DSC 212: Prob & Stats, F2025: Homework #1
Assigned: Thu. Oct. 09, 2025
Due: Thu. Oct 16, 2025

Instructor: A. Mazumdar

Problem 1 $(4+2=6)$
In a group of n unrelated individuals, none born on a leap year, what is the probability that at least two share a birthday? For what value of n this probability exceeds one-half? [Make natural assumptions for the probability space and show reasoning.]

Problem 2 (3+4=7)
There are 3 bags each containing 100 marbles. Bag 1 has 75 red and 25 blue marbles. Bag 2 has 60 red and 40 blue marbles. Bag 3 has 45 red and 55 blue marbles. Now a bag is chosen at random and a marble is also picked at random.
- What is the probability that the marble is blue?
- What happens when the first bag is chosen with probability 0.5 and other bags with equal probability each?

Problem 3 (6)
A gambler starts with k units of money and gambles in a sequence of trials. At each trial he either wins one unit or loses one unit of money, each of the possibilities having probability $1/2$ independent of past history. The gambler plays until his fortune reaches N at which point he leaves with his gains or until his fortune becomes 0 at which point he is ruined and leaves. Determine the probability q_k that he is bankrupted.

Problem 4 (5)
Suppose A and B are independently selected random subsets of $\Omega = \{1, 2, \ldots, n\}$ (not excluding the empty set $\emptyset$ or Ω itself). Show that $P(A \subseteq B) = (\frac{3}{4})^n$

Probelm 5 (2+2=4)
In a box there are 3 coins of which 2 are regular and 1 is fake (both sides heads).
1) What is the probability of getting a tail at a random pick up of a coin and tossing it?
2) At a random pick up and toss: the outcome is heads. What is the probability it is the fake coin?

Problem 6 $(3+3)$
In each of the following two cases determine whether the given function $F(x_1, x_2)$ is a distribution function on the plane. If this is the case, determine the marginal distribution functions $F_1(x_1)$ and $F_2(x_2)$ of X_1 and X_2, respectively. If this is not the case, explain why.
- $F(x_1, x_2) = 1 - e^{-x_1 - x_2}$ if $x_1, x_2 \geq 0$.
- $F(x_1, x_2) = 1 - e^{-\min(x_1, x_2)} - \min(x_1, x_2)e^{-\min(x_1, x_2)}$ if $x_1, x_2 \geq 0$.

The functions are to be assumed to be zero where not explicitly specified.

Problem 7 $(3+3)$
Suppose X and Y are independent random variables with distribution functions $F(x)$ and $G(y)$, respectively. Determine the distributions of $\max(X, Y)$ and $\min(X, Y)$.