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PEC UNIVERSITY OF TECHNOLOGY

Mid-term Examination (Feb 2019)

Programme: B.E. (ECE & CSE)

Course Name: Vector Calculus, Fourier Series and Laplace transform

Maximum Marks: 25

Year/Semester: 18192

Course Code: MAN-105

Time allowed: 1 hour 30 mins

NOTES:

- All questions are compulsory
- Unless stated otherwise, the symbol have their usual meanings in context with subject
- The candidates before starting to write the solutions should please check the question paper for any discrepancy.

Q.No.	Questions	Marks
1	Find the directional derivative of $\phi(x,y,z)=x^2yz+4xz^2$ at (1, -2, 1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$.	(4)
2	Apply Gauss Divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,dS$ where $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ and S is the surface of the cube bounded by $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=1$.	(4)
3	Prove that $(y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$ is both solenoidal and irrotational.	(2)
4	Evaluate $\iint_S \vec{A}.\hat{n}\,dS$ where $\vec{A}=(x+y^2)\hat{i}-2x\hat{j}+2yz\hat{k}$ and S is the surface of the plane $2x+y+2z=6$ in the first octant.	(4)
5	Using Green's theorem evaluate $\oint_C(\cos x\sin y-xy)dx+(\sin x\cos y)dy$ where C is the circle $x^2+y^2=1$	(2)
6	Verify Stoke's theorem for $\vec{A}=y^2\hat{i}+xy\hat{j}-xz\hat{k}$ where S is the hemisphere $x^2+y^2+z^2=a^2,\quad z\geq 0$	(4)
7	If $\vec{A}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$	
	(a) Prove that the line integral $\int_C \vec{A}.d\vec{r}$ is independent of curve C joining two given points $P_1(1,-2,1)$ and $P_2(3,1,4)$.	(2)
	(b) Show that there exists a scalar function f such that $\vec{A}=\nabla f$ and find f.	(2)
	(c) Find the work done in moving an object from P_1 to P_2.	(1)

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Punjab Engineering College (Deemed to be University), CHANDIGARH
End Term Examination(2019)

Programme: B.E. (ECE & CSE) **Year/Semester:** 18192
Course Name: Vector Calculus, Fourier Series and Laplace transform **Course Code:** MAN-105
Maximum Marks: 40 **Time allowed:** 3 Hours

NOTES:

- All questions are compulsory
- The candidates before starting to write the solutions, should please check the question paper for any discrepancy and also ensure that they have been delivered the question paper of right course code.

S.No	Questions	Marks
1	Prove that $\vec{A} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational. Find a scalar function f such that $\vec{A} = \nabla f$	3
2	Find the directional derivative of $f = xy + yz + zx$ in the direction of vector $\vec{i} + 2\vec{j} + 2\vec{k}$ at the point (1,2,0)	2
3	If $\vec{F} = (2x + y^2)\vec{i} + (3y - 4x)\vec{j}$, evaluate $\oint_C \vec{F}.d\vec{r}$ around a triangle ABC in the xy-plane with A(0,0), B(2,0), C(2,1)	3
4	Evaluate $\iint_S \vec{F}.\vec{n}dS$ where S is the surface of sphere $x^2 + y^2 + z^2 = a^2$ in the first octant where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$	3
5	Apply Green's theorem to evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the area enclosed by x-axis and upper half of circle $x^2 + y^2 = a^2$	3
6	Verify Stoke'stheorem for a vector field defined by $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region in xy-plane bounded by x=0, x=a, y=0, y=b	4
7	Apply Gauss Divergence theorem to evaluate $\iiint_V \vec{F}.\vec{n}dS$ where $\vec{F} = 4x^3\vec{i} - x^2y\vec{j} + x^2z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes z=0, z=b.	2
8	Find the Fourier Series expansion for $f(x) = x + x^2, \quad -\pi < x < \pi$. Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$	4

9	Find the Fourier series upto first harmonic to represent f(x) for given values x: 0 1 2 3 4 5 y: 9 18 24 28 26 20	4
10	Evaluate half-range cosine series for $f(x) = \begin{cases} \pi x, 0 < x < 1 \\ \pi(2-x), 1 < x < 2 \end{cases}$	3
11	Find Laplace transform of $f(z) = \int_0^t \frac{\cos az - \cos bz}{z} dz$ and inverse Laplace transform of $F(s) = \frac{1}{(s^2+a^2)^2}$	5
12	Solve the following differential equation using Laplace $y'' + 2y' + 5y = e^{-t} \sin t, y(0) = 0, y'(0) = 1$	4

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