Highlighted in pink = answer

PHILLIPS EXETER ACADEMY
MATHEMATICS DEPARTMENT

Placement Test 1/2/3

Name Tanish Tyagi

Please note the time you begin working on the test and the time you finish. At the end of the test, we will ask you to record the amount of time you spent working. ***We are interested in your analyses of these problems, not just your answers, so you must show your reasoning fully and clearly.*** *You are expected to work on your own. You may use any kind of calculator.*

1. The cost of a pizza varies directly with the area of the pizza. If a pizza that is 12" in diameter costs \$10.80, how much does a pizza that is 16" in diameter?

cost $\frac{\$10.80}{36\pi} = \frac{\$x}{64\pi} \Rightarrow \frac{\$10.80 \cdot 64\pi}{36\pi} = 19.20$

area

Areas
- 12" pizza: $\pi\left(\frac{12}{2}\right)^2 = 36\pi$
- 16" Pizza: $\pi\left(\frac{16}{2}\right)^2 = 64\pi$

$\$x = 19.20$

A pizza with a 16 inch diameter will cost \$19.20

To solve this question, I used a proportion when \$10.8 corresponds to 36π. I needed to find the cost of a 16 inch pie, so I set that value to a variable x. I knew the diameter of the second pie, so I constructed a proportion. From there, I cross multiplied.

2. The points (-97,-201), (3,-1), and (19,32) are graphed on the coordinate plane. Are they on the same line? Explain your reasoning.

$(3,-1) = x_1, y_1$

$(-97,-201) = x_2, y_2$

$\text{slope} = \frac{-201-(-1)}{-97-3} = \frac{-200}{-100} = 2$

IGNORE TY

slope $= \frac{y_2 - y_1}{x_2 - x_1}$

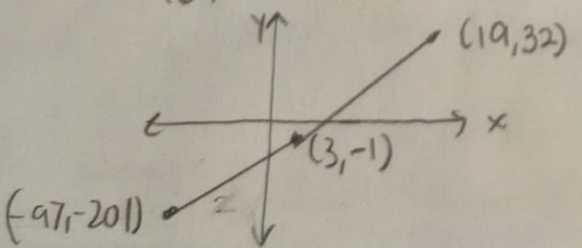
$(3,-1) = x_1, y_1$

$(19,32) = x_2, y_2$

$\text{slope} = \frac{32-(-1)}{19-3} = \frac{33}{16}$

$\frac{33}{16} \neq 2$

These 3 coordinates are not on the same line. In order for these coordinates to be on the same line, the slope between the coordinates has to be the same. The slope between (3, -1) & (-97, -201) is 2. The slope between (3,-1) & (19,32) is $\frac{33}{16}$, which is $\frac{1}{16}$ greater than 2.



In this graph, you can see the steepness of the line increases between (3,-1) & (19,32), proving that the points are not on the same line.

3. A hot-air balloon at 300 feet begins to rise at the rate of 100 feet per minute. At the same time, a second hot-air balloon at 2,000 feet starts to descend at the rate of 150 feet per minute.

a. When will the balloons be at the same height? Explain your answer.

m = minutes
y = height

1 $y = 300 + 100m$
2 $y = 2000 - 150m$

$2000 - 150m = 300 + 100m$
$1700 = 250m$
$m = 6.8$

After setting up equations for both balloon 1 & 2, you can set them equal to each other & solve, The solution says at 6.8 minutes after balloon 1 starts to ascend & balloon 2 start to descend, both balloons will be at the same height.

b. What is that height?

$y = 300 + 100(6.8)$
$y = 300 + 680$
$y = 980 \text{ ft.}$

$y = 2000 - 150(6.8)$
$Y = 2000 - 1020$
$y = 980 \text{ ft.}$

At 980 feet, both balloons will be at the same height.

c. What is the height of the ascending balloon when the descending balloon hits the ground?

$0 = 2000 - 150m$
$150m = 2000$
$m = 13\frac{1}{3}$

$y = 300 + 100(13\frac{1}{3})$
$y = 300 + 1333\frac{1}{3}$
$y = 1633\frac{1}{3}$

$\boxed{1633\frac{1}{3} \text{ feet}}$

4. The Ski Club has rented a luxury bus for their annual trip to Sugarloaf Mountain. The cost of the bus is $720 for a weekend. Each member going on the trip is going to pay an equal share of the expense. When eight members back out at the last minute, the expenses of the other members go up by $3 each.

a. Write an algebraic equation (or equations) which describe the situation. Please define your variables. m = original number of members, c = original cost, before member back out

1: $\frac{720}{m} = c$

2: $\frac{720}{m-8} = c + 3$

$\Rightarrow \frac{720}{m} + 3 = \frac{720}{m-8}$

b. How many members of the Ski Club go on the trip?

