Mathematical Induction

The process to establish the validity of an ordinary result involving natural numbers is the principle of mathematical induction.

Working Rule

Let n_0 be a fixed integer. Suppose P (n) is a statement involving the natural number n and we wish to prove that P (n) is true for all n $\geq n_0$.

1. Basic of Induction: P (n_0) is true i.e. P (n) is true for n = n_0.

2. Induction Step: Assume that the P (k) is true for n = k.
Then P (K+1) must also be true.
Then P (n) is true for all n $\geq n_0$.

Example 1:

Prove the follo2wing by Mathematical Induction:

$$1 + 3 + 5 + ... + 2n - 1 = n^2.$$

Solution: let us assume that.

$$P\ (n) = 1 + 3 + 5 + ... + 2n - 1 = n^2.$$

For n = 1, $P\ (1) = 1 = 1^2 = 1$

It is true for n = 1................ (i)

Induction Step: For n = r,

$$P\ (r) = 1 + 3 + 5 + ... +2r-1 = r^2 \text{ is true} .... \text{(ii)}$$

Adding 2r + 1 in both sides

$$P\ (r + 1) = 1 + 3 + 5 + ... +2r-1 + 2r +1$$

$$= r^2 + (2r + 1) = r^2 + 2r +1 = (r+1)^2 .... \text{(iii)}$$

As P(r) is true. Hence P (r+1) is also true.

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From (i), (ii) and (iii) we conclude that.
        1 + 3 + 5 +..... + 2n - 1 =n2 is true for n = 1, 2, 3, 4, 5 ....Hence Proved.
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Example 2:

$1^2 + 2^2 + 3^2 +.......+ n^2 =$

Solution: For n = 1,

P (1) = 1^2 = = 1

It is true for n = 1.

Induction Step: For n = r,.................. (i)

P (r) = $1^2 + 2^2 + 3^2 +........ + r^2 =$ is true........... (ii)

Adding $(r+1)^2$ on both sides, we get

P (r+1) = $1^2 + 2^2 + 3^2 +.......+ r^2+ (r+1)^2 = + (r+1)^2$

As P (r) is true, hence P (r+1) is true.

From (i), (ii) and (iii) we conclude that

$1^2 + 2^2 + 3^2 +......+ n^2=$ is true for n = 1, 2, 3, 4, 5 ..... Hence Proved.

Example3: Show that for any integer n

$11^{n+2} + 12^{2n+1}$ is divisible by 133.

Solution:

Let P (n) = $11^{n+2}+12^{2n+1}$

For n = 1,

P (1) = 11^3+12^3=3059=133 x 23

So, 133 divide P (1).................. (i)

Induction Step: For n = r,

P (r) = $11^{r+2}+12^{2r+1}$=133 x s............ (ii)

Now, for n = r + 1,

P (r+1) = $11^{r+2+1}+12^{2(r)+3}$=11[133s-12^{2r+1}] + 144. 12^{2r+1}

= 11 x 133s + 12^{2r+1}.133=133[11s+12^{2r+1}]=133 x t........... (iii)

As (i), (ii), and (iii) all are true, hence P (n) is divisible by 133.

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