PSET 3

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From class we know the hydrostatic balance equation:

$$\frac{dP}{dz} = -\rho g$$

Let the pressure at the surface (also our lower boundary) be P. Because the atmosphere is homogenous, we know ρ is constant. Thus, reorganizing our equation and integrating from the surface (0, P_0) to some height (H) (where $P = 0$) gives us:

$$\begin{aligned}
\int_{P_0}^{0} dP &= -\rho g \int_{0}^{H} dz \\
0 - P_0 &= -\rho g(H - 0) \\
-P_0 &= -\rho g H \\
H &= \frac{P_0}{\rho g}
\end{aligned}$$

At the surface, $P_0 = \rho R T_0$, where T_0 is our lower boundary temperature. Thus:

$$\begin{aligned}
H &= \frac{\rho R T_0}{\rho g} \\
H &= \frac{R T_0}{g}
\end{aligned}$$

Therefore, H must be a finite value as R and g are both constants, and T_0 is a constant determined at the lower boundary (surface), which is definitely not infinity.

For a homogenous atmosphere with $T_0 = 298$ K, the height is:

$$\begin{aligned}
H &= \frac{287 \text{ J kg}^{-1}\text{K}^{-1} \cdot 298\text{K}}{9.8\text{m s}^{-2}} \\
H &= 8727.14 \text{ m}
\end{aligned}$$

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A. Recall that the tropopause is the boundary between the troposphere and stratosphere where the temperature changes from decreasing to increasing with height. We also know that pressure decreases with height. Combining these two facts, we can conclude that the troposphere is represented by the dotted line in the figure. From the figure, we see that the pressure goes from 200 mb to 1000 mb, which results in around 13km and that the temperature changes from 300K to 200K. Thus, its lapse rate is:

$$\Gamma = -\frac{dT}{dz} = -\frac{200 \text{ K} - 300 \text{ K}}{13 \text{ km}} = 7.7 \text{ K km}^{-1}$$

This is slightly lower than the dry adiabatic lapse rate. Note that this uses the a general estimate (described in TA OH), and not one of pi's and 10's, which would result in a lapse rate of 10 K km^{-1}, equal to the dry adiabatic lapse rate.

B. The average temperature of the 700 mb to 300 mb layer is around 250K. Thus, the pressure scale height is:

$$H = \frac{RT_0}{g}$$
$$H = \frac{287 \text{ J kg}^{-1}\text{K}^{-1} \cdot 250\text{K}}{9.8\text{m s}^{-2}}$$
$$H \approx \frac{\pi \times 10^2 \times \pi \times 10^2}{10}\text{m}$$
$$H \approx 10^4 \text{ m}$$