Permutations and Combinations

What You'll Learn

- Determine probabilities using permutations.
- Determine probabilities using combinations.

Vocabulary

1) Permutation
2) Combination

Permutations and Combinations

An arrangement or listing in which **order or placement is important** is called a **permutation**.

Simple example: "combination lock"

31 – 5 – 17 is **NOT** the same as 17 – 31 – 5

Permutations and Combinations

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Simple example: "combination lock"

31 – 5 – 17 is **NOT** the same as 17 – 31 – 5

Though the same numbers are used, the **order** in which they are turned to, would mean the difference in the lock opening or not.

Thus, the order is very important.

Permutations and Combinations

The manager of a coffee shop needs to hire two employees, one to work at the counter and one to work at the drive-through window. Sara, Megen, Tricia and Jeff all applied for a job. How many possible ways are there for the manager to place the applicants?

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Counter	Drive-Through	Outcomes
Sara		
Megen		
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Sara	Megen	
	Tricia	
	Jeff	
Megen	Tricia	
	Jeff	
	Sara	
Tricia	Sara	
	Megen	
	Jeff	
Jeff	Sara	
	Megen	
	Tricia	

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Counter	Drive-Through	Outcomes
Sara	Megen	SM
	Tricia	ST
	Jeff	SJ
Megen	Tricia	MT
	Jeff	MJ
	Sara	MS
Tricia	Sara	TS
	Megen	TM
	Jeff	TJ
Jeff	Sara	JS
	Megen	JM
	Tricia	JT

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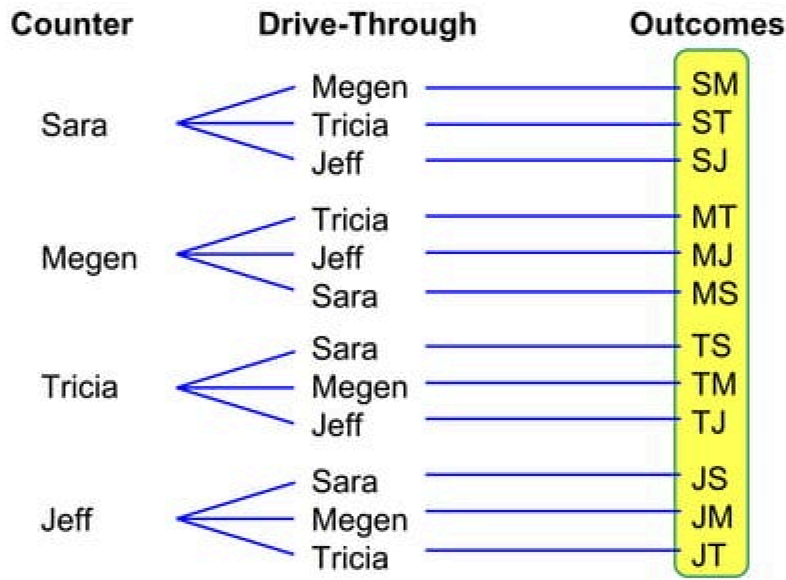

There are 12 different ways for the 4 applicants to hold the 2 positions.

Permutations and Combinations

In the previous example, the positions are in specific order, so each arrangement is unique.

The symbol $_4P_2$ denotes the number of permutations when arranging 4 applicants in two positions.

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The symbol $_4P_2$ denotes the number of permutations when arranging 4 applicants in two positions.

You can also use the Fundamental Counting Principle to determine the number of permutations.

$$_4P_2 = \underbrace{4}_{\text{ways to choose first employee}} \times \underbrace{3}_{\text{ways to choose second employee}}$$

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Permutations and Combinations

$$\underbrace{\text{ways to choose first employee}} \times \underbrace{\text{ways to choose second employee}}$$

$${}_4P_2 = 4 \times 3$$

$${}_4P_2 = \frac{4*3}{1}\left(\frac{2*1}{2*1}\right)$$

Note: $\boxed{\frac{2*1}{2*1}=1}$

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$${}_4P_2 = \frac{4!}{2!}$$

In general, ${}_nP_r$ is used to denote the number of permutations of n objects taken r at a time.

