HW 2

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1

(a) The gradient of hinge loss with respect to w:

$$\begin{aligned}\nabla_w \ell(w^T x, y) &= \nabla_w \max\{0, 1 - yw^T x\} \\ &= \begin{cases} 0 & \text{if } yw^T x \geq 1 \\ -yx & \text{if } yw^T x < 1 \end{cases}\end{aligned}$$

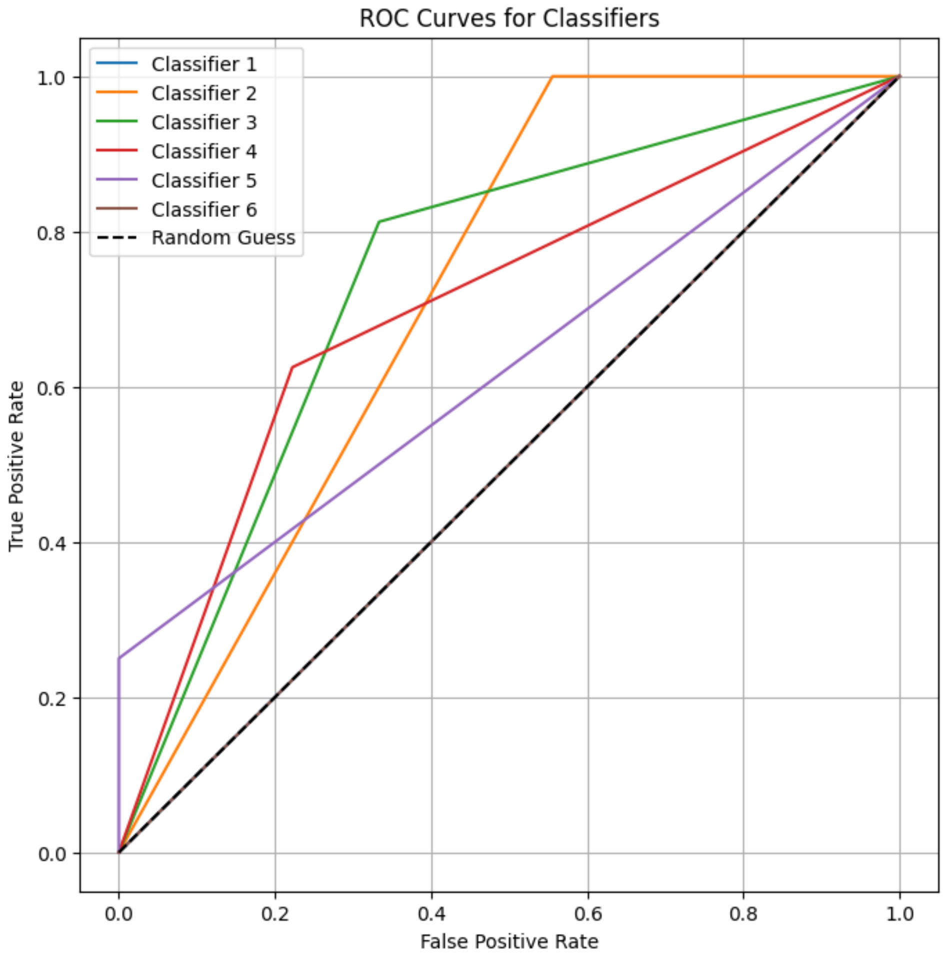
(b) The gradient of Perceptron loss w.r.t w:

$$\begin{aligned}\nabla_w \ell(w^T x, y) &= \nabla_w \max\{0, -yw^T x\} \\ &= \begin{cases} 0 & \text{if } yw^T x \geq 0 \\ -yx & \text{if } yw^T x < 0 \end{cases}\end{aligned}$$

(c) For hinge loss, w is updated only when the margin condition $yw^T x < 1$ is violated, meaning the prediction is not only incorrect but also not confident enough. However, for Perceptron loss, the update occurs whenever the prediction is incorrect (when $yw^T x < 0$). This means that hinge loss encourages a larger margin between classes while Perceptron loss focuses solely on correct classification.

2

(a) ROC plot (see Jupyter notebook for code):



(b) Classifier 2 has the highest accuracy of 0.8, while Classifier 6 has the lowest accuracy of 0.36. See Jupyter notebook for code.

(c) Classifier 5 has the highest precision of 1, while Classifier 1 has the lowest precision of 0.64. Classifier 6 has undefined precision as it has no true or false positives. See Jupyter notebook for code.

(d) Classifier 1 F1 score: 0.78
Classifier 2 F1 score: 0.86
Classifier 3 F1 score: 0.81
Classifier 4 F1 score: 0.71
Classifier 5 F1 score: 0.4
Classifier 6 F1 score: Undefined for the same reasons as precision.
See Jupyter notebook for code.

3

See Jupyter notebook for code.

