

hw3_code

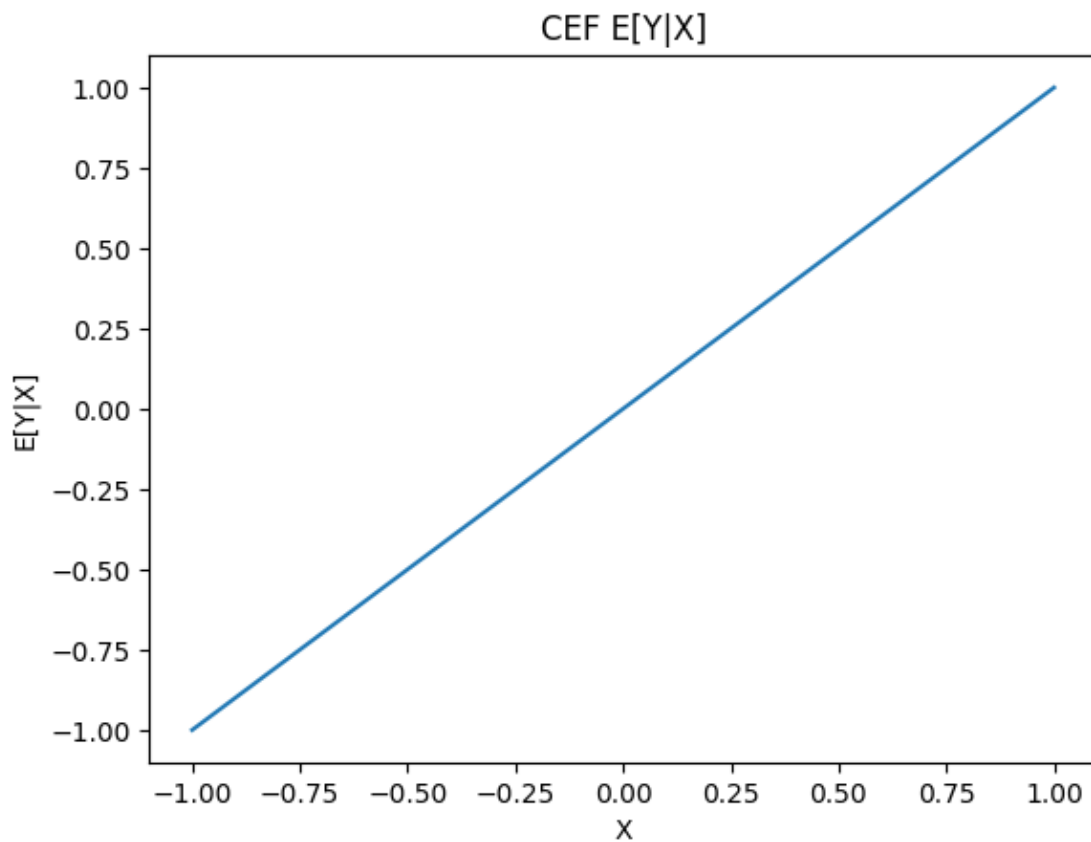
January 27, 2026

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[2]: import numpy as np
import matplotlib.pyplot as plt
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