HW 2

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10/23/2025

1

Define deviation $D = E[(X - c)^2]$ for some constant c. We want to find the value of c that minimizes D. We can expand D as follows:

$$D = E[(X - c)^2] = E[X^2 - 2cX + c^2] = E[X^2] - 2cE[X] + c^2$$

To find the minimum value of D, we can take the derivative of D with respect to c and set it equal to 0:

$$\frac{dD}{dc} = -2E[X] + 2c = 0 \therefore c = E[X]$$

Thus, the D is minimized when $c = E[X]$.

2

Note that each jump is independent and identically distributed. For the current particle's current position k, it can either hop left or right with probability p and $1 - p$ respectively. Thus, we can express the expected position $E[X_n]$ after n jumps as follows:

$$E[X_n] = \sum_{k=1}^{n} E[X_k] = nE[X_k]$$

$$E[X_k] = (-1)p + (1)(1 - p) = 1 - 2p$$

$$\therefore E[X_n] = n(1 - 2p)$$

Likewise, we can calculate the variance $Var(X_n)$ as:

$$Var(X_n) = \sum_{k=1}^{n} Var(X_k) = nVar(X_k)$$

$$Var(X_k) = E[X_k^2] - (E[X_k])^2$$

$$E[X_k^2] = (-1)^2 p + (1)^2(1 - p) = 1$$

$$Var(X_k) = 1 - (1 - 2p)^2 = 4p(1 - p)$$

$$\therefore Var(X_n) = 4np(1 - p)$$