(a)
```
Margins for each data point:
Point (2, 2, 3) with label 1: 0.74
Point (3, 3, 2) with label 1: 0.93
Point (1, 2, 3) with label 1: 0.19
Point (1, 4, 1) with label 1: -0.56
Point (4, 4, 4) with label 1: 3.34
Point (2, 2, 2) with label 1: 0.00
Point (3, 3, 1) with label -1: -0.19
Point (1, 1, 1) with label -1: 1.67
Point (3, 2, 2) with label -1: -0.56
Point (0, 4, 2) with label -1: 0.37
Point (4, 0, 0) with label -1: 1.11
Point (0, 0, 3) with label -1: 1.11
```

(b)
```
0-1 Loss for each data point:
Point (2, 2, 3) with label 1: 0
Point (3, 3, 2) with label 1: 0
Point (1, 2, 3) with label 1: 0
Point (1, 4, 1) with label 1: 1
Point (4, 4, 4) with label 1: 0
Point (2, 2, 2) with label 1: 0
Point (3, 3, 1) with label -1: 1
Point (1, 1, 1) with label -1: 0
Point (3, 2, 2) with label -1: 1
Point (0, 4, 2) with label -1: 0
Point (4, 0, 0) with label -1: 0
Point (0, 0, 3) with label -1: 0
```

(c)
```
Hinge Loss for each data point:
Point (2, 2, 3) with label 1: 0
Point (3, 3, 2) with label 1: 0
Point (1, 2, 3) with label 1: 0
Point (1, 4, 1) with label 1: 4
Point (4, 4, 4) with label 1: 0
Point (2, 2, 2) with label 1: 1
Point (3, 3, 1) with label -1: 2
Point (1, 1, 1) with label -1: 0
Point (3, 2, 2) with label -1: 4
Point (0, 4, 2) with label -1: 0
```

```
Point (4, 0, 0) with label -1: 0
Point (0, 0, 3) with label -1: 0
```

(d)

```
Squared Loss for each data point:
Point (2, 2, 3) with label 1: 9
Point (3, 3, 2) with label 1: 16
Point (1, 2, 3) with label 1: 0
Point (1, 4, 1) with label 1: 16
Point (4, 4, 4) with label 1: 289
Point (2, 2, 2) with label 1: 1
Point (3, 3, 1) with label -1: 4
Point (1, 1, 1) with label -1: 64
Point (3, 2, 2) with label -1: 16
Point (0, 4, 2) with label -1: 1
Point (4, 0, 0) with label -1: 25
Point (0, 0, 3) with label -1: 25
```

4

(a) We can derive the optimal $\mathbf{w}^*$ that minimizes $E_2\mathbf{w}$ as follows:

$$\begin{aligned} E_2(\mathbf{w}) &= \frac{1}{N}\|\mathbf{X}\mathbf{w}-\mathbf{y}\|_2^2 + \lambda\|\mathbf{w}\|_2^2 \\ &= \frac{1}{N}(\mathbf{X}\mathbf{w}-\mathrm{y})^T(\mathbf{X}\mathbf{w}-\mathbf{y}) + \lambda\mathbf{w}^T\mathbf{w} \\ &= \frac{1}{N}(\mathbf{w}^T\mathbf{X}^T\mathbf{X}\mathbf{w} - 2\mathbf{y}^T\mathbf{X}\mathbf{w} + \mathbf{y}^T\mathbf{y}) + \lambda\mathbf{w}^T\mathbf{w} \end{aligned}$$

Taking the gradient with respect to $\mathbf{w}$ and setting it to zero:

$$\begin{aligned} \nabla_{\mathbf{w}} E_2(\mathbf{w}) &= \frac{2}{N}\mathbf{X}^T\mathbf{X}\mathbf{w} - \frac{2}{N}\mathbf{X}^T\mathbf{y} + 2\lambda\mathbf{w} = 0 \\ &= (\mathbf{X}^T\mathbf{X} + N\lambda\mathbf{I})\mathbf{w} = \mathbf{X}^T\mathbf{y} \\ \mathbf{w}^* &= (\mathbf{X}^T\mathbf{X} + N\lambda\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y} \end{aligned}$$

(b) This new objective function overcomes the singularity issue by adding the term $\lambda\|\mathbf{w}\|_2^2$, which effectively adds $\lambda\mathbf{I}$ to the matrix $\mathbf{X}^T\mathbf{X}$. This addition ensures that the matrix $\mathbf{X}^T\mathbf{X} + N\lambda\mathbf{I}$ is positive definite and invertible, even if $\mathbf{X}^T\mathbf{X}$ is singular. The regularization term penalizes large weights, promoting stability and preventing overfitting.

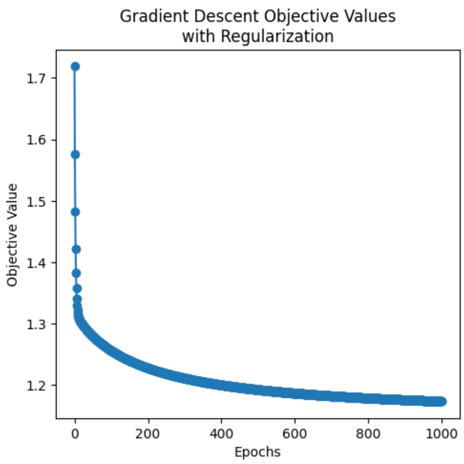
5

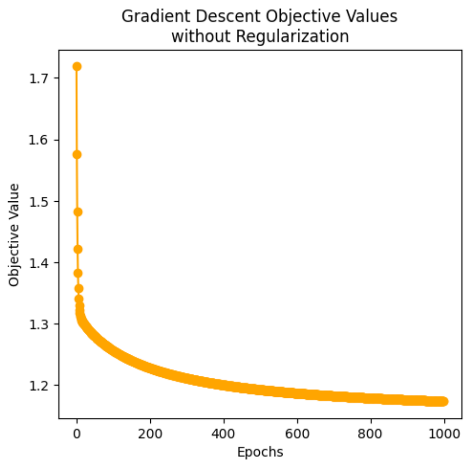
See Jupyter notebook for code.