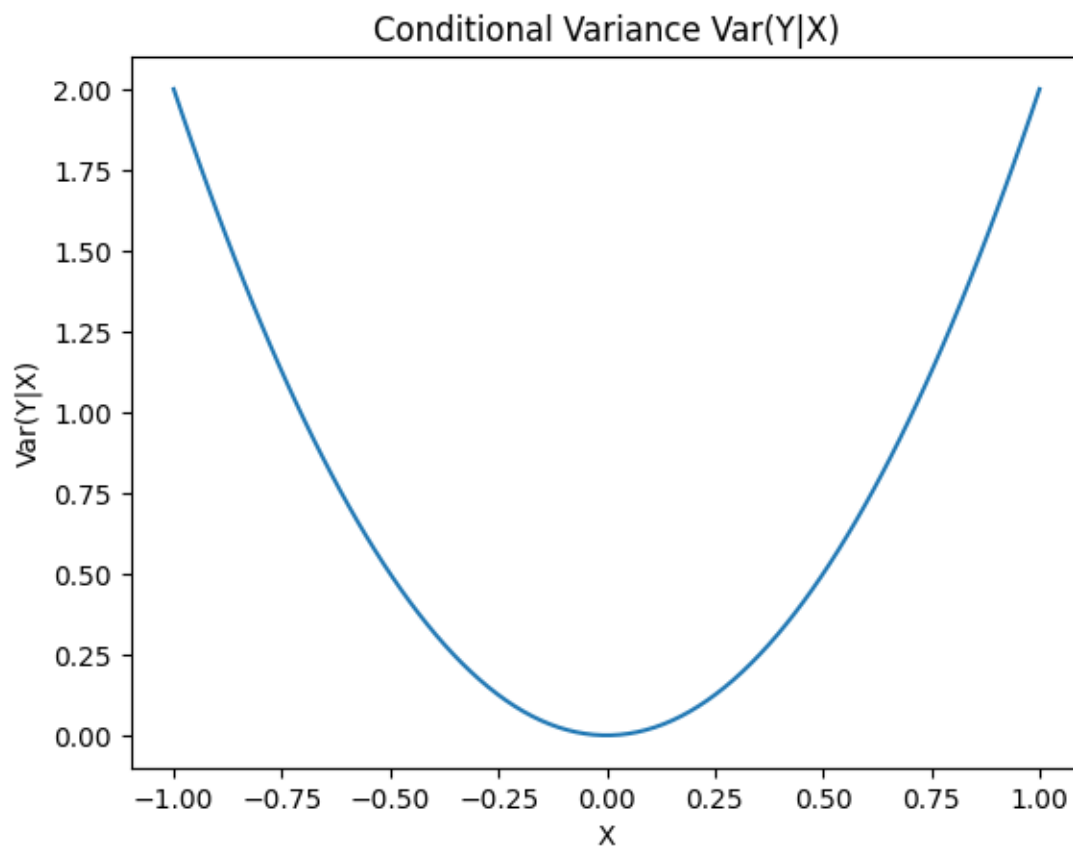
1 1

1.1 i

```
[ ]: plt.plot(np.linspace(-1, 1, 100), np.linspace(-1, 1, 100))
plt.title("CEF E[Y|X]")
plt.xlabel("X")
plt.ylabel("E[Y|X]")
plt.show()
```

