u; inf}
                                                                                          38 {u; inf}
                                                                                                         39 {u; inf}
start \{v; 0\}
                41 {u; c}
                              42 {u; inf}
                                                            44 {u; inf}
                                                                                          46 {u; inf}
                                                                                                         47 {u; inf}
                                             43 {u; inf}
                                                                          45 {u; inf}
```

Step 2: Visit node 32, assign cost to 24, 25

```
2 {u; inf}
 0 {u; inf}
                                              3 {u; inf}
                                                                           5 {u; inf}
                                                                                                        goal {u; inf}
                1 {u; inf}
                                                            4 {u; inf}
                                                                                          6 {u; inf}
 8 {u; inf}
                9 {u; inf}
                              10 {u; inf}
                                             11 {u; inf}
                                                            12 {u; inf}
                                                                           13 {u; inf}
                                                                                         14 {u; inf}
                                                                                                         15 {u; inf}
16 {u; inf}
               17 {u; inf}
                              18 {u; inf}
                                             19 {u; inf}
                                                            20 {u; inf}
                                                                           21 {u; inf}
                                                                                         22 {u; inf}
                                                                                                         23 {u; inf}
24 {u; 2c}
                25 {u; 2c}
                              26 {u; inf}
                                             27 {u; inf}
                                                            28 {u; inf}
                                                                           29 {u; inf}
                                                                                         30 {u; inf}
                                                                                                         31 {u; inf}
 32 {v; c}
                33 {u; c}
                              34 {u; inf}
                                             35 {u; inf}
                                                            36 {u; inf}
                                                                          37 {u; inf}
                                                                                         38 {u; inf}
                                                                                                         39 {u; inf}
start \{v; 0\}
                41 {u; c}
                              42 {u; inf}
                                             43 {u; inf}
                                                            44 {u; inf}
                                                                          45 {u; inf}
                                                                                         46 {u; inf}
                                                                                                         47 {u; inf}
```

Step 3: Visit node 33, assign cost to 26, 42

```
0 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                               5 {u; inf}
                    1 {u; inf}
                                                                                             6 {u; inf}
                                                                                                            goal \{u; inf\}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                                12 {u; inf}
                                                                              13 {u; inf}
                                                                                             14 {u; inf}
                                                                                                            15 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                 19 {u; inf}
                                                                20 {u; inf}
                                                                              21 {u; inf}
                                                                                             22 {u; inf}
                                                                                                            23 {u; inf}
    24 {u; 2c}
                    25 {u; 2c}
                                  26 {u; 2c}
                                                 27 {u; inf}
                                                                28 {u; inf}
                                                                              29 {u; inf}
                                                                                             30 {u; inf}
                                                                                                            31 {u; inf}
     32 \{v; c\}
                    33 {v; c}
                                  34 {u; inf}
                                                 35 {u; inf}
                                                                36 {u; inf}
                                                                              37 {u; inf}
                                                                                             38 {u; inf}
                                                                                                             39 {u; inf}
    start \{v; 0\}
                    41 {u; c}
                                  42 {u; 2c}
                                                                44 {u; inf}
                                                                                             46 {u; inf}
                                                                                                            47 {u; inf}
                                                 43 {u; inf}
                                                                              45 {u; inf}
Step 4: Visit node 41, no changes to any cost
     0 \{u; inf\}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                               5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                            goal \{u; inf\}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                                12 {u; inf}
                                                                              13 {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                                                                             14 {u; inf}
                                                                                                            15 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                 19 {u; inf}
                                                                20 {u; inf}
                                                                              21 {u; inf}
                                                                                             22 {u; inf}
                                                                                                            23 {u; inf}
    24 {u; 2c}
                    25 {u; 2c}
                                  26 {u; 2c}
                                                 27 {u; inf}
                                                                28 {u; inf}
                                                                              29 {u; inf}
                                                                                             30 \{u; inf\}
                                                                                                            31 {u; inf}
     32 {v; c}
                    33 {v; c}
                                                                36 {u; inf}
                                                                              37 {u; inf}
                                                                                             38 {u; inf}
                                                                                                            39 {u; inf}
                                  34 {u; inf}
                                                 35 {u; inf}
    start \{v; 0\}
                                                                                                            47 {u; inf}
                    41 {v; c}
                                  42 {u; 2c}
                                                 43 {u; inf}
                                                                44 {u; inf}
                                                                              45 {u; inf}
                                                                                             46 {u; inf}
Step 5:Visit node 24, no changes to any costs
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                               5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                            goal {u; inf}
     0 {u; inf}
