# hw5_project

February 11, 2026

## 1 1

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import statsmodels.api as sm
```

[2]:
```
insurance = pd.read_csv("../datasets/insurance.csv")
insurance.head()
```

[2]:

| | age | sex | bmi | children | smoker | region | charges |
|---|---|---|---|---|---|---|---|
| 0 | 19 | female | 27.900 | 0 | yes | southwest | 16884.92400 |
| 1 | 18 | male | 33.770 | 1 | no | southeast | 1725.55230 |
| 2 | 28 | male | 33.000 | 3 | no | southeast | 4449.46200 |
| 3 | 33 | male | 22.705 | 0 | no | northwest | 21984.47061 |
| 4 | 32 | male | 28.880 | 0 | no | northwest | 3866.85520 |

[3]:
```
X = insurance.drop(columns=['charges'])
X = pd.get_dummies(X, drop_first=True).astype(float)
y = insurance['charges']

X = sm.add_constant(X)
model = sm.OLS(y, X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                charges   R-squared:                       0.751
Model:                            OLS   Adj. R-squared:                  0.749
Method:                 Least Squares   F-statistic:                     500.8
Date:                Wed, 11 Feb 2026   Prob (F-statistic):               0.00
Time:                        16:00:17   Log-Likelihood:                -13548.
No. Observations:                1338   AIC:                         2.711e+04
Df Residuals:                    1329   BIC:                         2.716e+04
Df Model:                           8
Covariance Type:            nonrobust
================================================================================
====
```

```
                        coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
----
const             -1.194e+04    987.819    -12.086      0.000   -1.39e+04
-1e+04
age                 256.8564     11.899     21.587      0.000     233.514
280.199
bmi                 339.1935     28.599     11.860      0.000     283.088
395.298
children            475.5005    137.804      3.451      0.001     205.163
745.838
sex_male           -131.3144    332.945     -0.394      0.693    -784.470
521.842
smoker_yes         2.385e+04    413.153     57.723      0.000      2.3e+04
2.47e+04
region_northwest   -352.9639    476.276     -0.741      0.459   -1287.298
581.370
region_southeast -1035.0220    478.692     -2.162      0.031   -1974.097
-95.947
region_southwest   -960.0510    477.933     -2.009      0.045   -1897.636
-22.466
==============================================================================
Omnibus:                      300.366   Durbin-Watson:                   2.088
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              718.887
Skew:                           1.211   Prob(JB):                   7.86e-157
Kurtosis:                       5.651   Cond. No.                         311.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

**Intercept**: -11940, SE 987.819, $p < 0.0001$ This is the predicted insurance charge for a customer age 0, bmi 0, 0 children female, non smoking, and from the north east. Realistically this doesn't mean anything, but anchors the regression line.

**Age**: 256.8564, SE 11.899, $p < 0.0001$ Holding all other predictors constant, a 1 year increase in age is attributed to a $256.86 increase in medical insurance charge. The SE and p value are both low, indicating a well estimated and statistically significant predictor.

**BMI**: 339.1935, SE 28.599, $p < 0.0001$ Holding all other predictors constant, a 1-unit increase in BMI is associated with an average increase of $339.19 in insurance charges. The SE is modest relative to the coefficient, suggesting reasonable precision. The p-value is extremely small, indicating strong statistical significance.

**Children**: 475.5005, SE 137.804, p 0.001 Holding all other predictors constant, each additional child is associated with an average increase of $475.50 in insurance charges. The SE is somewhat larger relative to the coefficient than with age or BMI, indicating more variability in the estimate. However, the p-value (0.001) shows this predictor is still statistically significant.

**Sex (Male)**: -131.3144, SE 332.945, p 0.693 Holding all other predictors constant, being male (relative to female) is associated with an average decrease of $131.31 in insurance charges. However, the SE is large compared to the coefficient, and the p-value is very high (0.693). Thus, the predictor is not statistically significant.

