PHILLIPS EXETER ACADEMY
**MATHEMATICS DEPARTMENT**

Placement Test 1/2/3

Name _____

Please note the time you begin working on the test and the time you finish. At the end of the test, we will ask you to record the amount of time you spent working. **We are interested in your analyses of these problems, not just your answers, so you must show your reasoning fully and clearly.** *You are expected to work on your own. You may use any kind of calculator.*

1.   The cost of a pizza varies directly with the <u>area</u> of the pizza. If a pizza that is 12" in diameter costs $10.80, how much does a pizza that is 16" in diameter?

2.   The points (-97,-201), (3,-1), and (19,32) are graphed on the coordinate plane. Are they on the same line? Explain your reasoning.

3.   A hot-air balloon at 300 feet begins to rise at the rate of 100 feet per minute. At the same time, a second hot-air balloon at 2,000 feet starts to descend at the rate of 150 feet per minute.

    a.   When will the balloons be at the same height? Explain your answer.

    b.   What is that height?

    c.   What is the height of the ascending balloon when the descending balloon hits the ground?

4.   The Ski Club has rented a luxury bus for their annual trip to Sugarloaf Mountain. The cost of the bus is $720 for a weekend. Each member going on the trip is going to pay an equal share of the expense. When eight members back out at the last minute, the expenses of the other members go up by $3 each.

    a.   Write an algebraic equation (or equations) which describe the situation. Please define your variables.

   b.   How many members of the Ski Club go on the trip?

5.   The diagram at the right suggests an easy way of making a box with no top. Start with a square piece of cardboard, cut squares of equal sides from the four corners, and then fold up the sides. Here is the problem: We want to produce a box that is 8 cm. deep and whose capacity is exactly one liter (1000 cm^3). How large a square must we start with?
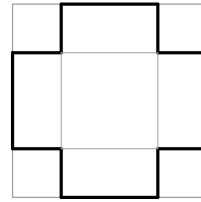
6.   A softball crosses home plate at a height of 4 feet, and the batter hits the ball. The path of the ball is described by $h = -\dfrac{1}{729}(x-162)^2 + 40$, where x represents the distance from home plate and h the height of the ball above the ground.

   a.   The outfield wall is 6 feet high and 318 feet from home plate. Will the ball go over the wall for a home run? If so, by how many feet will it clear the wall? Explain your reasoning and method.

   b.   Suppose the outfield wall is 326 feet from home plate. Would it be possible for an outfielder to catch the ball? If so, at what height above the ground would the ball be when she caught it with her back against the wall? Explain your reasoning and method.

7.   A right triangle is known to have a perimeter of 10 units and its hypotenuse is twice as long as one of its legs. Find the lengths of the sides, showing an algebraic solution.

8.   The figure at the right shows a sequence of squares inscribed under the line $y = \frac{x}{2}$ and above the x-axis. Every square has two vertices on the x-axis and one on the line $y = \frac{x}{2}$. The smallest square is 8 cm tall.
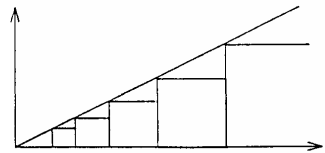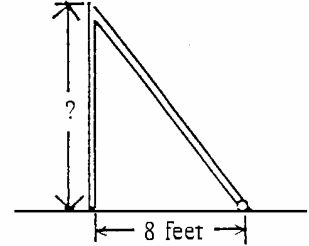

   a.   How tall are the next four squares?

   b.   How tall is the **nth** square?

9. A flagpole 24 feet high snapped in a storm, its top touching the ground 8 feet from its base.

   a. How far up the pole is the break?

   b. Find the angle between the ground and the top of the flagpole.