$\frac{720}{m-8} = \frac{720}{m} + 3$

$720 = \left(\frac{720}{m} + 3\right)(m-8)$

$720 = 720 - \frac{5760}{m} + 3m - 24$

$m(720) = \left(696 - \frac{5760}{m} + 3m\right)m$

$720m = 696m - 5760 + 3m^2$

$3m^2 - 24m - 5760 = 0$
$m^2 - 8m - 1920 = 0$

$(m-48)(m+40) = 0$
$m = 48, \underline{-40}$ extraneous

$48 - 8 = 40$

$\boxed{\text{40 members go on the trip}}$

3B Explaination

We already know the minute at which the balloons are at the same height, so we can substitute the value into the equations we derived in 3a. After we solve the equations, both values come around to 980 feet.

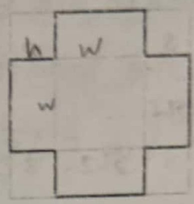
3C Explaination

When the desending balloon hits the ground, the height value must be 0 for that balloons equation. After solving for the minutes, they come out to $13\frac{1}{3}$. We can plug that minute value into the fistequation and get the height, which is $1{,}633\frac{1}{3}$ feet.

4B Explaination

We can use the equations derived in 4a to solve this equation. The first equation defines the value for variable c in terms of m, so we can substitute this value into the second equation. After solving this equation, we end up with $m^2-8m-1920=0$. We can factor this into $(m-48)(m+40)=0$. $M=48,-40$. -40 is extraneous because negative members cannot go on the trip. Therefore, members is equal to 48. But 8 members of the trip backed out, so $48-8=40$ members went on the trip.

5. The diagram at the right suggests an easy way of making a box with no top. Start with a square piece of cardboard, cut squares of equal sides from the four corners, and then fold up the sides. Here is the problem: We want to produce a box that is 8 cm. deep and whose capacity is exactly one liter (1000 cm^3). How large a square must we start with?



V=Volume
L= length
W= Width
H= Height

$V = l \cdot w \cdot h$

$1000 = l \cdot w \cdot 8$

$l \cdot w = 125$

$l \& w = 5\sqrt{5}$

$5\sqrt{5} + 2h \Rightarrow 5\sqrt{5} + 16$

Original Dimensions are $5\sqrt{5} + 16$ cm or ≈ 27.18 cm.

6. A softball crosses home plate at a height of 4 feet, and the batter hits the ball. The path of the ball is described by $h = -\frac{1}{729}(x-162)^2 + 40$, where x represents the distance from home plate and h the height of the ball above the ground.

a. The outfield wall is 6 feet high and 318 feet from home plate. Will the ball go over the wall for a home run? If so, by how many feet will it clear the wall? Explain your reasoning and method.

$h = -\frac{1}{729}(318-162)^2 + 40$, $h = 6.617$

The softball will clear the wall by 0.617 feet. Since x represents the distance from home plate, we can substitute 318 for x. When this equation is simplified, h=6.617, meaning that it will clear the wall by 0.617 feet.

b. Suppose the outfield wall is 326 feet from home plate. Would it be possible for an outfielder to catch the ball? If so, at what height above the ground would the ball be when she caught it with her back against the wall? Explain your reasoning and method.

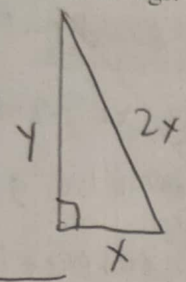
$h = -\frac{1}{729}(326-162)^2 + 40$, $h = 3.11$

The infielder will be able to catch the ball, and when she catches the ball with her back against, the wall, the ball will be 3.11 feet above the grand. I got this answer by substituting 326 for x. If the height was above 6 feet, then the outfielder would not be able to catch the ball.

