Supporting Online Material for

Optimally Interacting Minds

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Supplementary Online Material

Methods

Participants

Experiment 1. Participants were recruited from undergraduate, graduate and faculty members of Aarhus University, Denmark (22 participants; 11 male dyads) and University College London, UK (8 participants; 4 male dyads) so altogether, 30 male (mean age ± sd: 28.30±6.27) participants took part in the experiment after written informed consent was obtained from them.

Experiment 2-4. Participants were recruited from undergraduate, graduate and faculty members of Arhus University, Denmark (72 participants, 22 for experiment 2; 28 for experiment 3; 22 for experiment 4 all male dyads) (mean age ± sd: 25.5±5.6) after written informed consent was obtained from them.

All participants were healthy adults with normal or corrected to normal visual acuity. Members of each dyad knew each other. No participant was recruited for more than one experiment. All experiments were approved by the local ethics committee.

Display parameters and Response Mode

In all experiments, dyad members sat in the same testing room. Each viewed his own display. Display screens were placed on separate tables at right angle to each other. Participants could see each other by slightly turning around. The two displays were connected to the same graphic card via a video amplifier splitter and controlled by the Cogent toolbox (www.vislab.ucl.ac.uk/Cogent/) for MATLAB (Mathworks Inc).

Experiments 1, 3 and 4. Each participant viewed an LCD display at a distance of 57cm (resolution = 800×600 – Fujitsu Siemens AMILO SL 3220W, 22") for which a look-up table linearized the output luminance. Background luminance was 62.5 Cd/m^2 in both displays. The displays were connected to a personal computer through an output splitter that sent identical outputs to both of them. Within each session of the experiment, one participant responded with the keyboard and the other with the mouse. Both participants used their right hand to respond.

Experiment 2. In order to display stimuli with different levels of noise to the participants, each participant viewed one half of their screen: the left half of one display for the participant responding with keyboard, and the right half of the other display for the participant responding with mouse. A piece of thick black cardboard was used to occlude the other half from view. Two stimulus arrays were presented on both displays simultaneously, each on one-half of the display. Control over which one the participants saw was achieved by using the occluding cardboard. Stimulus eccentricity and size were identical to Experiment 1.

Task, Stimuli and Procedure

Experiment 1. A 2-Alternative Forced Choice (2AFC) design was employed. Two observation intervals were provided. A target stimulus always occurred either in the first or the second interval. Participants were instructed to choose the interval most likely to have contained the target. The stimulus set displayed in each interval consisted of six vertically oriented Gabor patches (standard deviation of the Gaussian envelope: 0.45

degrees; spatial frequency: 1.5 cycles/degree; contrast: 10%) organized around an imaginary circle (radius: 8 degrees) at equal distances from each other. The target stimulus was generated by elevating the contrast of one of the six patches, which produced a contrast oddball. The target location and interval were randomized across the experimental session. The stimulus duration in each interval was 85 ms. Oddball contrast was obtained by adding one of four possible values (1.5%, 3.5%, 7.0% or 15%) to the 10% contrast of the non-oddball items. Participants received identical visual stimuli, as described above.

Each trial was initiated by the participant responding with the keyboard after coordinating with their partner (see Figure 1, main text). A black central fixation cross (width: 0.75 degrees visual angle) appeared on the screen for a variable period, drawn uniformly from the range 500-1000 ms. The two observation intervals were separated by a blank display lasting 1000 ms. The fixation cross turned into a question mark after the second interval to prompt the participants to respond. The question mark stayed on the screen until both participants had responded. Each participant initially responded without consulting the other. The participant who used the keyboard responded by pressing "N" and "M" for the first and second interval, respectively; the participant who used the mouse responded with a left and right click for the first and second interval, respectively. Individual decisions were then displayed on the monitor (Figure 1A, main text), so both participants were informed about their own and their partner's choice of the target interval. Colour codes were used to denote keyboard (blue) and mouse (yellow) responses. Vertical locations of the blue and yellow text were randomised to avoid spatial biasing. If the partners disagreed, a joint decision was requested, with the request made in blue if the keyboard participant was to announce the decision and in yellow if the mouse participant was to announce the decision. The keyboard participant announced the joint decision in odd trials; the mouse participant on even trials.

Participants were free to verbally discuss their choice with each other as long as they wanted. They were also free to choose any strategy that they wished; for example, they could split the task spatially (e.g. left and right half of screen) or temporally (first and second intervals). Such strategies were frequently employed, but never sustained for more than a few trials; participants invariably went back to paying attention to both intervals in all locations. The experimenter was present in the testing room throughout all experiments to make sure that the instructions were observed.

