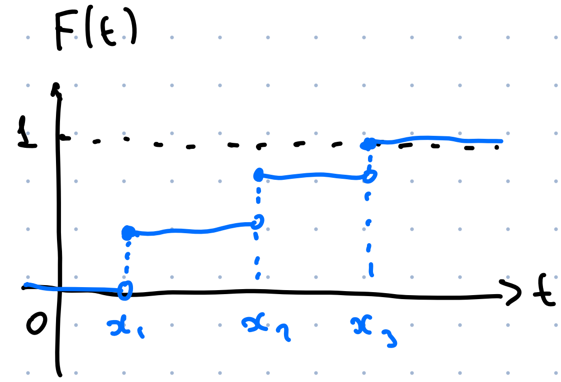
Непрерывные сл. вел.

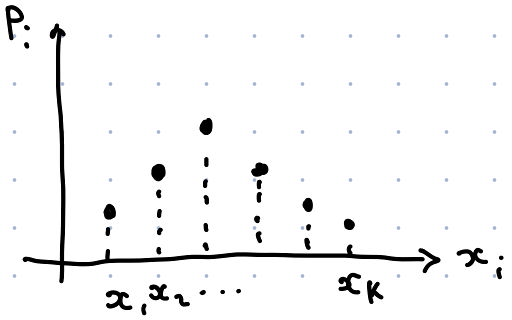
X	x_1	$\dots$	x_n
	p_1	$\dots$	p_n

$$\sum p_i = 1$$

$$F_X(t) = \mathbb{P}(X \le t) = \begin{cases} \vdots \\ \vdots \end{cases}$$

$$\mathbb{P}(X \le t) = \sum_{x_i \le t} \mathbb{P}(X = x_i)$$



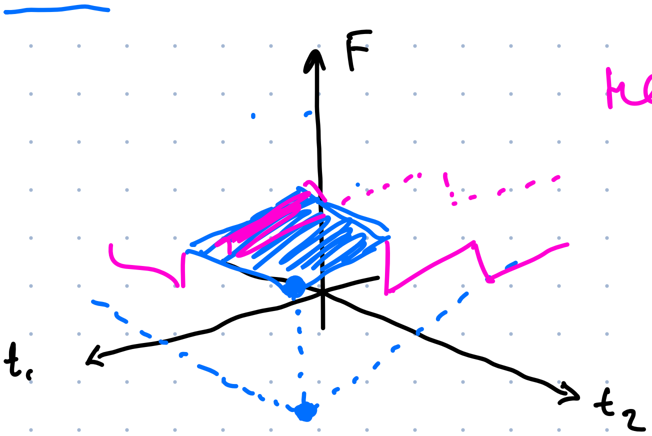


$Y \backslash X$	x_1	$\dots$	x_m
y_1		p_{ij}	
$\dots$			
y_k			

$$\sum_{ij} p_{ij} = 1$$

$$F(t_1, t_2) = \mathbb{P}(X \le t_1,\ Y \le t_2) = \begin{cases} \vdots \\ \vdots \end{cases}$$

не убывающ.



$$\mathbb{E}(X) = \sum x_i p_i$$

$$Var(X) = \mathbb{E}(X^2) - \mathbb{E}^2(X)$$

$$\sigma(X) = \sqrt{Var(X)}$$

$$Cov(X, Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X)\,\mathbb{E}(Y)$$

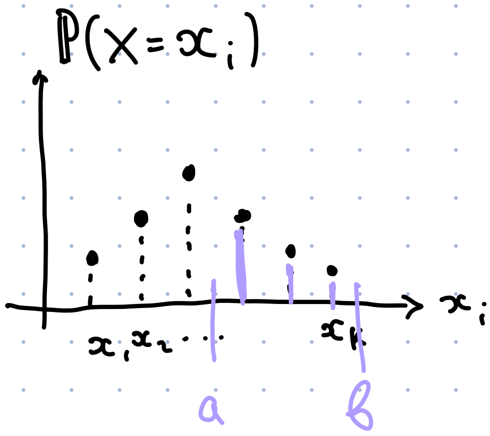
$$Corr(X, Y) = \frac{Cov(X, Y)}{\sigma(X) \cdot \sigma(Y)}$$