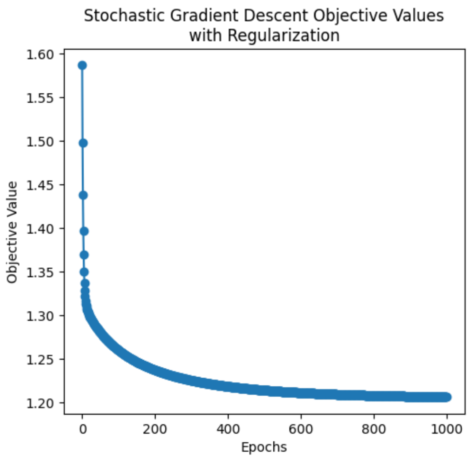
```
L2 Norm Differences of Least Squares Regression:
With Regularization:
GD vs SGD: 7.006176226969802
GD vs Closed Form: 7.021520447813614
SGD vs Closed Form: 0.08173535002944027
Without Regularization:
GD vs SGD: 5.282858727935199
GD vs Closed Form: 7.021521174825259
SGD vs Closed Form: 2.533883001083749
```

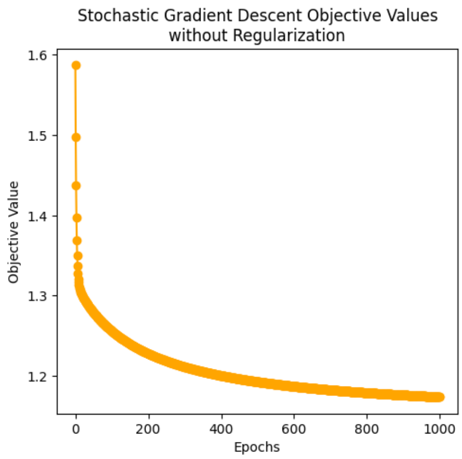
6

See Jupyter notebook for code (objective function values are also calculated in the same codeblock of Q5).



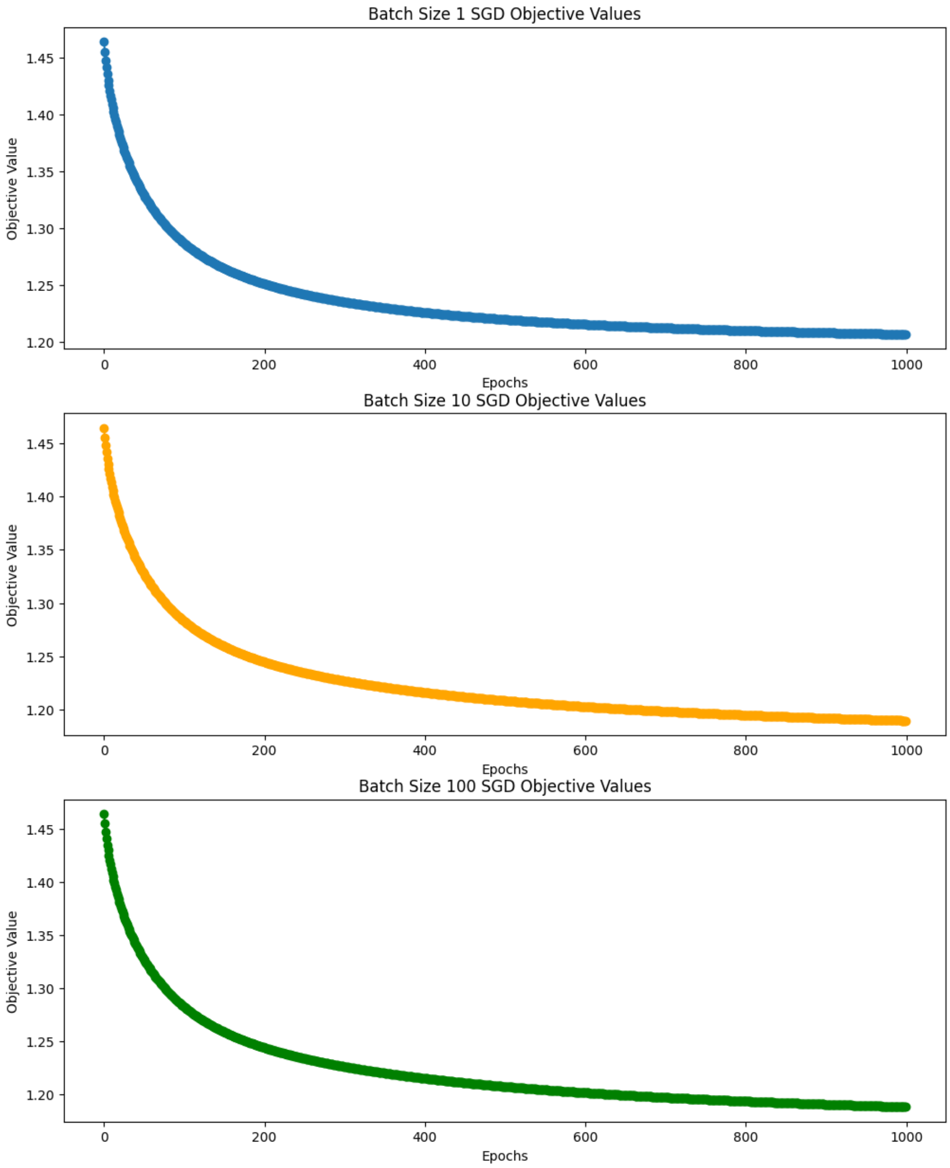






7

See Jupyter notebook for code.



8

We are given hat matrix $\mathbf{H} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$, where $\mathbf{X}$ is $N \times (d+1)$, and $\mathbf{X}^T\mathbf{X}$ is invertible.

