PSET 3

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1

From class we know the hydrostatic balance equation:

$$\frac{dP}{dz} = -\rho g$$

Let the pressure at the surface (also our lower boundary) be P. Because the atmosphere is homogenous, we know ρ is constant. Thus, reorganizing our equation and integrating from the surface (0, P_0) to some height (H) (where $P = 0$) gives us:

$$\begin{aligned}
\int_{P_0}^{0} dP &= -\rho g \int_{0}^{H} dz \\
0 - P_0 &= -\rho g(H - 0) \\
-P_0 &= -\rho g H \\
H &= \frac{P_0}{\rho g}
\end{aligned}$$

At the surface, $P_0 = \rho R T_0$, where T_0 is our lower boundary temperature. Thus:

$$\begin{aligned}
H &= \frac{\rho R T_0}{\rho g} \\
H &= \frac{R T_0}{g}
\end{aligned}$$

Therefore, H, which also represents the scale height of the atmosphere, must be a finite value as R and g are both constants, and T_0 is a constant determined at the lower boundary (surface), which is definitely not infinity.

2

A. Recall that the tropopause is the boundary between the troposphere and stratosphere where the temperature changes from decreasing to increasing with height. We also know that pressure decreases with height.

Combining these two facts, we can conclude that the troposphere is represented by the dotted line in the figure. We know that the troposphere is around 10 km in height, and that from the figure, its temperature changes from 300K to 200K. Thus, its lapse rate is:

$$\Gamma = -\frac{dT}{dz} = -\frac{200 \text{ K} - 300 \text{ K}}{10 \text{ km}} = 10 \text{ K km}^{-1}$$

This is the same as the dry adiabatic lapse rate.

B. The 700mbar to 300mbar layer is represented by the solid line in the figure, which is roughly linear. The average temperature of this layer is around 325K. Thus, the pressure scale height is:

$$H = \frac{RT_0}{g}$$
$$H = \frac{8.3\text{J mol}^{-1}\text{K}^{-1} \cdot 325\text{K}}{9.8\text{m s}^{-2}}$$
$$H \approx \frac{\pi^2 \times \pi \times 10^2}{10}\text{m}$$
$$H \approx \pi \times 10^2\text{m}$$