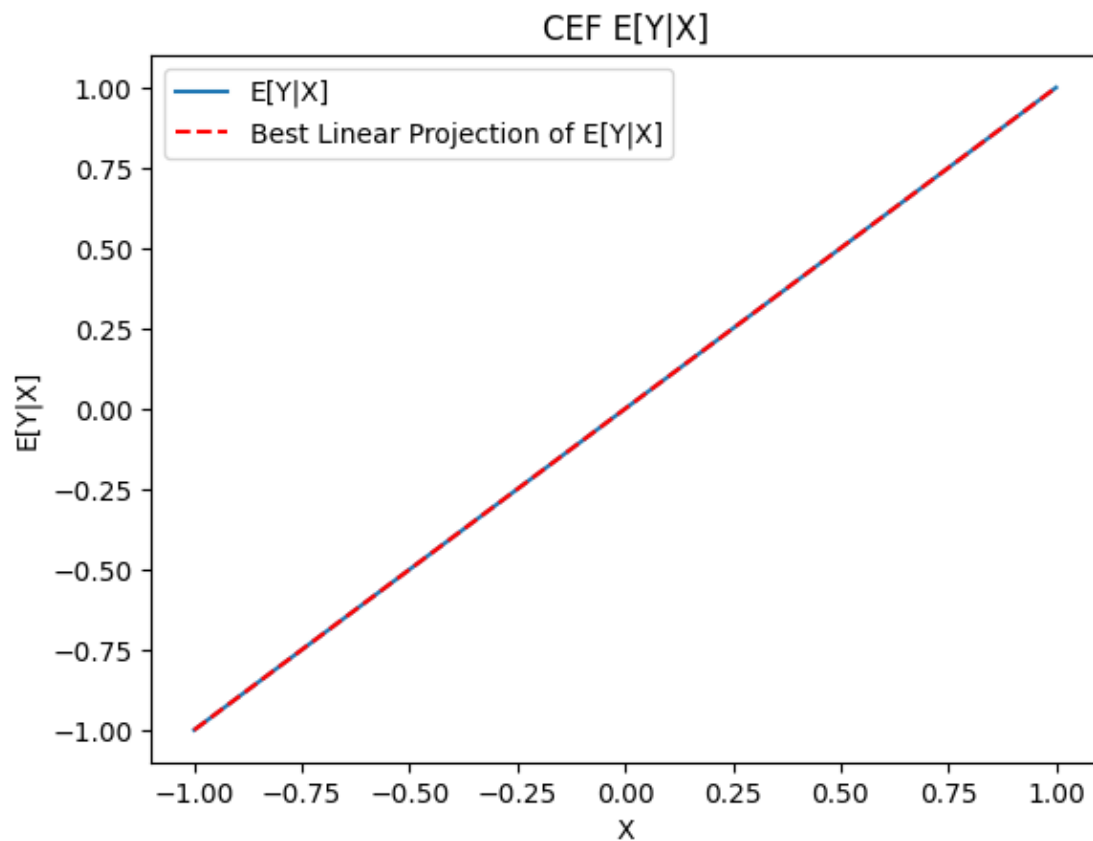


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[4]: plt.plot(np.linspace(-1, 1, 100), 2 * np.linspace(-1, 1, 100)**2)
plt.title("Conditional Variance Var(Y|X)")
plt.xlabel("X")
plt.ylabel("Var(Y|X)")
plt.show()
```



1.2 ii

```
[6]: plt.plot(np.linspace(-1, 1, 100), np.linspace(-1, 1, 100), label='E[Y|X]')
plt.plot(np.linspace(-1, 1, 100), np.linspace(-1, 1, 100), label='Best Linear
↳ Projection of E[Y|X]', color="red", linestyle="dashed")
plt.title("CEF E[Y|X]")
plt.xlabel("X")
plt.ylabel("E[Y|X]")
plt.legend()
plt.show()
```



2 2

3 i

```
[ ]: import statsmodels.api as sm
      from scipy.stats import norm

      def confBand(x, y, conf):
          x = np.asarray(x)
          y = np.asarray(y)

          X = sm.add_constant(x)
          model = sm.OLS(y, X).fit()

          xg = np.linspace(min(x), max(x), 100)
          Xg = sm.add_constant(xg)
          yhat_g = model.predict(Xg)

