LOGIC AND PROPOSITIONAL LOGIC

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Logic

Propositional Logic

Connectives

Tautology

Contradiction

Contingency

LOGIC

1. Free of emotion
2. Tool that develops reasonable conclusions based on a given set of data

2+1=3 (YES)

Man is mortal. President is a man.
So, President is mortal.

"Man is Mortal" TRUE

"$15 + 2 = 3 - 2$" FALSE

"Perhaps I am wrong"

Propositional logic

Propositional Logic is concerned with statements to which the truth values, “true” and “false”, can be assigned.

Propositional logic

A is equal to 2

Not a Proposition.

It is because unless we give a specific value of A, we cannot say whether the statement is true or false.

Propositional logic

Let’s Form the Definition

Definition- A proposition is a declarative statements that has either a truth value "true” or a truth value "false".

A proposition consists of propositional variables and connectives. *Variables are denoted by letter p, q etc.*

p: Two plus two equals four

Connectives

In propositional logic generally we use five connectives which are − OR (∨), AND (∧), Negation/ NOT (¬), Implication / if-then (→), If and only if (⇔).

Connectives

1. **OR (∨)** − The OR operation of two propositions A and B (written as A ∨ B) is true if at least any of the propositional variable A or B is true.

The truth table:

A	B	A ∨ B
True	True	True
True	False	True
False	True	True
False	False	False

Connectives

2. AND (∧) – The AND operation of two propositions A and B (written as A ∧ B) is true if both the propositional variable A and B is true.

The truth table:

A	B	A ∧ B
True	True	True
True	False	False
False	True	False
False	False	False

CONNECTIVES

3. Negation (¬) − The negation of a proposition A (written as ¬A) is false when A is true and is true when A is false.

The truth table:

A	B	A → B
True	True	True
True	False	False
False	True	True
False	False	True

CONNECTIVES

4. Implication / if-then (?) - An implication A → B is False if A is true and B is false. The rest cases are true.

The truth table:

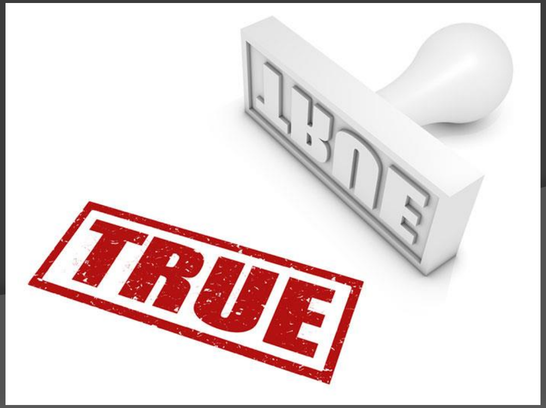
A	B	A → B
True	True	True
True	False	False
False	True	True
False	False	True

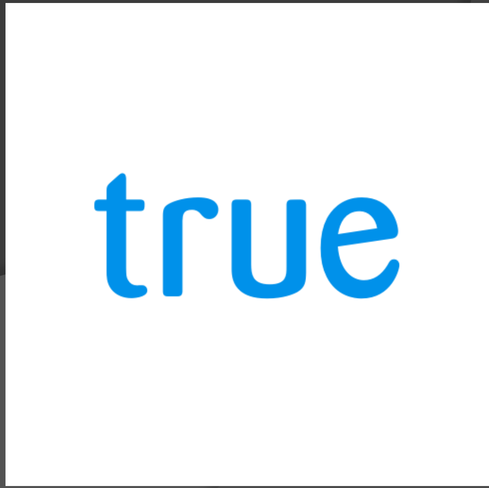
CONNECTIVES

- **5. If and only if (⇔)** − A⇔B is bi-conditional logical connective which is true when p and q are both false or both are true.
- The truth table:

A	B	A ⇔ B
True	True	True
True	False	False
False	True	False
False	False	True

A	B	$A \to B$	$(A \to B) \land A$	$[(A \to B) \land A] \to B$
True	True	True	True	True
True	False	False	False	True
False	True	True	False	True
False	False	True	False	True





Tautologies

A Tautology is a formula which is always true for every value of its propositional variables.

As we can see every value of $[(A \rightarrow B) \land A] \rightarrow B$ is “True”, it is a tautology.

What if it is opposite?

A	B	A ∨ B	¬A	¬B	(¬A) ∧ (¬B)	(A ∨ B) ∧ [(¬A) ∧ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	True	False	False
False	True	True	True	False	False	False
False	False	False	True	True	True	False

Contradictions

- A Contradiction is a formula which is always false for every value of its propositional variables.

As we can see every value of $(A \lor B) \land [(\neg A) \land (\neg B)]$ is “False”, it is a contradiction.

Hmm…What if both?

A	B	A ∨ B	¬A	(A ∨ B) ∧ (¬A)
True	True	True	False	False
True	False	True	False	False
False	True	True	True	True
False	False	False	True	False

Contingency

As we can see every value of (A ∨ B) ∧ (¬A) has both “True” and “False”, it is a contingency.

A Contingency is a formula which has both some true and some false values for every value of its propositional variables.


Thank you!