5 EXPLAINATION:

In order for the compacity to equal $1{,}000\,cm^3$, the length, width, & height of the box have to equal $1000\,cm^3$. The prompt tells us that the height equals 8 cm, so length · width = 125. Since the box is a square, the length & width are both congruent, so l & w = $5\sqrt{5}$. The height is congruent to the length & width of the 4 squares, so the dimensions are $5\sqrt{5}+16$ cm by $5\sqrt{5}+16$ cm.

7. A right triangle is known to have a perimeter of 10 units and its hypotenuse is twice as long as one of its legs. Find the lengths of the sides, showing an algebraic solution.



$x^2+y^2=4x^2$

$x+y+2x=10$

$3x+y=10$

$y=-3x+10$

$x^2+(-3x+10)^2=4x^2$

$x^2+(9x^2-60x+100)=4x^2$

$10x^2-60x+100=4x^2$

$6x^2-60x+100=0$

$3x^2-30x+50=0$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=3, b=-30, c=50$

$\frac{30\pm\sqrt{(-30)^2-4(3)(50)}}{6}$

$\frac{30\pm\sqrt{900-600}}{6}$

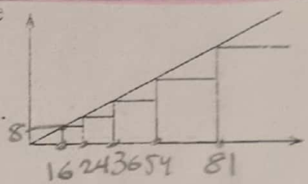
$\frac{30\pm 17.32}{6} = 7.89$ OR 2.113 extraneous

4.226
+2.113
6.339

$10-6.339=\approx 3.66$

Legs: 2.113 & 3.659 cm
Hypotenuse: 4.226 cm

8. The figure at the right shows a sequence of squares inscribed under the line $y = x/2$ and above the x-axis. Every square has two vertices on the x-axis and one on the line $y = x/2$. The smallest square is 8 cm tall.



a. How tall are the next four squares?

Height of Next 4 Squares: 12 cm, 18 cm, 27 cm, 40.5 cm

Since the height of the first square is 8cm, it means that the first vertice of the first square is 16. From then on, the heights start to increase by the x-value/2.

b. How tall is the **nth** square?

Formula for N^{th} Square: $8 \cdot 1.5^{(n-1)}$

A pattern among the heights of the squares is that they grow by 50%. Therefore, the heights follow a pattern of exponential growth. Also, the height of the first square is 8 cm.

Scanned with CamSca

7 Explaination

Using the Pythagorean Theorem, $x^2+y^2=2x^2$. However, we need another equation as we have 2 variables. The question tells us that the perimeter of the triangle is 10 units, so $x+y+2x=10$. With this system of equations, we can solve them and ultimately get $3x^2-30x+50=0$. We can use the quadratic formula on this quadratic & ultimately get $x=7.89$ or $x=2.113$. $x=7.89$ is extraneous because then the perimeter of the triangle would be greater than 10. So $x=2.113$, the hypotenuse is 4.226, & $y=3.659$ units respectively.

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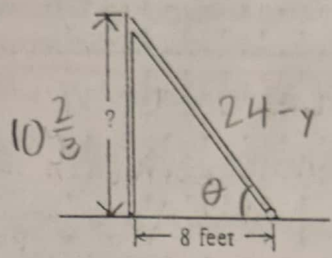
9. A flagpole 24 feet high snapped in a storm, its top touching the ground 8 feet from its base.



a. How far up the pole is the break?

$y^2+64=576-48y+y^2$

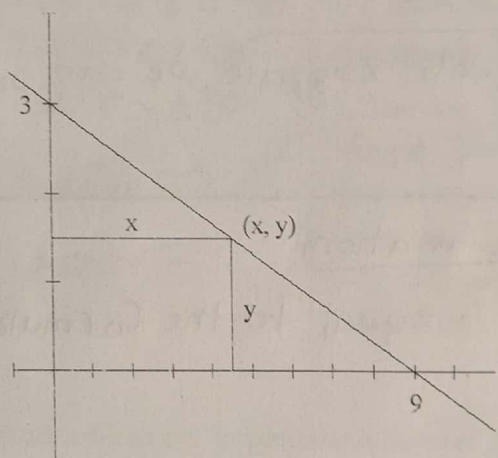
$48y=512 \quad y=10\frac{2}{3}$ $\boxed{10\frac{2}{3}\text{ feet}}$

b. Find the angle between the ground and the top of the flagpole.