(a) We can show $\mathbf{H}$ is symmetric:

$$\begin{aligned}\mathbf{H}^T &= (\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T)^T \\ &= \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \\ &= \mathbf{H}\end{aligned}$$

(b) We can show that $\mathbf{H}^K = \mathbf{H}$ for any integer $K \geq 1$ using induction:

$$\begin{aligned}HH &= \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \\ &= \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}(\mathbf{X}^T\mathbf{X})(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \\ &= \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T \\ &= \mathbf{H}\end{aligned}$$

Thus, by induction, $\mathbf{H}^K = \mathbf{H}$ for all integers $K \geq 1$.

(c) Given identity matrix $\mathbf{I}$ of size N, then $(\mathbf{I}-\mathbf{H})^K = \mathbf{I}-\mathbf{H}$ for any integer $K \geq 1$:

$$\begin{aligned}(\mathbf{I}-\mathbf{H})^2 &= \mathbf{I} - 2\mathbf{H} + \mathbf{H}^2 \\ &= \mathbf{I} - 2\mathbf{H} + \mathbf{H} \quad \text{(from part (b))} \\ &= \mathbf{I} - \mathbf{H}\end{aligned}$$

Thus, by induction, $(\mathbf{I}-\mathbf{H})^K = \mathbf{I}-\mathbf{H}$ for all integers $K \geq 1$.

(d) We can show that $\text{trace}(\mathbf{H}) = d + 1$:

$$\begin{aligned}\text{trace}(\mathbf{H}) &= \text{trace}(\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T) \\ &= \text{trace}((\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{X}) \quad \text{(by cyclic property of trace)} \\ &= \text{trace}(\mathbf{I}_{(d+1)\times(d+1)}) \\ &= d + 1\end{aligned}$$

hw2_code

February 1, 2026

[1]:
```python
import numpy as np
import matplotlib.pyplot as plt
```

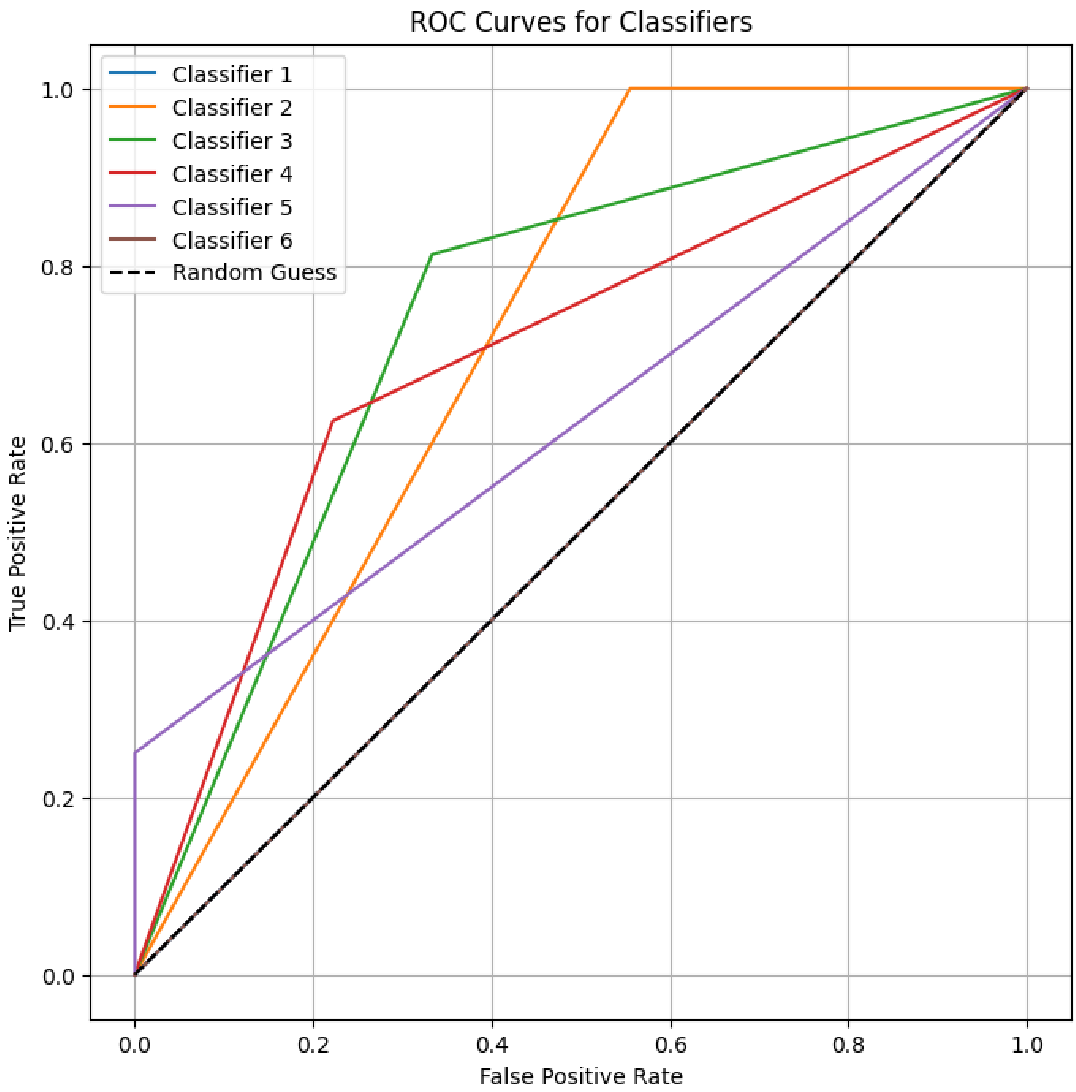
1 2

[2]:
```python
# total 25 samples
# 16 positive (filled)
# 9 negative (empty)

c1_tp = 16
c1_tn = 0
c1_fp = 9
c1_fn = 0

c2_tp = 16
c2_tn = 4
c2_fp = 5
c2_fn = 0

c3_tp = 13
c3_tn = 6
c3_fp = 3
c3_fn = 3

c4_tp = 10
c4_tn = 7
c4_fp = 2
c4_fn = 6

c5_tp = 4
c5_tn = 9
c5_fp = 0
c5_fn = 12

c6_tp = 0
c6_tn = 9
c6_fp = 0
```

```
c6_fn = 16
```

