HW 2

Kevin Lin

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1

Define deviation $D = E[(X-c)^2]$ for some constant c. We want to find the value of c that minimizes D. We can expand D as follows:

$$D = E[(X-c)^2] = E[X^2 - 2cX + c^2] = E[X^2] - 2cE[X] + c^2$$

To find the minimum value of D, we can take the derivative of D with respect to c and set it equal to 0:

$$\frac{dD}{dc} = -2E[X] + 2c = 0 \therefore c = E[X]$$

Thus, the D is minimized when $c = E[X]$.

2

For the particle's current position k, it can either hop left or right with probability p and $1-p$ respectively. Let $E[X_n]$ be the expected position of the particle after n hops. We can express $E[X_n]$ in terms of $E[X_{n-1}]$:

$$\begin{aligned} E[X_n] &= p(E[X_{n-1}] - 1) + (1-p)(E[X_{n-1}] + 1) \\ &= pE[X_{n-1}] - p + E[X_{n-1}] + 1 - pE[X_{n-1}] - p \\ &= E[X_{n-1}] + 1 - 2p \end{aligned}$$

Note that then:

$$\begin{gathered} E[X_n] = E[X_{n-2}] + 2(1-2p) \\ E[X_n] = E[X_{n-3}] + 3(1-2p) \\ \vdots \\ E[X_n] = E[X_0] + n(1-2p) \end{gathered}$$

Since the initial position $E[X_0] = 0$, we get $E[X_n] = n(1-2p)$.

Because each jump is independent and identically distributed, we can express