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A New Technique for Finding the Optimal Solution to Assignment Problems with Maximization Objective Function

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Abstract. The assignment problems (AP) are an important part of linear programming problems (LPP) that deal with the allocation of different resources for different activities based on one to one. The assignment problem is established in a variety positions when decision makers need to determine the optimal allocation and this means assigning only one task to one person to achieve maximum profits or imports or achieve less time or less cost based on the type of problem. In this work, a new technique has been provided to find an optimal solution for the assignment problems of maximization objective function. Comparing the proposed technique results with the Hungarian method indicates that the new technique has easier and less steps to find the optimal solution and thus the time is reduced and the effort is largely reduced.

Keywords. Linear Programming Problems. Mathematical Model. Maximization of Assignment Problems. Hungarian Method. Optimal Solution.

1. Introduction

Assignment problems are an integral part of optimization problems in operations research. In real life the application assignment problems can be used to allocate machines to jobs, vehicles to roads, products to factories, network computers, and airplanes for specific trips, etc. where the referrals have varying degrees of proficiency to get the tasks done [1, 2]. Previously, a large number of methods were presented so far to solve assignment problems, among which is the Hungarian method, which was derived by mathematicians D. Konig and E. Egervary [3, 4]. Although the name "Assignment Problem" appeared in 1952 in a paper of Votaw and Orden [1, 5, 6], that is the beginning of the development of practical solution methods and differences for the classic assignment problem. Assignment problems are concerned with finding a one-to-one distribution to achieve maximum profits and revenues or to reduce the cost or time to complete tasks, where the problem of assignment can be a problem of maximization or a problem of minimization [7, 8]. Various methods of solving these problems have been proposed by a number of researchers who have been interested in studying them. The authors published many papers in varied scientific fields such as conjugate gradient methods [9- 13], trust- region methods [14- 18], line search methods [19- 22], projection methods [23- 27], and reliability methods [28- 34], but in this research study, a new simple and effective technique is proposed to find the optimal solution for balanced and unbalanced assignment problems with an objective function of the type of maximization. The proposed



technique takes little effort and time when compared to the Hungarian method and other methods of solving assignment problems.

2. The Algorithm of the New Technique

The following solution steps are specific to the technique proposed in this paper:

Step 1: Build AP schedule if not given.

Step 2: Transform the *max* problem into the *min* problem by subtracting all items from the largest item in AP schedule.

Step 3: Convert the AP to balanced problem if it is unbalanced.

Step 4: At every row determine the two minimum values and output the difference between them (called the penalty), and at every column define the two maximum values and output the difference between them (called the penalty).

Step 5: Determine which row or column corresponds to the largest difference obtained in Step 4.

Step 6: Determine the lowest value in a special box in the column or row that corresponding to the larger difference particular in Step 5. If there is a repetition in the largest difference, choose the difference that corresponding to the column or row that contains the lowest value. If there is a repetition at the lowest value as well, choose the lowest value in the column or row corresponding to the biggest difference in which the sum is the largest to all values not deleted in the column and the row that both share the lowest value cell.

Step 7: Delete both the column and row that share the least value cell identified in the previous Step.

Step 8: Repeat Steps 4 through 7 until have only one value within a marked square for each column and row.

Step 9: Apply the objective function of AP with respect to the AP schedule in Step 1 after determining the corresponding values chosen in Step 6.

3. Numerical Examples

Example 1. Find the maximization of the assignment problem in the following table:

TABLE 1 AP matrix of example 1

	T_1	T_2	T_3	T_4
R_1	23	26	28	22
R_2	25	21	23	24
R_3	20	27	22	27

Solution:

TABLE 2 The AP matrix of example 1 converted into a minimization problem

	T_1	T_2	T_3	T_4
R_1	5	2	0	6
R_2	3	7	5	4
R_3	8	1	6	1

The AP in this example is unbalanced because the number of columns does not equal to the number of rows. So, the AP schedule will be balanced in Table 3.

TABLE 3 The Balanced AP Schedule

	T_1	T_2	T_3	T_4
R_1	5	2	0	6
R_2	3	7	5	4
R_3	8	1	6	1
R_4	0	0	0	0

The mathematical model for Example 1 is:

Objective Function:

$$Z = \sum_{i=1}^4 \sum_{j=1}^4 c_{ij} x_{ij}$$

Restrictions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$

$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$

$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$

According to the new algorithm, the following Table 4 is obtained:

TABLE 4 The AP matrix of example 1 according to the new technique

	T_1	T_2	T_3	T_4	Penalty		
R_1	5	2	0	6	2	5	-
R_2	3	7	5	4	1	1	1
R_3	8	1	6	1	0	5	7
R_4	0	0	0	0	0	-	-
Penalty	3	5	1	2			
	3	-	1	2			
	5	-	-	3			

$$\therefore Z = 28 + 25 + 27 + 0 = 80$$

Example 2. Find the maximization of the assignment problem in the following table:

TABLE 5 AP matrix of example 2

	T_1	T_2	T_3	T_4	T_5
R_1	15	12	4	8	19
R_2	14	18	7	16	10
R_3	16	8	19	11	5
R_4	11	6	15	18	9
R_5	17	14	12	17	13

Solution:

TABLE 6 The AP matrix of example 1 converted into a minimization problem

	T_1	T_2	T_3	T_4	T_5
R_1	4	7	15	11	0
R_2	5	1	12	3	9

R_3	3	11	0	8	14
R_4	8	13	4	1	10
R_5	2	5	7	2	6

AP in this example is balanced because the number of columns equals to the number of rows.

The mathematical model for Example 2 is:

Objective Function:

$$Z = \sum_{i=1}^5 \sum_{j=1}^5 c_{ij} x_{ij}$$

Restrictions:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 1$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 1$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 1$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} = 1$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 1$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 1$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 1$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 1$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 1$$

According to the new algorithm, the following Table 7 is obtained:

TABLE 7 The AP matrix of example 1 according to the new technique

	T_1	T_2	T_3	T_4	T_5	Penalty			
R_1	4	7	15	11	0	4	-	-	-
R_2	5	1	12	3	9	2	2	2	-
R_3	3	11	0	8	14	3	3	-	-
R_4	8	13	4	1	10	3	3	7	7
R_5	2	5	7	2	6	0	0	0	0

	3	2	3	3	4
Penalty	3	2	5	5	-
	3	8	-	1	-
	6	-	-	1	-

$$\therefore Z = 19 + 18 + 19 + 18 + 17 = 91$$

4. Comparison the Results

In this section, the results of solving the examples mentioned in this paper were compared using both of the proposed new technique and the Hungarian method to demonstrate the effectiveness and efficiency of the new technique. It has been observed that these results are equal because they represent the outcome of the optimal solution to the assignment problem. But what distinguishes is that the proposed technique has a clear idea, simple application and less steps comparing with Hungarian method.

TABLE 8 Comparison

Name	Hungarian method	The New Technique
Ex 1	80	80
Ex 2	91	91

5. Conclusion

In this work, a new technique is presented to solve assignment problems with an objective function of the type of maximization. This new technique can be used to solve both of balanced and unbalanced assignment problems. Many examples were solved using the new technique, and most of them the optimal solution is equal to the optimal solution obtained by Hungarian method as we saw in the previous section of comparing the results. It has been realized that this proposed technique leads to the optimal solution in fewer steps compared to the Hungarian method because it includes few and simple solution steps and thus time and effort are significantly saved.

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