1.1 a

[9]:
```
plt.figure(figsize=(8, 8))
plt.plot([0, c1_fp/(c1_fp + c1_tn), 1], [0, c1_tp/(c1_tp + c1_fn), 1],␣
↪label='Classifier 1')
plt.plot([0, c2_fp/(c2_fp + c2_tn), 1], [0, c2_tp/(c2_tp + c2_fn), 1],␣
↪label='Classifier 2')
plt.plot([0, c3_fp/(c3_fp + c3_tn), 1], [0, c3_tp/(c3_tp + c3_fn), 1],␣
↪label='Classifier 3')
plt.plot([0, c4_fp/(c4_fp + c4_tn), 1], [0, c4_tp/(c4_tp + c4_fn), 1],␣
↪label='Classifier 4')
plt.plot([0, c5_fp/(c5_fp + c5_tn), 1], [0, c5_tp/(c5_tp + c5_fn), 1],␣
↪label='Classifier 5')
plt.plot([0, c6_fp/(c6_fp + c6_tn), 1], [0, c6_tp/(c6_tp + c6_fn), 1],␣
↪label='Classifier 6')
plt.plot([0, 1], [0, 1], 'k--', label='Random Guess')
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate')
plt.title('ROC Curves for Classifiers')
plt.legend()
plt.grid()
plt.show()
```



1.2 b

[5]:
```
acc_c1 = (c1_tp + c1_tn) / 25
acc_c2 = (c2_tp + c2_tn) / 25
acc_c3 = (c3_tp + c3_tn) / 25
acc_c4 = (c4_tp + c4_tn) / 25
acc_c5 = (c5_tp + c5_tn) / 25
acc_c6 = (c6_tp + c6_tn) / 25

print(f'Classifier 1 Accuracy: {acc_c1:.2f}')
print(f'Classifier 2 Accuracy: {acc_c2:.2f}')
print(f'Classifier 3 Accuracy: {acc_c3:.2f}')
```

```
print(f'Classifier 4 Accuracy: {acc_c4:.2f}')
print(f'Classifier 5 Accuracy: {acc_c5:.2f}')
print(f'Classifier 6 Accuracy: {acc_c6:.2f}')
```

```
Classifier 1 Accuracy: 0.64
Classifier 2 Accuracy: 0.80
Classifier 3 Accuracy: 0.76
Classifier 4 Accuracy: 0.68
Classifier 5 Accuracy: 0.52
Classifier 6 Accuracy: 0.36
```

1.3 c

[ ]:
```
prec_c1 = c1_tp / (c1_tp + c1_fp)
prec_c2 = c2_tp / (c2_tp + c2_fp)
prec_c3 = c3_tp / (c3_tp + c3_fp)
prec_c4 = c4_tp / (c4_tp + c4_fp)
prec_c5 = c5_tp / (c5_tp + c5_fp)
prec_c6 = 0 # undefined

print(f'Classifier 1 Precision: {prec_c1:.2f}')
print(f'Classifier 2 Precision: {prec_c2:.2f}')
print(f'Classifier 3 Precision: {prec_c3:.2f}')
print(f'Classifier 4 Precision: {prec_c4:.2f}')
print(f'Classifier 5 Precision: {prec_c5:.2f}')
print(f'Classifier 6 Precision: Undefined')
```

```
Classifier 1 Precision: 0.64
Classifier 2 Precision: 0.76
Classifier 3 Precision: 0.81
Classifier 4 Precision: 0.83
Classifier 5 Precision: 1.00
Classifier 6 Precision: Undefined
```

1.4 d

[7]:
```
f1_c1 = 2 * (prec_c1 * (c1_tp / (c1_tp + c1_fn))) / (prec_c1 + (c1_tp / (c1_tp␣
↪+ c1_fn)))
f1_c2 = 2 * (prec_c2 * (c2_tp / (c2_tp + c2_fn))) / (prec_c2 + (c2_tp / (c2_tp␣
↪+ c2_fn)))
f1_c3 = 2 * (prec_c3 * (c3_tp / (c3_tp + c3_fn))) / (prec_c3 + (c3_tp / (c3_tp␣
↪+ c3_fn)))
f1_c4 = 2 * (prec_c4 * (c4_tp / (c4_tp + c4_fn))) / (prec_c4 + (c4_tp / (c4_tp␣
↪+ c4_fn)))
f1_c5 = 2 * (prec_c5 * (c5_tp / (c5_tp + c5_fn))) / (prec_c5 + (c5_tp / (c5_tp␣
↪+ c5_fn)))
f1_c6 = 0 # undefined since precision is undefined
```

```python
print(f'Classifier 1 F1 Score: {f1_c1:.2f}')
print(f'Classifier 2 F1 Score: {f1_c2:.2f}')
print(f'Classifier 3 F1 Score: {f1_c3:.2f}')
print(f'Classifier 4 F1 Score: {f1_c4:.2f}')
print(f'Classifier 5 F1 Score: {f1_c5:.2f}')
print(f'Classifier 6 F1 Score: Undefined')
```

```
Classifier 1 F1 Score: 0.78
Classifier 2 F1 Score: 0.86
Classifier 3 F1 Score: 0.81
Classifier 4 F1 Score: 0.71
Classifier 5 F1 Score: 0.40
Classifier 6 F1 Score: Undefined
```

