Midterm

● Graded

Student

PO-YUN) 鄭博允 (CHENG

Total Points

76 / 100 pts

Question 1

Question 1

8 / 8 pts

✔ **+ 8 pts** Correct

+ 6 pts One minor mistake

+ 4 pts Multiple minor mistakes

+ 2 pts Some reasonable effort (incorrect proof)

+ 0 pts Totally wrong or empty

Question 2

Question 2

8 / 8 pts

- ✔ **+ 8 pts** Correct
- **+ 7 pts** 1 mistake
- **+ 6 pts** 2 mistakes
- **+ 5 pts** 3 mistakes
- **+ 4 pts** 4 mistakes
- **+ 3 pts** 5 mistakes
- **+ 2 pts** 6 or more mistakes
- **+ 0 pts** Totally wrong or empty

Question 3

Question 3

16 / 16 pts

✔ **+ 8 pts** 3.1. Correct

+ 6 pts 3.1. One minor mistake

+ 4 pts 3.1. Two minor mistakes or one major mistake

+ 2 pts 3.1. Some reasonable effort

+ 0 pts 3.1. Totally wrong or empty

✔ **+ 8 pts** 3.2. Correct

+ 6 pts 3.2. One minor mistake

+ 4 pts 3.2. Two minor mistakes or one major mistake

+ 2 pts 3.2. Some reasonable effort

+ 0 pts 3.2. Totally wrong or empty

Question 4

Question 4

8 / 8 pts

✔ **+ 8 pts** Correct (#1,#3,#4,#6,#9) and minimum

+ 7 pts Correct (#1,#3,#4,#6,#9) with one incorrect answer

+ 6 pts Correct but not minimum

+ 4 pts Incorrect with #1 and #3

+ 2 pts Some reasonable effort

+ 0 pts Totally wrong or empty

Question 5

Question 5

Resolved 4 / 8 pts

+ 8 pts Correct (0, A'B, AB', AB, A, B, A'B+AB', A+B).

+ 6 pts 1 mistake (e.g., missing 1 correct expression)

+ 4 pts 2 mistakes (e.g., missing 2 correct expressions or having 2 incorrect expressions)

✔ + 4 pts 1 major mistake (e.g., listing all 16 expressions, list all 8 expressions without minimization)

+ 3 pts 3 mistakes (e.g., missing 3 correct expressions)

+ 2 pts At least 1 correct expression

+ 1 pt Some reasonable effort

+ 0 pts Totally wrong or empty

Regrade Request

Submitted on: Apr 17

教授好，打擾了。
關於第五題，我認為我的算法並沒有錯，答案也是對的，只是沒有經過簡化，然而即便沒有簡化，結果也會是一樣的，例如我的答案 AB+A'B+AB' 和正確的答案 A+B 實際上兩者是等價的。
因此我認為此題應該視為忘記簡化而斟酌扣分，
而不是視為缺少正確答案或是給出錯誤答案而只有2分。
感謝教授。

We have decided to give 4 points for correct functions without minimization.

Reviewed on: Apr 20

Question 6

Question 6

8 / 8 pts

✔ **+ 8 pts** Correct

+ 6 pts 1 mistake

+ 4 pts 2 mistakes

+ 2 pts 3 mistakes

Question 7

Question 7

8 / 8 pts

✔ **+ 8 pts** Correct

+ 6 pts 1 mistake

+ 4 pts 2 mistakes

+ 2 pts Some reasonable effort

+ 0 pts Totally wrong or empty

Question 8

Question 8

6 / 16 pts

+ 8 pts 8.1. Correct

+ 6 pts 8.1. One minor mistake

✔ **+ 4 pts** 8.1. Two minor mistakes or one major mistake

+ 2 pts 8.1. Some reasonable effort

+ 0 pts 8.1. Totally wrong or empty

+ 8 pts 8.2. Correct

+ 6 pts 8.2. One minor mistake

+ 4 pts 8.2. Two minor mistakes or one major mistake

✔ **+ 2 pts** 8.2. Some reasonable effort

+ 0 pts 8.2. Totally wrong or empty

Question 9

Question 9 — 2 / 10 pts

- **+ 10 pts** Correct
- **+ 8 pts** Almost correct (no generalization)
- **+ 6 pts** True with a correct (3-gate) but non-generalizable counterexample
- **+ 4 pts** True with a wrong counterexample
- **+ 2 pts** True (only)
- ✔ **+ 2 pts** False with some reasonable effort
- **+ 0 pts** Totally wrong or empty
- **– 2 pts** A minor mistake (check comments below)

