DSC 212: Prob & Stats, F2025: Homework #3
Assigned: Thu. Nov. 13, 2024
Due: Sat. Nov. 22, 2025

Instructor: A. Mazumdar

Problem 1 $(6 + 6 = 12)$
Let $X_1, X_2, \ldots, X_n$ be IID such that X_i is uniform in $[0, \theta]$. For the following two estimators, find bias, standard error, and mean square error:

- $\widehat{\theta} = \max\{X_1, X_2, \ldots, X_n\}$
- $\widehat{\theta} = 2\bar{X}_n = \frac{2}{n}\sum_{i=1}^{n} X_i$

Problem 2 (4+4=8)
Let $X_1, \ldots, X_n$ be IID with X_i being uniform in $[a, b]$.
- Find the method of moments estimator for a and b
- Find the Maximum Likelihood Estimator for a and b

Problem 3 $(2 + 4 = 6)$
Give example of a biased estimator which has a better MSE than an unbiased estimator for the same parameter. Show your calculation. (Hint: We saw this estimator in class).

Problem 4 $(3 + 3 + 3 = 9)$
Suppose $P(X = 1) = P(X = -1) = 0.5$. Define,

$$X_n = \begin{cases} X, \text{ with probability } 1 - \frac{1}{n} \\ e^n \text{ with probability } \frac{1}{n}. \end{cases}$$

Does X_n converges to X in probability? Does X_n converges to X in distribution? Does $E((X_n - X)^2)$ converge to 0?