Permutations and Combinations

Permutation

The number of permutations of n objects taken r at a time is the **quotient** of $n!$ and $(n-r)!$

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Permutations and Combinations

Permutation: (Order is important!)

Find ${}_{10}P_6$

Permutations and Combinations

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Permutations and Combinations

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$${}_{10}P_6 = \frac{10!}{(10-6)!}$$

$${}_{10}P_6 = \frac{10!}{4!} = \frac{10*9*8*7*6*5*4*3*2*1}{4*3*2*1}$$

Permutations and Combinations

Permutation: (Order is important!)

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$${}_{10}P_6 = \frac{10!}{4!} = \frac{10*9*8*7*6*5*4*3*2*1}{4*3*2*1}$$

$${}_{10}P_6 = 10*9*8*7*6*5 \quad \text{or} \quad 151{,}200$$

There are 151,200 permutations of 10 objects taken 6 at a time.

Permutations and Combinations

Permutation and Probability:

A computer program requires the user to enter a **7-digit** registration code made up of the digits 1, 2, 4, 5, 6, 7, and 9.
Each number has to be used, and no number can be used more than once.

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$${}_nP_r = {}_7P_7$$

n = 7, r = 7; recall that 0! = 1.

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$${}_7P_7 = \frac{7*6*5*4*3*2*1}{1}$$

or 5040

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There are 5040 possible codes with the digits 1, 2, 4, 5, 6, 7, and 9.

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Q2) What is the **probability** that the first three digits of the code are even numbers?

$$\text{Probability} = \frac{\#\text{ of favorable outcomes}}{\#\text{ of total outcomes}}$$

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Use the **Fundamental Counting Principle** to determine the number of ways for the first three digits to be even.

__ __ __ __ __ __ __

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<u>3</u> ___ ___ ___ ___ ___ ___

There are three even numbers to choose from.
So, there are three ways that the first digit could be even.

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3 __ 2 __ __ __ __ __ __

Now there are only two even numbers to choose from.
So, there are two ways that the second digit could be even.

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3 2 1 4 ___ ___ ___

Now we come to the fourth digit, and there are four odd numbers to choose from.
So, there are four ways that the fourth digit could be odd.

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3 __ **2** __ **1** __ **4** __ **3** __ **2** __ **1** __

Using this same logic, we can determine the different possibilities for the remaining digits.

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$\underline{3}$ $\underline{2}$ $\underline{1}$ $\underline{4}$ $\underline{3}$ $\underline{2}$ $\underline{1}$

So, the number of favorable outcomes is **3 * 2 * 1 * 4 * 3 * 2 * 1** or **144**.

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There are **144** ways for this **event** to occur out of the **5040** possible **permutations**.

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$$P(\text{first 3 digits even}) = \frac{144}{5040}$$

144 ← favorable outcomes

5040 ← possible outcomes

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The probability that the first three digits of the code are even is $\frac{1}{35}$ or about **3%**.

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Combination

The number of combinations of n objects taken r at a time is the **quotient** of $n!$ and $(n-r)! * r!$

$$_nC_r = \frac{n!}{(n-r)!r!}$$

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The students of Mr. Fant's Seminar class had to choose 4 out of the 7 people who were nominated to serve on the Student Council.
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$${}_nC_r = {}_7C_4$$

$${}_7C_4 = \frac{7!}{(7-4)!4!} = \frac{7*6*5*\cancel{4*3*2*1}}{3*2*1*\cancel{4*3*2*1}}$$

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$$= \quad \frac{7*6*5}{3*2*1} \quad \text{or} \quad 35$$

There are 35 different groups of students that could be selected.

Permutations and Combinations

Tip!

When working with **permutations** and **combinations**, it is vital that you are able to distinguish when the counting **order** is important, or not.

This is only recognizable after a considerable amount of practice.

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However, if Mr. Fant's class was choosing 4 out of 7 students to be president, vice-president, secretary, and treasurer of the student council, then the order in which they are chosen would matter. **(Permutation)**

End of Lesson!