HW 5

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1

We are given $X_i \in \{0, 1\}$ (0 means control, 1 means treatment) and Y_i be the outcome for observation i. Let $n_0 = \sum_{i=1}^{n} \mathbf{1}\{X_i = 0\}$ and $n_1 = \sum_{i=1}^{n} \mathbf{1}\{X_i = 1\}$ be the number of control and treatment observations, respectively. Let $\bar{Y}_0$ and $\bar{Y}_1$ be the sample means of the control and treatment groups, respectively, and $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ the sample variance, defined as

$$\hat{\sigma}_g^2 = \frac{1}{n_g - 1} \sum_{i:X_i=g} (Y_i - \bar{Y}_g)^2, \quad g \in \{0, 1\}$$

The pooled variance two sample t-statistic is given by:

$$t = \frac{\bar{Y}_1 - \bar{Y}_0}{s_p \sqrt{\frac{1}{n_0} + \frac{1}{n_1}}}$$

where the pooled standard deviation is:

$$s_p = \sqrt{\frac{(n_0 - 1)\hat{\sigma}_0^2 + (n_1 - 1)\hat{\sigma}_1^2}{n_0 + n_1 - 2}}$$

Lastly, we are given the linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$.

(a) We can show that the OLS slope estimate equals the difference between the sample mean of the two groups as follows:
OLS satisfies the normal equations:

$$\sum_{i=1}^{n} (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) = 0, \quad \sum_{i=1}^{n} X_i (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) = 0$$

We know the group means should be:

$$\bar{Y}_0 = \frac{1}{n_0} \sum_{i:X_i=0} Y_i, \quad \bar{Y}_1 = \frac{1}{n_1} \sum_{i:X_i=1} Y_i$$

Using the second normal equation and that X_i is binary ($X_i^2 = X_i$), we have:

$$\sum_{i=1}^{n} X_i Y_i - \hat{\beta}_0 \sum_{i=1}^{n} X_i - \hat{\beta}_1 \sum_{i=1}^{n} X_i = 0$$

Again, because X_i is binary, any summation terms with $X_i = 0$ will vanish, so we can simplify the above to:

$$\begin{aligned}
\sum_{i:X_i=1} Y_i - \hat{\beta}_0 \sum_{i:X_i=1} 1 - \hat{\beta}_1 \sum_{i:X_i=1} 1 = 0 \\
\sum_{i:X_i=1} Y_i - \hat{\beta}_0 n_1 - \hat{\beta}_1 n_1 = 0 \\
\frac{1}{n_1} \sum_{i:X_i=1} Y_i = \hat{\beta}_0 + \hat{\beta}_1 \\
\bar{Y}_1 = \hat{\beta}_0 + \hat{\beta}_1
\end{aligned}$$

Now we go back to the first equation and simplify by splitting into their respective groups:

$$\begin{aligned}
\sum_{i:X_i=0} (Y_i - \hat{\beta}_0) + \sum_{i:X_i=1} (Y_i - \hat{\beta}_0 - \hat{\beta}_1) = 0 \\
\sum_{i:X_i=0} Y_i - n_0 \hat{\beta}_0 + \sum_{i:X_i=1} Y_i - n_1 \hat{\beta}_0 - n_1 \hat{\beta}_1 = 0 \\
\sum_{i:X_i=0} Y_i + \sum_{i:X_i=1} Y_i = (n_0 + n_1)\hat{\beta}_0 + n_1 \hat{\beta}_1
\end{aligned}$$

Substitute in the group means and the equation for $\bar{Y}_1$:

$$\begin{aligned}
n_0 \bar{Y}_0 + n_1 \bar{Y}_1 = (n_0 + n_1)\hat{\beta}_0 + n_1 \hat{\beta}_1 \\
n_0 \bar{Y}_0 + n_1 (\hat{\beta}_0 + \hat{\beta}_1) = (n_0 + n_1)\hat{\beta}_0 + n_1 \hat{\beta}_1 \\
n_0 \bar{Y}_0 = n_0 \hat{\beta}_0 \\
\bar{Y}_0 = \hat{\beta}_0
\end{aligned}$$

Finally, we can substitute $\hat{\beta}_0$ back into the equation for $\bar{Y}_1$ to get:

$$\begin{aligned}
\bar{Y}_1 = \bar{Y}_0 + \hat{\beta}_1 \\
\hat{\beta}_1 = \bar{Y}_1 - \bar{Y}_0
\end{aligned}$$

which proves that the OLS slope estimate is the difference between the sample means of the two groups.

