Mathematical Proofs

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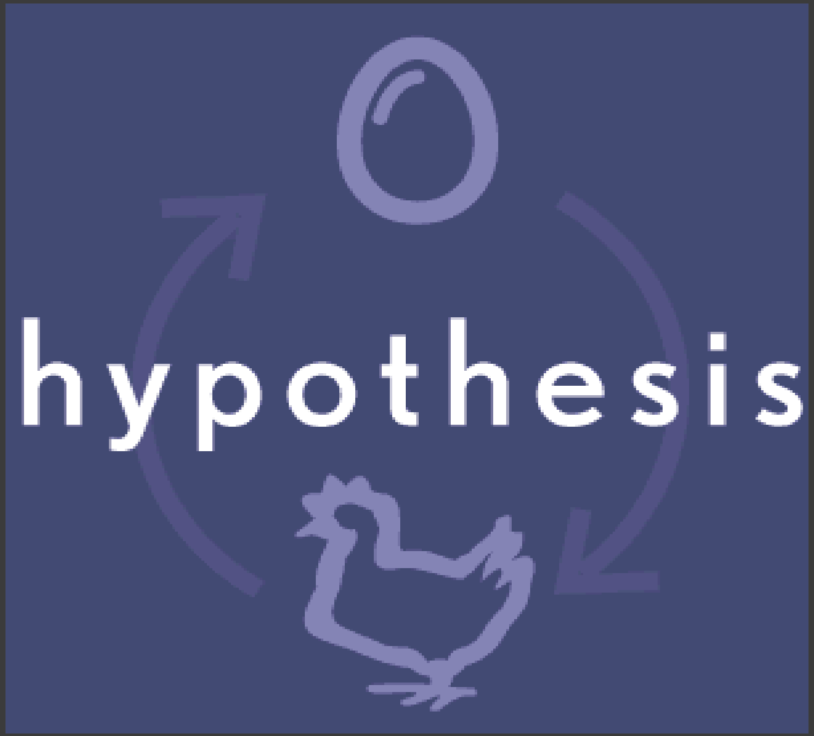
- Conditional Statements
- Propositional Equivalence
- Inference

“If you do your homework, you will not be punished.”



Conditional Statement

Another Name: Implication

A conditional statement has two parts

- **Hypothesis** and **Conclusion**.

"you do your homework" is the hypothesis and

"you will not be punished" is the conclusion.

From Implication we know…

An implication $A \rightarrow B$ is False if A is true and B is false

If Hypothesis is true and Conclusion is false, it is false

Conditional Statement

Inverse – An inverse of the conditional statement is the negation of both the hypothesis and the conclusion.

If the statement is “If p, then q”, the inverse will be “If not p, then not q”.

The inverse of “If you do your homework, you will not be punished” is…

“If you do not do your homework, you will be punished.”

Conditional Statement

Converse – The converse of the conditional statement is computed by interchanging the hypothesis and the conclusion.

The converse of "If you do your homework, you will not be punished" is…..

"If you will not be punished, you do not do your homework”

Conditional Statement

Contra-positive – The contra-positive of the conditional is computed by interchanging the hypothesis and the conclusion of the inverse statement.

If the statement is “If p, then q”, the contrapositive will be “If not *q*, then not *p*”.

The Contra-positive of "If you do your homework, you will not be punished” is…

" If you do your homework, you will not be punished”

Propositional Equivalences

Two statements X and Y are logically equivalent if −

The truth tables of each statement have the same truth values.

The bi-conditional statement $X \Leftrightarrow Y$ is a tautology.

How to construct Truth Table?

- Write the truth table for P : $r \vee (p \wedge q)$

Step 1: First, list all possible “T/F” outcomes for the input triple: p, q, r . To do this, list twice all possible “T/F” outcomes for the pair p, q,