$\tan^{-1}(\theta)=\frac{10^{2/3}}{8}$ $\boxed{\theta=53.13^\circ}$

$\theta=\tan^{-1}\left(\frac{10^{2/3}}{8}\right)$

10. The diagram shows a line with intercepts 3 and 9 and one of many rectangles which can be inscribed under this line and within the first quadrant.



a. Write an equation for the line.

$\frac{3}{9}=\frac{3-y}{x} \quad \frac{3}{9}=\frac{y}{9-x}$

$27-3x=9y$ $\boxed{y=-\frac{1}{3}x+3}$

$27-3y=3x$

b. Determine if the rectangle can have $x=4$ and $y=2$ as its dimensions.

$2=-\frac{1}{3}(4)+3,\ y=-\frac{4}{3}+3,\ y=1\frac{2}{3}$

These dimensions are not possible.

c. Find x and y if the rectangle is a square.

$x=y$, for rectangle = square

$x=-\frac{1}{3}x+3$ $\boxed{x \,\&\, y=\frac{9}{4}\text{ OR } 2.25}$

$\frac{4}{3}x=3,\ x=9/4$

d. Write an expression for A, the area of the rectangle, using x as the only variable.

$\text{Area}=x\cdot y,\ y=-\frac{1}{3}x+3,$ $\boxed{\text{Area}=x\cdot\left(-\frac{1}{3}x+3\right)}$

e. Find x if the area of the rectangle is 6.

$x=3,6$

$6=x\cdot\left(-\frac{1}{3}x+3\right) \quad -\frac{1}{3}x^2+3x-6=0$ $\boxed{x=3,6}$

$-x^2+9x-18=0$

$6=-\frac{1}{3}x^2+3x$

$(-x+6)(x-3)=0$

f. Which x gives the rectangle its largest area?

After simplifying the formula derived in 10d, you get $-\frac{x^2}{3}+3x+0$. In order to find the max value of the x-coordinate, you can use $\frac{-b}{2a}$, since this is a quadratic equation.

$-\frac{x^2}{3}+3x+0=0,\ a=-\frac{1}{3},\ b=3,\ c=0.\ \frac{-b}{2a}=X_{max}$

$\frac{-3}{-2/3}=X_{max}$ $\boxed{X_{max}=-3\cdot-\frac{3}{2}=\frac{9}{2}=4.5}$

10 A EXPLAINATION

Both of the triangles shown in the diagram are similar to the big triangle b/c of Angle-Angle Similarity. Based on this we can create a proportion based off of corresponding sides.

10 B EXPLAINATION:

Using the formula derived in 10a, we can set $y=2$ & $x=4$ and see if both sides are equal. Both sides are not equal, meaning the dimensions are not possible.

10 C EXPLAINATION

Since we're looking for the coordinates of a square, x & y have to be equal. We can substitute x in place of y & solve.

10 D EXPLAINATION

The area equals $x \cdot y$, are we can substitute the equation used in 10c for y.

10 E Explanation

We can set 6 equal to the formula we derived in 10c.

10F Continued: You can also use deriatives to solve this problem. Since derivatives represent slope, at the vertex the slope would be 0. $\frac{d}{dx}\left(-\frac{x^2}{3}+3x\right)=0 = \left(-\frac{1}{3}\cdot 2x\right)+3=0$

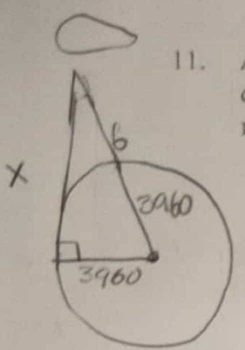
$-\frac{2x}{3}+3=0$

$\frac{-2x}{3}=-3$

$-2x=-9$

$\boxed{x=\frac{9}{2} \text{ OR } 4.5}$

I was also interested in solving this problem using a computer program, and you can check it out here: repl.it/@TanishTyagi123/area



11. A plane is flying 6 miles above the surface of the earth. A passenger looks out the window to the distant horizon. On a clear day, how far away is that horizon, to the nearest mile? Assume the radius of the earth is 3960 miles.

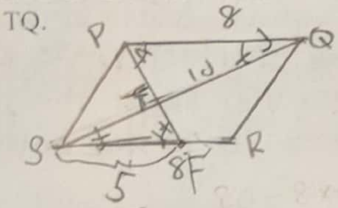
$3960^2 + x^2 = 3966^2$

$x^2 = 47556$

$\boxed{x = 218.073 \text{ miles}}$

In order to use the Pythagorean Theorem, a triangle needs to be right. In order to make this a right triangle, the horizon has to be drawn tangent to the circle. Also, the horizon is the line from which the line of sight is tangent to the Earth.

