PEC UNIVERSITY OF TECHNOLOGY, CHANDIGARH

Mid-Term Examination September, 2019

Programme: B.E. — Year/Semester: 2nd /4th sem

Course Name: Communication Theory — Course Code: ECN - 210

Maximum Marks: 25 — Time Allowed: 1.5 Hours

Note:

1. All questions are compulsory.
2. Unless stated otherwise, the symbols have their usual meanings in context with subject.

Q.No.	Questions	Marks
Q1.	If X is any random variable with E(X) =2 and E(X(X-1)) =6, find Var(X).	(2)
Q2.	The mean and variance of binomial variate X are 16 and 8. Find	
	(a) $P(X=1)$	(2)
	(b) $P(X\geq 2)$	(2)
Q3.	Let $X = N(0;\sigma^2)$. Find	
	(a) $E(X\mid X>0)$	(2)
	(b) $Var(X\mid X>0)$	(3)
Q4.	The joint pmf of a bivariate r.v. (X, Y) is given by $p_{XY}(x_i, y_j) = \begin{cases} k(2x_i + y_j) & x_i = 1, 2;\ y_j = 1, 2 \\ 0 & \text{otherwise} \end{cases}$ where k is a constant.	
	(a) Find the value of k.	(1)
	(b) Find the marginal pmf's of X and Y.	(2)
	(c) Are X and Y independent?	(1)
Q5.	The joint pmf of a bivariate r.v. (X, Y) is given by $p_{XY}(x_i, y_j) = \begin{cases} kx_i^2 y_j & x_i = 1, 2,\ y_j = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$ Find the conditional pmf's $p_{Y\mid X}(y_j\mid x_i)$ and $p_{X\mid Y}(x_i\mid y_j)$ for the bivariate r.v. (X, Y)	(4)
Q6.	Let (X, Y) be a bivariate r.v. If X and Y are independent, show that X and Y are uncorrelated.	(2)
Q7.	Find the mean and variance of a Rayleigh random variable.	(4)

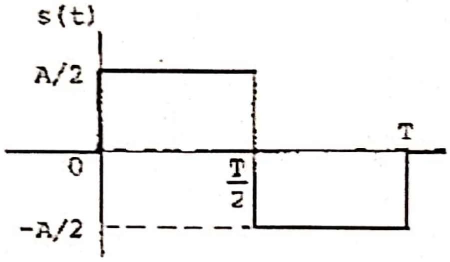
PUNJAB ENGINEERING COLLEGE
(Deemed to be University) CHANDIGARH
End-Term Examination Nov, 2019

Programme: B.E. | Year/Semester: 2^{nd} /3^{th} sem
Course Name: **Communication Theory** | Course Code: **ECN - 210**
Maximum Marks: **45** | Time Allowed: **3 Hours**

Note: 4

1. All questions are compulsory.
2. Unless stated otherwise, the symbols have their usual meanings in context with subject.
3. The candidates, before starting to write the solutions, should please check the question paper for any discrepancy and also ensure that they have been delivered the question paper of right course code.

<table>
<tr><th>Q.No.</th><th>Questions</th><th>Marks</th></tr>
<tr><td>Q1.</td><td>
a) Why do we use matched filter as an essential component in detection and estimation of signals? (2)
b) Consider the signal s (t) shown in Figure