(b) We can show that the OLS intercept estimate equals the sample mean of the control group in the same steps we took above with the first normal equation:

$$\sum_{i:X_i=0}(Y_i - \hat{\beta}_0) + \sum_{i:X_i=1}(Y_i - \hat{\beta}_0 - \hat{\beta}_1) = 0$$
$$\sum_{i:X_i=0} Y_i - n_0\hat{\beta}_0 + \sum_{i:X_i=1} Y_i - n_1\hat{\beta}_0 - n_1\hat{\beta}_1 = 0$$
$$\sum_{i:X_i=0} Y_i + \sum_{i:X_i=1} Y_i = (n_0 + n_1)\hat{\beta}_0 + n_1\hat{\beta}_1$$

Substitute in the group means and the equation for $\bar{Y}_1$:

$$n_0\bar{Y}_0 + n_1\bar{Y}_1 = (n_0 + n_1)\hat{\beta}_0 + n_1\hat{\beta}_1$$
$$n_0\bar{Y}_0 + n_1(\hat{\beta}_0 + \hat{\beta}_1) = (n_0 + n_1)\hat{\beta}_0 + n_1\hat{\beta}_1$$
$$n_0\bar{Y}_0 = n_0\hat{\beta}_0$$
$$\bar{Y}_0 = \hat{\beta}_0$$

(c) We can show that the homoscedastic OLS estimator of the error standard deviation equals the pooled standard deviation as follows: We know $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$, so:

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2$$

We can split the above summation into the control and treatment groups:

$$\hat{\sigma}^2 = \frac{1}{n-2}\left(\sum_{i:X_i=0}(Y_i - \hat{\beta}_0)^2 + \sum_{i:X_i=1}(Y_i - \hat{\beta}_0 - \hat{\beta}_1)^2\right)$$

Substitute in the equations for $\hat{\beta}_0$ and $\hat{\beta}_1$:

$$\hat{\sigma}^2 = \frac{1}{n-2}\left(\sum_{i:X_i=0}(Y_i - \bar{Y}_0)^2 + \sum_{i:X_i=1}(Y_i - \bar{Y}_1)^2\right)$$

Recall that the corresponding within-group sample variances are:

$$\hat{\sigma}_0^2 = \frac{1}{n_0 - 1}\sum_{i:X_i=0}(Y_i - \bar{Y}_0)^2, \quad \hat{\sigma}_1^2 = \frac{1}{n_1 - 1}\sum_{i:X_i=1}(Y_i - \bar{Y}_1)^2$$

We can rearrange the above to get:

$$\sum_{i:X_i=0}(Y_i - \bar{Y}_0)^2 = (n_0 - 1)\hat{\sigma}_0^2, \quad \sum_{i:X_i=1}(Y_i - \bar{Y}_1)^2 = (n_1 - 1)\hat{\sigma}_1^2$$

Substitute the above into the equation for $\hat{\sigma}^2$:

$$\hat{\sigma}^2 = \frac{1}{n-2}\left((n_0-1)\hat{\sigma}_0^2 + (n_1-1)\hat{\sigma}_1^2\right)$$
$$\hat{\sigma} = \sqrt{\frac{(n_0-1)\hat{\sigma}_0^2 + (n_1-1)\hat{\sigma}_1^2}{n-2}} = s_p$$

which proves that the homoscedastic OLS estimator of the error standard deviation equals the pooled standard deviation.

(d) We can show that the OLS t-statistic for the slope estimate equals the two-sample t-statistic as follows:
We know our null hypothesis $H_0 : \beta_1 = 0$ and the OLS t-statistic is given by:

$$t_0 = \frac{\hat{\beta}_1}{\hat{se}(\hat{\beta}_1)}$$

We need to prove that:

$$\frac{\hat{\beta}_1}{\hat{se}(\hat{\beta}_1)} = \frac{\bar{Y}_1 - \bar{Y}_0}{s_p\sqrt{\frac{1}{n_0}+\frac{1}{n_1}}}$$