2 3

[3]:
```python
data = [
    (2, 2, 3),
    (3, 3, 2),
    (1, 2, 3),
    (1, 4, 1),
    (4, 4, 4),
    (2, 2, 2),
    (3, 3, 1),
    (1, 1, 1),
    (3, 2, 2),
    (0, 4, 2),
    (4, 0, 0),
    (0, 0, 3)
]

labels = [
    1,
    1,
    1,
    1,
    1,
    1,
    -1,
    -1,
    -1,
    -1,
    -1,
    -1
]
```

2.1 a

[4]:
```
print("Margins for each data point:")
for point, label in zip(data, labels):
    x1, x2, x3 = point
    decision_value = 3*x1 + 2*x2 + 4*x3
    margin = label * (decision_value - 18) / np.sqrt(3**2 + 2**2 + 4**2)
    print(f'Point {point} with label {label}: {margin:.2f}')
```

```
Margins for each data point:
Point (2, 2, 3) with label 1: 0.74
Point (3, 3, 2) with label 1: 0.93
Point (1, 2, 3) with label 1: 0.19
Point (1, 4, 1) with label 1: -0.56
Point (4, 4, 4) with label 1: 3.34
Point (2, 2, 2) with label 1: 0.00
Point (3, 3, 1) with label -1: -0.19
Point (1, 1, 1) with label -1: 1.67
Point (3, 2, 2) with label -1: -0.56
Point (0, 4, 2) with label -1: 0.37
Point (4, 0, 0) with label -1: 1.11
Point (0, 0, 3) with label -1: 1.11
```

2.2 b

[5]:
```
print("0-1 Loss for each data point:")
for point, label in zip(data, labels):
    x1, x2, x3 = point
    decision_value = 3*x1 + 2*x2 + 4*x3
    prediction = 1 if decision_value >= 18 else -1
    loss = 0 if prediction == label else 1
    print(f'Point {point} with label {label}: {loss}')
```

```
0-1 Loss for each data point:
Point (2, 2, 3) with label 1: 0
Point (3, 3, 2) with label 1: 0
Point (1, 2, 3) with label 1: 0
Point (1, 4, 1) with label 1: 1
Point (4, 4, 4) with label 1: 0
Point (2, 2, 2) with label 1: 0
Point (3, 3, 1) with label -1: 1
Point (1, 1, 1) with label -1: 0
Point (3, 2, 2) with label -1: 1
Point (0, 4, 2) with label -1: 0
Point (4, 0, 0) with label -1: 0
Point (0, 0, 3) with label -1: 0
```

2.3 c

[7]:
```
print("Hinge Loss for each data point:")
for point, label in zip(data, labels):
    x1, x2, x3 = point
    decision_value = 3*x1 + 2*x2 + 4*x3
    hinge_loss = max(0, 1 - label * (decision_value - 18))
    print(f'Point {point} with label {label}: {hinge_loss}')
```

```
Hinge Loss for each data point:
Point (2, 2, 3) with label 1: 0
Point (3, 3, 2) with label 1: 0
Point (1, 2, 3) with label 1: 0
Point (1, 4, 1) with label 1: 4
Point (4, 4, 4) with label 1: 0
Point (2, 2, 2) with label 1: 1
Point (3, 3, 1) with label -1: 2
Point (1, 1, 1) with label -1: 0
Point (3, 2, 2) with label -1: 4
Point (0, 4, 2) with label -1: 0
Point (4, 0, 0) with label -1: 0
Point (0, 0, 3) with label -1: 0
```

2.4 d

[8]:
```
print("Squared Loss for each data point:")
for point, label in zip(data, labels):
    x1, x2, x3 = point
    decision_value = 3*x1 + 2*x2 + 4*x3
    squared_loss = (label - (decision_value - 18))**2
    print(f'Point {point} with label {label}: {squared_loss}')
```

```
Squared Loss for each data point:
Point (2, 2, 3) with label 1: 9
Point (3, 3, 2) with label 1: 16
Point (1, 2, 3) with label 1: 0
Point (1, 4, 1) with label 1: 16
Point (4, 4, 4) with label 1: 289
Point (2, 2, 2) with label 1: 1
Point (3, 3, 1) with label -1: 4
Point (1, 1, 1) with label -1: 64
Point (3, 2, 2) with label -1: 16
Point (0, 4, 2) with label -1: 1
Point (4, 0, 0) with label -1: 25
Point (0, 0, 3) with label -1: 25
```

3 5

[11]:
```
import time

def gradient_descent(X, y, learning_rate=0.1, epochs=1000, regularization=1e-6):
    start_time = time.time()
    m, n = X.shape
    weights = np.zeros(n)
    obj_values = []
    for epoch in range(epochs):
        predictions = X @ weights
        errors = predictions - y
        gradient = (2/m) * (X.T @ errors) + 2 * regularization * weights
        if np.linalg.norm(gradient) < 1e-6:
            if regularization > 0:
                print("Completed GD w/ regularization in epoch:", epoch, "in",␣
 ↪time.time() - start_time, "seconds")
            else:
                print("Completed GD w/o regularization in epoch:", epoch, "in",␣
 ↪time.time() - start_time, "seconds")
            return weights, obj_values
        weights -= learning_rate * gradient
        obj_values.append((errors @ errors) / m + regularization * (weights @␣
 ↪weights))
    if regularization > 0:
        print("Completed GD w/ regularization in epoch:", epoch, "in", time.
 ↪time() - start_time, "seconds")
    else:
        print("Completed GD w/o regularization in epoch:", epoch, "in", time.
 ↪time() - start_time, "seconds")
    return weights, obj_values

def stochastic_gradient_descent(X, y, learning_rate=0.1, epochs=1000,␣
 ↪regularization=1e-6):
    start_time = time.time()
    m, n = X.shape
    weights = np.zeros(n)
    obj_values = []
    for epoch in range(epochs):
        for i in range(m):
            xi = X[i]
            yi = y[i]
            prediction = xi @ weights
            error = prediction - yi
            gradient = (2/m) * xi * error + 2 * regularization * weights
            weights -= learning_rate * gradient
        predictions = X @ weights
```