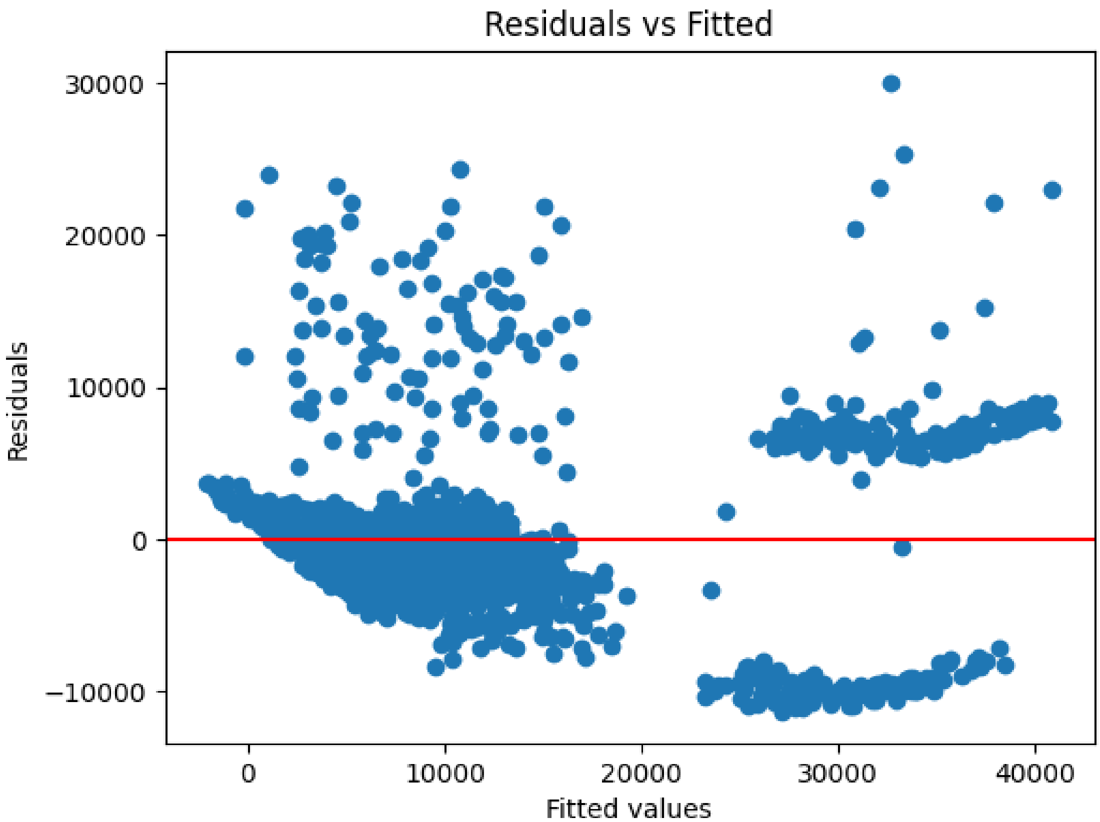
**Smoker (Yes)**: 23850, SE 413.153, $p < 0.0001$ Holding all other predictors constant, being a smoker is associated with an average increase of approximately $23,850 in insurance charges compared to non-smokers. The SE is small relative to the coefficient, and the p-value is extremely small, indicating statistical significance. This is by far the strongest predictor in the model due to its effect size and being precisely estimated.

**Region (Northwest)**: -352.9639, SE 476.276, p 0.459 Compared to the Northeast (reference region), living in the Northwest is associated with an average decrease of $352.96 in charges, holding other variables constant. The SE is larger than the coefficient magnitude and the p-value is high, thus the predictor is not statistically significant.

**Region (Southeast)**: -1035.0220, SE 478.692, p 0.031 Compared to the Northeast, living in the Southeast is associated with an average decrease of $1,035.02 in charges. The SE is moderate relative to the coefficient, and the p-value (0.031) indicates statistical significance at the 5% level.

