Class : 12

Register Number | | | | | | |

COMMON QUARTERLY EXAMINATION-2024-25

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART – I

$20 \times 1 = 20$

1. Answer all the questions by choosing the correct answer from the given 4 alternatives
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$

(1) A (2) B (3) I_3 (4) B^T

2. If $A=\begin{bmatrix}3 & 1 & -1\\ 2 & -2 & 0\\ 1 & 2 & -1\end{bmatrix}$, and $A^{-1} = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ then the value of a_{23} is

(1) 0 (2) -2 (3) -3 (4) -1

3. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(1) 0 (2) 1 (3) 2 (4) 3

4. If z is a complex number such that $z \in C \setminus R$ and $z + \frac{1}{z} \in R$, then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

5. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(1) 2 (2) 4 (3) 1 (4) ∞

6. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

(1) 0 (2) n (3) $< n$ (4) r

7. If $\sin^{-1} x = 2\sin^{-1}\alpha$ has a solution, then

(1) $|\alpha| \le \frac{1}{\sqrt{2}}$ (2) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

8. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1} x + 2\sin^{-1} x)$ is

(1) $-\sqrt{\frac{24}{25}}$ (2) $\sqrt{\frac{24}{25}}$ (3) $\frac{1}{5}$ (4) $-\frac{1}{5}$

9. If $|x| \le 1$, then $2\tan^{-1} x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to

(1) $\tan^{-1} x$ (2) $\sin^{-1} x$ (3) 0 (4) π

10. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

11. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(1) 2ab (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

12. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$

13. If $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

(1) $|\vec{a}||\vec{b}||\vec{c}|$ (2) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (3) 1 (4) -1

14. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

(1) 0 (2) 1 (3) 6 (4) 3

15. The angle between the lines $\frac{x-2}{3} = \frac{y-1}{-2}$, $z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

16. If $[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a}] = 64$ then $[\vec{a}\;\vec{b}\;\vec{c}] =$

(1) 32 (2) 8 (3) 128 (4) -4

17. Equation of parabola having vertex at (0,0), directrix $y = -2$ is

(1) $y^2 = 8x$ (2) $y^2 = -8x$ (3) $x^2 = 8y$ (4) $x^2 = -8y$

18. Value of $\sin\left[\pi - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

(1) $-\frac{1}{2}$ (2) -1 (3) 1 (4) $\frac{1}{2}$

19. If $x = \frac{-1+i\sqrt{3}}{2}$ then value of $x^4 + x^2 + 1$ is

(1) 2 (2) $\frac{1}{2}$ (3) 0 (4) 1

20. The system of equation $x + 2y + 3z = 1$, $x - y + 4z = 0$, $2x + y + 7z = 1$ has

(1) one solution (2) Two solutions (3) Infinitely many solutions (4) No solution

PART - II

1. Answer any 7 questions **7 x 2 = 14**
2. Each question carries 2 marks
3. Question number 30 is compulsory

21. Solve by Cramer's rule: $\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$ (1)

22. If $|z| = 2$ show that $3 \le |z + 3 + 4i| \le 7$ (2)

23. Simplify : $\sum_{n=1}^{10} i^{n+50}$ (2)

24. Find a polynomial equation of minimum degree with rational coefficients having $2 + \sqrt{3}i$ as a root. (3)

25. Discuss the nature of the roots of the following polynomials: $x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$ (3)

26. State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq -\frac{\pi}{6}$. (4)

27. Find centre and radius of the following circles : $x^2 + y^2 - x + 2y - 3 = 0$ (5)

28. Prove that $\left(\vec{a} \cdot (\vec{b} \times \vec{c})\right)\vec{a} = (\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c})$ (6)

29. Find the angles between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with coordinate axes. (6)

30. Find the point of intersection of tangents at $t = 2$ and $t = 3$ to the parabola $y^2 = 8x$. (5)

PART - III

1. Answer any 7 questions $\underline{7 \times 3 = 21}$
2. Each question carries 3 marks
3. Question number 40 is compulsory

31. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

32. Find the rank of the following matrices by row reduction method: $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

33. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. into rectangular form

34. Obtain the Cartesian form of the locus of $z = x + iy$ in each of the following cases: $\text{Im}[(1-i)z+1] = 0$

35. If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid, then the volume in increased by 52 cubic units. Find the volume of the cuboid.

36. Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

37. Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1, -3)$.

38. Find the equation of the hyperbola passing through $(5, -2)$ and length of the transverse axis along x axis and of length 8 units.

39. Show that the points $(2,3,4)$, $(-1,4,5)$ and $(8,1,2)$ are collinear.

40. Find the image of the point whose position vector is $\hat{i} + 2\hat{j} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + 4\hat{k}) = 38$.

PART - IV

1. **Answer all the questions** $\underline{7 \times 5 = 35}$
2. **Each question carries 5 marks**

41. a) The prices of three commodities A, B and C are ₹ x, y, and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q, and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B, and C. (Use matrix inversion method to solve the problem.)

(OR)

b) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

42. a) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. 2

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$. 2

43. a) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros. 3

(OR)

b) Solve the following equations : $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. 3

44. a) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}$$ 4

(OR)

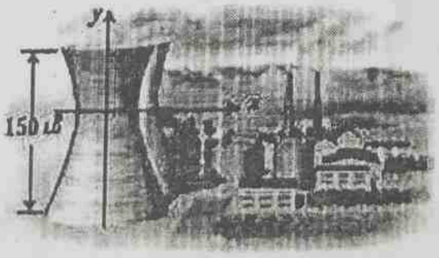
b) Find the domain of the following : $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$ 4

45. a) Find the foci, vertices and length of major and minor axis of the conic $4x^2 + 36y^2 + 40x - 288y + 532 = 0$. 5

(OR)

b) Find the equation of the circle passing through the points (1,1), (2, − 1) and (3,2). 5

46. a) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. 5



(OR)

b) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. 6

47. a) If $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} - \hat{j} - 4\hat{k}, \vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$ 6

(OR)

b) Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point (1,-2,4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$. 6