From the previous parts, we've already proved that $\hat{\beta}_1 = \bar{Y}_1 - \bar{Y}_0$ and $\hat{\sigma} = s_p$. We just need to show that $\hat{se}(\hat{\beta}_1) = s_p\sqrt{\frac{1}{n_0}+\frac{1}{n_1}}$. We know that the OLS standard error of the slope estimate is given by:

$$\hat{se}(\hat{\beta}_1) = \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

Here, we know that $\bar{X} = \frac{n_1}{n_0+n_1}$ since X_i is binary. Then we have:

$$X_i = 0: \quad X_i - \bar{X} = 0 - \frac{n_1}{n_0+n_1} = -\frac{n_1}{n_0+n_1}$$
$$X_i = 1: \quad X_i - \bar{X} = 1 - \frac{n_1}{n_0+n_1} = \frac{n_0}{n_0+n_1}$$

Thus,

$$\begin{aligned}
\sum_{i=1}^{n}(X_i - \bar{X})^2 &= \sum_{i:X_i=0}\left(-\frac{n_1}{n_0+n_1}\right)^2 + \sum_{i:X_i=1}\left(\frac{n_0}{n_0+n_1}\right)^2 \\
&= n_0\left(\frac{n_1}{n_0+n_1}\right)^2 + n_1\left(\frac{n_0}{n_0+n_1}\right)^2 \\
&= \frac{n_0 n_1^2 + n_1 n_0^2}{(n_0+n_1)^2} \\
&= \frac{n_0 n_1 (n_0+n_1)}{(n_0+n_1)^2} \\
&= \frac{n_0 n_1}{n_0+n_1}
\end{aligned}$$

Plugging this back into our equation for $\hat{se}(\hat{\beta}_1)$, and applying our previous proof that $\hat{\sigma} = s_p$ we have:

$$\hat{se}(\hat{\beta}_1) = \frac{\hat{\sigma}}{\sqrt{\frac{n_0 n_1}{n_0+n_1}}} = \hat{\sigma}\sqrt{\frac{n_0+n_1}{n_0 n_1}} = s_p\sqrt{\frac{1}{n_0}+\frac{1}{n_1}}$$

(e) In any finite sample, the OLS slope and intercept will always equal the difference in group means and the control group mean, respectively, regardless of the distribution of the error term ϵ. This comes algebraicly from the linear model, the fact that X is binary, and the OLS normal equations. However, the OLS estimator of the error standard deviation will only equal the pooled standard deviation if the error term ϵ is homoscedastic. This is because the pooled variance formula assumes a common population variance across the two groups, which the homoscedastic OLS estimator of the error standard deviation also assumes. If the error term (grouped variances) differ, then the pooled t-test is invalid and the homoscedastic OLS SE is wrong, thus breaking the equality. Likewise, the t-statistic for the slope estimate will only equal the two-sample t-statistic if the error term is homoscedastic, since the OLS t-statistic relies on the pooled SE. It also requires that the correct degrees of freedom (in this case, $n-2$) is used to calculate the OLS SE, which is the same as the degrees of freedom used to calculate the pooled SE (two degrees of freedom for β_0, β_1, and two degrees of freedom considering X_0, X_1 where $(n_0-1)+(n_1-1) = n-2$). If these assumptions are violated, then the OLS t-statistic will not equal the two-sample t-statistic.

2

See attached Jupyter notebook for code and plots.

In [4]:
```
import pandas as pd
import numpy as np

raw = pd.read_csv("boston.txt", sep=r"\s+", skiprows=22, header=None)
X = np.hstack([raw.values[::2, :], raw.values[1::2, :2]])
y = raw.values[1::2, 2]
columns = [ "crim", "zn", "indus", "chas", "nox", "rm", "age",
"dis", "rad", "tax", "ptratio", "b", "lstat"]
df = pd.DataFrame(X, columns=columns)
df["medv"] = y
```