Опр. сл. вел. X наз. **непрерывной** если $\exists$ неотриц. ф. $f_X(t)$ такая что $\forall\ [a; b]$ выполняется

$$\mathbb{P}(X \in [a; b]) = \int_a^b f_X(t)\,dt$$

$$a \le X \le b$$

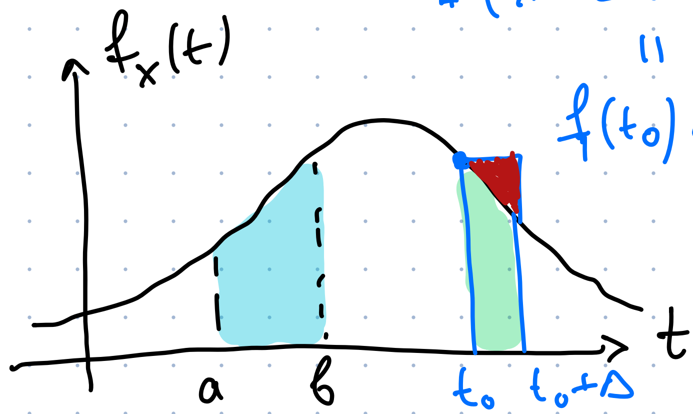
$f_X(t)$ – **плотность р.-я**



$$\mathbb{P}(a \le X \le b) = \sum_{a \le x_i \le b} \mathbb{P}(X = x_i)$$

$\mathbb{P}(X \in [t_0; t_0+\Delta])$
$\shortparallel$
$f(t_0)\cdot\Delta + o(\Delta)$

$\Delta \to 0$



$$\mathbb{P}(a \le X \le b) = \int_a^b f_X(t)$$

$f_X(t)$ - вероятностная масса

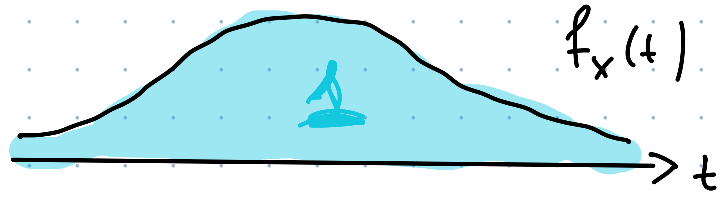
пётотина вероятности

Св-ва плотности:

1) $\mathbb{P}(X = a) = \int_a^a f_X(t)\,dt = 0$

2) $f_X(t) \ge 0$

3) $\int\limits_{-\infty}^{+\infty} f_x(t)\,dt = 1$



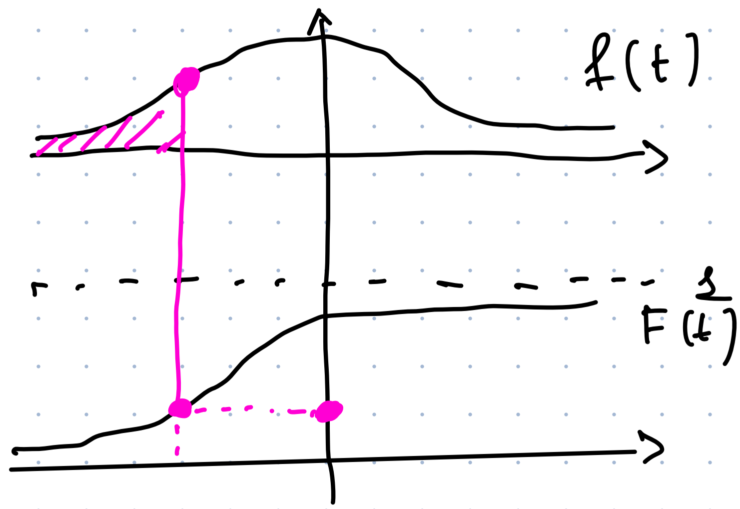
4) $\mathbb{P}(a \le X \le b) = \int\limits_a^b f_x(t)\,dt$