                    1 {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                                12 {u; inf}
                                                                              13 {u; inf}
                                                                                             14 {u; inf}
                                                                                                            15 {u; inf}
                                                                                             22 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                 19 {u; inf}
                                                                20 {u; inf}
                                                                              21 {u; inf}
                                                                                                            23 {u; inf}
    24 {v; 2c}
                    25 {u; 2c}
                                  26 {u; 2c}
                                                 27 {u; inf}
                                                                28 {u; inf}
                                                                              29 {u; inf}
                                                                                             30 {u; inf}
                                                                                                            31 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                 35 {u; inf}
                                                                36 {u; inf}
                                                                              37 {u; inf}
                                                                                             38 {u; inf}
                                                                                                            39 {u; inf}
    start \{v; 0\}
                                                                44 {u; inf}
                                                                              45 {u; inf}
                                                                                             46 {u; inf}
                                                                                                            47 {u; inf}
                    41 {v; c}
                                  42 {u; 2c}
                                                 43 {u; inf}
Step 6: Visit node 25, no changes to any cost
     0 {u; inf}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                               5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                            goal {u; inf}
                                                                                                            15 {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                                12 {u; inf}
                                                                              13 {u; inf}
                                                                                             14 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                 19 {u; inf}
                                                                20 {u; inf}
                                                                              21 {u; inf}
                                                                                             22 {u; inf}
                                                                                                            23 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                  26 {u; 2c}
                                                 27 {u; inf}
                                                                28 {u; inf}
                                                                              29 {u; inf}
                                                                                             30 {u; inf}
                                                                                                            31 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                 35 {u; inf}
                                                                36 {u; inf}
                                                                              37 {u; inf}
                                                                                             38 {u; inf}
                                                                                                            39 {u; inf}
    start \{v; 0\}
                    41 {v; c}
                                  42 {u; 2c}
                                                 43 {u; inf}
                                                                44 {u; inf}
                                                                              45 {u; inf}
                                                                                             46 {u; inf}
                                                                                                            47 {u; inf}
Step 7: Visit node 42, assign cost to 43
                                                                4 {u; inf}
                                                                                             6 {u; inf}
                                                                                                            goal {u; inf}
     0 \{u; inf\}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                               5 {u; inf}
     8 {u; inf}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                                12 {u; inf}
                                                                              13 {u; inf}
                                                                                             14 {u; inf}
                                                                                                            15 {u; inf}
                    9 {u; inf}
                                                                              21 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                 19 {u; inf}
                                                                20 {u; inf}
                                                                                             22 {u; inf}
                                                                                                            23 {u; inf}
                                                                                             30 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                  26 {u; 2c}
                                                 27 {u; inf}
                                                                28 {u; inf}
                                                                              29 {u; inf}
                                                                                                            31 {u; inf}
     32 \{v; c\}
                    33 {v; c}
                                  34 {u; inf}
                                                 35 {u; inf}
                                                                36 {u; inf}
                                                                              37 {u; inf}
                                                                                             38 {u; inf}
                                                                                                            39 {u; inf}
    start \{v; 0\}
                                  42 {v; 2c}
                                                 43 {u; 3c}
                                                                44 {u; inf}
                                                                              45 {u; inf}
                                                                                             46 {u; inf}
                                                                                                            47 {u; inf}
                    41 {v; c}
Step 8: Visit node 26, assign cost to 27
     0 {u; inf}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                               5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                            goal {u; inf}
                                                                                                            15 {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                                12 {u; inf}
                                                                              13 {u; inf}
                                                                                             14 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                 19 {u; inf}
                                                                20 {u; inf}
                                                                              21 {u; inf}
                                                                                             22 {u; inf}
                                                                                                            23 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                  26 {v; 2c}
                                                 27 {u; 3c}
                                                                28 {u; inf}
                                                                              29 {u; inf}
                                                                                             30 {u; inf}
                                                                                                            31 {u; inf}
                                                                36 {u; inf}
