Event: The possible outcomes of a trial are called events.

Equally likely events: The events are said to be equally likely if there is no reason to expect any one in preference to any other

Exhaustive events: It is the total no. of all possible outcomes of any trial

Mutually Exclusive Events: Two or more events are said to be mutually exclusive if they cannot happen simultaneously in a trial.

Favourable events: The cases which ensure the occurrence of the events are called favourable

Sample Space: The set of all possible outcomes of an experiment is called a sample space.

Probability of occurrences of event A, denoted by P(A), is defined as

$$P(A) = \frac{\text{No. of favourable cases}}{\text{No. of exhaustive cases}} = \frac{n(A)}{n(S)}$$

Theorems: In a random experiment, if S be a sample space and A is an event, then

i) $P(A) \geq 0$

ii) $P(\phi) = 0$,

iii) $P(S) = 1$

ii). If A and B are mutually exclusive events, then $P(A \cap B) = 0$

iii). If A and B are two mutually exclusive events, then $P(A) + P(B) \leq 1$

iv). If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

v). For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(vi). For each event A, $P(\bar{A}) = 1 - P(A)$ where $\bar{A}$ is the complimentary event.

(vii) $0 \leq P(A) \leq 1$.

Compound Event: The simultaneous happening of

i) two or more events is called a compound event if they occur in connection with each other.

ii) **Conditional Probability:**

Let A and B be two events associated with the sample space, then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Ex: Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E|F)$ and $P(F|E)$.

Sol: i) $P(E|F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{0.2}{0.3} = 2/3$

ii) $P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{2}{6} = \frac{1}{3}$

Ex: Compute $P(A|B)$ if $P(B) = 0.5$ and $P(A \cap B) = 0.32$

Sol: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.32}{0.5} = \frac{16}{25}$.

Ex: If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find

i). $P(A \cap B)$, ii). $P(A|B)$, iii) $P(A \cup B)$.

Sol: Given $P(B|A) = 0.4$

$\therefore \frac{P(A \cap B)}{P(A)} = P(B|A)$

$\Rightarrow \frac{P(A \cap B)}{0.8} = 0.4$

$P(A \cap B) = 0.4 \times 0.8$

$= 0.32$

ii). $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$

iii). $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.5 - 0.32$

$= 1.30 - 0.32 = 0.98$

Ex: If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find

i). $P(A \cap B)$, ii). $P(A|B)$, iii). $P(B|A)$

Sol: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11}$

$= \frac{4}{11}$

ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$

iii). $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{11}}{\frac{6}{11}}$

$= \frac{4}{6} = 2/3$

Ex: Determine $P(E|F)$ when A coin is tossed three times, where

i). E: head on third toss; F: heads on first two tosses.

ii). E: at least two heads; F: at most two heads

iii). E: at most two tails; F: at least one tail.

Sol: i) E: head on third toss

$\therefore$ Sample Space: $\{HHH, HTH, THH, TTH\}$

F: heads on first two tosses

$\therefore S = \{HHH, HHT\}$

Total No of Cases $= 2^3 = 8$

$\therefore P(F) = \frac{2}{8} = \frac{1}{4}$

$P(E \cap F) = \frac{1}{8}$

$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$

ii) E: at least two heads

$\therefore S = \{HHT, HTH, THH, HHH\}$

F: at most two heads

$\therefore S = \{TTT, HTT, THT, TTH, HHT, HTH, THH\}$

$\therefore P(F) = 7/8$

$P(E \cap F) = 3/8$

$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{7/8} = 3/7$

iii). E: at most two tails

$S = \{HHH, THH, HTH, HHT, HTT, THT, TTH\}$

F: at least one tail

$S = \{THH, HTH, HHT, HTT, THT, TTH, TTT\}$

$P(E) = 7/8$, $P(E \cap F) = 6/8$

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{6/8}{7/8} = 6/7$