12. Parallelogram PQRS has PQ = RS = 8 cm. and diagonal QS = 10 cm. Point F is on RS, exactly 5 cm. from S. Let T be the intersection of PF and QS. Draw a diagram and find the lengths of TS and TQ.



$\overleftrightarrow{QT} = x$

$\overleftrightarrow{TS} = 10 - x$

S	R
1. $\angle TSF \cong \angle PQT$	1. Alternate Interior Angles
2. $\angle TFS \cong \angle QPT$	2. Alternate Interior Angles
3. $\triangle TSF \sim \triangle TQP$	3. Similar Triangle by Angle Angle Theorem
4. $\overleftrightarrow{QP} \sim \overleftrightarrow{FS}$	4. Corresponding parts of Similar Triangles
5. $\overleftrightarrow{QT} \sim \overleftrightarrow{TS}$	5. Corresponding parts of Similar Triangles

$\dfrac{\overleftrightarrow{QP}}{\overleftrightarrow{FS}} = \dfrac{\overleftrightarrow{QT}}{\overleftrightarrow{TS}}$

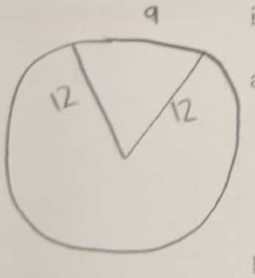
$\dfrac{8}{5} = \dfrac{x}{10-x} \Rightarrow 80 - 8x = 5x$

$\Rightarrow x = \dfrac{80}{13}$

$\overleftrightarrow{TQ} = \dfrac{80}{13}$ or 6.15 cm

$\overleftrightarrow{TS} = \dfrac{50}{13}$ or 3.85 cm

13. A sector of a circle is enclosed by two 12-inch radii and a 9-inch arc. Its perimeter is therefore 33 inches.



a. What is the area of this sector, to the nearest 0.1 square inch?

Total Circumference: 24π

Total Area: 144π

$\dfrac{9}{24\pi} = \dfrac{x}{144\pi}$

$1296\pi = 24\pi x$

$\boxed{x = 54 \text{ in}^2}$

b. What is the central angle, to the nearest tenth of a degree?

$\dfrac{54}{144\pi} = \dfrac{x}{360}$

$144\pi x = 19440$

$x = \dfrac{135}{\pi}$

$\boxed{x = 43°}$

There is another circular sector that has the same 33-inch perimeter and that encloses the same area. However, it has a different radius and a different arc length.

c. Find its radius and arc length.

r = radius

a = arc length

$2r + a = 33$

$\dfrac{a}{2\pi r} \cdot \pi r^2 = 54$

$a\pi r^2 = 108\pi r$

$a = \dfrac{108r}{r^2} = \dfrac{108}{r}$

$2r + \left(\dfrac{108}{r}\right) = 33$

$2r^2 + 108 = 33r$

$2r^2 - 33r + 108 = 0$

$(2r - 9)(r - 12) = 0$

$r = \dfrac{9}{2}$, 12 → already used

$9 + a = 33$

$a = 24$

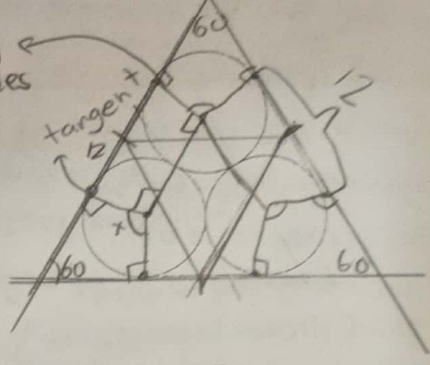
Radius $\dfrac{9}{2}$ in

Arc: 24 in

12 Explaination

The first step to solving this problem is to prove that Δ'S TSF & TQP are similar. After using Angle-Angle similarity to prove similarity, use can set up a proportion using corresponding parts of both triangles. We know that QS= 10, and QT & TS = QS. So we can associate x to QT & 10 -x to TS and solve the new proportion.