Participants received feedback either immediately after they made their decision, in cases where they initially agreed, or after the joint decision was announced, in cases where they initially disagreed. The feedback word was either "CORRECT" or "WRONG", one for each participant (keyboard: blue; mouse: yellow) and one for the dyad (white), and it remained on the screen until the next trial was initiated by the keyboard (Figure 1A, main text). Vertical order of the blue and yellow was randomized and the dyad feedback always appeared in the centre. Participants often took time to discuss (and sometimes chuckle over) their decisions and reflect on what had just happened.

After one practice block of 16 trials, two main experimental sessions were conducted. Each main session consisted of 8 blocks of 16 trials. Participants switched places (and thereby response device) at the end of session one. The pace of the experiment's progress was set by the participants.

Experiment 2. This experiment was similar to *Experiment* 1, except that noise was added to all the Gabor patches of either both, one, or neither of the participants on some of the

trials. Specifically, in 25% of the trials, both participants received extra noise ('Equal' condition); in 50% of the trials, only one of the two participant, chosen randomly, received extra noise ('Unequal' condition); and in 25% of the trials, neither participant received extra noise ('None' condition). These conditions can be summarized as

Equal: (Participant 1 – stimulus+noise); (Participant 2 – stimulus+noise)
None: (Participant 1 – stimulus); (Participant 2 – stimulus)
Unequal: (Participant 1 – stimulus+noise); (Participant 2 – stimulus)
Unequal: (Participant 1 – stimulus); (Participant 2 – stimulus+noise)

Participants were not told that they received extra noise. To create the white noise, each pixel's grey value was drawn, on each update, from a random uniform distribution ranging from 0 to 30% of the monitor's maximum luminance.

The experiment consisted of one practice block, which was not analysed and four main sessions, each consisting of 6 blocks of 32 trials, resulting in a total of 768 trials that were included in the analysis. Participants switched places (and thereby response devices) at the end of each session. The pace of the experiment's progress was set by the participants.

Experiment 3. Participants did not communicate with each other. They were instructed not to talk, and were given earphones to eliminate any meaningful auditory communication through chuckles or unintentional utterances. A screen was placed between them to prevent any visual contact. The experimenter was present in the testing room throughout all experiments to make sure that the instructions were observed.

Experiment 4. No feedback was provided. The sentence "START NEXT TRIAL" appeared at the fixation point once the joint decision was determined, and no information was given about who was right or wrong. The participant using the keyboard then initiated the next trial by a button press. All other aspects of the experiment were identical to Experiment 1.

Data Analysis In experiments 1 and 3-4 psychometric functions were constructed for each observer and for the dyad by plotting the proportion of trials in which the oddball was seen in the second interval against the contrast difference at the oddball location (the contrast in the second interval minus the contrast in the first; see Figure 1B, main text). One dyad's data were discarded from Experiment 3 due to poor the behavioural data from one of the participants in the first session that did not permit a reliable fit of the cumulative Gaussian.

For experiment 2, four psychometric functions were constructed for each participant and for the dyad. These 4 functions corresponded to 'Eequal, 'None', 'Unequal' (noise for one participant) and 'Unequal' (noise for the other participant). Examples of estimated slopes for 3 dyads are shown in Figure S2.

The psychometric curves were fit to a cumulative Gaussian function whose parameters were bias, b, and variance, σ^2. To estimate these parameters a probit regression model was employed using the *glmfit* function in Matlab (Mathworks Inc). A participant with bias b and variance σ^2 would have a psychometric curve, denoted $P(\Delta c)$ where Δc is the contrast difference between the second and first presentations, given by

$$P(\Delta c) = H\left(\frac{\Delta c + b}{\sigma}\right), \tag{S1}$$

where $H(z)$ is the cumulative Normal function,

$$H(z) \equiv \int_{-\infty}^{z} \frac{dt}{(2\pi)^{1/2}} \exp\left[-t^2/2\right]. \tag{S2}$$

As usual, the psychometric curve, $P(\Delta c)$, corresponds to the probability of saying that the second interval had the higher contrast. Thus, a positive bias indicates an increase probability of saying that the second interval had higher contrast (and thus corresponds to a negative mean for the underlying Gaussian distribution)
Given the above definitions for $P(\Delta c)$, we see that the variance is related to the maximum slope of the psychometric curve, denote s, via

$$s = \frac{1}{\left(2\pi\sigma^2\right)^{1/2}}. \tag{S3}$$

A large slope indicates small variance and thus highly sensitive performance.