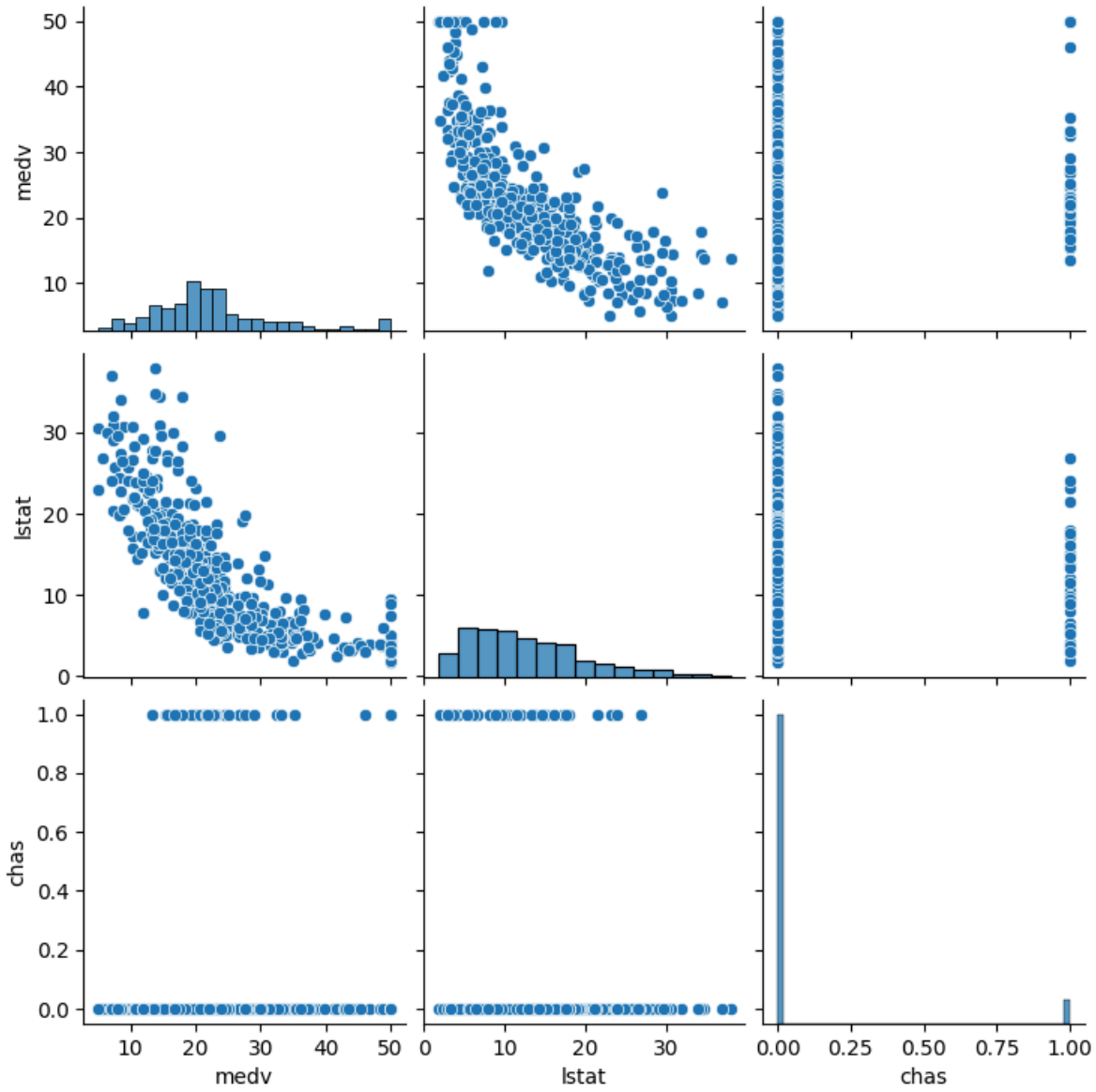
a

In [5]:
```
import matplotlib.pyplot as plt
from seaborn import pairplot
import statsmodels.api as sm

pairplot(df[["medv", "lstat", "chas"]])
X = sm.add_constant(df[["lstat", "chas"]])
model = sm.OLS(df["medv"], X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.563
Model:                            OLS   Adj. R-squared:                  0.561
Method:                 Least Squares   F-statistic:                     323.4
Date:                Fri, 13 Feb 2026   Prob (F-statistic):           4.93e-91
Time:                        12:22:43   Log-Likelihood:                -1631.1
No. Observations:                 506   AIC:                             3268.
Df Residuals:                     503   BIC:                             3281.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         34.0941      0.561     60.809      0.000      32.993      35.196
lstat         -0.9406      0.038    -24.729      0.000      -1.015      -0.866
chas           4.9200      1.069      4.601      0.000       2.819       7.021
==============================================================================
Omnibus:                      131.896   Durbin-Watson:                   0.975
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              275.510
Skew:                           1.406   Prob(JB):                     1.49e-60
Kurtosis:                       5.272   Cond. No.                         57.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```



The pair plots indicate a non linear relationship between `lstat` and `medv`, `medv` drops quickly for low-mid `lstat`, and then flattens out. Usually taking the log, square root, or inverse of `lstat` can help fix this. For this problem, we will use a log transformation.

Intercept: 34.0941, SE 0.561, p < 0.0001
This is the predicted `medv` for an `lstat` and `chas` value of 0. This isn't meaningful realistically as `lstat` 0 isn't possible, but this anchors the regression line. The value is statistically significant and well estimated from the low SE and p value.