          V_std = model.cov_params()
```

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V_rob = model.get_robustcov_results(cov_type='HC1').cov_params()

se_std = np.sqrt(np.sum(Xg @ V_std * Xg, axis=1))
se_rob = np.sqrt(np.sum(Xg @ V_rob * Xg, axis=1))

z_point = norm.ppf(1 - (1 - conf) / 2)
z_bonf = norm.ppf(1 - (1 - conf) / (2 * len(xg)))

pw_std_u = yhat_g + z_point * se_std
pw_std_l = yhat_g - z_point * se_std

pw_rob_u = yhat_g + z_point * se_rob
pw_rob_l = yhat_g - z_point * se_rob

bonf_std_u = yhat_g + z_bonf * se_std
bonf_std_l = yhat_g - z_bonf * se_std

bonf_rob_u = yhat_g + z_bonf * se_rob
bonf_rob_l = yhat_g - z_bonf * se_rob

plt.scatter(x, y, label='Data Points')
plt.plot(xg, yhat_g, color='black', label='OLS Regression Line')

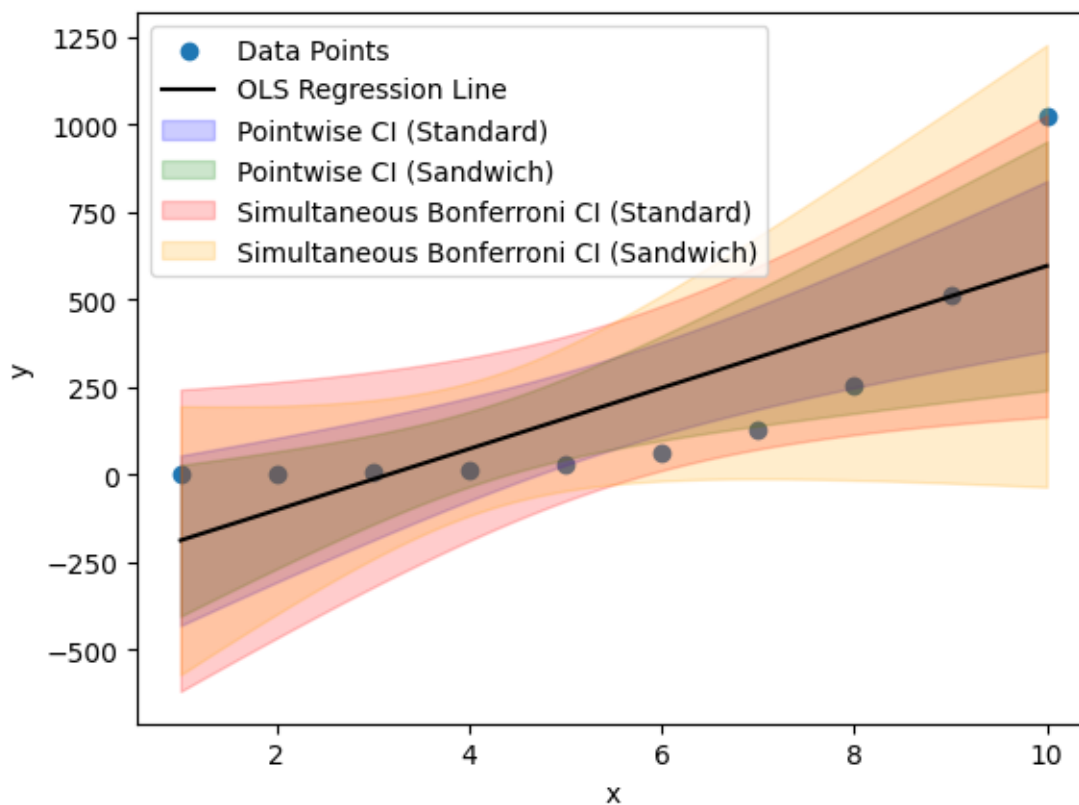
plt.fill_between(xg, pw_std_l, pw_std_u, color='blue', alpha=0.2,
↳label='Pointwise CI (Standard)')
plt.fill_between(xg, pw_rob_l, pw_rob_u, color='green', alpha=0.2,
↳label='Pointwise CI (Sandwich)')
plt.fill_between(xg, bonf_std_l, bonf_std_u, color='red', alpha=0.2,
↳label='Simultaneous Bonferroni CI (Standard)')
plt.fill_between(xg, bonf_rob_l, bonf_rob_u, color='orange', alpha=0.2,
↳label='Simultaneous Bonferroni CI (Sandwich)')

plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()

n = 10
x = np.arange(1, n + 1)
y = 2 ** x

confBand(x, y, 0.95)