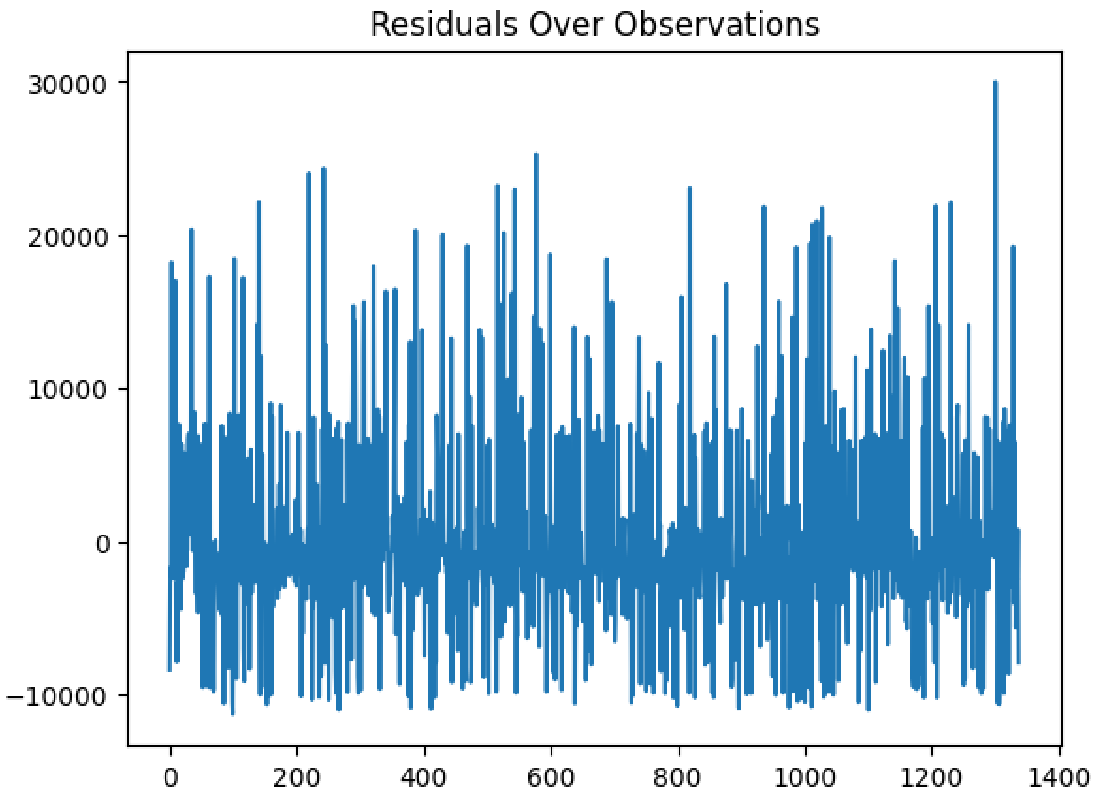
**Region (Southwest)**: -960.0510, SE 477.933, p 0.045 Compared to the Northeast, living in the Southwest is associated with an average decrease of $960.05 in charges. The SE is about half the magnitude of the coefficient. The p-value (0.045) indicates marginal statistical significance at the 5% level.

[4]:
```
plt.scatter(model.fittedvalues, model.resid)
plt.axhline(0, color='red')
plt.xlabel("Fitted values")
plt.ylabel("Residuals")
plt.title("Residuals vs Fitted")
plt.show()
```




Residuals are clearly not random, indicating a non linear relationship in the model. This is expected as we can tell from the omnibus / skew statistics.

[6]:
```
plt.plot(model.resid)
plt.title("Residuals Over Observations")
plt.show()
```




This is a good indication that there is no autocorrelation, as also indicated by the DW statistic near 2. Insurance data should not be related to time.

# 2 2

## 2.1 a

[8]:
```
X = insurance.drop(columns=['charges'])
X = pd.get_dummies(X, drop_first=True).astype(float)
X = X[['age', 'bmi', 'children', 'smoker_yes']]

X['obese'] = (X['bmi'] > 30).astype(float)
X = X.drop(columns=['bmi'])

X_smoker = X[X['smoker_yes'] == 1].drop(columns=['smoker_yes'])
y_smoker = y[X['smoker_yes'] == 1]
X_smoker = sm.add_constant(X_smoker)
model_smoker = sm.OLS(y_smoker, X_smoker).fit()
print(model_smoker.summary())

X_non_smoker = X[X['smoker_yes'] == 0].drop(columns=['smoker_yes'])
```