`lstat` **coefficient**: -0.9406, SE 0.038, p < 0.0001
Holding `chas` constant, a 1 percentage point increase in `lstat` is attributed to about a $940.6 decrease in `medv`. The value is statistically significant and well estimated from the low SE and p value.

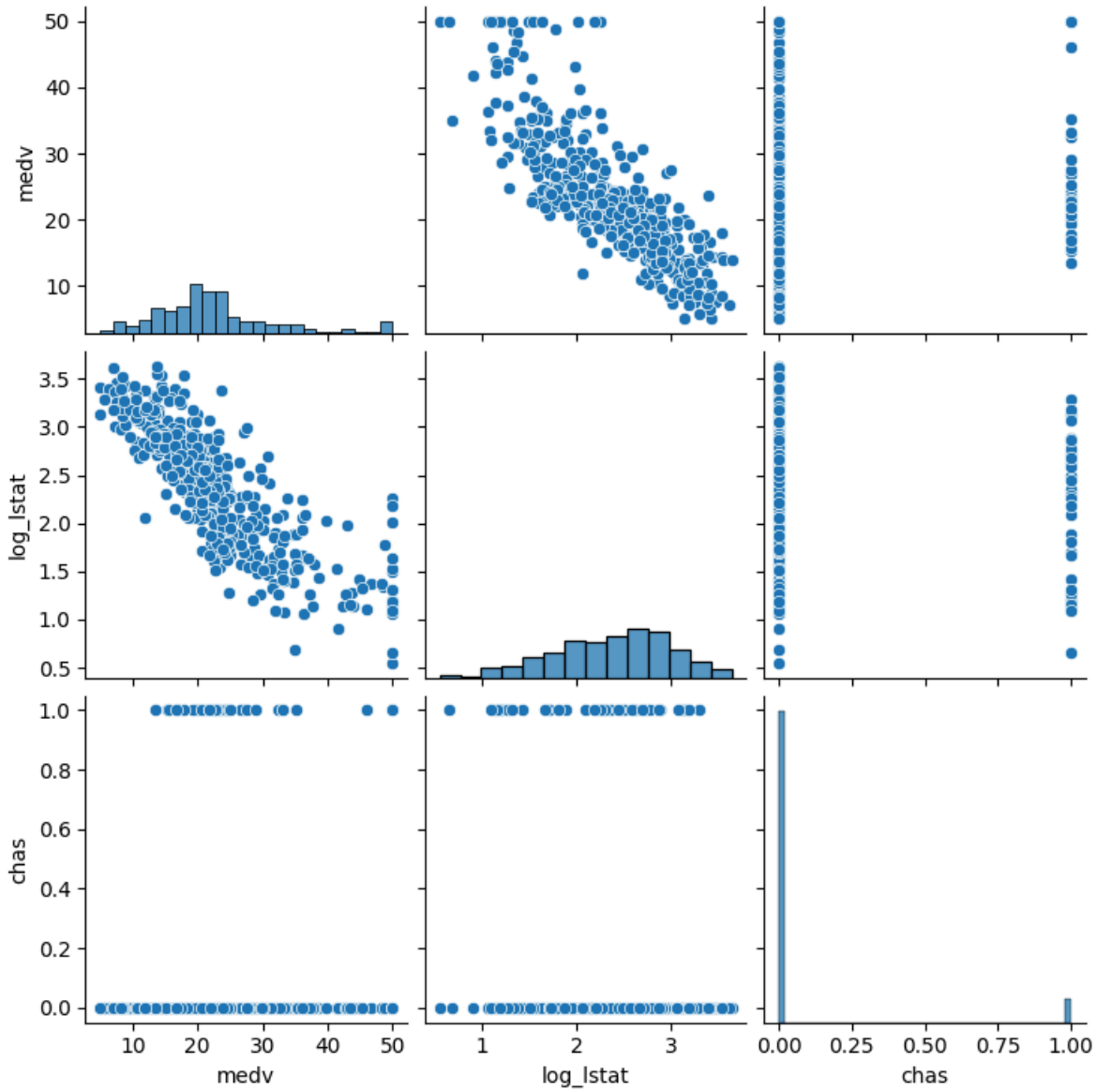
`chas` **coefficient**: 4.92, SE 1.069, p < 0.0001 Holding `lstat` constant, properties that border the Charles River are attributed to a $4,920 increase in `medv`. The value is

statistically significant and well estimated from the low SE and p value.

In [6]:
```
df["log_lstat"] = np.log(df["lstat"])
pairplot(df[["medv", "log_lstat", "chas"]])
X = sm.add_constant(df[["log_lstat", "chas"]])
model = sm.OLS(df["medv"], X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.678
Model:                            OLS   Adj. R-squared:                  0.677
Method:                 Least Squares   F-statistic:                     530.1
Date:                Fri, 13 Feb 2026   Prob (F-statistic):          1.43e-124
Time:                        12:22:54   Log-Likelihood:                -1553.4
No. Observations:                 506   AIC:                             3113.
Df Residuals:                     503   BIC:                             3125.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         51.5250      0.956     53.899      0.000      49.647      53.403
log_lstat    -12.3500      0.388    -31.814      0.000     -13.113     -11.587
chas           4.1819      0.918      4.554      0.000       2.378       5.986
==============================================================================
Omnibus:                      126.805   Durbin-Watson:                   0.990
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              335.582
Skew:                           1.230   Prob(JB):                     1.35e-73
Kurtosis:                       6.140   Cond. No.                         11.9
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```