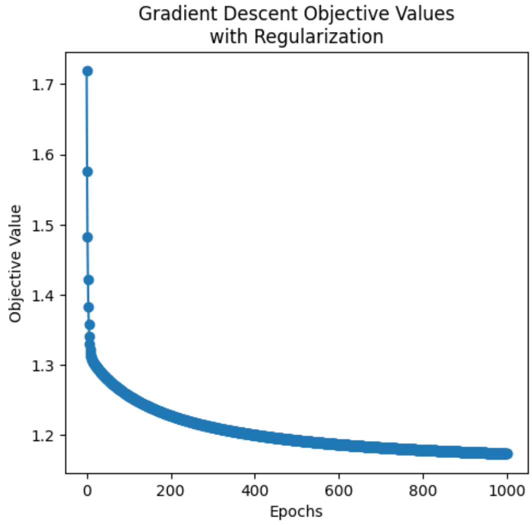
```
        errors = predictions - y
        obj_values.append((errors @ errors) / m + regularization * (weights @␣
↪weights))
    if regularization > 0:
        print("Completed SGD w/ regularization in epoch:", epoch, "in", time.
↪time() - start_time, "seconds")
    else:
        print("Completed SGD w/o regularization in epoch:", epoch, "in", time.
↪time() - start_time, "seconds")
    return weights, obj_values

def closed_form_solution(X, y, regularization=1e-6):
    n = X.shape[1]
    return np.linalg.inv(X.T @ X + regularization * np.eye(n)) @ X.T @ y

X = np.load('data_X_Q5Q6.npy')
y = np.load('data_y_Q5Q6.npy').reshape(-1)

weights_gd, obj_gd = gradient_descent(X, y)
weights_sgd, obj_sgd = stochastic_gradient_descent(X, y)
weights_closed_form = closed_form_solution(X, y)

weights_gd_noReg, obj_gd_noReg = gradient_descent(X, y, regularization=0)
weights_sgd_noReg, obj_sgd_noReg = stochastic_gradient_descent(X, y,␣
↪regularization=0)
weights_closed_form_noReg = closed_form_solution(X, y, regularization=0)

def l2_norm_diff(w1, w2):
    return np.linalg.norm(w1 - w2)

print("L2 Norm Differences of Least Squares Regression:")
print("With Regularization:")
print("GD vs SGD:", l2_norm_diff(weights_gd, weights_sgd))
print("GD vs Closed Form:", l2_norm_diff(weights_gd, weights_closed_form))
print("SGD vs Closed Form:", l2_norm_diff(weights_sgd, weights_closed_form))
print("Without Regularization:")
print("GD vs SGD:", l2_norm_diff(weights_gd_noReg, weights_sgd_noReg))
print("GD vs Closed Form:", l2_norm_diff(weights_gd_noReg,␣
↪weights_closed_form_noReg))
print("SGD vs Closed Form:", l2_norm_diff(weights_sgd_noReg,␣
↪weights_closed_form_noReg))
```

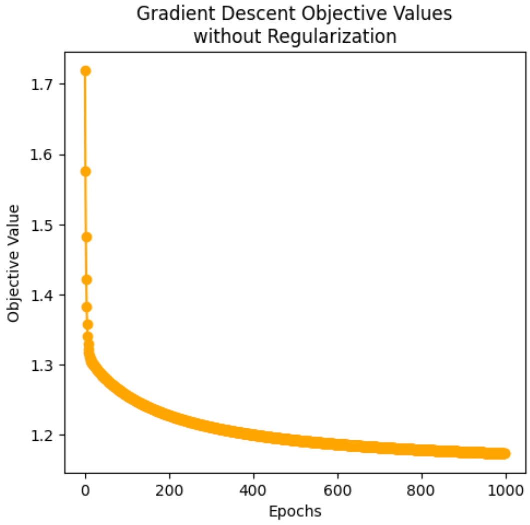
```
Completed GD w/ regularization in epoch: 999 in 0.24744129180908203 seconds
Completed SGD w/ regularization in epoch: 999 in 79.89985132217407 seconds
Completed GD w/o regularization in epoch: 999 in 0.25933051109313965 seconds
Completed SGD w/o regularization in epoch: 999 in 87.29134202003479 seconds
L2 Norm Differences of Least Squares Regression:
```