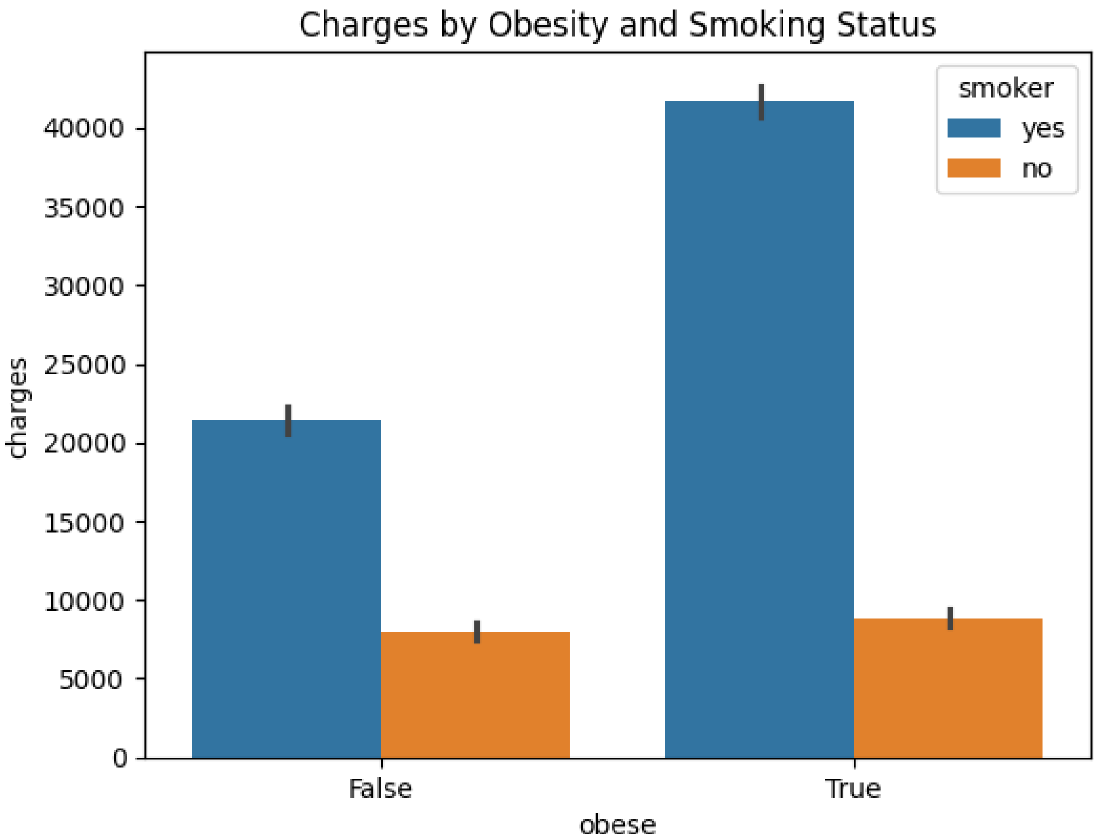
```python
y_non_smoker = y[X['smoker_yes'] == 0]
X_non_smoker = sm.add_constant(X_non_smoker)
model_non_smoker = sm.OLS(y_non_smoker, X_non_smoker).fit()
print(model_non_smoker.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                charges   R-squared:                       0.885
Model:                            OLS   Adj. R-squared:                  0.883
Method:                 Least Squares   F-statistic:                     690.3
Date:                Wed, 11 Feb 2026   Prob (F-statistic):         2.88e-126
Time:                        16:22:26   Log-Likelihood:                -2655.3
No. Observations:                 274   AIC:                             5319.
Df Residuals:                     270   BIC:                             5333.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       1.088e+04    757.111     14.377      0.000    9394.127    1.24e+04
age          272.0695     17.208     15.811      0.000     238.192     305.948
children     163.4200    206.862      0.790      0.430    -243.847     570.687
obese       1.999e+04    477.353     41.874      0.000      1.9e+04    2.09e+04
==============================================================================
Omnibus:                      187.780   Durbin-Watson:                   1.796
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1424.355
Skew:                           2.858   Prob(JB):                    5.07e-310
Kurtosis:                      12.596   Cond. No.                         134.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                charges   R-squared:                       0.408
Model:                            OLS   Adj. R-squared:                  0.407
Method:                 Least Squares   F-statistic:                     243.9
Date:                Wed, 11 Feb 2026   Prob (F-statistic):         2.55e-120
Time:                        16:22:26   Log-Likelihood:                -10485.
No. Observations:                1064   AIC:                         2.098e+04
Df Residuals:                    1060   BIC:                         2.100e+04
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -2710.9336    449.266     -6.034      0.000   -3592.486   -1829.381
```

