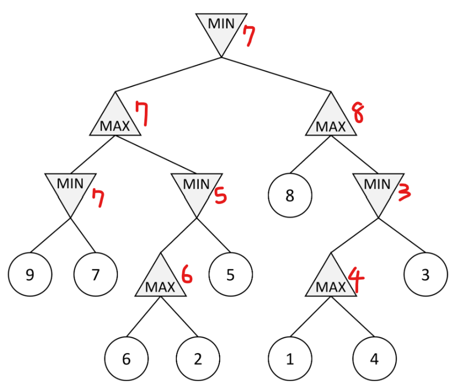
Foundations of Artificial Intelligence: Homework 2

b11902038 鄭博允

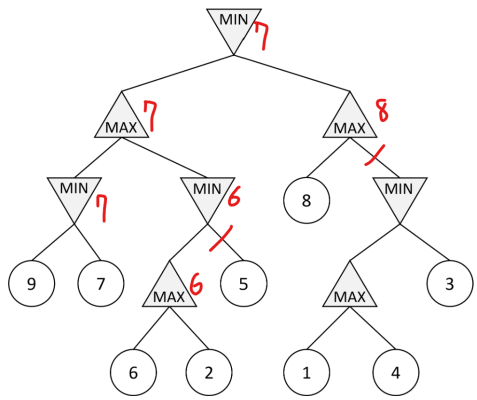
April 3, 2025

Problem 1

a)

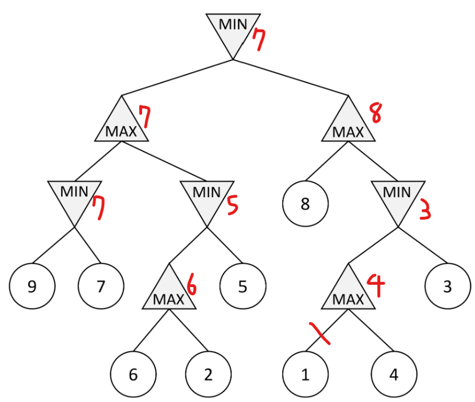


b)



MIN
7
MAX
7
MAX
8
MIN
7
MIN
6
8
MIN
9
7
MAX
6
5
MAX
3
6
2
1
4

c)



MIN
7
MAX
7
MAX
8
MIN
7
MIN
5
8
MIN
3
9
7
MAX
6
5
MAX
4
3
6
2
1
4

Problem 2

a)

" 當官要掙錢，必須跪著"

$$(O \wedge K) \implies E$$

" 拿槍想掙錢，必須在山裡"

$$(G \wedge M) \implies E$$

" 我是想站著，還把錢掙了"

$$\neg K \wedge E$$

b)

For the left-hand side and right-hand side:

$$\begin{aligned}
&((O \wedge K) \implies E) \vee ((G \wedge M) \implies E) \\
&= (E \vee \neg(O \wedge K)) \vee (E \vee \neg(G \wedge M)) \\
&= E \vee \neg(O \wedge K)) \vee \neg(G \wedge M) \\
&= E \vee (\neg O \vee \neg K) \vee (\neg G \vee \neg M) \\
&= E \vee \neg O \vee \neg K \vee \neg G \vee \neg M
\end{aligned}
\qquad
\begin{aligned}
&(O \wedge G \wedge \neg K \wedge \neg M) \implies E \\
&= E \vee \neg(O \wedge G \wedge \neg K \wedge \neg M) \\
&= E \vee (\neg O \vee G \vee K \vee M) \\
&= E \vee \neg O \vee G \vee K \vee M
\end{aligned}$$

Then we have the final CNF:

$$(E \vee \neg O \vee \neg K \vee \neg G \vee \neg M) \implies (E \vee \neg O \vee K \vee G \vee M)$$

Since there exist exactly one assignment let the sentence become false:

$$E = \text{false}, O = \text{true}, K = \text{false}, G = \text{false}, M = \text{false}$$

While the other assignments let the sentence remain true. Thus the sentence is **Satisfiable**.

c)