Intercept: 51.5250, SE 0.956, p < 0.0001
This is the predicted `medv` for an `lstat` and `chas` value of 0. This isn't meaningful realistically as `lstat` 0 isn't possible, but this anchors the regression line. The value is statistically significant and well estimated from the low SE and p value.

`log_lstat` **coefficient**: -12.3500, SE 0.388, p < 0.0001
Holding `chas` constant, a 1 percentage point increase in `lstat` is attributed to about a $123.50 decrease in `medv`. The value is statistically significant and well estimated from the low SE and p value.

`chas` **coefficient**: 4.1819, SE .0918, p < 0.0001 Holding `lstat` constant, properties that border the Charles River are attributed to a $4,181.90 increase in `medv`. The value is statistically significant and well estimated from the low SE and p value.

The adjusted R^2 of the model is much higher than the original model, already showing an improvement. From the pairplot, we can see the relationship between `log_lstat` and

`medv` is linear now, showing the log transformation helped.

b

In [7]:
```
df["loglstat_chas"] = df["log_lstat"] * df["chas"]
X = sm.add_constant(df[["log_lstat", "chas", "loglstat_chas"]])
model_interaction = sm.OLS(df["medv"], X).fit()
print(model_interaction.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.680
Model:                            OLS   Adj. R-squared:                  0.678
Method:                 Least Squares   F-statistic:                     354.8
Date:                Fri, 13 Feb 2026   Prob (F-statistic):          1.35e-123
Time:                        12:22:57   Log-Likelihood:                -1552.4
No. Observations:                 506   AIC:                             3113.
Df Residuals:                     502   BIC:                             3130.
Df Model:                           3
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
const           51.0995      1.000     51.075      0.000      49.134      53.065
log_lstat      -12.1715      0.408    -29.868      0.000     -12.972     -11.371
chas             8.3909      3.090      2.715      0.007       2.320      14.462
loglstat_chas   -1.8922      1.327     -1.426      0.154      -4.498       0.714
==============================================================================
Omnibus:                      128.919   Durbin-Watson:                   1.002
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              351.383
Skew:                           1.239   Prob(JB):                     4.99e-77
Kurtosis:                       6.245   Cond. No.                         38.2
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

Intercept: 51.0995, SE 1, p < 0.0001
This is the predicted `medv` for an `lstat` and `chas` value of 0. This isn't meaningful realistically as `lstat` 0 isn't possible, but this anchors the regression line. The value is statistically significant and well estimated from the low SE and p value.

`log_lstat` coefficient: -12.1715, SE 0.408, p < 0.0001
Holding `chas` constant, a 1 percentage point increase in `lstat` is attributed to about a $121.72 decrease in `medv`. The value is statistically significant and well estimated from the low SE and p value. The coefficient is slightly smaller than the model without the interaction term.

`chas` coefficient: 8.3909, SE 3.090, p 0.007 Holding `lstat` constant, properties that border the Charles River are attributed to a $9,825.10 increase in `medv`. The value is

statistically significant and well estimated from the low SE and p value. The coefficient is significantly larger than the model without the interaction term.

`lstat:chas` **coefficient**: -1.8922, SE 1.327, p 0.154
The negative effect of `lstat` is stronger for properties bordering the Charles River. The value is not statistically significant nor well estimated from the high SE and p value. Specifically, for properties that border the Charles River, a 1 percentage point increase in `lstat` decreased `medv` by $14.06.

Given that the new R squared value of this model 0.680 is barely an improvement from the previous model of 0.678, and the interaction term is not statistically significat, interactions do not improve the model.

C

In [8]:
```
df0 = df[df["chas"] == 0]
df1 = df[df["chas"] == 1]