$\quad\quad \parallel$

$\mathbb{P}(X \le b) - \mathbb{P}(X \le a) = F_X(b) - F_X(a)$

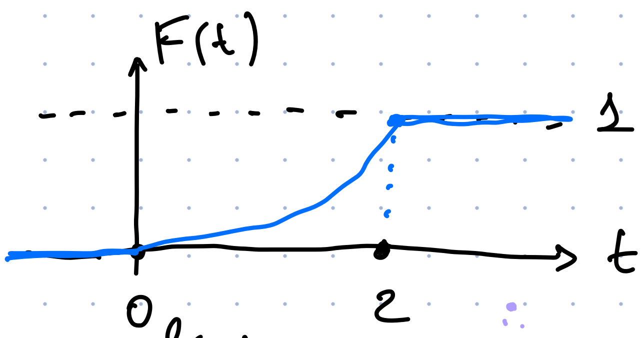
5) $F_X'(t) = f_X(t)$

$F_X(t) = \int\limits_{-\infty}^{t} f_X(t)\,dt$

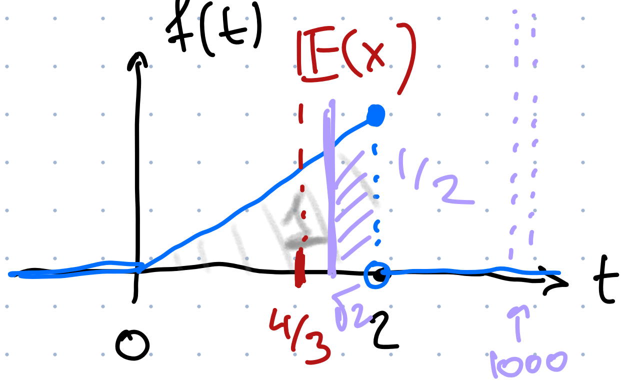


Упражнение

$X \quad [0;2] \qquad F_X(t) = c\cdot t^2 = \begin{cases} 1, & t > 2 \\ \overset{1/4}{c}\cdot t^2, & t \in (0;2] \\ 0, & t \le 0 \end{cases}$



$f_X(t) = F_X'(t) = \begin{cases} 2\overset{1/4}{\overset{\parallel}{c}}t, & t \in [0;2] \\ 0, & \text{иначе} \end{cases}$



а) c – ?

б) $f_X(t)$ + графики

в) $\mathbb{P}(X < 1)$

2) $\mathbb{P}(X \in [1; 1.5])$

g) $F_X(-5)$ $F_X(10)$ $F_X(1)$

e) $\mathbb{E}(X)$ $Var(X)$ $\sigma(X)$

ë) $Med(X)$ $Mod(X)$

$$\int_0^2 2 \cdot c \cdot t \, dt = 1 \qquad \frac{2ct^2}{2}\Big|_0^2 = c2^2 - 0 = 1$$

$$4c = 1$$

$$c = 1/4$$

$$\mathbb{P}(X < 1) = \mathbb{P}(X \leq 1) = F_X(1) = \frac{1}{4}$$

$$\mathbb{P}(X = 1) = 0$$

$$\mathbb{P}(X < 1) = \int_{-\infty}^{1} f_X(t)\, dt = \int_0^1 \frac{1}{2} t \, dt = \frac{t^2}{4}\Big|_0^1 = \frac{1}{4}$$

$$\mathbb{P}(1 \leq X \leq 1.5) = \int_1^{1.5} \frac{1}{2} t \, dt = F_X(1.5) - F_X(1) = \frac{1.5^2}{4} - \frac{1^2}{4}$$

$$F_X(1) = \frac{1^2}{4}$$

$$F_X(-5) = 0$$

$$F_X(10) = \mathbb{P}(X \leq 10) = 1$$

$$\mathbb{E}(X) = \int_{-\infty}^{+\infty} t \cdot f_X(t)\, dt = \int_0^2 t \cdot \frac{t}{2}\, dt = \frac{t^3}{3 \cdot 2}\Big|_0^2 =$$

$$\sum_{k \in \mathbb{X}} k \cdot \mathbb{P}(X=k)$$

$$= \frac{8}{6} - 0 = 4/3$$

$$Med(X): \ \mathbb{P}(X \leq Med) = \frac{1}{2}$$

$$F_X(Med) = \frac{1}{2}$$

$$\frac{1}{4} med^2 = \frac{1}{2} \Rightarrow med^2 = 2 \qquad med = \sqrt{2}$$

$$\sqrt{2} \ \overset{>}{vs} \ 4/3$$

$$2 \ \overset{>}{vs} \ \frac{16}{9}$$

$$2 > 1.65\ldots$$

$$Mod(X) = \arg\max_t f_X(t) \qquad f_X(t) \to \max_t$$

надо искать экстремум

$$Mod(X) = 2$$

$$\mathbb{E}(X^2) = \int_0^2 t^2 \cdot \frac{t}{2}\, dt = \frac{t^4}{4 \cdot 2}\Big|_0^2 = 2$$

