Due: 04/06/2025 23:59

Foundations of Artificial Intelligence: Homework 2

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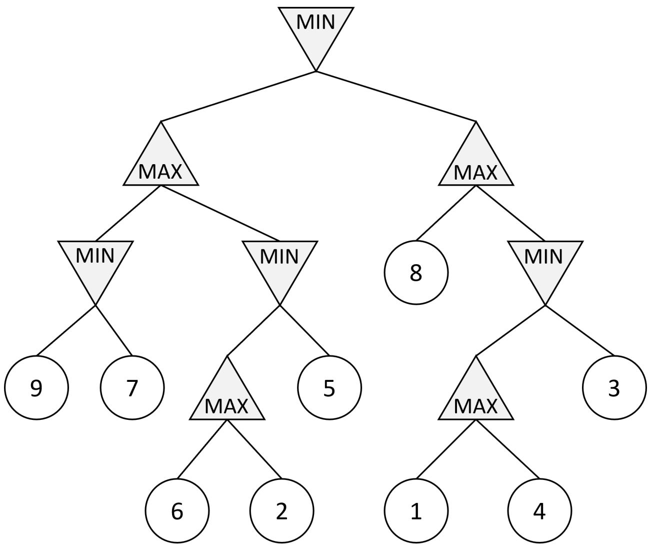
- Any form of cheating, lying, or plagiarism is strictly prohibited. Violations will result in a zero score and failure of the course.
- To create a discussion environment, if you have any questions, please discuss them on NTUCOOL instead of sending a letter.
- Please label each question with its corresponding number and also indicate the answer clearly.
- You may consult ChatGPT, books, and Internet resources, but you must not copy directly from them.
- Ensure that your file is clearly readable. If using a camera to scan, check the resolution for clarity.
- Version 2 (2025.03.25): revised parts are highlighted in red.

Problem 1 (8 points)

Consider the `MAX-MIN` game tree shown below where the numbers underneath the leaves of the tree are utility values from the first player's point of view (`MIN`).



a) Draw a copy of the tree on paper and perform the **minimax** algorithm on it by hand. Write the resulting minimax values next to every node.

b) Do the same, but with **left-to-right alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning. You may simply show which edges are cut.

c) Do the same, but with **right-to-left alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

Problem 2 (12 points)

Based on Let the Bullets Fly, we introduce the following atomic propositions:

Figure 1: 《讓子彈飛》 (Let the Bullets Fly)

- **O**: Possessing an official position. （擁有官位）
- **K**: Kneeling. （跪著）
- **G**: Possessing a gun. （擁有槍）
- **M**: Being in the mountains. （在山裡）
- **E**: Earning money. （掙錢）

a) Based on the atomic propositions, please formalize the following statements into logical sentences (Hint: standing $\equiv \neg$ kneeling):

1. "Anyone who holds an official position and earns money must kneel." （當官要掙錢，必須跪著）
2. "Anyone who owns a gun and earns money is definitely in the mountains." （拿槍想掙錢，必須在山裡）
3. "I want to earn money while standing" （我是想站著，還把錢掙了）

b) Based on the atomic propositions, we construct the following logical sentence:

$$\Big[((O \land K) \Rightarrow E) \lor ((G \land M) \Rightarrow E)\Big] \Rightarrow \Big[(O \land G \land \neg K \land \neg M) \Rightarrow E\Big].$$

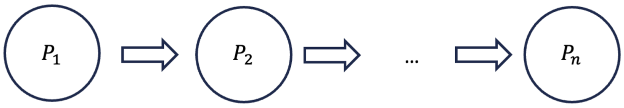
Please convert both the left-hand side and right-hand side of the above sentence into CNF. Additionally, determine whether the sentence is **Valid** (a tautology), **Satisfiable**, or Neither.

c) Please complete the following "connect the dots" task. Specifically, connect the logically equivalent sentence. Note that a point may be connected to multiple points on the other side or none.

