hw3_project

January 29, 2026

[12]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import statsmodels.api as sm
from scipy.stats import norm
```

[4]:
```
insurance = pd.read_csv("../datasets/insurance.csv")
insurance.head()
```

[4]:

	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

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[8]:
```
# continuous predictor variables only
X = insurance[['age', 'bmi', 'children']]
y = insurance['charges']

X = sm.add_constant(X)
model = sm.OLS(y, X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                charges   R-squared:                       0.120
Model:                            OLS   Adj. R-squared:                  0.118
Method:                 Least Squares   F-statistic:                     60.69
Date:                Thu, 29 Jan 2026   Prob (F-statistic):           8.80e-37
Time:                        19:32:28   Log-Likelihood:                -14392.
No. Observations:                1338   AIC:                         2.879e+04
Df Residuals:                    1334   BIC:                         2.881e+04
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
```

```
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -6916.2433   1757.480     -3.935      0.000   -1.04e+04   -3468.518
age          239.9945     22.289     10.767      0.000     196.269     283.720
bmi          332.0834     51.310      6.472      0.000     231.425     432.741
children     542.8647    258.241      2.102      0.036      36.261    1049.468
==============================================================================
Omnibus:                      325.395   Durbin-Watson:                   2.012
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              603.372
Skew:                           1.520   Prob(JB):                    9.54e-132
Kurtosis:                       4.255   Cond. No.                         290.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

Intercept: -6916.2433, SE 1757.480, $p < 0.0001$
This is the predicted charge for a newborn (age 0) with a bmi of 0, no children, female, non-smoker, from the northeast region. This has no meaning in it of itself, however given its extremely low p-value, is obviously extremely relevant to the model as it anchors the regression line.

Age: 239.9945, SE 22.289, $p < 0.0001$ Holding all other variables constant, each year a patient is older is associated with an increase of roughly $240 in insurance charge. The SE is low and p-value is also extremely low, indicating a decently estimated and statistically significant predictor.

BMI: 332.0834, SE 51.31, $p < 0.0001$ Holding all other variables constant, a one unit increase in BMI is associated with an increase of roughly $332 in insurance charge. The SE is high and p-value is extremely low, indicating an average estimate but still statistically significant predictor.

Children: 542.8647, SE 258.241, p 0.036 Holding all other variables constant, every additional child is associated with an increase of roughly $542 in insurance charge. The SE is higher than both coefficient estimates for *age* and *bmi*, but still statistically signifcant at an alpha level of 0.05.

The adjusted R-squared value of the model is 0.118. This means that after accounting for model complexity, only about 11.8% of the variation in insurnace charges is expalined by age, BMI, and number of children. This indicates that most variation in charges is driven by non-continuous predictor variables, which Lantz does wish to show.

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[11]:
```
# recompute standard error and p values with sandwich formula

robust_cov = model.get_robustcov_results(cov_type='HC1')
print(robust_cov.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                charges   R-squared:                       0.120
Model:                            OLS   Adj. R-squared:                  0.118
```

```
Method:                 Least Squares   F-statistic:                     60.13
Date:                Thu, 29 Jan 2026   Prob (F-statistic):           1.85e-36
Time:                        19:40:39   Log-Likelihood:                -14392.
No. Observations:                1338   AIC:                         2.879e+04
Df Residuals:                    1334   BIC:                         2.881e+04
Df Model:                           3
Covariance Type:                  HC1
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -6916.2433   1703.922     -4.059      0.000   -1.03e+04   -3573.585
age          239.9945     22.355     10.736      0.000     196.140     283.849
bmi          332.0834     56.384      5.890      0.000     221.472     442.695
children     542.8647    243.126      2.233      0.026      65.913    1019.816
==============================================================================
Omnibus:                      325.395   Durbin-Watson:                   2.012
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              603.372
Skew:                           1.520   Prob(JB):                    9.54e-132
Kurtosis:                       4.255   Cond. No.                         290.
==============================================================================

Notes:
[1] Standard Errors are heteroscedasticity robust (HC1)
```

All coefficient values are the same, but this is expected as the sandwich formula does not change the fitted coefficients, only the estimated covariance matrix.

Intercept: The SE decreased slightly, however p-value still indicates a statistically significant value. This means that heteroskedasticity correction slightly tightens uncertainty aroudn the intercept.

Age: Value remains relatively unchanged, thus the effect of age is very stable and not sensitive to heteroskedasticity.

BMI: The SE has increased, however p-value still indicates a statistically significant predictor variable. This means that OLS underestimated uncertainty for BMI. After correcting, BMI is still significant but less precisely estimated.

Children: The SE decreased slightly, and p-value has decreased as well. This means that once heteroskedasticity is accounted for, the effect of children is actually slightly more statistically significant.

Because all coefficient values are the same, the adjusted R-squared is still 0.118.

