

① 60 рублей $X = U_1 + U_2 + \dots + U_{60}$ ($U_i \sim U(0, 1)$)

1) Согласно ЦПТ: $S_n = X_1 + \dots + X_n \rightarrow N(n \cdot E(X_i); n \cdot \text{Var}(X_i))$

т.е. $nE(X) = \cancel{60} 60 E(U) = 60 \cdot \frac{1}{2} = \underline{30}$

$$E(U) = \frac{0+1}{2} = \frac{1}{2}$$

$$n\text{Var}(X) = 60 \text{Var}(U) = 60 \cdot \frac{1}{12} = 5$$

$$\text{Var}(U) = \frac{(1-0)^2}{12} = \frac{1}{12}$$

$$\Rightarrow X \sim N(30, 5)$$

$$2) Z = \frac{X - \mu}{\sigma} = \frac{X - 30}{\sqrt{5}} \Rightarrow P(X > 20) = P\left(\frac{X - 30}{\sqrt{5}} > \frac{20 - 30}{\sqrt{5}}\right)$$

т.е. $P(Z > -2.236) = 1$

$$② Z_n = \frac{X_1 + X_2 + \dots + X_n}{Y_1 + Y_2 + \dots + Y_n}; E(X_i) = 2; E(Y_i) = 4$$

По 3Б4: $\frac{1}{n} \sum_{i=1}^n X_i = E(X_i) = 2$ и $\frac{1}{n} \sum_{i=1}^n Y_i = E(Y_i) = 4$

т.е. $\lim_{n \rightarrow \infty} Z_n = \frac{2}{4} = \frac{1}{2}$

Ответ: 0,5

$$③ E(X_i) = 20; \text{Var}(X_i) = 1; E(Y_i) = 18; \text{Var}(Y_i) = 4; 408 \text{ руб}$$

1) Согласно ЦПТ: $S_n \rightarrow N(n \cdot E(X_i + Y_i); n \cdot \text{Var}(X_i + Y_i))$

$$E(X_i + Y_i) = E(X_i) + E(Y_i) = 20 + 18 = 38$$

$$E(S) = 40 \cdot 38 = 1520$$

$$\text{Var}(X_i + Y_i) = \text{Var}(X_i) + \text{Var}(Y_i) = 1 + 4 = 5$$

$$\text{Var}(S) = 40 \cdot 5 = 200 \Rightarrow \sigma = 10\sqrt{2}$$

т.е. $S_n \rightarrow N(1520; 200)$

$$P(S > 1550) = P\left(\frac{S - 1520}{10\sqrt{2}} > \frac{1550 - 1520}{10\sqrt{2}}\right) \Rightarrow$$

$$\Rightarrow P(Z > \frac{3}{\sqrt{2}}) \approx \underline{0,017}$$

$$④ 10000 \text{ г. } P(4900 \leq X \leq 5100)$$

Вероятность ответа равновероятна, т.е. $p = 0,5$

$$\Rightarrow E(X) = 10000 \cdot 0,5 = 5000, \text{ а } \text{Var}(X) = 10000 \cdot 0,5(1 - 0,5) =$$

$$= 2500, \text{ i.e. } \sigma = 50$$

$$Z = \frac{X - 5000}{50} \Rightarrow P(\frac{4800 - 5000}{50} \leq Z \leq \frac{5200 - 5000}{50})$$

$$\Rightarrow P(-2 \leq Z \leq 2) \approx 0.9544$$

~~Normal distribution~~