$$\mathbb{E}(\varphi(X)) = \int_{-\infty}^{+\infty} \varphi(t) \cdot f_X(t)\, dt$$

$$Var(X) = \mathbb{E}(X^2) - \mathbb{E}^2(X) = 2 - (4/3)^2 = 2/9$$

$\sigma(x) = \sqrt{2/9} = \sqrt{2}/3$

Опр. (X, Y) – многомерная сл. вел.

$$F(t_1, t_2) = \mathbb{P}(X \le t_1 \cap Y \le t_2)$$

$$f(t_1, t_2) = (F'_{t_1})'_{t_2} = (F_{t_2})'_{t_1}$$

$$F(t_1, t_2) = \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} f(t_1, t_2)\, dt_1\, dt_2$$

$$\mathbb{P}((X, Y) \in \mathcal{D}) = \iint_{\mathcal{D}} f(t_1, t_2)\, dt_1\, dt_2$$

Св-ва:

1) $F(+\infty, +\infty) = \lim_{\substack{t_1 \to +\infty \\ t_2 \to +\infty}} F(t_1, t_2) =$

$= \mathbb{P}(X \le +\infty \cap Y \le +\infty) = 1$

$$F(t_1, -\infty) = \mathbb{P}(X \le t_1 \cap Y \le -\infty) = 0$$

$$F(-\infty, t_2) = 0$$

$$F(-\infty, -\infty) = 0$$

$$F(t_1, +\infty) = \mathbb{P}(X \le t_1 \cap Y \le +\infty) = \mathbb{P}(X \le t_1) = F_X(t_1)$$

$$0 \le F(t_1, t_2) \le 1$$

$F(t_1, t_2)$ не убывает по t_1 и t_2

$$\lim_{y \to +\infty} F(x, y) = F_x(x)$$

2) $f(x, y) \sim f(t_1, t_2)$

$$f(x, y) \geq 0$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y)\, dx\, dy = 1$$

$$\mathbb{P}((X, Y) \in D) = \iint_D f(x, y)\, dx\, dy$$

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$$

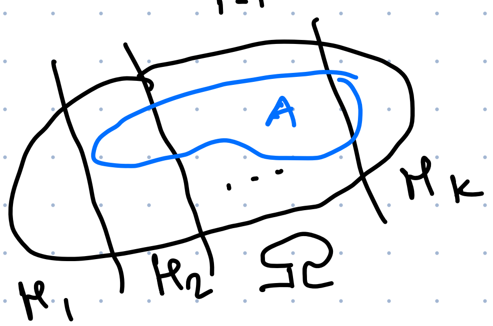
3) Тут работают все те же самые формулы, что и для вероятностей

$$\mathbb{P}(A | B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \qquad f(x | y) = \frac{f(x, y)}{f(y)}$$

$$\mathbb{P}(A | B) = \frac{\mathbb{P}(B|A) \cdot \mathbb{P}(A)}{\mathbb{P}(B)} \qquad f(x | y) = \frac{f(y | x) \cdot f(x)}{f(y)}$$

$$\mathbb{P}(A) = \sum_{i=1}^{k} \mathbb{P}(A | H_i) \cdot \mathbb{P}(H_i) \qquad f(y) = \int_{-\infty}^{+\infty} f(y | x) \cdot f(x)\, dx$$

$$\int_{-\infty}^{+\infty} f(y | x) \cdot f(x)\, dx = \int_{-\infty}^{+\infty} f(y, x)\, dx = \mathbb{E}_X[f(y | x)]$$