Question 10

Question 10

8 / 10 pts

+ 10 pts Correct

✔ **+ 8 pts** Almost correct (show a correct counterexcample and the understanding of PI/EPI but consider "<=" instead of "<")

+ 6 pts False with some reasonable effort

+ 4 pts True with a good direction

+ 2 pts True with some reasonable effort

+ 2 pts False with a totally wrong counterexample

+ 0 pts Totally wrong or empty

CSIE 2344, Spring 2023: Midterm

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1 Boolean Algebra (8pts)

Prove $A(B \oplus C) = (AB) \oplus (AC)$, where $\oplus$ is XOR.

$A(B \oplus C) = A(B'C + BC')$

$= AB'C + ABC'$

$= A'AC + AB'C + A'AB + ABC'$

$= AC(A' + B') + AB(A' + C')$

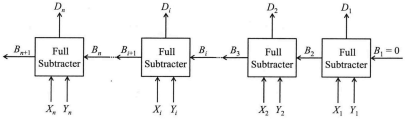
$= (AC)(AB)' + (AB)(AC)'$

$= (AB) \oplus (AC)$

Q.E.D.

2 Full Subtracter (8pts)

A parallel subtracter for $(X - Y)$ is shown below. A full subtracter has three inputs X_i, Y_i, B_i and two outputs D_i, B_{i+1}. Complete the following truth table. No explanation is required.

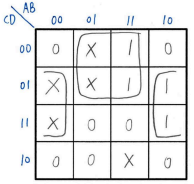


X_i	Y_i	B_i	B_{i+1}	D_i
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

3 Karnaugh Maps (16pts)

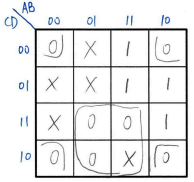
Given a Boolean function $F(A, B, C, D) = \sum m(9, 11, 12, 13) + \sum d(1, 3, 4, 5, 14)$.

1. (8pts) Find a minimum sum-of-products expression for F. Only the Karnaugh Map and the final expression are required.



A: $F = BC' + B'D$

2. (8pts) Find a minimum product-of-sums expression for F. Only the Karnaugh Map and the final expression are required.



$F' = BC + B'D'$

A: $F = (B'+C')(B+D)$

4 Quine-McCluskey Method (8pts)

Given all prime implicants (PIs) in the following table, list the labels of the selected prime implicants (PIs) for a minimum sum-of-products expression. No explanation is required.

Label	PI	0	4	8	10	12	16	17	21	23	26	30	31
#1	(0,4,8,12)	×	×	×		×							
#2	(0,16)	×					×						
#3	(10,26)				×						×		
#4	(16,17)						×	×					
#5	(17,21)							×	×				
#6	(21,23)								×	×			
#7	(23,31)									×			×
#8	(26,30)										×	×	
#9	(30,31)											×	×

A: #1 #3 #4 #6 #9

5 Boolean Functions (8pts)

Given a Boolean function $F(A, B)$, where $F(0, 0) = 0$, list the minimum sum-of-products expressions for *all* possible Boolean functions $F(A, B)$. No explanation is required.

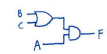
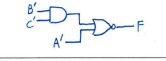
A	B								
0	0	0	0	0	0	0	0	0	
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1
		0	AB	AB′	AB+AB′	A′B	A′B+AB	AB+AB′	AB+A′B+AB′

A:

0	AB+AB′
AB	AB+A′B
AB′	A′B+AB′
A′B	AB+A′B+AB′

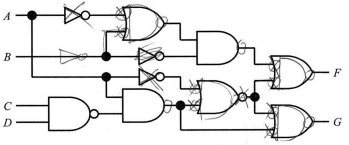
6 Two-Level Circuit Conversion (8pts)

Draw the following four different two-level gate circuits to realize $F(A, B, C) = A(B+C)$. No explanation is required.