14. The figure shows three circular pipes, all with 12-inch diameters, that have been strapped together by a metal band. What is the length of the band?



$x = 360 - (180 + 60)$
$x = 360 - 240$
$x = 120°$

rectangle, opposite sides are congruent.

Vague

Circumference of Each Circle: 24π
$12\pi / 3 = 4\pi$ → Length of curved sides
Flat sides $= 2r = 12$ in
$36 + 12\pi = \boxed{73.699 \text{ in} = \text{Length of Band}}$

15. Terry just bought a used car with only 28,500 miles on it and it cost $8,200. Every year Terry expects to put another 14,000 miles on the car. Unfortunately, the car's value will depreciate by 15% per year. How much will the car be worth when it has 100,000 miles on it? Explain your work.

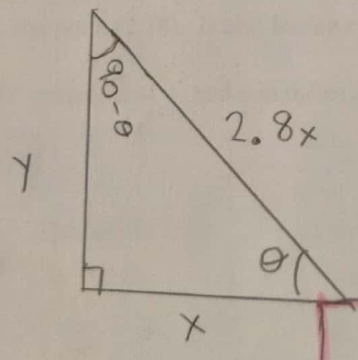
assumming 15% of previous value

$100{,}000 - 28{,}500 = 71{,}500$
$71{,}500 / 14{,}000 = 5.10714285\overline{7}$ years
$8{,}200 \cdot (0.85)^{5.107...} = \boxed{\$3{,}575.58}$

In order to solve this problem, I first calculated the number of years it would take Terry's car to reach 100,000 miles. That number came out to be 5.107, and since the car value depreciates exponentially, I knew that the exponential decay would occur. 8200 was the starting value of the car, the car's value

continued on back

16. To the nearest tenth of a degree, find the sizes of the acute angles in the right triangle whose hypotenuse is 2.8 times as long as its short leg.



$\cos(\theta) = \frac{1}{2.8}$
$\theta = \cos^{-1}\left(\frac{1}{2.8}\right)$
$\theta \approx 69.1°$
$90 - \theta = 90 - 69.075 = 20.92°$

Measures of 2 Acute Triangles
$\approx 69.1°$ & $\approx 20.9°$

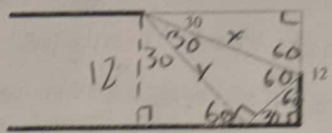
11 Explainations

In order to solve this question, I first extended the lengths of the bands to form a triangle that the circles are inscribed in. The next step was to find the angle measures for each angle of the triangle, and you can use the triangle midsegments theorem for this. The triangle midsegments theorem says that the 3 midsegments of a triangle will split the triangle into 4 congruent triangles. The angle measures for the big triangle are corresponding for every small triangle, which means all the measures are congruent. 180/3=60°. With this, we can look for the central angle by drawing 2 radii from the center to a point on the circle, creating tangent lines, which give angle measures of 90°. Therefore, the central angle = 120°. The circumference of the circle is 12π, and the part is 1/3 of circumference, so it is 4π. 4π · 3 = 12π, & the straight lines are just 2 radii combined, so they equal 12in. 36 + 12π = 73.699 in.

15 Cont.

Went down 15% per year, so 1 − 0.15 = 0.85, and in ≈ 5.107 years, the car's mileage will reach 100,000, so that gets raised to 0.85.

17. a. The figure at the right shows a long rectangular strip of paper, one corner of which has been folded over to meet the opposite edge, thereby creating a 30-degree angle. Given the width of the strip is 12 inches, find the length of the crease.



$\sin(60°) = \frac{12}{y}$

$y = \frac{12}{\sin(60°)}$

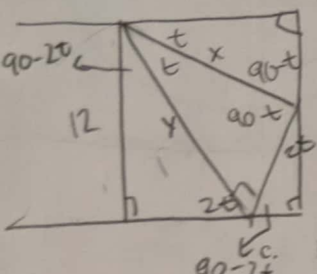
$y = 13.856$

$\sin(60°) = \frac{13.856}{x}$

$x = \frac{13.856}{\sin(60°)}$

$\boxed{x = 16 \text{ inches}}$

b. Instead of a 30-degree angle, suppose that the angle has an unspecified size t. Use trigonometry to find the length of the crease, expressed in terms of t.