$K \Rightarrow (G \Rightarrow E)$ •	• $K \land G \Rightarrow E$
$((K \Rightarrow G) \land (G \Rightarrow E) \land (E \Rightarrow K))$ •	• $G \Rightarrow (K \Rightarrow E)$
$(K \land G) \Rightarrow E$ •	• $\neg((K \land G) \land \neg E)$
$\Big[(K \Rightarrow G) \land (\neg G \Rightarrow E)\Big] \lor \Big[(K \land \neg E) \lor \neg K\Big]$ •	• $(E \Rightarrow K) \land (K \Leftrightarrow G)$

Problem 3 (10 points)

Consider a chain of n binary variables $P_1, P_2, \ldots, P_n$.



To find $\Pr(P_n)$ via **enumeration**, we can marginalize out the other variables:

$$\Pr(P_n = p_n) \; = \sum_{p_1=0}^{1} \sum_{p_2=0}^{1} \cdots \sum_{p_{n-1}=0}^{1} \left[\Pr(P_1 = p_1) \prod_{i=2}^{n} \Pr(P_i = p_i \mid P_{i-1} = p_{i-1}) \right].$$

a) Suppose $n = 3$ and each P_i is binary. Let

$$\Pr(P_1 = 1) = 0.6, \quad \Pr(P_2 = 1 \mid P_1 = 1) = 0.7, \quad \Pr(P_2 = 1 \mid P_1 = 0) = 0.2,$$

$$\Pr(P_3 = 1 \mid P_2 = 1) = 0.8, \quad \Pr(P_3 = 1 \mid P_2 = 0) = 0.3.$$

Calculate $\Pr(P_3 = 1)$ by explicitly summing over P_1 and P_2. Show your steps and final numerical result.

b) In the above $n = 3$ example, the computation required 8 multiplications and 3 additions. In general, for a chain with n binary variables, what is the complexity (in tightest Big-O notation) for the number of multiplications and additions separately required to compute $\Pr(P_n)$ by **naive enumeration**?

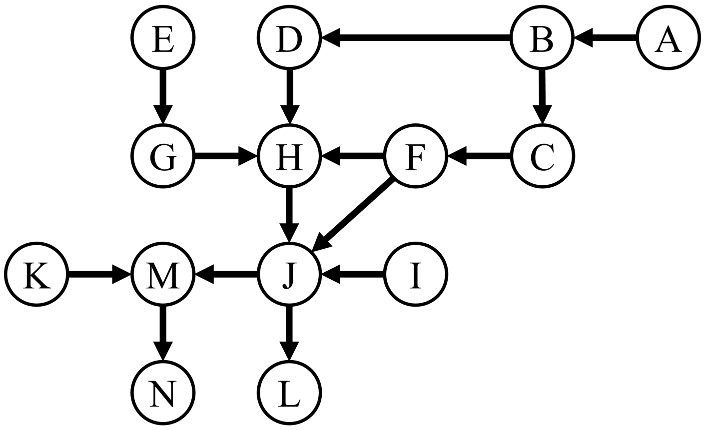
c) Now, consider computing $\Pr(P_n)$ by **variable elimination**, what is the complexity (in tightest Big-O notation) for the number of multiplications and additions separately required in this method?

Problem 4 (10 points)

Recall that we can determine whether a variable X is d-separated (i.e. guaranteed to be conditionally independent) from Y given an observation set Z by using the following criteria:

- X is d-separated from its non-descendant nodes given its parents,
- X is d-separated from all other nodes given its Markov blankets (i.e. its parents, children, and the other parents of its children),
- Analyze whether there is an active trail from X to Y without Z blocking the paths (refer to the slides, textbook, or MIT's d-separation resource).

Given the following Bayesian network:



a) Write the factored joint probability distribution represented by the network.

b) List F's Markov blankets. Then, list all variables that are d-separated from F when F's Markov blanket is observed.

c) For each of the following statements, indicate whether the conditional independence is guaranteed:

- $D \perp\!\!\!\perp I \mid \{J\}$
- $H \perp\!\!\!\perp C \mid \{\}$
- $H \perp\!\!\!\perp K \mid \{\}$
- $B \perp\!\!\!\perp E \mid \{N\}$

d) List all variables that are d-separated from B when J is observed.

e) List all variables that are d-separated from B when H and M are observed.