

BTECH
(SEM III) THEORY EXAMINATION 2022-23
MATHEMATICS-III

Time: 3 Hours**Total Marks: 100****Note: 1.** Attempt all Sections. If require any missing data; then choose suitably.**SECTION A****1. Attempt all questions in brief.****2 x 10 = 20**

- (a) Find $L\{t \cos at\}$.
- (b) Evaluate $L^{-1}\left\{\frac{1}{s-1} + \frac{2}{s+3} + \frac{2}{s^3}\right\}$.
- (c) Write the formula for Fourier integral representation of a function $f(x)$ with appropriate properties of $f(x)$.
- (d) Find Fourier transform of $f(x) = \begin{cases} 1, & -1 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$
- (e) Define a Ring.
- (f) State Lagrange's theorem.
- (g) Write the conditions for a relation to be an equivalence relation.
- (h) If $f(x) = \cos(\log x)$, then prove that $f(a)f(b) - \frac{1}{2}\left\{f\left(\frac{a}{b}\right) + f(ab)\right\} = 0$.
- (i) Draw Hasse diagram for the POSET (A, \leq) , where $A = \{1, 2, 3, 9, 18\}$ and relation " \leq " is the "divides" relation.
- (j) Let $A = \{2, 3, 4, 6, 8, 24, 48\}$ with partial ordering of divisibility. Determine all the maximal and minimal elements of A .

SECTION B**2. Attempt any three of the following:****10x3=30**

- (a) If $L\{f(t)\} = F(s)$, then prove that $L\left\{\int_0^t f(u)du\right\} = \frac{1}{s}F(s)$.
- (b) Find inverse Fourier sine transform of $\frac{s}{1+s^2}$.
- (c) Show that the set $G = \{1, -1, i, -i\}$ of all 4th roots of unity forms a multiplicative abelian group. Also find the order of each element of G .
- (d) Establish the following formula by the method of mathematical induction:

$$1 + 3 + 5 + \dots + (2n - 1) = n^2.$$
- (e) Determine the disjunctive normal form of the following Boolean expression:

$$x \wedge (y \vee z).$$

SECTION C**3. Attempt any one part of the following:****10x1=10**

- (a) By using the method of Laplace transform, solve the initial value problem

$$y'' + 2y - 3y = 3, y(0) = 4, y'(0) = -7.$$
- (b) Use convolution theorem to find inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$.

4. Attempt any *one* part of the following:

10x1=10

- (a) If $f(x) = e^{-|x|}$, then prove that the Fourier transform $F\{f(x)\} = \frac{2}{1+s^2}$.
- (b) Solve the following difference equation by using Z- transform:
 $y_{n+2} - 3y_{n+1} + 2y_n = 0, \quad y_0 = -1, y_1 = 2.$

5. Attempt any *one* part of the following:

10x1=10

- (a) Show that the proposition $p \vee \sim (p \wedge q)$ is tautology.
- (b) Find the congruent solutions of the following:
(i) $2x + 1 \equiv 4 \pmod{5}$,
(ii) $51x \equiv 32 \pmod{7}$.

6. Attempt any *one* part of the following:

10x1=10

- (a) Find the solution for the recurrence relation:
$$\begin{cases} x_n = 2x_{n-1} - 5x_{n-2}, & n \geq 2 \\ x_0 = 1 \\ x_1 = 5 \end{cases}$$
- (b) Find the coefficient of x^{2005} in the generating function $G(x) = \frac{1}{(1-x)^2(1+x)^2}$.

7. Attempt any *one* part of the following:

10x1=10

- (a) Given the Boolean expression $f = ABC + B\bar{C}D + \bar{A}BC$. Then,
(i) Make a truth table,
(ii) Simplify using k -map,
(iii) Make a switching circuit of the expression.
- (b) Define distributive lattices. Prove that in a distributive lattice $(L, \wedge, \vee), (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ holds for all $a, b, c \in L$.