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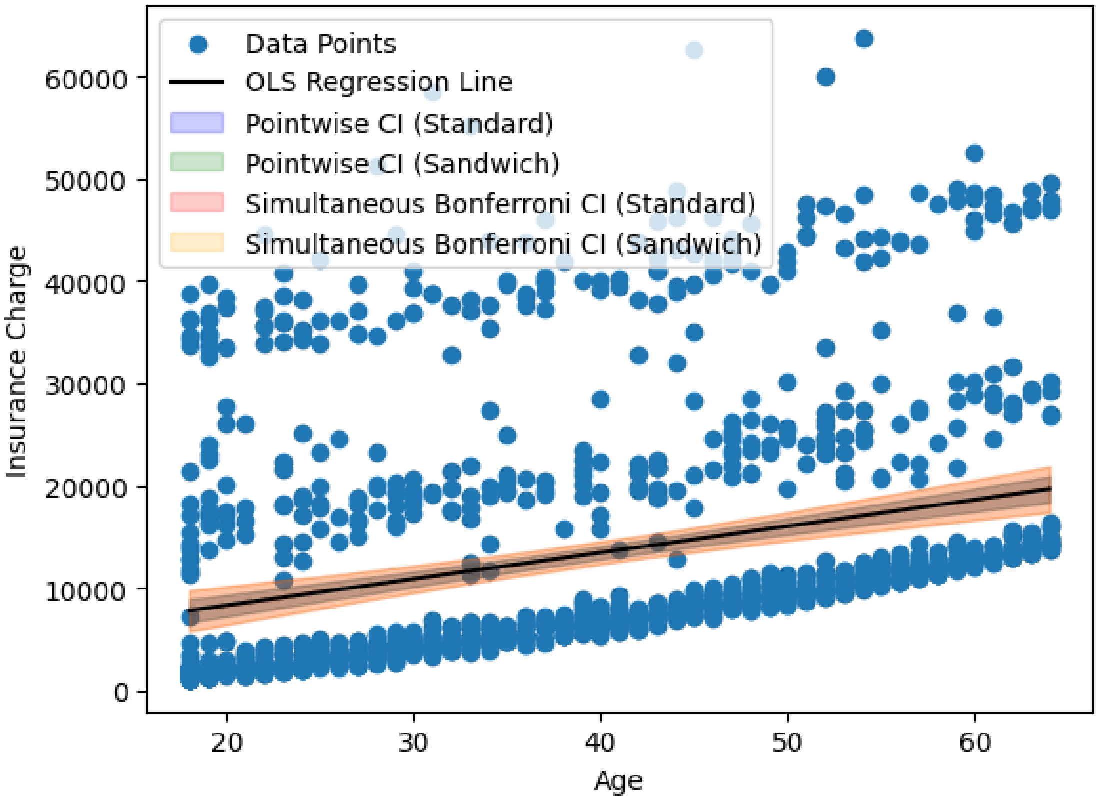
[13]:
```
x = insurance['age']
y = insurance['charges']
conf = 0.95

x = np.asarray(x)
```

```python
y = np.asarray(y)

X = sm.add_constant(x)
model = sm.OLS(y, X).fit()

xg = np.linspace(min(x), max(x), 100)
Xg = sm.add_constant(xg)
yhat_g = model.predict(Xg)

V_std = model.cov_params()
V_rob = model.get_robustcov_results(cov_type='HC1').cov_params()

se_std = np.sqrt(np.sum(Xg @ V_std * Xg, axis=1))
se_rob = np.sqrt(np.sum(Xg @ V_rob * Xg, axis=1))

z_point = norm.ppf(1 - (1 - conf) / 2)
z_bonf = norm.ppf(1 - (1 - conf) / (2 * len(xg)))

pw_std_u = yhat_g + z_point * se_std
pw_std_l = yhat_g - z_point * se_std

pw_rob_u = yhat_g + z_point * se_rob
pw_rob_l = yhat_g - z_point * se_rob

bonf_std_u = yhat_g + z_bonf * se_std
bonf_std_l = yhat_g - z_bonf * se_std

bonf_rob_u = yhat_g + z_bonf * se_rob
bonf_rob_l = yhat_g - z_bonf * se_rob

plt.scatter(x, y, label='Data Points')
plt.plot(xg, yhat_g, color='black', label='OLS Regression Line')

plt.fill_between(xg, pw_std_l, pw_std_u, color='blue', alpha=0.2,
 ↪label='Pointwise CI (Standard)')
plt.fill_between(xg, pw_rob_l, pw_rob_u, color='green', alpha=0.2,
 ↪label='Pointwise CI (Sandwich)')
plt.fill_between(xg, bonf_std_l, bonf_std_u, color='red', alpha=0.2,
 ↪label='Simultaneous Bonferroni CI (Standard)')
plt.fill_between(xg, bonf_rob_l, bonf_rob_u, color='orange', alpha=0.2,
 ↪label='Simultaneous Bonferroni CI (Sandwich)')

plt.xlabel('Age')
plt.ylabel('Insurance Charge')
plt.legend()
plt.show()
```



LLM Usage: All work was done by myself in VSCode with GitHub Copilot integration. The integration "provides code suggestions, explanations, and automated implementations based on natural language prompts and existing code context," and also offers autonomous coding and an in-IDE chat interface that is able to interact with the current codebase. Only the Copilot provided automatic inline suggestions for both LaTex and Python in `.tex` and `.ipynb` Jupyter notebook files respectively were taken into account / used.