PSET 5

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A. Given that the Moon has an albedo of 0.07, we can calculate the amount of absorbed flux as: $F_{\text{abs}} = (1 - A) \times 1360 \text{ Wm}^{-2} = (1 - 0.07) \times 1360 \text{ Wm}^{-2} = 1264.8 \text{ Wm}^{-2}$. Using the Stefan-Boltzmann law, we can find the temperature at the hottest point on the Moon to be:

$$F_{\text{abs}} = \sigma T^4$$
$$T = \left(\frac{F_{\text{abs}}}{\sigma}\right)^{1/4}$$
$$T = \left(\frac{1264.8 \text{ Wm}^{-2}}{5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}}\right)^{1/4}$$
$$T = 386.465 \text{ K} = 235.967 \text{ °F}$$

B. Astronaut's wear space suits that are white to reflect sunlight and minimize the amount of heat absorbed.

C. If it is noon and the equinox, the Sun is directly overhead the equator. Knowing Chicago is 42°N, then the amount of flux striking is $F = \cos(42°) \times 1360 \text{ Wm}^{-2} = 1010.68 \text{ Wm}^{-2}$. For noon on the winter solstice, given that Earth's obliquity is 23°, the amount of flux striking is $F = \cos(42° + 23°) \times 1360 = 574.76 \text{ Wm}^{-2}$. For the summer solstice, the flux striking is $F = \cos(42° - 23°) \times 1360 = 1285.91 \text{ Wm}^{-2}$.

D. We know that the sun strikes a circular area of the Earth, thus the total power of sunlight striking the Earth is:

$$W_{\text{total}} = \pi R^2 \times F$$
$$W_{\text{total}} = \pi(6.4 \times 10^6 \text{ m})^2 \times 1360 \text{ Wm}^{-2}$$

Averaged over the Earth's entire surface, the flux is:

$$F_{\text{avg}} = \frac{W_{\text{total}}}{4\pi R^2}$$
$$F_{\text{avg}} = \frac{\cancel{\pi(6.4 \times 10^6 \text{ m})^2} \times 1360 \text{ Wm}^{-2}}{4 \times \cancel{\pi(6.4 \times 10^6 \text{ m})^2}}$$
$$F_{\text{avg}} = \frac{1360}{4} \text{ Wm}^{-2} = 340 \text{ Wm}^{-2}$$

E. If Earth had no greenhouse gases, the surface temperature would be:

$$F = \sigma T^4$$
$$T = \left(\frac{F}{\sigma}\right)^{1/4}$$
$$T = \left(\frac{340 \text{ Wm}^{-2}}{5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}}\right)^{1/4}$$
$$T = 278.275 \text{ K}$$

If the planet had an average albedo of 0.3, then the surface temperature would be:

$$(1 - A) \times F = \sigma T^4$$
$$(1 - 0.3) \times 340 \text{ Wm}^{-2} = \sigma T^4$$
$$0.7 \times 340 \text{ Wm}^{-2} = \sigma T^4$$
$$T = \left(\frac{0.7 \times 340 \text{ Wm}^{-2}}{5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}}\right)^{1/4}$$
$$T = 254.536 \text{ K}$$

F. From class, we calculated the equation for the surface temperature of the Earth with a single-layer atmosphere with emissivity ϵ to be:

$$T_s = \left(\frac{S(1 - A)}{\sigma(4 - 2\epsilon)}\right)^{1/4}$$

Thus, for an albedo of 0.3 and surface temperature of 278 K, the emissivity is:

$$287K = \left(\frac{1360 \text{ Wm}^{-2}(1 - 0.3)}{5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}(4 - 2\epsilon)}\right)^{1/4}$$
$$\epsilon = 0.76$$

G. Using Kirchhoff's law, the longwave absorptivity of this single-layer atmosphere is equal to the emissivity, thus $\alpha = \epsilon = 0.76$. Assuming no scatter, the longwave transmissivity is $T = 1 - \alpha = 0.24$. Using Beer's law, the optical thickness of the atmosphere is simply:

$$\tau = -\ln T$$
$$\tau = -\ln(0.24)$$
$$\tau = 1.427$$