X0 = sm.add_constant(df0["log_lstat"])
model0 = sm.OLS(df0["medv"], X0).fit()
print(model0.summary())

X1 = sm.add_constant(df1["log_lstat"])
model1 = sm.OLS(df1["medv"], X1).fit()
print(model1.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.664
Model:                            OLS   Adj. R-squared:                  0.663
Method:                 Least Squares   F-statistic:                     925.6
Date:                Fri, 13 Feb 2026   Prob (F-statistic):          4.68e-113
Time:                        12:23:00   Log-Likelihood:                -1437.2
No. Observations:                 471   AIC:                             2878.
Df Residuals:                     469   BIC:                             2887.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         51.0995      0.982     52.025      0.000      49.169      53.030
log_lstat    -12.1715      0.400    -30.424      0.000     -12.958     -11.385
==============================================================================
Omnibus:                      127.277   Durbin-Watson:                   0.996
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              347.256
Skew:                           1.306   Prob(JB):                     3.93e-76
Kurtosis:                       6.297   Cond. No.                         11.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.713
Model:                            OLS   Adj. R-squared:                  0.704
Method:                 Least Squares   F-statistic:                     81.96
Date:                Fri, 13 Feb 2026   Prob (F-statistic):           1.84e-10
Time:                        12:23:00   Log-Likelihood:                -113.75
No. Observations:                  35   AIC:                             231.5
Df Residuals:                      33   BIC:                             234.6
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         59.4904      3.598     16.536      0.000      52.171      66.810
log_lstat    -14.0637      1.553     -9.053      0.000     -17.224     -10.903
==============================================================================
Omnibus:                        9.133   Durbin-Watson:                   1.242
Prob(Omnibus):                  0.010   Jarque-Bera (JB):               10.478
Skew:                           0.686   Prob(JB):                      0.00531
Kurtosis:                       5.303   Cond. No.                         8.99
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

Model	n	R^2	Adj R^2	AIC	BIC
Interaction	506	0.680	0.678	3113	3130

Model	n	R^2	Adj R^2	AIC	BIC
`chas` = 0	471	0.664	0.664	2878	2887
`chas` = 1	35	0.713	0.704	231.5	234.6

Note that the slopes for each `chas` specific model are the same as the indicated slopes in the interaction model when accounting for the interaction term. Additionally, the `chas` = 1 model only has 35 samples. Thus, we prefer the original log `lstat` model due to it using all 506 observations, and more importantly, because the interaction term is not statistically significant, using a common slope reduces variance, increases efficiency (in future predictions), simplifies interpretation, and avoids overfitting the smaller group (`chas` = 1). We can see this already occurring as the R^2 for the `chas` = 1 model is higher.