                                                                              37 {u; inf}
                                                                                                             39 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                 35 {u; inf}
                                                                                             38 {u; inf}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {u; 3c}
                                                                44 {u; inf}
                                                                              45 {u; inf}
                                                                                             46 {u; inf}
                                                                                                            47 {u; inf}
Step 9: Visit node 43, assign cost to 44
     0 {u; inf}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                               5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                            goal {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                                12 {u; inf}
                                                                              13 {u; inf}
                                                                                             14 {u; inf}
                                                                                                            15 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                 19 {u; inf}
                                                                20 {u; inf}
                                                                              21 {u; inf}
                                                                                             22 {u; inf}
                                                                                                            23 {u; inf}
     24 {v; 2c}
                    25 {v; 2c}
                                                                                                            31 {u; inf}
                                  26 {v; 2c}
                                                 27 {u; 3c}
                                                                28 {u; inf}
                                                                              29 {u; inf}
                                                                                             30 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                 35 {u; inf}
                                                                36 {u; inf}
                                                                              37 {u; inf}
                                                                                             38 {u; inf}
                                                                                                            39 {u: inf}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                                44 {u; 4c}
                                                                              45 {u; inf}
                                                                                             46 {u; inf}
                                                                                                            47 {u; inf}
```

```
Step 10: Visit node 27, assign cost to 20, 28
     0 \{u; inf\}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                              5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                           goal {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                               12 {u; inf}
                                                                              13 {u; inf}
                                                                                            14 {u; inf}
                                                                                                           15 {u; inf}
                                                11 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                19 {u; inf}
                                                               20 {u; 4c}
                                                                              21 {u; inf}
                                                                                            22 {u; inf}
                                                                                                            23 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                  26 {v; 2c}
                                                 27 {v; 3c}
                                                               28 {u; 4c}
                                                                              29 {u; inf}
                                                                                            30 {u; inf}
                                                                                                           31 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                35 {u; inf}
                                                               36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; inf}
                                                                                                            39 {u; inf}
                                                                                                            47 {u; inf}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                               44 {u; 4c}
                                                                             45 {u; inf}
                                                                                            46 {u; inf}
Step 11: Visit node 44, assign cost to 45
     0 {u; inf}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                              5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                           goal {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                11 {u; inf}
                                                               12 {u; inf}
                                                                             13 {u; inf}
                                                                                            14 {u; inf}
                                                                                                            15 {u; inf}
                                                19 {u; inf}
    16 {u; inf}
                    17 {u; inf}
                                  18 {u; inf}
                                                               20 {u; 4c}
                                                                             21 {u; inf}
                                                                                            22 {u; inf}
                                                                                                           23 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                  26 {v; 2c}
                                                 27 {v; 3c}
                                                               28 {u; 4c}
                                                                             29 {u; inf}
                                                                                            30 {u; inf}
                                                                                                            31 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                               36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; inf}
                                                                                                           39 {u; inf}
                                                35 \{u; inf\}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                               44 {v; 4c}
                                                                              45 {u; 5c}
                                                                                            46 {u; inf}
                                                                                                           47 {u; inf}
Step 12: Visit node 20, assign cost to 12, 21, 29
     0 \{u; inf\}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                              5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                           goal \{u; inf\}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                11 {u; inf}
                                                               12 {u; 5c}
                                                                              13 {u; inf}
                                                                                            14 {u; inf}
                                                                                                           15 {u; inf}
    16 {u; inf}
                   17 {u; inf}
                                  18 {u; inf}
                                                19 {u; inf}
                                                               20 {v; 4c}
                                                                              21 {u; 5c}
                                                                                            22 {u; inf}
                                                                                                           23 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                  26 {v; 2c}
                                                 27 {v; 3c}
                                                               28 {u; 4c}
                                                                              29 {u; 5c}
