Rules of Inference

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Argument: Premises | Hypothesis, Conclusion

- Consider a set of proposition p1, p2, p3, … pn and q.

$(p1 \wedge p2 \wedge p3 \wedge \ldots \wedge pn) \rightarrow q$ is called an argument

p1, p2, p3, … pn – Premises or hypothesis of the argument

q - conclusion

Valid Argument:

Representation:

$$\begin{array}{c} p1 \\ p2 \\ p3 \\ \vdots \\ pn \\ \hline \therefore\ q \end{array}$$

If p1, p2, p3, … pn are all true, then the conclusion is true.

Rules of Inference

Rule of Inference	Logical Implication	Name of the rule
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Rule of Conjunction
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Rule of Conjunctive Simplification
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Rule of Disjunctive Amplification
p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus Ponens or Rule of Detachment

Rules of Inference

$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus Tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Law of syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Rule of Disjunctive syllogism
$\neg p \rightarrow F$ $\therefore p$	$(\neg p \rightarrow F) \rightarrow p$	Rule of Contradiction

Rules of Inference

1. Test whether the following argument is valid

If Sachin hits a century, then he gets a free car.

Sachin hits a century

∴ Sachin gets a free car.

Solution:

Let p: Sachin hits a century

q: Sachin gets a free car

The given argument is symbolically represented as

$p \rightarrow q$

p

$\therefore q$

In the view of Modus Ponens rule, this is a valid argument

2. Test whether the following argument is valid

If Sachin hits a century, then he gets a free car.

Sachin gets a free car

∴ Sachin has hit a century

Solution:

Let p: Sachin hits a century

q: Sachin gets a free car

The given argument is symbolically represented as

$p \rightarrow q$

q

$\therefore p$

There is no rule of inference that asserts p is true.

∴ It is not a valid argument.

3. Test whether the following argument is valid

I will study calculus or I will not become a software engineer

I will become a software engineer

∴ I will study calculus

Solution:

Let p: I will study calculus

q: I will become a software engineer

The given argument can be symbolically represented as

$$\begin{array}{l} p \vee \neg q \\ \underline{q} \\ \therefore p \end{array} \quad \Leftrightarrow \quad \begin{array}{l} \neg q \vee p \\ \underline{q} \\ \therefore p \end{array} \quad \Leftrightarrow \quad \begin{array}{l} q \to p \\ \underline{q} \\ \therefore p \end{array}$$

In the view of Modus Ponens rule, this is a valid argument

4. Test the validity of the following argument

Rita is driving.

If Rita is driving, then she is not reading.

If Rita is not reading, then she will not top the class.

Therefore, Rita will not top the class.

Solution:

Let p: Rita is driving

q: Rita is reading

r: Rita will top the class

The argument can be symbolically represented as:

$$\begin{array}{l} p \\ p \to \neg q \\ \neg q \to \neg r \\ \hline \therefore \neg r \end{array}$$

p
$p \to \neg q$
$\neg q \to \neg r$

$\therefore \neg r$

Steps	*Reasons*
1. $p \to \neg q$	Premise
2. $\neg q \to \neg r$	Premise
3. $p \to \neg r$	Step (1) and (2), law of syllogism
4. p	Premise
5. $\therefore \neg r$	Step (3) and (4), Modus Ponens

Therefore, the given argument is valid

5. Test the validity of the following argument

$\neg p \leftrightarrow q$

$q \rightarrow r$

$\neg r$

$\therefore p$

Solution:

Steps	Reasons
1. $\neg p \leftrightarrow q$	*Premise*
2. $(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$	*Step (1) , $\neg p \leftrightarrow q \Leftrightarrow ((\neg p \rightarrow q) \wedge (q \rightarrow \neg p))$*
3. $\neg p \rightarrow q$	*Step (2), rule of conjunctive simplification*
4. $q \rightarrow r$	*Premise*
5. $\neg p \rightarrow r$	*Steps (3) and (4), law of syllogism*
6. $\neg r$	*Premise*
7. $\neg(\neg p)$	*Steps (5) and (6), Modus Tollens*
8. $\therefore p$	*Step (7), Law of double negation*