Models
If two participants observe the same stimulus but disagree on what it was, how do they resolve their differences? The answer depends on what information they communicate. We considered four possibilities: the coin flip (CF) model, the behaviour and feedback (BF) model, the weighted confidence sharing (WCS) model, and the direct signal sharing (DSS) model. These are described in detail below, where we also compute the slope and bias of the dyad given the individual slopes and biases.

Coin flip model
For the coin flip model, in case of disagreement the participants effectively flip a coin. Thus, given our interpretation of the psychometric curve as the probability of being correct, the dyad probability under the coin flip model, denoted $P_{dyad}^{CF}(\Delta c)$, is given by

$$P_{dyad}^{CF}(\Delta c) = P_1(\Delta c)P_2(\Delta c) + \frac{P_1(\Delta c)\left(1 - P_2(\Delta c)\right) + P_2(\Delta c)\left(1 - P_1(\Delta c)\right)}{2} \tag{S4}$$

where $P_1(\Delta c)$ and $P_2(\Delta c)$ are the psychometric curves for participants 1 and 2, respectively. After a very small amount of algebra, and using Eq. (S1), this simplifies to

$$P_{dyad}^{CF}(\Delta c) = \frac{1}{2}\left[H\left(\frac{\Delta c + b_1}{\sigma_1}\right) + H\left(\frac{\Delta c + b_2}{\sigma_2}\right)\right]. \tag{S5}$$

An immediate outcome of this analysis is that for the coin flip model, the dyad psychometric curve is not a cumulative Gaussian. Nevertheless, we may characterize it by its slope and bias, although these are somewhat harder to compute than in the

confidence sharing model. Let us start with the bias, denoted b_{dyad}^{CF} and defined implicitly via $P_{dyad}^{CF}\left(-b_{dyad}^{CF}\right)=1/2$. There is no closed form solution for the bias, but we can find an approximate value in the limit that b_1 and b_2 are small compared to the standard deviation. We start by defining

$$b_0 \equiv \frac{\sigma_1 b_2 + \sigma_2 b_1}{\sigma_1 + \sigma_2}. \tag{S6}$$

Then, after a small amount of algebra, we find that Eq. (S5) can be written

$$P_{dyad}^{CF}(\Delta c) = \frac{1}{2}\left[H\left(\frac{\Delta c + b_0}{\sigma_1} + \frac{b_1 - b_2}{\sigma_1 + \sigma_2} \right) + H\left(\frac{\Delta c + b_0}{\sigma_2} - \frac{b_1 - b_2}{\sigma_1 + \sigma_2} \right) \right]. \tag{S7}$$

Taylor expanding around $\Delta c = -b_0$, we find that

$$P_{dyad}^{CF}(-b_0) = \frac{1}{2} + order\left((b_1 - b_2)^2 / (\sigma_1 + \sigma_2)^2 \right). \tag{S8}$$

Thus, to lowest order in $(b_1-b_2)^2/(\sigma_1+\sigma_2)^2$, the bias is b_0. Using the fact that the standard deviation is the inverse of the slope (Eq. (S3)), this gives us

$$b_{dyad}^{CF} \approx \frac{s_1 b_1 + s_2 b_2}{s_1 + s_2}, \tag{S9}$$

exactly the bias we find below for the weighted confidence sharing model, Eq. (S16a).
 Given Eq. (S7), the slope is easy to compute: we simply differentiate the right hand side with respect to Δc, evaluate the expression at $\Delta c=-b_0$, and use Eq. (S3) to express the standard deviations in terms of the slopes. When we do that, we find that the dyad slope, denoted s_{dyad}^{CF}, is given by

$$s_{dyad}^{CF} = \frac{s_1 + s_2}{2} \exp\left[-\frac{(b_1 - b_2)^2}{2(\sigma_1 + \sigma_2)^2} \right]. \tag{S10}$$

Again assuming the bias is small compared to the standard deviation, we may write

$$s_{dyad}^{CF} \approx \frac{s_1 + s_2}{2}. \tag{S11}$$

This is the expression in Eq. 1 of the main text.

Behaviour and feedback model
For the behaviour and feedback model, the participants cannot communicate anything besides their choice during the trial. Thus, the best they can do is learn which subject performs better on average, and use that subject's choice. Consequently, the dyad bias and slope are given by the bias and slope of the best participant, the latter denoted b_{best},

$$b_{dyad}^{BF} = b_{best} \tag{S12a}$$

$$s_{dyad}^{BF} = \max(s_1, s_2) \tag{S12b}$$

Equation (S12b) is reproduced in Eq. 2 of the main text.

