HW 2

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Let X and Y be bivariate Gaussian with mean (μ_X, μ_Y), common variance σ^2, and correlation coefficient $\rho > 0$. The bivariate mean is random and determined by Z, where

$$(\mu_X, \mu_Y) = \begin{cases} (-\mu, \mu) & Z = -1, \\ (\mu, -\mu) & Z = 1 \end{cases}$$

with equal probability for $Z = -1$ and $Z = 1$.

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(a) The conditional expectation $E[Y|X, Z]$ can be calculated as:

$$E[Y|X, Z] = \mu_Y + \frac{Cov(X, Y)}{Var(X)}(X - \mu_X)$$

Since $\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$, we have:

$$\rho = \frac{Cov(X, Y)}{\sqrt{\sigma^2 \dot{\sigma}^2}}$$
$$Cov(X, Y) = \rho\sigma^2$$

Substituting this back into our equation:

$$E[Y|X, Z] = \mu_Y + \frac{\rho\sigma^2}{\sigma^2}(X - \mu_X)$$
$$E[Y|X, Z] = \mu_Y + \rho(X - \mu_X)$$

$\therefore$

$$E[Y|X, Z] = \begin{cases} \mu - \rho(X + \mu) & Z = -1, \\ -\mu + \rho(X - \mu) & Z = 1 \end{cases}$$

The CEF consequently is linear in X for each fixed Z.
The conditional expectation $E[Y|Z]$ is:

$$
\begin{aligned}
&E[Y|Z] = \mu_Y + \frac{Cov(X,Y)}{Var(X)}(E[X|Z] - \mu_X) \\
&E[Y|Z] = \mu_Y + \rho(\mu_X - mu_X) \\
&E[Y|Z] = \mu_Y \\
&E[Y|Z] = \begin{cases} \mu & Z = -1, \\ -\mu & Z = 1 \end{cases}
\end{aligned}
$$

The CEF consequently is constant for each fixed Z.
The conditional expectation $E[Y|X]$ is:

$$
\begin{aligned}
&E[Y|X] = E[E[Y|X,Z]|X] = \frac{1}{2}\left(\mu - \rho(X+\mu)\right) + \frac{1}{2}\left(-\mu + \rho(X-\mu)\right) \\
&E[Y|X] = \rho X - (1-\rho)\mu
\end{aligned}
$$

The CEF consequently is linear in X.