Therefore, the given argument is valid

6. Test the validity of the following argument

$p \rightarrow r$

$r \rightarrow s$

$t \vee \neg s$

$\neg t \vee u$

$\neg u$

$\therefore \neg p$

Steps	Reasons
1. $p \rightarrow r$	Premise
2. $r \rightarrow s$	Premise
3. $p \rightarrow s$	Steps (1) and (2), law of syllogism
4. $t \vee \neg s$	Premise
5. $\neg s \vee t$	Step (4), commutative law
6. $s \rightarrow t$	Step (5) and the fact $p \rightarrow q \Leftrightarrow \neg p \vee q$
7. $p \rightarrow t$	Step (3) and (6), law of syllogism
8. $\neg t \vee u$	Premise
9. $t \rightarrow u$	Step (8) and the fact $p \rightarrow q \Leftrightarrow \neg p \vee q$
10. $p \rightarrow u$	Steps (7) and (9), law of syllogism
11. $\neg u$	Premise
12. $\therefore \neg p$	Steps (10) and (11), Modus Tollens

$$\begin{array}{l} p \rightarrow r \\ r \rightarrow s \\ t \vee \neg s \\ \neg t \vee u \\ \neg u \\ \hline \therefore \neg p \end{array}$$

Therefore, the given argument is valid

7. Test the validity of the following argument

$(\neg p \vee \neg q) \rightarrow (r \wedge s)$
$r \rightarrow t$
$\neg t$
$\therefore p$

	Steps	Reasons
1.	$r \rightarrow t$	*Premise*
2.	$\neg t$	*Premise*
3.	$\neg r$	*Steps(1) and (2), Modus Tollens*
4.	$\neg r \vee \neg s$	*Step(3), rule of disjunctive amplification*
5.	$\neg(r \wedge s)$	*Step(4), DeMorgan's Law*
6.	$(\neg p \vee \neg q) \rightarrow (r \wedge s)$	*Premise*
7.	$\neg(\neg p \vee \neg q)$	*Step(6) and (5), Modus Tollens*
8.	$(p \wedge q)$	*Step(7), Demorgan's Law*
9.	$\therefore p$	*Step(8), rule of conjunctive simplification*

8. Test the validity of the following argument

$a \rightarrow b$

$a \vee (c \wedge d)$

$\neg b \wedge \neg e$

$\therefore c$

Steps	Reasons
1. $\neg b \wedge \neg e$	Premise
2. $\neg b$	Step (1), law of conjunctive simplification
3. $a \rightarrow b$	Premise
4. $\neg a$	Step(2), (3), Modus Tollens
5. $a \vee (c \wedge d)$	Premise
6. $(c \wedge d)$	Step (4), (5), disjunctive syllogism
7. $\therefore c$	Step(6), conjunctive simplification

8. Test the validity of the following argument

$(t \to e) \wedge (a \to l)$

$\therefore (t \wedge a) \to (e \wedge l)$

Steps	Reasons
1) $(t \to e) \wedge (a \to l)$	Premise
2) $t \to e$	Step (1) and conjunctive simplification
3) $\neg t \vee e$	Step (2) and the fact that $p \to q \Leftrightarrow \neg p \vee q$
4) $(\neg t \vee e) \vee \neg a$	Step (3) and disjunctive amplification
5) $(\neg a \vee \neg t) \vee e$	Step (4), commutative & associative laws
6) $\neg(t \wedge a) \vee e$	Step (5), DeMorgan's & commutative laws.
7) $a \to l$	Step (1), rule of conjunctive simplification
8) $\neg a \vee l$	Step (7), & the fact that $p \to q \Leftrightarrow \neg p \vee q$
9) $(\neg a \vee l) \vee \neg t$	Step (8), & rule of disjunctive amplification
10) $(\neg a \vee \neg t) \vee l$	Step (9) & associative laws.
11) $\neg(t \wedge a) \vee l$	Step (10) & DeMorgan's and commutative laws.
12) $[\neg(t \wedge a) \vee e] \wedge [\neg(t \wedge a) \vee l]$	Steps (6) & (11) and conjunction rule
13) $\neg(t \wedge a) \vee (e \wedge l)$	Step (12) & distributive law
14) $\therefore (t \wedge a) \to (e \wedge l)$	Step (13) & the fact that $p \to q \Leftrightarrow \neg p \vee q$