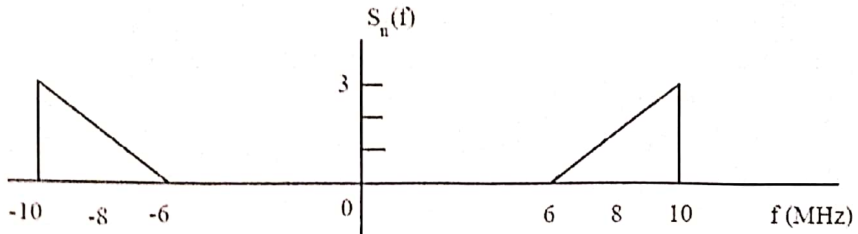
I. Determine the impulse response of a filter matched to this signal and sketch it as a function of time
II. Plot the matched filter output as a function of time.
III. What is the peak value of output?
</td><td>(2)
(1)
(1)
(1)</td></tr>
<tr><td>Q2.</td><td>How the minimum mean square error (MMSE) equalizer is implemented to assess optimum linear receiver.</td><td>(4)</td></tr>
<tr><td>Q3.</td><td>The impulse response of the circuit is given as h(t) = $e^{-2t}\ u(t)$. This circuit is excited by an input of x(t) = $e^{-4t}\ [u(t) - u(t-2)]$. Determine the output of the circuit.</td><td>(4)</td></tr>
<tr><td>Q4.</td><td>On which conditions the random process defined its ergodicity and stationarity.</td><td>(2)</td></tr>
<tr><td>Q5.</td><td>The gain of the three stages of an amplifier are 8.45 dB, 10.79 dB, and 20 dB. The noise figures associated with these stages are 2.04 dB, 3.0 dB, and 9.29 dB. What is the overall Noise Figure and Noise Temperature for this cascade of amplifiers?</td><td>(4)</td></tr>
<tr><td>Q6.</td><td>Discuss the types, causes and effects of the various forms of noise which may be created within a receiver or an amplifier.</td><td>(2)</td></tr>
<tr><td>Q7.</td><td>Let $Y(n) = X(n) + W(n)$, where $X(n) = A$ (for all n) and A is a random variable (r.v.) with zero mean and variance σ_A^2 , and $W(n)$ is a discrete-time white noise with average power σ^2. It is also assumed that $X(n)$ and $W(n)$ are independent.
(a) Find that $Y(n)$ is Wide Sense Stationary(WSS) or not.
(b) Find the power spectral density of $Y(n)$.</td><td>(2)
(2)</td></tr>
</table>

Q8. Consider a random process $X(t)$ defined by

$$X(t) = U \cos t + V \sin t \qquad -\infty < t < \infty$$

where U and V are independent r.v.'s, each of which assumes the values -2 and 1 with the probabilities $\frac{1}{3}$ and $\frac{2}{3}$, respectively.
Show that $X(t)$ is WSS but not strict-sense stationary. (3)

Q9. A bandpass noise signal $n(t)$ can be expressed as $n(t) = n_c(t) \cos \omega_c t + n_s(t) \cos \omega_c t$. (2)
Consider bandpass noise $n(t)$ having the power spectral density shown below in figure.
Draw the power spectral density of $n_s(t)$ if the center frequency $\omega_c / 2\pi$ is 8 MHz.



Power spectral density of $n(t)$.

Q10. Let X be a Poisson r.v. with parameter λ.

(a) Show that pmf of poission r.v. satisfies Eq. $\sum_x P_x(x_k) = 1$ (2)

(b) Find $P(X > 2)$ with $\lambda = 4$. (2)

Q11. Let $Z_1, Z_2, \ldots$ be independent identically distributed r.v.'s with $P(Z_n=1) = p$ and $P(Z_n = -1) = q = 1 - p$ for all n. Let

$$X_n = \sum_{i=1}^{n} Z_i \qquad n = 1, 2, \ldots$$

and $X_0 = 0$. The collection of r.v.'s $\{X_n, n \geq 0\}$ is a random process, and it is called the *simple random walk* $X(n)$ in one dimension.

(a) Find the probability that $X(n) = -2$ after four steps. (2)

(b) Find the mean and variance of the simple random walk $X(n)$ (4)

Q12. Suppose we select one point at random from within the circle with radius R. If we let the center of the circle denote the origin and define X and Y to be the coordinates of the point chosen then (X, Y) is a uniform bivariate r.v. with joint pdf given by

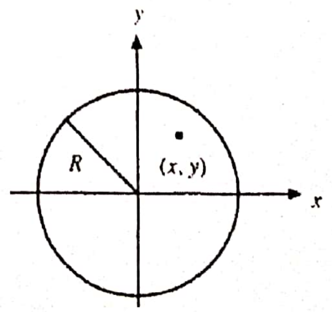
$$f_{XY}(x, y) = \begin{cases} k & x^2 + y^2 \leq R^2 \\ 0 & x^2 + y^2 > R^2 \end{cases}$$

where k is a constant.

(a) Determine the value of k. (1)

(b) Find the marginal pdf's of X and Y. (2)

(c) Find the probability that the distance from the origin of the point selected is not greater than a. (2)



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