                                                                                            30 {u; inf}
                                                                                                           31 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                35 {u; inf}
                                                               36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; inf}
                                                                                                           39 {u; inf}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                               44 {v; 4c}
                                                                              45 {u; 5c}
                                                                                            46 {u; inf}
                                                                                                           47 {u; inf}
Step 13: Visit node 28, no changes to any costs
                                                                4 {u; inf}
     0 {u; inf}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                              5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                           goal {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                11 {u; inf}
                                                               12 {u; 5c}
                                                                              13 {u; inf}
                                                                                            14 {u; inf}
                                                                                                           15 {u; inf}
    16 {u; inf}
                   17 {u; inf}
                                  18 {u; inf}
                                                19 {u; inf}
                                                               20 {v; 4c}
                                                                              21 {u; 5c}
                                                                                            22 {u; inf}
                                                                                                           23 {u; inf}
                    25 {v; 2c}
                                                               28 {v; 4c}
    24 {v; 2c}
                                  26 {v; 2c}
                                                 27 {v; 3c}
                                                                              29 {u; 5c}
                                                                                            30 {u; inf}
                                                                                                           31 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                35 {u; inf}
                                                               36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; inf}
                                                                                                            39 {u; inf}
                                                                                            46 {u; inf}
                                                                                                           47 {u; inf}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                               44 {v; 4c}
                                                                              45 {u; 5c}
Step 14: Visit node 45, assign cost to 38, 46
     0 \{u; inf\}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                              5 {u; inf}
                                                                                             6 {u; inf}
                                                                                                           goal \{u; inf\}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                11 {u; inf}
                                                               12 {u; 5c}
                                                                              13 {u; inf}
                                                                                            14 {u; inf}
                                                                                                           15 {u; inf}
    16 {u; inf}
                   17 {u; inf}
                                                19 {u; inf}
                                                               20 {v; 4c}
                                                                              21 {u; 5c}
                                                                                            22 {u; inf}
                                                                                                           23 {u; inf}
                                  18 {u; inf}
                                                                              29 {u; 5c}
                                                                                                           31 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                                               28 {v; 4c}
                                                                                            30 {u; inf}
                                  26 {v; 2c}
                                                 27 {v; 3c}
     32 \{v; c\}
                    33 {v; c}
                                  34 {u; inf}
                                                35 {u; inf}
                                                               36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; 6c}
                                                                                                           39 {u; inf}
    start \{v; 0\}
                                                               44 {v; 4c}
                                                                                            46 {u; 6c}
                                                                                                           47 {u; inf}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                                              45 {v; 5c}
Step 15: Visit node 21, assign cost to 14, 22
                    1 {u; inf}
                                   2 {u; inf}
                                                                              5 {u; inf}
                                                                                             6 {u; inf}
     0 {u; inf}
                                                 3 {u; inf}
                                                                4 {u; inf}
                                                                                                           goal \{u; inf\}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                 11 {u; inf}
                                                               12 {u; 5c}
                                                                              13 {u; inf}
                                                                                            14 {u; 6c}
                                                                                                           15 {u; inf}
                                                               20 {v; 4c}
    16 {u; inf}
                   17 {u; inf}
                                  18 {u; inf}
                                                19 {u; inf}
                                                                              21 {v; 5c}
                                                                                            22 {u; 6c}
                                                                                                           23 {u; inf}
    24 {v; 2c}
                    25 {v; 2c}
                                                               28 {v; 4c}
                                                                                            30 {u; inf}
                                                                                                           31 {u; inf}
                                  26 {v; 2c}
                                                 27 {v; 3c}
                                                                              29 {u; 5c}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                35 {u; inf}
                                                               36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; 6c}
                                                                                                           39 {u; inf}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                               44 {v; 4c}
                                                                              45 {v; 5c}
                                                                                            46 {u; 6c}
                                                                                                            47 {u; inf}
Step 16: Visit node 12, assign cost to 3, 4, 5
     0 {u; inf}
                    1 {u; inf}
                                   2 {u; inf}
                                                 3 {u; 6c}
                                                                4 {u; 6c}
                                                                              5 {u; 6c}
                                                                                             6 {u; inf}
                                                                                                           goal {u; inf}
     8 {u; inf}
                    9 {u; inf}
                                  10 {u; inf}
                                                11 {u; inf}