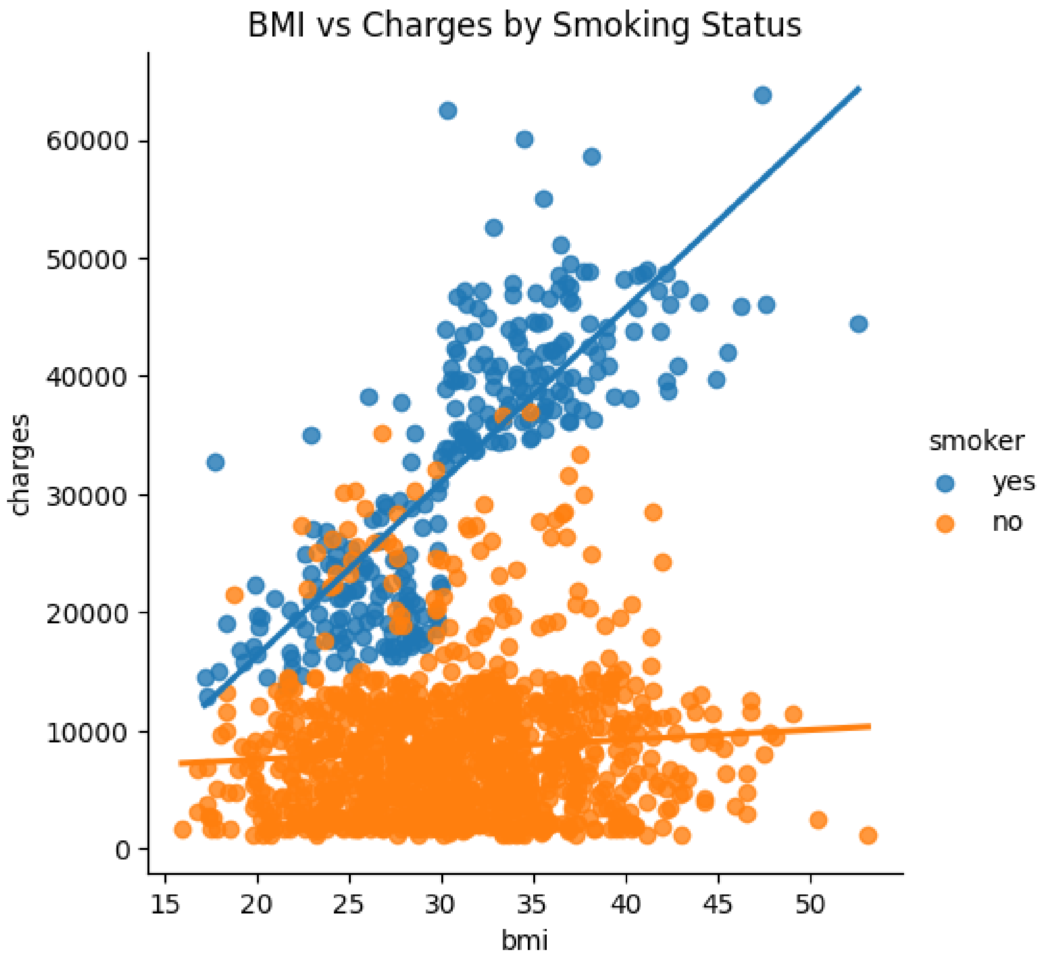
```
age           265.1057     10.108     26.228      0.000     245.272     284.939
children      580.3462    116.320      4.989      0.000     352.102     808.590
obese         135.0719    284.874      0.474      0.635    -423.909     694.053
==============================================================================
Omnibus:                      708.588   Durbin-Watson:                   2.058
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5640.544
Skew:                           3.182   Prob(JB):                         0.00
