DSC 212: Prob & Stats, F2025: Homework #2
Assigned: Thu. Oct. 23, 2025
Due: Thu. Oct 30, 2025

Instructor: A. Mazumdar

Problem 1 (4)
Define deviation $D = E(X - c)^2$ where X is a random variable and c is a real valued number. Prove that deviation D is minimum when $c = E[X]$.

Problem 2 $(3 + 3)$
A particle starts at the origin of the real line and moves along the line in jumps of one unit. For each jump the probability is p that the particle will jump one unit to the left and the probability is $1 - p$ that the particle will jump one unit to the right. Let X_n be the position of the particle after n jumps. Find $\mathrm{E}(X_n)$ and $\mathrm{var}(X_n)$.

Problem 3 $(4 + 5 + 5 = 14)$
Computer Program: Use `numpy.random` or any other of your favorite library to generate uniform random variables between $[0, 1]$. Suppose $\bar{X}_n$ be the sample mean of n such random variables. Find mean μ and variance of $\bar{X}_n$. Let $Z_n = \frac{\bar{X}_n - \mu}{\sqrt{\mathrm{Var}(\bar{X}_n)}}$. For $n = 100$, do 100 trials of this experiment, and plot the number of times $Z_n \leq a$ for $a = -2, -1, -0.5, -0.25.0, 0.25, 0.5, 1, 2$. Plot the cumulative distribution function of $\mathcal{N}(0, 1)$ and compare.

Problem 4 $(3 + 3 + 2)$
Let $X_1, X_2, \ldots, X_n$ be $\mathcal{N}(0, 1)$ IID random variables and let $\bar{X}_n$ be the sample mean. Plot $\bar{X}_n$ vs n for $n = 1, 2, \ldots 1000$ Repeat for $X_1, X_2, \ldots, X_n$ each being IID Cauchy (density $\frac{1}{\pi(x^2+1)}$). Explain why there is such a difference.