                                                               12 {v; 5c}
                                                                              13 {u; inf}
                                                                                            14 {u; 6c}
                                                                                                           15 {u; inf}
    16 {u; inf}
                   17 {u; inf}
                                  18 {u; inf}
                                                19 {u; inf}
                                                               20 {v: 4c}
                                                                              21 {v; 5c}
                                                                                            22 {u; 6c}
                                                                                                           23 {u; inf}
     24 {v; 2c}
                    25 {v; 2c}
                                                               28 {v; 4c}
                                                                                                           31 {u; inf}
                                  26 {v; 2c}
                                                 27 {v; 3c}
                                                                              29 {u; 5c}
                                                                                            30 {u; inf}
     32 {v; c}
                    33 {v; c}
                                  34 {u; inf}
                                                35 {u; inf}
                                                               36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; 6c}
                                                                                                           39 {u; inf}
                                                               44 {v; 4c}
    start \{v; 0\}
                    41 {v; c}
                                  42 {v; 2c}
                                                 43 {v; 3c}
                                                                              45 {v; 5c}
                                                                                            46 {u; 6c}
                                                                                                           47 {u; inf}
```

Step 17: Visit node 29, no changes to any costs										
0 {u; inf} 8 {u; inf} 16 {u; inf} 24 {v; 2c} 32 {v; c} start {v; 0}	1 {u; inf} 9 {u; inf} 17 {u; inf} 25 {v; 2c} 33 {v; c} 41 {v; c}	2 {u; inf} 10 {u; inf} 18 {u; inf} 26 {v; 2c} 34 {u; inf} 42 {v; 2c}	3 {u; 6c} 11 {u; inf} 19 {u; inf} 27 {v; 3c} 35 {u; inf} 43 {v; 3c}	4 {u; 6c} 12 {v; 5c} 20 {v; 4c} 28 {v; 4c} 36 {u; inf} 44 {v; 4c}	5 {u; 6c} 13 {u; inf} 21 {v; 5c} 29 {v; 5c} 37 {u; inf} 45 {v; 5c}	6 {u; inf} 14 {u; 6c} 22 {u; 6c} 30 {u; inf} 38 {u; 6c} 46 {u; 6c}	goal {u; inf} 15 {u; inf} 23 {u; inf} 31 {u; inf} 39 {u; inf} 47 {u; inf}			
Step 18: Visit	Step 18: Visit node 14, assign cost to goal, 5, 6, 15									
0 {u; inf} 8 {u; inf} 16 {u; inf} 24 {v; 2c} 32 {v; c} start {v; 0}	1 {u; inf} 9 {u; inf} 17 {u; inf} 25 {v; 2c} 33 {v; c} 41 {v; c}	2 {u; inf} 10 {u; inf} 18 {u; inf} 26 {v; 2c} 34 {u; inf} 42 {v; 2c}	3 {u; 6c} 11 {u; inf} 19 {u; inf} 27 {v; 3c} 35 {u; inf} 43 {v; 3c}	4 {u; 6c} 12 {v; 5c} 20 {v; 4c} 28 {v; 4c} 36 {u; inf} 44 {v; 4c}	5 {u; 6c} 13 {u; inf} 21 {v; 5c} 29 {v; 5c} 37 {u; inf} 45 {v; 5c}	6 {u; 7c} 14 {v; 6c} 22 {u; 6c} 30 {u; inf} 38 {u; 6c} 46 {u; 6c}	goal {u; 7c} 15 {u; 7c} 23 {u; inf} 31 {u; inf} 39 {u; inf} 47 {u; inf}			
Step 19: Visit	node 46, as	sign cost to								
0 {u; inf} 8 {u; inf} 16 {u; inf} 24 {v; 2c} 32 {v; c} start {v; 0}	1 {u; inf} 9 {u; inf} 17 {u; inf} 25 {v; 2c} 33 {v; c} 41 {v; c}	2 {u; inf} 10 {u; inf} 18 {u; inf} 26 {v; 2c} 34 {u; inf} 42 {v; 2c}	3 {u; 6c} 11 {u; inf} 19 {u; inf} 27 {v; 3c} 35 {u; inf} 43 {v; 3c}	4 {u; 6c} 12 {v; 5c} 20 {v; 4c} 28 {v; 4c} 36 {u; inf} 44 {v; 4c}	5 {u; 6c} 13 {u; inf} 21 {v; 5c} 29 {v; 5c} 37 {u; inf} 45 {v; 5c}	6 {u; 7c} 14 {v; 6c} 22 {u; 6c} 30 {u; inf} 38 {u; 6c} 46 {v; 6c}	goal {u; 7c} 15 {u; 7c} 23 {u; inf} 31 {u; inf} 39 {u; 7c} 47 {u; 7c}			
Step 20: Visit	node 38, as	sign cost to	31							
0 {u; inf} 8 {u; inf} 16 {u; inf} 24 {v; 2c} 32 {v; c} start {v; 0}	1 {u; inf} 9 {u; inf} 17 {u; inf} 25 {v; 2c} 33 {v; c} 41 {v; c}	2 {u; inf} 10 {u; inf} 18 {u; inf} 26 {v; 2c} 34 {u; inf} 42 {v; 2c}	3 {u; 6c} 11 {u; inf} 19 {u; inf} 27 {v; 3c} 35 {u; inf} 43 {v; 3c}	4 {u; 6c} 12 {v; 5c} 20 {v; 4c} 28 {v; 4c} 36 {u; inf} 44 {v; 4c}	5 {u; 6c} 13 {u; inf} 21 {v; 5c} 29 {v; 5c} 37 {u; inf} 45 {v; 5c}	6 {u; 7c} 14 {v; 6c} 22 {u; 6c} 30 {u; inf} 38 {v; 6c} 46 {v; 6c}	goal {u; 7c} 15 {u; 7c} 23 {u; inf} 31 {u; 7c} 39 {u; 7c} 47 {u; 7c}			
Step 21: Visit		ign cost to 2								
0 {u; inf} 8 {u; inf} 16 {u; inf} 24 {v; 2c} 32 {v; c} start {v; 0}	1 {u; inf} 9 {u; inf} 17 {u; inf} 25 {v; 2c} 33 {v; c} 41 {v; c}	2 {u; 7c} 10 {u; inf} 18 {u; inf} 26 {v; 2c} 34 {u; inf} 42 {v; 2c}	3 {v; 6c} 11 {u; inf} 19 {u; inf} 27 {v; 3c} 35 {u; inf} 43 {v; 3c}	4 {u; 6c} 12 {v; 5c} 20 {v; 4c} 28 {v; 4c} 36 {u; inf} 44 {v; 4c}	5 {u; 6c} 13 {u; inf} 21 {v; 5c} 29 {v; 5c} 37 {u; inf} 45 {v; 5c}	6 {u; 7c} 14 {v; 6c} 22 {u; 6c} 30 {u; inf} 38 {v; 6c} 46 {v; 6c}	goal {u; 7c} 15 {u; 7c} 23 {u; inf} 31 {u; 7c} 39 {u; 7c} 47 {u; 7c}			
Step 22: Visit node 4, no changes to any cost										
0 {u; inf} 8 {u; inf} 16 {u; inf} 24 {v; 2c} 32 {v; c} start {v; 0}	1 {u; inf} 9 {u; inf} 17 {u; inf} 25 {v; 2c} 33 {v; c} 41 {v; c}	2 {u; 7c} 10 {u; inf} 18 {u; inf} 26 {v; 2c} 34 {u; inf} 42 {v; 2c}	3 {v; 6c} 11 {u; inf} 19 {u; inf} 27 {v; 3c} 35 {u; inf} 43 {v; 3c}	4 {v; 6c} 12 {v; 5c} 20 {v; 4c} 28 {v; 4c} 36 {u; inf} 44 {v; 4c}	5 {u; 6c} 13 {u; inf} 21 {v; 5c} 29 {v; 5c} 37 {u; inf} 45 {v; 5c}	6 {u; 7c} 14 {v; 6c} 22 {u; 6c} 30 {u; inf} 38 {v; 6c} 46 {v; 6c}	goal {u; 7c} 15 {u; 7c} 23 {u; inf} 31 {u; 7c} 39 {u; 7c} 47 {u; 7c}			
Step 23: Visit node 5, no changes to any costs										
0 {u; inf} 8 {u; inf} 16 {u; inf} 24 {v; 2c} 32 {v; c} start {v; 0}	1 {u; inf} 9 {u; inf} 17 {u; inf} 25 {v; 2c} 33 {v; c} 41 {v; c}	2 {u; 7c} 10 {u; inf} 18 {u; inf} 26 {v; 2c} 34 {u; inf} 42 {v; 2c}	3 {v; 6c} 11 {u; inf} 19 {u; inf} 27 {v; 3c} 35 {u; inf} 43 {v; 3c}	4 {v; 6c} 12 {v; 5c} 20 {v; 4c} 28 {v; 4c} 36 {u; inf} 44 {v; 4c}	5 {v; 6c} 13 {u; inf} 21 {v; 5c} 29 {v; 5c} 37 {u; inf} 45 {v; 5c}	6 {u; 7c} 14 {v; 6c} 22 {v; 6c} 30 {u; inf} 38 {v; 6c} 46 {v; 6c}	goal {u; 7c} 15 {u; 7c} 23 {u; inf} 31 {u; 7c} 39 {u; 7c} 47 {u; 7c}			

## Step 24: Visit goal, terminate algorithm

```
0 \{u; inf\}
               1 {u; inf}
                              2 {u; 7c}
                                             3 {v; 6c}
                                                           4 {v; 6c}
                                                                         5 {v; 6c}
                                                                                        6 {u; 7c}
                                                                                                     goal {v; 7c}
8 \{u; inf\}
               9 {u; inf}
                             10 {u; inf}
                                            11 {u; inf}
                                                          12 {v; 5c}
                                                                        13 {u; inf}
                                                                                       14 {v; 6c}
                                                                                                      15 {u; 7c}
               17 {u; inf}
                                                                                       22 {v; 6c}
                                                                                                      23 {u; inf}
16 {u; inf}
                             18 {u; inf}
                                            19 {u; inf}
                                                          20 {v; 4c}
                                                                         21 {v; 5c}
24 {v; 2c}
               25 {v; 2c}
                              26 {v; 2c}
                                            27 {v; 3c}
                                                          28 {v; 4c}
                                                                         29 {v; 5c}
                                                                                       30 {u; inf}
                                                                                                      31 {u; 7c}
                                                                                                      39 {u; 7c}
 32 \{v; c\}
                33 {v; c}
                             34 {u; inf}
                                            35 {u; inf}
                                                          36 {u; inf}
                                                                        37 {u; inf}
                                                                                       38 {v; 6c}
start \{v; 0\}
                41 {v; c}
                              42 {v; 2c}
                                            43 {v; 3c}
                                                          44 {v; 4c}
                                                                         45 {v; 5c}
                                                                                       46 {v; 6c}
                                                                                                      47 {u; 7c}
```

The shortest path to goal from start using Dijkstra's algorithm is through nodes {start, 33, 26, 27, 28, 21, 14, goal} listed here sequentially. Total cost of travel is 7c.