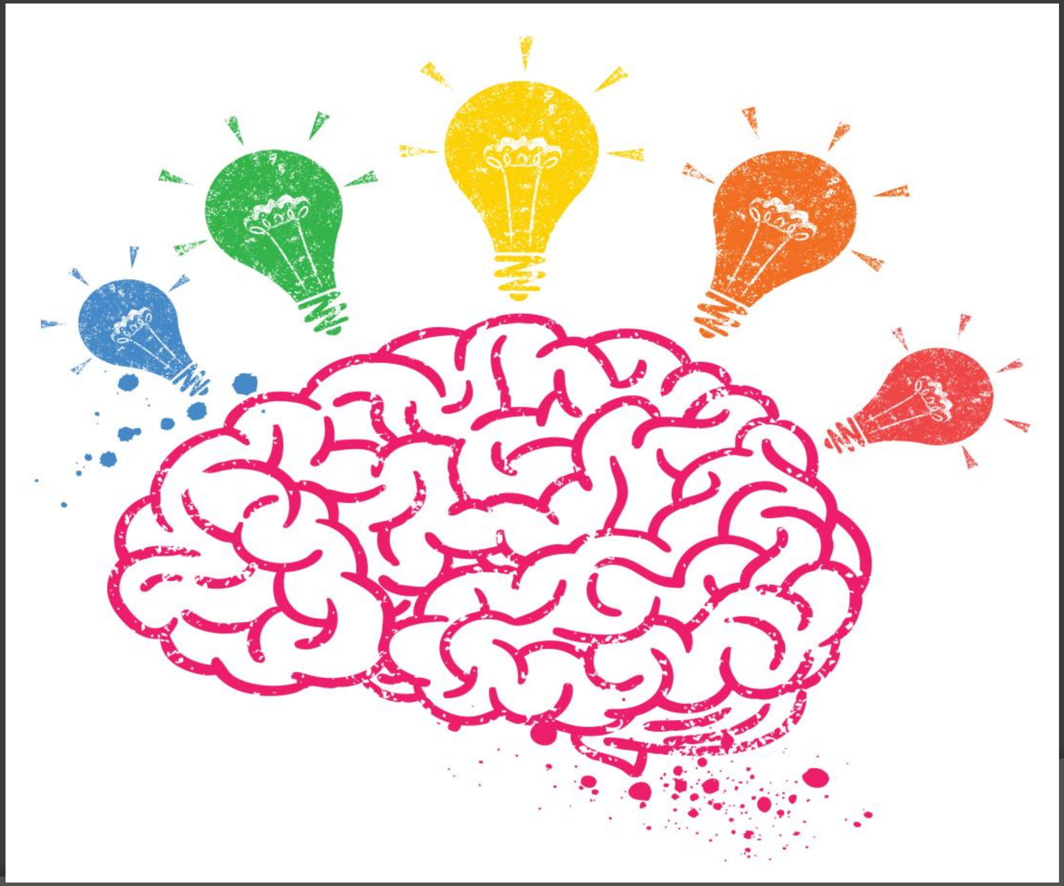
p	q	r	$p \wedge q$	$r \vee (p \wedge q)$
T	T			
T	F			
F	T			
F	F			
T	T			
T	F			
F	T			
F	F			

Then, fill in the “r” column: first with four “T ”then with four “F”:

p	q	r	$p \wedge q$	$r \vee (p \wedge q)$
T	T	T		
T	F	T		
F	T	T		
F	F	T		
T	T	F		
T	F	F		
F	T	F		
F	F	F		

Thus, the truth table of a compound statement using three input statements contains eight output statements

- How is 8 related to 2 (i.e. T and F) and 3 (i.e. p, q and r)?



Answer: $8 = 2^3$.....

Step 2: Next, fill in the “p ^ q” input column using the truth table for ^ (ignore the r column!)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$\Rightarrow$

p	q	r	$p \wedge q$	$r \vee (p \wedge q)$
T	T	T	T	
T	F	T	F	
F	T	T	F	
F	F	T	F	
T	T	F	T	
T	F	F	F	
F	T	F	F	
F	F	F	F	

Step 3: At last, fill in the output statement “r v (p ^ q)” using the input column for r together with the input column for s : p ^ q and the truth table for v (ignore the p and q input columns!)

r	s	$r \vee s$
T	T	T
T	F	T
F	T	T
F	F	F

$\Rightarrow$

p	q	r	$p \wedge q$	$r \vee (p \wedge q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T
T	T	F	T	T
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

Propositional Equivalences

Testing by 1st method (Matching truth table)

A	B	A ∨ B	¬ (A ∨ B)	¬A	¬B	[(¬A) ∧ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	False	True	False
False	True	True	False	True	False	False
False	False	False	True	True	True	True

Here, we can see the truth values of ¬ (A ∨ B) and [(¬A) ∧ (¬B)] are same, hence the statements are equivalent.

Propositional Equivalences

Testing by 2nd method (Bi-conditionality)

A	B	¬ (A ∨ B)	[(¬A) ∧ (¬B)]	[¬ (A ∨ B)] ⇔ [(¬A) ∧ (¬B)]
True	True	False	False	True
True	False	False	False	True
False	True	False	False	True
False	False	True	True	True

As [¬ (A ∨ B)] ⇔ [(¬A) ∧ (¬B)] is a tautology, the statements are equivalent.


Thank you!