Weighted confidence sharing model

For the weighted confidence sharing model we assume, consistent with numerous behavioural studies e.g. (*1, 2*), that participants have access to their variance on the task, and that they believe they are unbiased. Thus, participants assume that the probability that they are correct is $H(\Delta c/\sigma)$, which we define as confidence. To communicate this, we assume (for convenience) that they communicate the ratio $\Delta c/\sigma$, rather than their probability of being correct, $H(\Delta c/\sigma)$. This is just their z-score, so, in mathematical terms, the weighted confidence sharing model assumes that a decision is made based on the z-scores of the two participants, which we denote z_1 and z_2.

Given the two z-scores, what is the Bayes optimal decision? To answer this, we need the posterior distribution of contrast given z_1 and z_2, which we denote $P(\Delta c|z_1, z_2)$. Fortunately, we don't need the full posterior; we just need some of its properties. Note first of all that, by symmetry, if $z_1=-z_2$, then the probability that Δc is positive is 1/2. Second, we can safely assume that $P(\Delta c|z_1, z_2)$ is a monotonic function of both z_1 and z_2. Consequently, the decision boundary for $P(\Delta c|z_1, z_2)$ is $z_1+ z_2=0$, and the Bayes optimal decision is to choose the second interval (corresponding to Δc positive) if $z_1+ z_2>0$, and the first interval (corresponding to Δc negative) if $z_1+ z_2<0$.

Given this decision boundary, we can easily compute the probability correct for the dyad, denoted $P_{dyad}^{WCS}(c)$; it is given by

$$P_{dyad}^{WCS}(\Delta c) = \int\limits_{x_1/\sigma_1+x_2/\sigma_2>0} \frac{dx_1 dx_2}{2\pi\sigma_1\sigma_2} \exp\left[-\frac{(x_1-b_1-\Delta c)}{2\sigma_1^2} - \frac{(x_2-b_2-\Delta c)}{2\sigma_2^2} \right]. \tag{S13}$$

The integral is straightforward, and we arrive at

$$P_{dyad}^{WCS}(\Delta c) = H\left(\frac{\Delta c + b_{dyad}^{WCS}}{\sigma_{dyad}^{WCS}} \right), \tag{S14}$$

where

$$b_{dyad}^{WCS} = \frac{\sigma_2 b_1 + \sigma_1 b_2}{\sigma_1 + \sigma_2} \tag{S15a}$$

$$\sigma_{dyad}^{WCS} = 2^{1/2}\frac{\sigma_1\sigma_2}{\sigma_1 + \sigma_2}. \tag{S15b}$$

Expressed in terms of slopes (which are proportional to the inverse of the standard deviations) rather than standard deviations, these equations become

$$b_{dyad}^{WCS} = \frac{s_1 b_1 + s_2 b_2}{s_1 + s_2} \tag{S16a}$$

$$s_{dyad}^{WCS} = \frac{s_1 + s_2}{2^{1/2}}.$$ (S16b)

Equation (S16b) is reproduced in Eq. 3 of the main text.

Direct signal sharing model
For the direct signal sharing model, we can take over the standard results for a Gaussian distribution (*1, 3*), for which the dyad bias is given in terms of the individual biases as
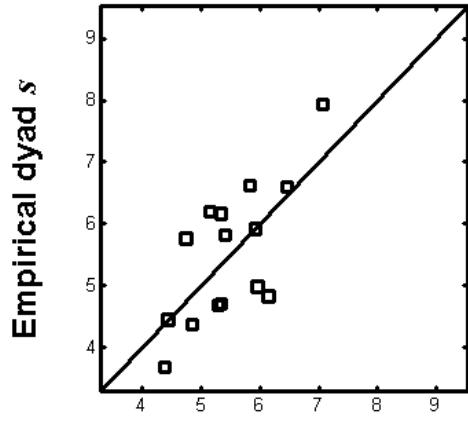
$$b_{dyad}^{DSS} = \frac{s_1^2 b_1 + s_2^2 b_2}{s_1^2 + s_2^2},$$ (S17a)

And dyad slope is
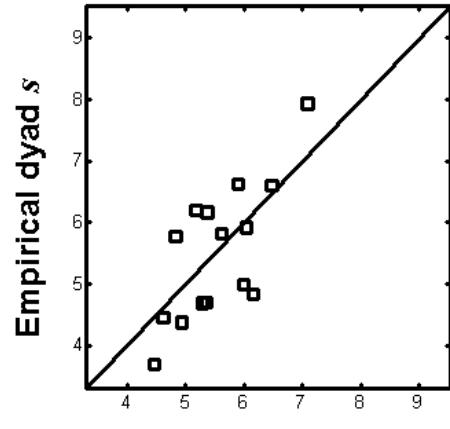
$$s_{dyad}^{DSS} = \left(s_1^2 + s_2^2 \right)^{1/2}.$$ (S17b)

Equation (S17b) is reproduced in Eq. 4 of the main text.