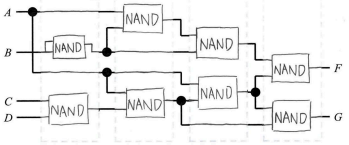
1. (2pts) OR-AND	2. (2pts) NOR-NOR
	
3. (2pts) NAND-AND	4. (2pts) AND-NOR
	

7 Multi-Level Circuit Conversion (8pts)

Convert the following circuit *directly* (not from a Karnaugh Map) to a four-level circuit containing only NAND gates (NOT gates are not allowed) and circuit inputs A, B, C, D (A', B', C', D' are not allowed as circuit inputs). The number of NAND gates should be 8. No explanation is required.



Draw Your Circuit Below

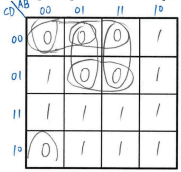


$F' = BC'D' + BC'D + A'B'D'$

8 Static Hazards (16pts)

Given a Boolean function $F(A, B, C, D) = (B' + C + D)(B' + C + D')(A + B + D)$.

1. (8pts) Find a minimum product-of-sums expression for F, where the corresponding gate circuit has no static-0 hazard. Only the Karnaugh Map and the final expression are required.

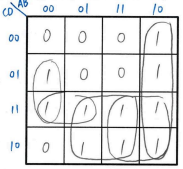


BB'C DD'A
BB'D DD'B DD'B'
BB'D' DD'C

$F' = A'C'D' + A'B'D' + BC'$

A: $F = (A+C+D)(B'+C+D)(B'+C+D')(A+B'+C)(A'+B'+C)(A+B+D)$

2. (8pts) Find a minimum product-of-sums expression for F, where the corresponding gate circuit has no static-1 hazard. Only the Karnaugh Map and the final expression are required.



$F = A'B'D + A'CD +$
$BC + AC + AB'$

=

9 Dynamic Hazards (10pts)

In the examples of dynamic hazards in the lecture and the discussion, there are three changes of the circuit output. Here, we are considering $2n + 1$ changes of the circuit output.

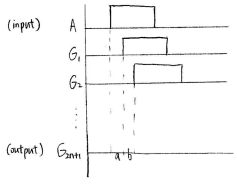
Assumptions:

- There is only one circuit input.
- There is only one circuit output.
- There are only four types of gates: {NOT, AND, OR, XOR}.
- A NOT gate has exactly one gate input.
- An AND gate, an OR gate, or an XOR gate has exactly two gate inputs.
- All gates have the same positive propagation delay.

Statement: Given the assumptions, it is possible to use only $2n+1$ gates to let one change of the circuit input results in $2n + 1$ changes of the circuit output.

Answer if the statement is True or False and then draw a corresponding gate circuit (if True) or prove it (if False).

False.



Since there only $2n$ gates output (exclude final output), there are at most $2n$ possible 組合 and one 組合 correspond to one changes, so there are at most $2n$ changes.

10 Prime Implicants and Essential Prime Implicants (10pts)

The *size* of an implicant is defined as the number of minterms covered by the implicant. For example, given a Boolean function $F(A, B, C, D) = ABCD + A'B'C'$, the size of $ABCD$ is 1, and the size of $A'B'C'$ is 2.

Also, the *total-size* of a function is defined as the summation of the sizes of all prime implicants of the function. For example, given a Boolean function $F(A, B, C, D) = ABCD + A'B'C'$, the total-size of F is 3.

Statement: given a Boolean function $F(A, B, C, D)$, if all of its prime implicants are essential prime implicants, then the total-size of F is smaller than 32.

Answer if the statement is True or False and then prove it (if True) or find a counterexample (if False).

True.

The max size of an implicant is 8, such implicant are $A, A', B, B', C, C', D, D'$

Since $A + A' = 1$ can be ingnore,

the max total-size of $F(A,B,C,D)$ happened when $F = (A/A') + (B/B') + (C/C') + (D/D')$,

and it's 32.

so the statement is true.