Kurtosis:                      12.312   Cond. No.                         136.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

We can see that for a smoker who is also obese (bmi $>$ 30), the coefficient for BMI is 19990, with an SE of 477 and a p-value $<$ 0.0001. However, for a non smoker and the coefficient for obese is 135 with high SE and high p-value. The difference in coefficient and statistical significance clearly indicates the presence of an interaction term that is needed.

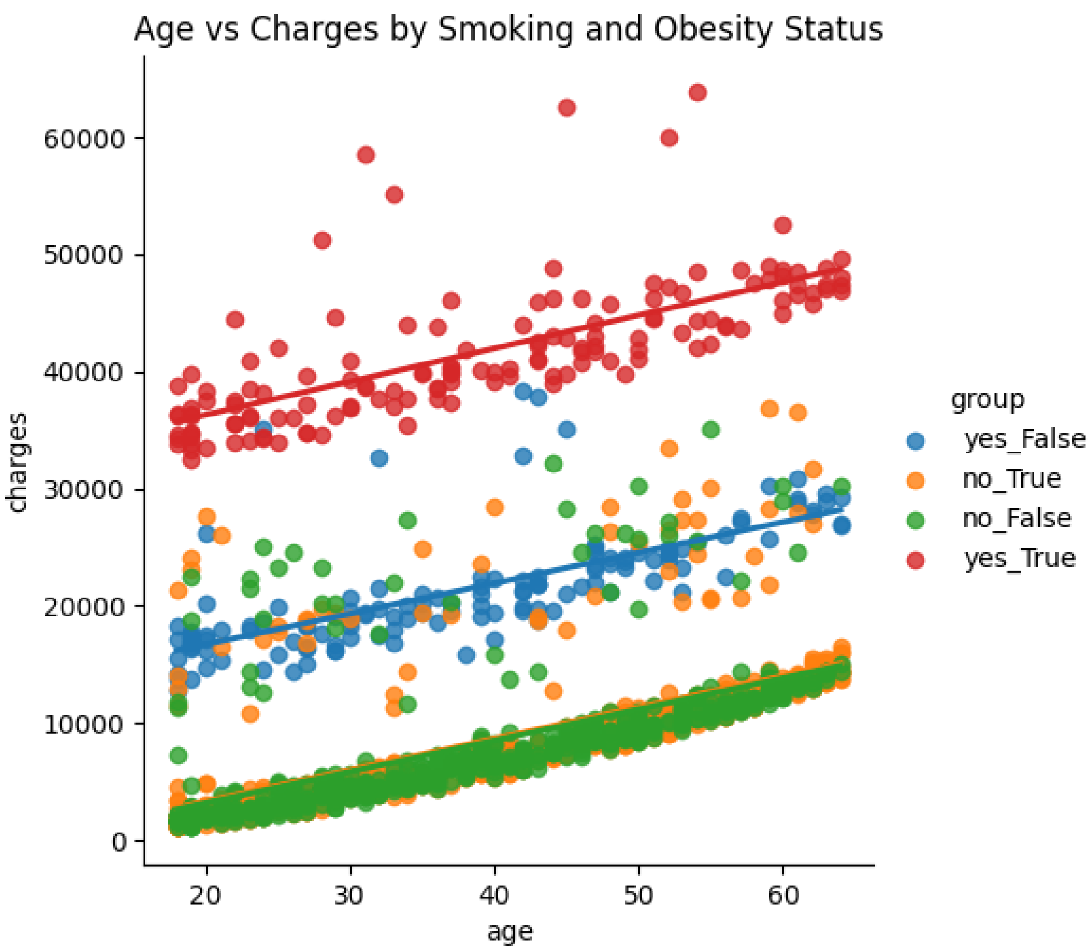
[9]:
```python
import seaborn as sns