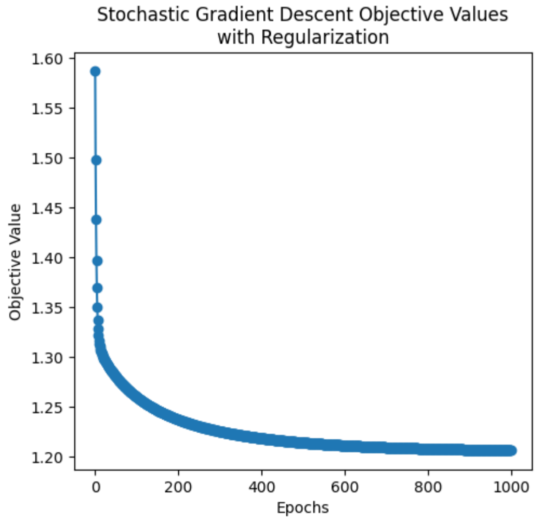
```
With Regularization:
GD vs SGD: 1.3603089942927995
GD vs Closed Form: 5.191738520295133
SGD vs Closed Form: 5.998876146468824
Without Regularization:
GD vs SGD: 0.0009529716509423173
GD vs Closed Form: 5.191620708668827
SGD vs Closed Form: 5.191819050801421
```

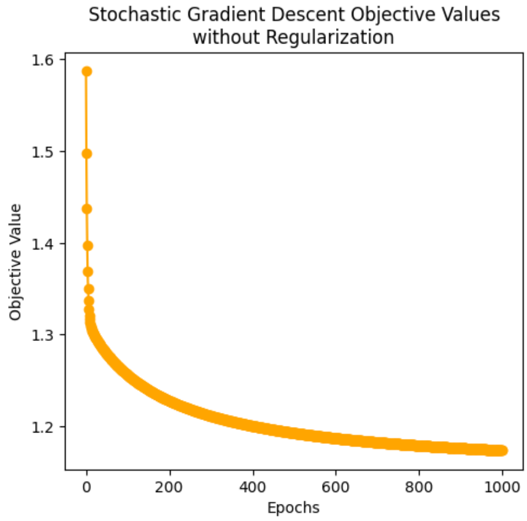
4 6

[12]:
```python
plt.figure(figsize=(12, 5))
plt.subplot(1, 2, 1)
plt.plot(obj_gd, label='With Regularization', linestyle='-', marker='o')
plt.xlabel('Epochs')
plt.ylabel('Objective Value')
plt.title('Gradient Descent Objective Values\nwith Regularization')
plt.subplot(1, 2, 2)
plt.plot(obj_gd_noReg, label='Without Regularization', color='orange',
    linestyle='-', marker='o')
plt.xlabel('Epochs')
plt.ylabel('Objective Value')
plt.title('Gradient Descent Objective Values\nwithout Regularization')
plt.show()

plt.figure(figsize=(12, 5))
plt.subplot(1, 2, 1)
plt.plot(obj_sgd, label='With Regularization', linestyle='-', marker='o')
plt.xlabel('Epochs')
plt.ylabel('Objective Value')
plt.title('Stochastic Gradient Descent Objective Values\nwith Regularization')
plt.subplot(1, 2, 2)
plt.plot(obj_sgd_noReg, label='Without Regularization', color='orange',
    linestyle='-', marker='o')
plt.xlabel('Epochs')
plt.ylabel('Objective Value')
plt.title('Stochastic Gradient Descent Objective Values\nwithout
    Regularization')
plt.show()
```









5 7

[17]:
```
def batch_stochastic_gradient_descent(X, y, learning_rate=0.1, epochs=1000, batch_size=32, regularization=1e-6):
    start_time = time.time()
    m, n = X.shape
    weights = np.zeros(n)
    obj_values = []
    for epoch in range(epochs):
        perm = np.random.permutation(m)
```

```
        X_shuffled = X[perm]
        y_shuffled = y[perm]
        for i in range(0, m, batch_size):
            xi = X_shuffled[i:i+batch_size]
            yi = y_shuffled[i:i+batch_size]
            prediction = xi @ weights
            error = prediction - yi
            gradient = (2/m) * (xi.T @ error) + 2 * regularization * weights
            weights -= learning_rate * gradient
        predictions = X @ weights
        errors = predictions - y
        obj_values.append((errors @ errors) / m + regularization * (weights @␣
 ↪weights))
    print(f"Completed Batch {batch_size} SGD w/ regularization in epoch:␣
 ↪{epoch} in {time.time() - start_time} seconds")
    return weights, obj_values

X = np.load('data_X_Q7.npy')
y = np.load('data_y_Q7.npy').reshape(-1)
batch_sgd_1, batch_obj_1 = batch_stochastic_gradient_descent(X, y, batch_size=1)
batch_sgd_10, batch_obj_10 = batch_stochastic_gradient_descent(X, y,␣
 ↪batch_size=10)
batch_sgd_100, batch_obj_100 = batch_stochastic_gradient_descent(X, y,␣
 ↪batch_size=100)
```

```
Completed Batch 1 SGD w/ regularization in epoch: 999 in 78.41542434692383
seconds
Completed Batch 10 SGD w/ regularization in epoch: 999 in 24.432676076889038
seconds
Completed Batch 100 SGD w/ regularization in epoch: 999 in 20.637977838516235
seconds
```

[19]:
```
plt.figure(figsize=(12, 15))
plt.subplot(3, 1, 1)
plt.plot(batch_obj_1, linestyle='-', marker='o')
plt.xlabel('Epochs')
plt.ylabel('Objective Value')
plt.title('Batch Size 1 SGD Objective Values')
plt.subplot(3, 1, 2)
plt.plot(batch_obj_10, color='orange', linestyle='-', marker='o')
plt.xlabel('Epochs')
plt.ylabel('Objective Value')
plt.title('Batch Size 10 SGD Objective Values')
plt.subplot(3, 1, 3)
plt.plot(batch_obj_100, color='green', linestyle='-', marker='o')
plt.xlabel('Epochs')
plt.ylabel('Objective Value')
```

```
plt.title('Batch Size 100 SGD Objective Values')
plt.show()
```