$K \Rightarrow (G \Rightarrow E)$	$K \wedge G \Rightarrow E$
$((K \Rightarrow G) \wedge (G \Rightarrow E) \wedge (E \Rightarrow K))$	$G \Rightarrow (K \Rightarrow E)$
$(K \wedge G) \Rightarrow E$	$\neg((K \wedge G) \wedge \neg E)$
$\Big[(K \Rightarrow G) \wedge (\neg G \Rightarrow E)\Big] \vee \Big[(K \wedge \neg E) \vee \neg K\Big]$	$(E \Rightarrow K) \wedge (K \Leftrightarrow G)$

Problem 3

a)

$$
\begin{aligned}
\Pr(P_3 = 1) &= \sum_{p_1=0}^{1} \sum_{p_2=0}^{1} \left[\Pr(P_1 = p_1) \Pr(P_2 = p_2 \mid P_1 = p_1) \Pr(P_3 = 1 \mid P_2 = p_2)\right] \\
&= \Pr(P_1 = 0) \Pr(P_2 = 0 \mid P_1 = 0) \Pr(P_3 = 1 \mid P_2 = 0) \\
&\quad + \Pr(P_1 = 0) \Pr(P_2 = 1 \mid P_1 = 0) \Pr(P_3 = 1 \mid P_2 = 1) \\
&\quad + \Pr(P_1 = 1) \Pr(P_2 = 0 \mid P_1 = 1) \Pr(P_3 = 1 \mid P_2 = 0) \\
&\quad + \Pr(P_1 = 1) \Pr(P_2 = 1 \mid P_1 = 1) \Pr(P_3 = 1 \mid P_2 = 1) \\
&= (0.4 \cdot 0.8 \cdot 0.3) + (0.4 \cdot 0.2 \cdot 0.8) \\
&\quad + (0.6 \cdot 0.3 \cdot 0.3) + (0.6 \cdot 0.7 \cdot 0.8) \\
&= 0.55
\end{aligned}
$$

b)

The additions are used to sum over all possible combinations of variables. Since the variable we want to compute (P_n) has a fixed value, we only need to consider the combinations of the $n-1$ preceding variables, each of which has 2 possible values. Therefore, there are 2^{n-1} such combinations, and the number of additions required is $O(2^{n-1})$.

The multiplications are used to compute the probability of each variable for every combination. Since each combination involves n probability terms, we need $n-1$ multiplications per combination. Given 2^{n-1} combinations, the toal number of multiplications required is $2^{n-1} \cdot (n-1)$, which implies a complexity of $O(n2^{n-1})$.

c)

Using variable elimination, we can subsitute $\Pr(P_{n+1} = p_{n+1})$ with

$$\sum_{p_n} \Pr(P_n = p_n) \Pr(P_{n+1} = p_{n+1} \mid P_n = p_n)$$

This allows us to break down the problem involving n variables into $n-1$ smaller problems, each involving only 2 variables.

For the case of $n = 2$, using naive enumeration, the number of additions and multiplications required is 2 and 4, respectively. Thus, for general n, the total number of operations follows:

- additions: $O(2 \cdot (n-1)) = O(n)$
- multiplications: $O(4 \cdot (n-1)) = O(n)$

Therefore, the computational complexity of variable elimination is $O(n)$ for both additions and multiplications.

Problem 4

a)

$$\Pr(A)\Pr(B \mid A)\Pr(C \mid B)\Pr(D \mid B)\Pr(E)$$
$$\Pr(F \mid C)\Pr(G \mid E)\Pr(H \mid D,F,G)\Pr(I)$$
$$\Pr(J \mid F,H,I)\Pr(K)\Pr(L \mid J)\Pr(M \mid J,K)\Pr(N \mid M)$$

b)

F's Markov blankets: $[C,D,F,G,H,I,J]$
Variables that are d-separated from F: $[A,B,E,K,L,M,N]$

c)

- $D \perp\!\!\!\perp I \mid \{J\}$: false
- $H \perp\!\!\!\perp C|\{\}$: false
- $H \perp\!\!\!\perp K \mid \{\}$: true
- $B \perp\!\!\!\perp E \mid \{N\}$: false

d)

$[K,L,M,N]$

e)

$[I,N]$