$180-(180-2t)$
$2t$

$\sin(2t) = \frac{12}{y}$

$y = \frac{12}{\sin(2t)}$

$\sin(90-t) = \frac{12}{x(\sin(2t))}$

$x(\sin(2t)) = \frac{12}{\sin(90-t)}$

$\boxed{x = \frac{12}{\sin(90-t) \cdot \sin(2t)}}$

DEGREE MODE

c. Find the approximate value of t that makes the crease as short as it can be. Restrict your attention to values of t that are less than 45 degrees. Explain your method.

After graphing the equation, we can see the lowest x & y-value. The lowest y-value is 15.588 and the corresponding x-value is 35.264.

18. A parabola, with an equation of the form $y = ax^2 + bx + c$, has as its maximum point, often called the vertex, the point (2,18). It also has an x-intercept the point (-1,0).

Find the values for a, b, and c in the equation $y = ax^2 + bx + c$.

$\frac{-b}{2a} = 2, f\left(\frac{-b}{2a}\right) = 18$

$y = a(x-h)^2 + k$

$(h,k) =$ vertex

$(x,y) =$ coordinates of x-intercept

$0 = a(-1-2)^2 + 18$

$0 = 9a + 18$

$a = -2$

Substituting b into $\frac{-b}{2a} = 2$

$\frac{-b}{-4} = 2$

$-b = -8$

$b = 8$

$f(2) = 18$

$18 = -2x^2 + 8x + c$

$18 = -2(2)^2 + 8(2) + c$

$18 = -8 + 16 + c$

$18 = 8 + c$

$c = 10$

$a = -2$
$b = 8$
$c = 10$

$y = -2x^2 + 8x + 10$

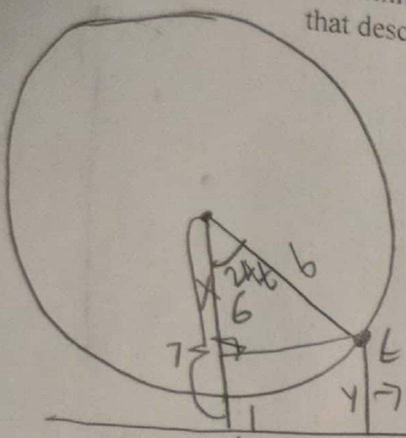
17 Explaination

You can use 30-60-90 triangles to solve this question. You can also use the special properties of 30-60-90 triangles to find the value of the initial crease. To solve the second part of the question, we can use the fact that triangles add up to 180°, and start substituting variables in for angle measures. Then we can use trigonometry to get a variable solution for the crease.

This is a very fast Ferris Wheel! I would advise the police to check it out!

19. Centered 7 meters above the ground, a Ferris wheel of radius 6 meters is rotating with angular speed 24 degrees per second.

a. Assuming that you begin at time $t = 0$ seconds at the lowest point on the wheel, find a formula that describes the distance h (in meters) from you to the ground after t seconds of riding.



$\cos(24t) = \frac{x}{6}$

$x = 6(\cos(24t))$

$\boxed{7 - 6(\cos(24t)) = y}$

Since the total height from the center to the ground is 7, in order to find the height @ any given time, we need to subtract 7 from something. By drawing a perpendicular line from point t, we can create a right triangle & use trigonometry to solve.

b. At what times are you 10 meters above the ground? Please explain clearly how you got your solution.

$10 = 7 - 6(\cos(24t))$

$3 = -6(\cos(24t))$

$\cos(24t) = -\frac{1}{2}$

$24t = \cos^{-1}(-\frac{1}{2})$

$24t = 120 \text{ or } 240$

$\boxed{t = 5 \ \& \ 10 \text{ seconds}}$

In order to solve this problem, I used the formula derived in part a. I substituted 10 for y and solved the equation for t. After getting $24t = \cos^{-1}(-\frac{1}{2})$, we have to realize that the ferris wheel will reach a height of 10 meters at 2 spots, so this equation will have 2 solutions. Since $\cos^{-1}(-\frac{1}{2}) = 120°$, we have to subtract $360 - 120 = 240°$. to get the second solution. As a result, t = 5 & 10 seconds.

PLEASE RETURN THIS TEST WITHIN TWO WEEKS.

How much time did you spend? 1 hours, 45 minutes, 41 seconds

Your signature confirms that you have done this work on your own.

Tanish Tyagi

Signature