10. The diagram shows a line with intercepts 3 and 9 and one of many rectangles which can be inscribed under this line and within the first quadrant.

   a. Write an equation for the line.

   b. Determine if the rectangle can have $x = 4$ and $y = 2$ as its dimensions.

   c. Find x and y if the rectangle is a square.
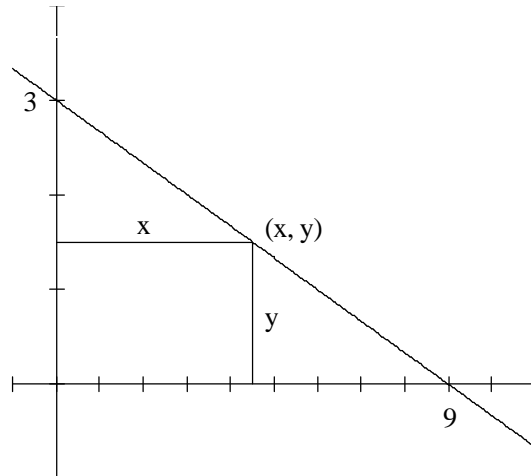
   d. Write an expression for A, the area of the rectangle, using x as the only variable.

   e. Find x if the area of the rectangle is 6.

   f. Which x gives the rectangle its largest area?

11.   A plane is flying 6 miles above the surface of the earth. A passenger looks out the window to the distant horizon. On a clear day, how far away is that horizon, to the nearest mile? Assume the radius of the earth is 3960 miles.

12.   Parallelogram PQRS has PQ = RS = 8 cm. and diagonal QS = 10 cm. Point F is on RS, exactly 5 cm. from S. Let T be the intersection of PF and QS. Draw a diagram and find the lengths of TS and TQ.

13.   A sector of a circle is enclosed by two 12-inch radii and a 9-inch arc. Its perimeter is therefore 33 inches.

    a.   What is the area of this sector, to the nearest 0.1 square inch?

    b.   What is the central angle, to the nearest tenth of a degree?

    There is another circular sector that has the same 33-inch perimeter and that encloses the same area. However, it has a different radius and a different arc length.

    c.   Find its radius and arc length.

14. The figure shows three circular pipes, all with 12-inch diameters, that have been strapped together by a metal band. What is the length of the band?
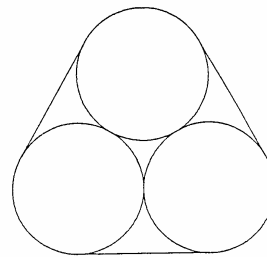
15. Terry just bought a used car with only 28,500 miles on it and it cost $8,200. Every year Terry expects to put another 14,000 miles on the car. Unfortunately, the car's value will depreciate by 15% per year. How much will the car be worth when it has 100,000 miles on it? Explain your work.

16. To the nearest tenth of a degree, find the sizes of the acute angles in the right triangle whose hypotenuse is 2.8 times as long as its short leg.

17. a. The figure at the right shows a long rectangular strip of paper, one corner of which has been folded over to meet the opposite edge, thereby creating a 30-degree angle. Given the width of the strip is 12 inches, find the length of the crease.
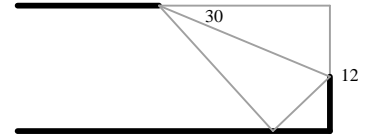
b. Instead of a 30-degree angle, suppose that the angle has an unspecified size t. Use trigonometry to find the length of the crease, expressed in terms of t.

c. Find the approximate value of t that makes the crease as short as it can be. Restrict your attention to values of t that are less than 45 degrees. Explain your method.

18. A parabola, with an equation of the form $y = ax^2 + bx + c$, has as its maximum point, often called the vertex, the point (2,18). It also has an x-intercept the point (-1,0).

Find the values for a, b, and c in the equation $y = ax^2 + bx + c$.

19. Centered 7 meters above the ground, a Ferris wheel of radius 6 meters is rotating with angular speed 24 degrees per second.

    a.   Assuming that you begin at time $t = 0$ seconds at the lowest point on the wheel, find a formula that describes the distance h (in meters) from you to the ground after t seconds of riding.

    b.   At what times are you 10 meters above the ground? Please explain clearly how you got your solution.

PLEASE RETURN THIS TEST WITHIN TWO WEEKS.

How much time did you spend? _____

Your signature confirms that you have done this work on your own.

_____
Signature