## 1.2 Grid world planning with A\*

Using figure 1, write out steps for A\* (8-connected, assume uniform cost for each action). At each step, list the grid cells in the open set with their f-value (use Manhattan distance to the goal as the heuristic function) and the grid cells in the visited set. Write the final path found as a list of grid cell ids.

#### A\* Path Planning

For each step, the state of every node is shown in the form {visited(v) or unvisited(u); running cost} next to the node number. Since uniform cost is used for any move, we take c to be the normal cost of a single move to any of the 8-connected neighbors of a node. We also use the unit cost c to calculate Manhattan distance (where total cost of travel to that node is represented as a sum of normal cost and cost of heuristic function).

**Key**: inf - infinity, u - unvisited, v - visited

Step 1:Start from start node, assign cost to 32, 33, 41

```
0 \{u; inf\}
                        1 {u; inf}
                                          2 {u; inf}
                                                         3 {u; inf}
                                                                        4 {u; inf}
                                                                                        5 {u; inf}
                                                                                                       6 {u; inf}
                                                                                                                      goal {u; inf}
   8 {u; inf}
                        9 {u; inf}
                                         10 {u; inf}
                                                         11 {u; inf}
                                                                        12 {u; inf}
                                                                                       13 {u; inf}
                                                                                                      14 {u; inf}
                                                                                                                       15 {u; inf}
   16 {u; inf}
                        17 {u; inf}
                                         18 {u; inf}
                                                         19 {u; inf}
                                                                        20 {u; inf}
                                                                                       21 {u; inf}
                                                                                                      22 {u; inf}
                                                                                                                       23 {u; inf}
                       25 {u; inf}
   24 {u; inf}
                                         26 {u; inf}
                                                         27 {u; inf}
                                                                        28 {u; inf}
                                                                                       29 {u; inf}
                                                                                                      30 {u; inf}
                                                                                                                       31 {u; inf}
32 \{u; c + 11c\}
                     33 \{u; c + 10c\}
                                         34 {u; inf}
                                                         35 {u; inf}
                                                                        36 {u; inf}
                                                                                       37 {u; inf}
                                                                                                      38 {u; inf}
                                                                                                                       39 {u; inf}
  start \{v; 0\}
                     41 \{u; c + 11c\}
                                                                                                      46 {u; inf}
                                                                                                                       47 {u; inf}
                                         42 {u; inf}
                                                         43 {u; inf}
                                                                        44 {u; inf}
                                                                                       45 \{u; inf\}
```

Step 2: Visit 33, assign cost to 24, 25, 26, 42

```
0 \{u; inf\}
                                              2 {u; inf}
                                                                 3 {u; inf}
                         1 {u; inf}
                                                                                4 {u; inf}
                                                                                               5 {u; inf}
                                                                                                              6 {u; inf}
                                                                                                                             goal {u; inf}
   8 {u; inf}
                         9 {u; inf}
                                              10 {u; inf}
                                                                11 {u; inf}
                                                                               12 {u; inf}
                                                                                              13 {u; inf}
                                                                                                              14 {u; inf}
                                                                                                                             15 {u; inf}
   16 {u; inf}
                        17 {u; inf}
                                              18 {u; inf}
                                                                19 {u; inf}
                                                                               20 {u; inf}
                                                                                              21 {u; inf}
                                                                                                             22 {u; inf}
                                                                                                                             23 {u; inf}
24 \{u; 2c + 10c\}
                     25 \{u; 2c + 9c\}
                                           26 \{u; 2c + 8c\}
                                                                27 {u; inf}
                                                                               28 {u; inf}
                                                                                              29 {u; inf}
                                                                                                             30 {u; inf}
                                                                                                                             31 {u; inf}
  32 {u; 12c}
                        33 {v; 11c}
                                              34 {u; inf}
                                                                35 {u; inf}
                                                                               36 {u; inf}
                                                                                              37 {u; inf}
                                                                                                             38 {u; inf}
                                                                                                                             39 {u; inf}
  start \{v; 0\}
                        41 {u; 12c}
                                          42 \{u; 2c + 10c\}
                                                                43 {u; inf}
                                                                               44 {u; inf}
                                                                                              45 {u; inf}
                                                                                                             46 {u; inf}
                                                                                                                             47 {u; inf}
```

Step 3: Visit 26, assign cost to 27

```
0 {u; inf}
                 1 {u; inf}
                                2 {u; inf}
                                                  3 {u; inf}