```



4 3

4.1 i

```
[12]: np.random.seed(0)

N = 1000
n = 100
conf = 0.95

pw_std_cov = 0
pw_rob_cov = 0
bonf_std_cov = 0
bonf_rob_cov = 0

pw_std_cov_i = 0
pw_rob_cov_i = 0
bonf_std_cov_i = 0
bonf_rob_cov_i = 0

for i in range(N):
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X = np.random.uniform(0, 1, n)
Z = np.random.normal(0, 1, n)
Y = X**3 + 0.5 * X * Z

grid = np.sort(X)
true_ef = grid**3

x_data = np.asarray(X)
y_data = np.asarray(Y)

X_model = sm.add_constant(x_data)
model = sm.OLS(y_data, X_model).fit()

Xg = sm.add_constant(grid)
yhat_g = model.predict(Xg)

V_std = model.cov_params()
V_rob = model.get_robustcov_results(cov_type='HC1').cov_params()

se_std = np.sqrt(np.sum(Xg @ V_std * Xg, axis=1))
se_rob = np.sqrt(np.sum(Xg @ V_rob * Xg, axis=1))

z_point = norm.ppf(1 - (1 - conf) / 2)
z_bonf = norm.ppf(1 - (1 - conf) / (2 * len(grid)))

pw_std_u = yhat_g + z_point * se_std
pw_std_l = yhat_g - z_point * se_std

if np.all((pw_std_l <= true_ef) & (true_ef <= pw_std_u)):
    pw_std_cov += 1

pw_std_cov_i += np.mean((pw_std_l <= true_ef) & (true_ef <= pw_std_u))

pw_rob_u = yhat_g + z_point * se_rob
pw_rob_l = yhat_g - z_point * se_rob

if np.all((pw_rob_l <= true_ef) & (true_ef <= pw_rob_u)):
    pw_rob_cov += 1

pw_rob_cov_i += np.mean((pw_rob_l <= true_ef) & (true_ef <= pw_rob_u))

bonf_std_u = yhat_g + z_bonf * se_std
bonf_std_l = yhat_g - z_bonf * se_std

if np.all((bonf_std_l <= true_ef) & (true_ef <= bonf_std_u)):
    bonf_std_cov += 1

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bonf_std_cov_i += np.mean((bonf_std_l <= true_ef) & (true_ef <= bonf_std_u))

bonf_rob_u = yhat_g + z_bonf * se_rob
bonf_rob_l = yhat_g - z_bonf * se_rob

if np.all((bonf_rob_l <= true_ef) & (true_ef <= bonf_rob_u)):
    bonf_rob_cov += 1

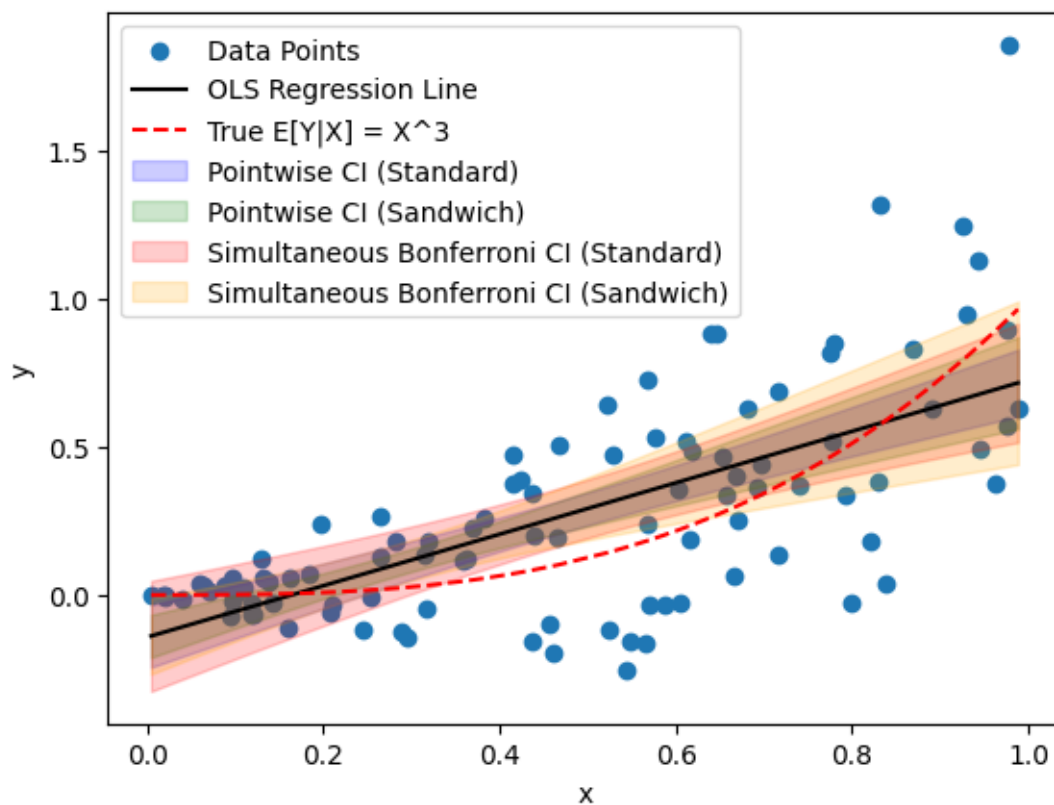