df_plot = insurance.copy()
df_plot["obese"] = (df_plot["bmi"] > 30)

sns.barplot(
    data=df_plot,
    x="obese",
    y="charges",
    hue="smoker"
)

plt.title("Charges by Obesity and Smoking Status")
plt.show()
```




[10]:
```
sns.lmplot(
    data=df_plot,
    x="bmi",
    y="charges",
    hue="smoker",
    ci=None
)

plt.title("BMI vs Charges by Smoking Status")
plt.show()
```




```
[12]: df_plot["group"] = (
          df_plot["smoker"] + "_" + df_plot["obese"].astype(str)
      )

      sns.lmplot(
          data=df_plot,
          x="age",
          y="charges",
          hue="group",
          ci=None
      )
      plt.title("Age vs Charges by Smoking and Obesity Status")
      plt.show()
```




The graphs distinctly show the separation between people who smoke and are obese and their respective insurance charges. The interaction term between smoking and obesity is crucial to the model.

### 2.2 b

[13]:
```
X = insurance.drop(columns=['charges'])
X = pd.get_dummies(X, drop_first=True).astype(float)
X = X[['age', 'bmi', 'children', 'smoker_yes']]

X['obese'] = (X['bmi'] > 30).astype(float)
X = X.drop(columns=['bmi'])

# Add interaction term
X['smoker_obese'] = X['smoker_yes'] * X['obese']
X = sm.add_constant(X)
model_interaction = sm.OLS(y, X).fit()
```

```python
print(model_interaction.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                charges   R-squared:                       0.863
Model:                            OLS   Adj. R-squared:                  0.863
Method:                 Least Squares   F-statistic:                     1680.
Date:                Wed, 11 Feb 2026   Prob (F-statistic):               0.00
Time:                        16:27:18   Log-Likelihood:                -13147.
No. Observations:                1338   AIC:                         2.631e+04
Df Residuals:                    1332   BIC:                         2.634e+04
Df Model:                           5
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const        -2670.0560    399.933     -6.676      0.000   -3454.623   -1885.489
age            266.2188      8.783     30.310      0.000     248.988     283.449
children       502.6480    101.941      4.931      0.000     302.666     702.630
smoker_yes     1.34e+04    441.692     30.327      0.000    1.25e+04    1.43e+04
obese          135.0550    276.711      0.488      0.626    -407.781     677.891
smoker_obese  1.987e+04    609.192     32.612      0.000    1.87e+04    2.11e+04
==============================================================================
Omnibus:                      885.600   Durbin-Watson:                   2.060
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7358.552
Skew:                           3.167   Prob(JB):                         0.00
Kurtosis:                      12.585   Cond. No.                         246.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

The interaction effect model is stronger, with an R^2 of 0.863 compared to the separate fitted models R^2 of 0.885 and 0.408. Although the R^2 is slightly lower, we still prefer the interaction model. The split models showed that obesity effect is massive for smokers, while negligible for non smokers. However, the models reduce sample size and prevent formal testing of a slope difference. For non-obese individuals a person who smokes (smoking coefficient) on average is predicted to have $13400 more in insurance charges. This predictor is statistically significant. For non-smokers and obese person on average is predicted only to have a $135 increase in charge. With a p-value of 0.626, this is not statistically significant. But, we can see that the interaction term has a coefficient of 19870, the most impactful predictor in the model, and has a low SE and extremely low p-value indicating statistical significance. While we can tell from the smaller smoking and obese coefficients that these factors alone matter, the interaction term is what affects the model the strongest. Thus, we prefer the interaction term model.

### 2.3 c

The interaction term specifically represents the predicted increase in insurance charges associated with obesity, which is $19,870 larger for smokers than for non-smokers, while holding age and number of children constant.

**LLM Usage**: All work was done by myself in VSCode with GitHub Copilot integration. The integration "provides code suggestions, explanations, and automated implementations based on natural language prompts and existing code context," and also offers autonomous coding and an in-IDE chat interface that is able to interact with the current codebase. Only the Copilot provided automatic inline suggestions for both LaTex and Python in `.tex` and `.ipynb` Jupyter notebook files respectively were taken into account / used.