DSC 212: Prob & Stats, F2025: Homework #4
Assigned: Tue. Nov. 25, 2025
Due: Thu. Dec 4, 2025

Instructor: A. Mazumdar

Problem 1 $(3+3+3=9)$
Let $X_1, \ldots, X_n$ are IID Poisson(λ). Find the method of moments estimator, the maximum likelihood estimator and the Fisher information $J(\lambda)$.

Problem 2 $(4+4+2=10)$
Find the Fisher information for the following families:

- $f(x;\theta) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{x^2}{2\theta}}$
- $f(x;\theta) = \theta e^{-\theta x}, x \geq 0$

Find the Cramer-Rao bounds on the MSE for each of the two families.

Problem 3 (5)
Let $\widehat{F}_n(x)$ be the empirical distribution function. For two distinct points x, y, find $Cov(\widehat{F}_n(x), \widehat{F}_n(y))$.

Problem 4 $(4+4+4=12)$
Let $f_1(x)$ and $f_0(x)$ be two given probability densities. Let Z be Bernoulli(θ), where θ is unknown. Let $X \sim f_1(x)$ if $Z = 1$ and $X \sim f_0(x)$ if $Z = 0$.

- Find the density $f_\theta(x)$ of the observed X. Find the Fisher information $J(\theta)$.
- What is the Cramer-Rao lower bound on the mean-squared error of an unbiased estimate of θ?
- Can you exhibit an unbiased estimator of θ?