HW 1

Kevin Lin

1/26/2026

1

Let:

$$A = \begin{bmatrix} 4 & 1 & 3 & 6 \\ 2 & 7 & 5 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ 7 & 6 \\ 5 & 8 \\ 3 & 11 \end{bmatrix}, C = \begin{bmatrix} -13 & 0 & 2 \\ 5 & 2 & 10 \\ 0 & 7 & 9 \end{bmatrix}, D = \begin{bmatrix} 5 & -3 & -7 \\ 4 & 0 & 10 \\ 7 & 3 & 11 \end{bmatrix}, E = \begin{bmatrix} -4 & 6 \\ 12 & 7 \end{bmatrix}$$

(a) $(3B)^T$:

$$\begin{aligned} (3B)^T &= 3 \cdot B^T \\ &= 3 \cdot \begin{bmatrix} 0 & 7 & 5 & 3 \\ 4 & 6 & 8 & 11 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 21 & 15 & 9 \\ 12 & 18 & 24 & 33 \end{bmatrix} \end{aligned}$$

(b) $(A - B)^T$ is not possible due to dimension mismatch. A is 2×4 while B is 4×2.

(c) $(2B^T - A)^T$:

$$\begin{aligned} (2B^T - A)^T &= 2B - A^T \\ &= 2 \cdot \begin{bmatrix} 0 & 4 \\ 7 & 6 \\ 5 & 8 \\ 3 & 11 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 1 & 7 \\ 3 & 5 \\ 6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 8 \\ 14 & 12 \\ 10 & 16 \\ 6 & 22 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 1 & 7 \\ 3 & 5 \\ 6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -4 & 6 \\ 13 & 5 \\ 7 & 11 \\ 0 & 19 \end{bmatrix} \end{aligned}$$

(d) $(C + 2D^T + E)^T$ is not possible due to dimension mismatch. C and D are both 3×3 while E is 2×2.

(e) $(-A)^T E$:

$$\begin{aligned}
(-A)^T E &= -A^T E \\
&= -\begin{bmatrix} 4 & 2 \\ 1 & 7 \\ 3 & 5 \\ 6 & 3 \end{bmatrix} \cdot \begin{bmatrix} -4 & 6 \\ 12 & 7 \end{bmatrix} \\
&= -\begin{bmatrix} 4 \cdot -4 + 2 \cdot 12 & 4 \cdot 6 + 2 \cdot 7 \\ 1 \cdot -4 + 7 \cdot 12 & 1 \cdot 6 + 7 \cdot 7 \\ 3 \cdot -4 + 5 \cdot 12 & 3 \cdot 6 + 5 \cdot 7 \\ 6 \cdot -4 + 3 \cdot 12 & 6 \cdot 6 + 3 \cdot 7 \end{bmatrix} \\
&= -\begin{bmatrix} -16 + 24 & 24 + 14 \\ -4 + 84 & 6 + 49 \\ -12 + 60 & 18 + 35 \\ -24 + 36 & 36 + 21 \end{bmatrix} \\
&= -\begin{bmatrix} 8 & 38 \\ 80 & 55 \\ 48 & 53 \\ 12 & 57 \end{bmatrix} \\
&= \begin{bmatrix} -8 & -38 \\ -80 & -55 \\ -48 & -53 \\ -12 & -57 \end{bmatrix}
\end{aligned}$$

2

No, $AB \neq BA$. Matrix multiplication is not commutative. We can prove this by calculating both AB and BA:

$$AB = \begin{bmatrix} 2 & 7 & 3 \\ 1 & 0 & 9 \\ -1 & 2 & 10 \end{bmatrix} \begin{bmatrix} -2 & 0 & 3 \\ 2 & -1 & 7 \\ 6 & 4 & -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 \cdot -2 + 7 \cdot 2 + 3 \cdot 6 & 2 \cdot 0 + 7 \cdot -1 + 3 \cdot 4 & 2 \cdot 3 + 7 \cdot 7 + 3 \cdot -3 \\ 1 \cdot -2 + 0 \cdot 2 + 9 \cdot 6 & 1 \cdot 0 + 0 \cdot -1 + 9 \cdot 4 & 1 \cdot 3 + 0 \cdot 7 + 9 \cdot -3 \\ -1 \cdot -2 + 2 \cdot 2 + 10 \cdot 6 & -1 \cdot 0 + 2 \cdot -1 + 10 \cdot 4 & -1 \cdot 3 + 2 \cdot 7 + 10 \cdot -3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -4 + 14 + 18 & 0 - 7 + 12 & 6 + 49 - 9 \\ -2 + 0 + 54 & 0 + 0 + 36 & 3 + 0 - 27 \\ 2 + 4 + 60 & 0 - 2 + 40 & -3 + 14 - 30 \end{bmatrix}$$

$$AB = \begin{bmatrix} 28 & 5 & 46 \\ 52 & 36 & -24 \\ 66 & 38 & -19 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 0 & 3 \\ 2 & -1 & 7 \\ 6 & 4 & -3 \end{bmatrix} \begin{bmatrix} 2 & 7 & 3 \\ 1 & 0 & 9 \\ -1 & 2 & 10 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 \cdot 2 + 0 \cdot 1 + 3 \cdot -1 & -2 \cdot 7 + 0 \cdot 0 + 3 \cdot 2 & -2 \cdot 3 + 0 \cdot 9 + 3 \cdot 10 \\ 2 \cdot 2 + -1 \cdot 1 + 7 \cdot -1 & 2 \cdot 7 + -1 \cdot 0 + 7 \cdot 2 & 2 \cdot 3 + -1 \cdot 9 + 7 \cdot 10 \\ 6 \cdot 2 + 4 \cdot 1 + -3 \cdot -1 & 6 \cdot 7 + 4 \cdot 0 + -3 \cdot 2 & 6 \cdot 3 + 4 \cdot 9 + -3 \cdot 10 \end{bmatrix}$$

$$BA = \begin{bmatrix} -4 + 0 - 3 & -14 + 0 + 6 & -6 + 0 + 30 \\ 4 - 1 - 7 & 14 + 0 + 14 & 6 - 9 + 70 \\ 12 + 4 + 3 & 42 + 0 - 6 & 18 + 36 - 30 \end{bmatrix}$$

$$BA = \begin{bmatrix} -7 & -8 & 24 \\ -4 & 28 & 67 \\ 19 & 36 & 24 \end{bmatrix}$$

Thus, $AB \neq BA$.

3

- ℓ_1 norm of $[0, 0, 0]$:

  $$||[0, 0, 0]||_1 = |0| + |0| + |0| = 0$$

  ℓ_2 norm of $[0, 0, 0]$:

  $$||[0, 0, 0]||_2 = \sqrt{0^2 + 0^2 + 0^2} = 0$$

  ℓ_∞ norm of $[0, 0, 0]$:

- ℓ_1 norm of $[1,2,3]$:

$$||[1,2,3]||_1 = |1| + |2| + |3| = 6$$

  ℓ_2 norm of $[1,2,3]$:

$$||[1,2,3]||_2 = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

- ℓ_1 norm of $[2,4,6]$:

$$||[2,4,6]||_1 = |2| + |4| + |6| = 12$$

  ℓ_2 norm of $[2,4,6]$:

$$||[2,4,6]||_2 = \sqrt{2^2 + 4^2 + 6^2} = \sqrt{56} = 2\sqrt{14}$$

  These norms are related to those of $[1,2,3]$ by a factor of 2.

-