                                                                    4 {u; inf}
                                                                                   5 {u; inf}
                                                                                                  6 {u; inf}
                                                                                                                goal {u; inf}
                 9 {u; inf}
                                10 {u; inf}
                                                  11 {u; inf}
                                                                    12 {u; inf}
                                                                                   13 {u; inf}
                                                                                                 14 {u; inf}
                                                                                                                 15 {u; inf}
 8 {u; inf}
16 {u; inf}
                17 {u; inf}
                                18 {u; inf}
                                                  19 {u; inf}
                                                                   20 {u; inf}
                                                                                  21 {u; inf}
                                                                                                 22 {u; inf}
                                                                                                                 23 {u; inf}
24 {u; 12c}
                25 {u; 11c}
                               26 {v; 10c}
                                               27 \{u; 3c + 7c\}
                                                                   28 {u; inf}
                                                                                   29 {u; inf}
                                                                                                 30 {u; inf}
                                                                                                                 31 {u; inf}
                33 {v: 11c}
32 {u; 12c}
                                34 {u; inf}
                                                  35 {u; inf}
                                                                   36 {u; inf}
                                                                                  37 {u; inf}
                                                                                                 38 {u; inf}
                                                                                                                 39 {u; inf}
                                                  43 {u; inf}
                                                                    44 {u; inf}
                                                                                                 46 {u; inf}
                                                                                                                 47 {u; inf}
start \{v; 0\}
                41 {u; 12c}
                               42 {u; 12c}
                                                                                  45 {u; inf}
```

Step 4: Visit 27, assign cost to 20, 28

```
0 \{u; inf\}
                1 {u; inf}
                                2 {u; inf}
                                                3 {u; inf}
                                                                  4 {u; inf}
                                                                                    5 {u; inf}
                                                                                                   6 {u; inf}
                                                                                                                 goal {u; inf}
 8 {u; inf}
                9 {u; inf}
                                10 {u; inf}
                                               11 {u; inf}
                                                                 12 {u; inf}
                                                                                   13 {u; inf}
                                                                                                  14 {u; inf}
                                                                                                                  15 {u; inf}
                17 {u; inf}
                                18 {u; inf}
                                               19 {u; inf}
                                                               20 \{u; 4c + 5c\}
                                                                                   21 {u; inf}
                                                                                                  22 {u; inf}
                                                                                                                  23 {u; inf}
16 {u; inf}
24 {u; 12c}
               25 {u; 11c}
                               26 {v; 10c}
                                               27 {v; 10c}
                                                               28 \{u: 4c + 6c\}
                                                                                   29 {u; inf}
                                                                                                  30 {u; inf}
                                                                                                                  31 {u; inf}
32 {u; 12c}
               33 {v; 11c}
                               34 {u; inf}
                                               35 {u; inf}
                                                                 36 {u; inf}
                                                                                   37 {u; inf}
                                                                                                  38 {u; inf}
                                                                                                                  39 {u; inf}
start \{v; 0\}
               41 {u; 12c}
                               42 {u; 12c}
                                               43 {u; inf}
                                                                 44 {u; inf}
                                                                                   45 {u; inf}
                                                                                                  46 {u; inf}
                                                                                                                  47 {u; inf}
```

Step 5: Visit 20, assign cost to 12, 21, 29

```
0 \{u; inf\}
                 1 {u; inf}
                                2 {u; inf}
                                                3 {u; inf}
                                                                  4 {u; inf}
                                                                                      5 {u; inf}
                                                                                                        6 {u; inf}
                                                                                                                      goal {u; inf}
 8 {u; inf}
                 9 {u; inf}
                                10 {u; inf}
                                               11 {u; inf}
                                                               12 \{u; 5c + 4c\}
                                                                                      13 {u; inf}
                                                                                                        14 {u; inf}
                                                                                                                       15 {u; inf}
16 {u; inf}
                17 {u; inf}
                                18 {u; inf}
                                               19 {u; inf}
                                                                  20 {v; 9c}
                                                                                   21 \{u: 5c + 4c\}
                                                                                                       22 {u; inf}
                                                                                                                       23 {u; inf}
                                               27 {v; 10c}
                                                                 28 {u; 10c}
                                                                                   29 \{u; 5c + 5c\}
                                                                                                                       31 {u; inf}
24 {u; 12c}
                25 {u; 11c}