bonf_rob_cov_i += np.mean((bonf_rob_l <= true_ef) & (true_ef <= bonf_rob_u))

if i == 0:
    plt.scatter(x_data, y_data, label='Data Points')
    plt.plot(grid, yhat_g, color='black', label='OLS Regression Line')
    plt.plot(grid, true_ef, color='red', linestyle='--', label='True E[Y|X]')
    plt.fill_between(grid, pw_std_l, pw_std_u, color='blue', alpha=0.2,
        label='Pointwise CI (Standard)')
    plt.fill_between(grid, pw_rob_l, pw_rob_u, color='green', alpha=0.2,
        label='Pointwise CI (Sandwich)')
    plt.fill_between(grid, bonf_std_l, bonf_std_u, color='red', alpha=0.2,
        label='Simultaneous Bonferroni CI (Standard)')
    plt.fill_between(grid, bonf_rob_l, bonf_rob_u, color='orange', alpha=0.2,
        label='Simultaneous Bonferroni CI (Sandwich)')

    plt.xlabel('x')
    plt.ylabel('y')
    plt.legend()
    plt.show()

print("Empirical Coverage Probabilities:")
print(f"Pointwise CI (Standard): {pw_std_cov / N:.3f}")
print(f"Pointwise CI (Sandwich): {pw_rob_cov / N:.3f}")
print(f"Simultaneous Bonferroni CI (Standard): {bonf_std_cov / N:.3f}")
print(f"Simultaneous Bonferroni CI (Sandwich): {bonf_rob_cov / N:.3f}")
print()
print("Average Pointwise Coverage:")
print(f"Pointwise CI (Standard): {pw_std_cov_i / N:.3f}")
print(f"Pointwise CI (Sandwich): {pw_rob_cov_i / N:.3f}")
print(f"Simultaneous Bonferroni CI (Standard): {bonf_std_cov_i / N:.3f}")
print(f"Simultaneous Bonferroni CI (Sandwich): {bonf_rob_cov_i / N:.3f}")

```



Empirical Coverage Probabilities:

Pointwise CI (Standard): 0.000

Pointwise CI (Sandwich): 0.000

Simultaneous Bonferroni CI (Standard): 0.003

Simultaneous Bonferroni CI (Sandwich): 0.003

Average Pointwise Coverage:

Pointwise CI (Standard): 0.420

Pointwise CI (Sandwich): 0.381

Simultaneous Bonferroni CI (Standard): 0.760

Simultaneous Bonferroni CI (Sandwich): 0.706