Supplementary Figures

Figure S1. Quantitative assessment of dyad behaviour against the weighted confidence sharing (WCS) and direct signal sharing (DSS) models for *Experiment* 1. The individual slopes, s_1 and s_2, were used for each dyad to predict, from Eqs. 3 and 4 of the main text, the dyad slopes (units are contrast^{-1}). These predictions were then compared to the empirically obtained dyad behaviour using least squares regression. The 45° line corresponds to perfect agreement between predicted and empirical dyad slope. The weighted confidence sharing (WCS) and direct signal sharing (DSS) models make very similar predictions. For both, the slope and intercept of the regression line were statistically indistinguishable from one and zero, respectively (WCS: regression slope = 1.06±0.31 and intercept = -0.31±1.72, $F_{(13,1)}=13.54$, $R^2=0.6594$, $p<0.001$; DSS: regression slope = 1.11±0.32 and intercept = -0.67±1.81, $F_{(13,1)}=13.54$, $R^2=0.6594$, $p<0.001$) .
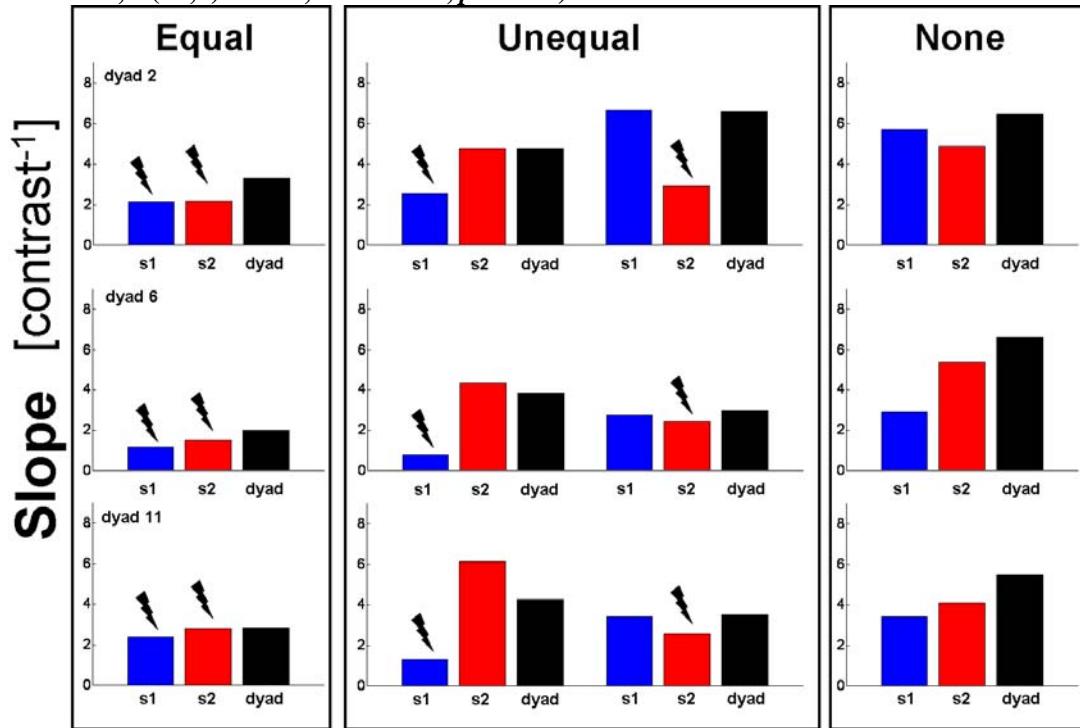
Figure S2. Example data from three dyads in Experiment 2. The slopes of the psychometric functions for each noise condition (indicated at the top) are plotted for participants (blue and red) and dyad (black). The black flash symbol indicates the participant receiving the noisy stimulus. In Equal and None conditions, black bars are generally larger than both blue and red indicating that collective benefit is achieved. Under unequal condition though, collective benefit is much less evident.

Reference List

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