                               26 {v; 10c}
                                                                                                       30 {u; inf}
32 {u: 12c}
                33 {v: 11c}
                                34 {u; inf}
                                               35 {u; inf}
                                                                 36 {u; inf}
                                                                                      37 {u; inf}
                                                                                                       38 {u; inf}
                                                                                                                       39 {u; inf}
start \{v; 0\}
                41 {u; 12c}
                               42 {u; 12c}
                                               43 {u; inf}
                                                                 44 {u; inf}
                                                                                      45 {u; inf}
                                                                                                       46 {u; inf}
                                                                                                                       47 {u; inf}
```

### Step 6: Visit 21, assign cost to 22, 14

```
0 {u; inf}
                1 {u; inf}
                                2 {u; inf}
                                               3 {u; inf}
                                                              4 {u; inf}
                                                                              5 {u; inf}
                                                                                               6 {u; inf}
                                                                                                                goal \{u; inf\}
 8 {u; inf}
                9 {u; inf}
                               10 {u; inf}
                                               11 {u; inf}
                                                              12 {u; 9c}
                                                                             13 {u; inf}
                                                                                            14 \{u; 6c + 2c\}
                                                                                                                 15 {u; inf}
                                                              20 {v; 9c}
                                                                             21 {v; 9c}
16 {u; inf}
                17 {u; inf}
                               18 {u; inf}
                                               19 {u; inf}
                                                                                            22 \{u; 6c + 3c\}
                                                                                                                 23 {u; inf}
24 {u; 12c}
               25 {u; 11c}
                               26 {v; 10c}
                                              27 {v; 10c}
                                                              28 {u; 10c}
                                                                             29 {u; 10c}
                                                                                               30 {u; inf}
                                                                                                                 31 {u; inf}
                                                                                               38 {u; inf}
32 {u; 12c}
               33 {v; 11c}
                               34 {u; inf}
                                              35 {u; inf}
                                                              36 {u; inf}
                                                                             37 {u; inf}
                                                                                                                 39 {u; inf}
start \{v; 0\}
               41 {u; 12c}
                               42 {u; 12c}
                                              43 {u; inf}
                                                              44 {u; inf}
                                                                             45 {u; inf}
                                                                                               46 {u; inf}
                                                                                                                 47 {u; inf}
```

### Step 7: Visit 14, assign cost to 5, 6, goal, 15

$0 \{u; inf\}$	$1 \{u; inf\}$	$2 \{u; inf\}$	$3 \{u; inf\}$	$4 \{u; inf\}$	$5 \{u; 7c + 2c\}$	$6 \{u; 7c + c\}$	goal $\{u; 7c + 0\}$
8 {u; inf}	9 {u; inf}	$10 \{u; inf\}$	$11 \{u; inf\}$	$12 \{u; 9c\}$	$13 \{u; inf\}$	14 {v; 8c}	$15 \{u; 7c + c\}$
$16 \{u; inf\}$	$17 \{u; inf\}$	18 {u; inf}	$19 \{u; inf\}$	$20 \{v; 9c\}$	$21 \{v; 9c\}$	$22 \{u; 9c\}$	$23 \{u; inf\}$
24 {u; 12c}	25 {u; 11c}	26 {v; 10c}	27 {v; 10c}	28 {u; 10c}	29 {u; 10c}	$30 \{u; inf\}$	$31 \{u; inf\}$
32 {u; 12c}	33 {v; 11c}	$34 \{u; inf\}$	$35 \{u; inf\}$	$36 \{u; inf\}$	$37 \{u; inf\}$	$38 \{u; inf\}$	$39 \{u; inf\}$
start $\{v; 0\}$	41 {u; 12c}	42 {u; 12c}	$43 \{u; inf\}$	$44 \{u; inf\}$	$45 \{u; inf\}$	$46 \{u; inf\}$	$47 \{u; inf\}$

#### Step 8: Visit goal, terminate algorithm

```
0 {u; inf}
                1 {u; inf}
                               2 {u; inf}
                                               3 {u; inf}
                                                              4 {u; inf}
                                                                             5 {u; 9c}
                                                                                            6 {u; 8c}
                                                                                                          goal \{v; 7c\}
 8 {u; inf}
                9 {u; inf}
                               10 {u; inf}
                                              11 {u; inf}
                                                              12 {u; 9c}
                                                                             13 {u; inf}
                                                                                            14 {v; 8c}
                                                                                                           15 {u; 8c}
16 {u; inf}
               17 {u; inf}
                               18 {u; inf}
                                              19 {u; inf}
                                                              20 {v; 9c}
                                                                             21 {v; 9c}
                                                                                            22 {u; 9c}
                                                                                                           23 {u; inf}
               25 {u; 11c}
                              26 {v; 10c}
                                                                                            30 {u; inf}
                                                                                                           31 {u; inf}
24 {u; 12c}
                                              27 {v; 10c}
                                                             28 {u; 10c}
                                                                            29 {u; 10c}
32 {u; 12c}
               33 {v; 11c}
                               34 {u; inf}
                                              35 {u; inf}
                                                             36 {u; inf}
                                                                             37 {u; inf}
                                                                                            38 {u; inf}
                                                                                                           39 {u; inf}
start \{v; 0\}
               41 {u; 12c}
                                              43 {u; inf}
                                                                             45 {u; inf}
                                                                                                           47 {u; inf}
                              42 {u; 12c}
                                                             44 {u; inf}
                                                                                            46 {u; inf}
```

The shortest path to goal from start using A\* algorithm is through nodes {start, 33, 26, 27, 20, 21, 14, goal} listed here sequentially. Total cost of travel is 7c.