d

In [9]:
```
df_restrict = df[df["rad"] <= 8]

X_num = sm.add_constant(df_restrict[["log_lstat", "rad"]].astype(float))
model_num = sm.OLS(df_restrict["medv"].astype(float), X_num).fit()
print(model_num.summary())

rad_dummies = pd.get_dummies(df_restrict["rad"], prefix="rad", drop_first=True)
X_cat = sm.add_constant(pd.concat([df_restrict["log_lstat"].astype(float), rad_dumm
model_cat = sm.OLS(df_restrict["medv"].astype(float), X_cat).fit()
print(model_cat.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.606
Model:                            OLS   Adj. R-squared:                  0.603
Method:                 Least Squares   F-statistic:                     284.8
Date:                Fri, 13 Feb 2026   Prob (F-statistic):           1.12e-75
Time:                        12:23:08   Log-Likelihood:                -1153.1
No. Observations:                 374   AIC:                             2312.
Df Residuals:                     371   BIC:                             2324.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         48.4863      1.337     36.263      0.000      45.857      51.115
log_lstat    -11.6251      0.489    -23.757      0.000     -12.587     -10.663
rad            0.4140      0.168      2.462      0.014       0.083       0.745
==============================================================================
Omnibus:                       71.571   Durbin-Watson:                   0.945
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              125.233
Skew:                           1.088   Prob(JB):                     6.40e-28
Kurtosis:                       4.816   Cond. No.                         26.9
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
                            OLS Regression Results
==============================================================================
Dep. Variable:                   medv   R-squared:                       0.634
Model:                            OLS   Adj. R-squared:                  0.626
Method:                 Least Squares   F-statistic:                     78.89
Date:                Fri, 13 Feb 2026   Prob (F-statistic):           7.15e-75
Time:                        12:23:08   Log-Likelihood:                -1139.4
No. Observations:                 374   AIC:                             2297.
Df Residuals:                     365   BIC:                             2332.
Df Model:                           8
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         46.2099      1.501     30.788      0.000      43.258      49.161
log_lstat    -11.2540      0.495    -22.715      0.000     -12.228     -10.280
rad_2.0        4.4640      1.563      2.857      0.005       1.391       7.537
rad_3.0        4.7632      1.425      3.344      0.001       1.962       7.565
rad_4.0        1.6991      1.270      1.338      0.182      -0.797       4.196
rad_5.0        4.3335      1.255      3.452      0.001       1.865       6.802
rad_6.0        2.4319      1.554      1.565      0.118      -0.624       5.488
rad_7.0        3.1987      1.700      1.881      0.061      -0.145       6.542
rad_8.0        5.9636      1.560      3.822      0.000       2.895       9.032
==============================================================================
Omnibus:                       77.028   Durbin-Watson:                   1.016
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              143.714
Skew:                           1.131   Prob(JB):                     6.21e-32
Kurtosis:                       5.027   Cond. No.                         31.9
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

Model	n	R^2	Adj R^2	AIC	BIC
Numerical `rad`	374	0.606	0.603	2312	2324
Categorical `rad`	374	0.634	0.626	2297	2332

Using `rad` as categorical significantly improves R^2 by 0.23. The AIC strongly favors the categorical specification, as the delta of 15 shows that allowing non-linear, level-specific effects of rad improves the fit. BIC slightly favors numerical specification however, as rad is heavily penalized with extra parameters as a categorical variable. The numerical model indicates that the rad coefficient is statistically significant. This does make sense, however it also implies that the movement from different rads has the same effect, which isn't necessarily realistic or true. The categorical model on the other hand, shows that different rads have varying statistical significance levels. This matches how rad is constructed realistically as it is an index, not a continuous economic quantity. Thus overall, we prefer the categorical specification due to higher adjusted R^2 and better representation of the predictor variables.

e

In [10]:
```
b3 = model_cat.params["rad_3.0"]
b4 = model_cat.params["rad_4.0"]
diff = b3 - b4
print(f"Estimated difference in expected value of medv between houses w/ rad = 3 an

cov = model_cat.cov_params()
var_diff = (
    cov.loc["rad_3.0", "rad_3.0"]
    + cov.loc["rad_4.0", "rad_4.0"]
    - 2 * cov.loc["rad_3.0", "rad_4.0"]
)
se_diff = np.sqrt(var_diff)
print(f"Estimated standard error of the difference: {se_diff}")
```

```
Estimated difference in expected value of medv between houses w/ rad = 3 and rad =
4: 3.0640954476181186
Estimated standard error of the difference: 0.9816951849191523
```

LLM Usage: All work was done by myself in VSCode with GitHub Copilot integration. The integration "provides code suggestions, explanations, and automated implementations based on natural language prompts and existing code context," and also offers autonomous coding and an in-IDE chat interface that is able to interact with the current codebase. Only the Copilot provided automatic inline suggestions for both LaTex and Python in `.tex` and `.ipynb` Jupyter notebook files respectively were taken into account / used.