Маргинальные р.-я:

$$f_X(x) = \int_{-\infty}^{+\infty} f(x, y)\, dy$$

4) A и B нез. если $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$

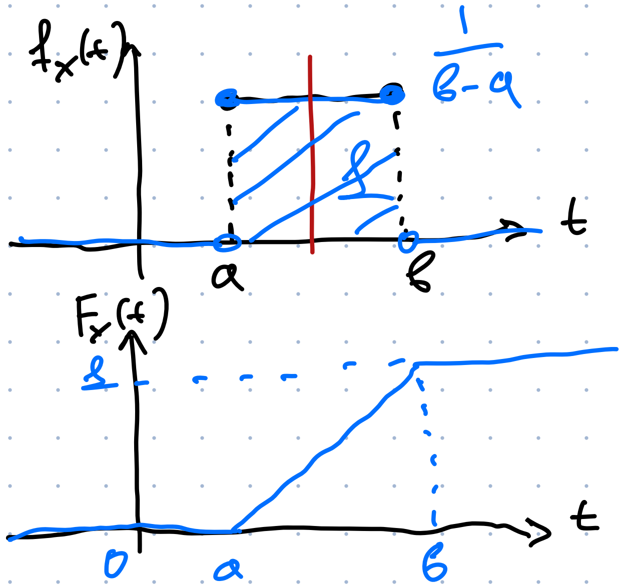
сл. вел. X и Y нез. $f(y, x) = f_X(x) \cdot f_Y(y)$

$F(y, x) = F_X(x) \cdot F_Y(y)$

$\forall t_1, t_2 \quad \mathbb{P}(X \leq t_1 \cap Y \leq t_2) = \mathbb{P}(X \leq t_1) \cdot \mathbb{P}(Y \leq t_2)$

Опр. $X \sim U[a; b]$

$$f_X(t) = \begin{cases} \frac{1}{b-a}, & t \in [a; b] \\ 0, & \text{иначе} \end{cases}$$



$$\mathbb{E}(X) = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

Упражнение

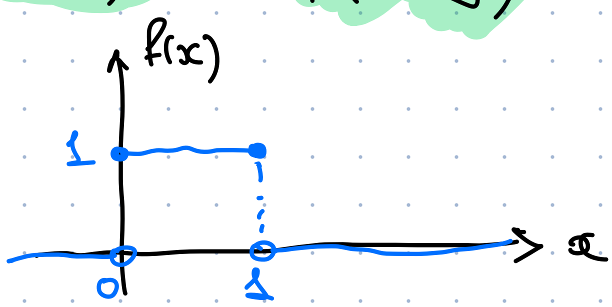
$X \sim U[0; 1]$

$Y \sim U[0; X]$

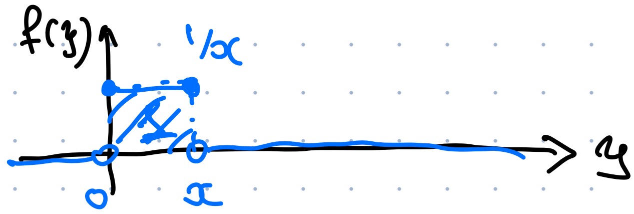
$f(y|x)$ $f(x, y)$ $f(y)$

$f(x)$ $f(x|y)$

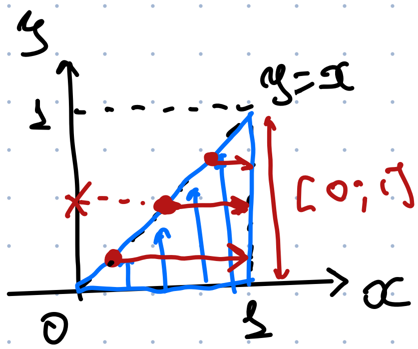
$$f(x) = \begin{cases} 1, & x \in [0; 1] \\ 0, & x \notin [0; 1] \end{cases}$$



$$f(y|x) = \begin{cases} \frac{1}{x}, & y \in [0;x] \\ 0, & y \notin [0;x] \end{cases}$$



$$f(x,y) = f(y|x) \cdot f(x) = \begin{cases} \frac{1}{x}, & x \in [0;1],\ y \in [0,x] \\ 0, & \text{иначе} \end{cases}$$



$$f(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \int_{y}^{1} \frac{1}{x}\,dx = \ln x \Big|_{y}^{1} = -\ln y$$

$$f(y) = \begin{cases} -\ln y, & y \in [0;1] \\ 0, & \text{иначе} \end{cases}$$

$$f(x|y) = \frac{f(x,y)}{f(y)} = \begin{cases} \frac{1}{x} \cdot \frac{1}{-\ln y} & x \in [y;1] \\ 0 & \text{иначе} \end{cases}$$