OPTIMIZATION TECHNIQUES REPORT
ASSIGNMENT PROBLEM

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INTRODUCTION

An assignment problem is a particular case of transportation problem. The objective is to assign a number of resources to an equal number of activities . So as to minimize total cost or maximize total profit of allocation.
The problem of assignment arises because available resources such as men, machines etc. have varying degrees of efficiency for performing different activities, therefore, cost, profit or loss of performing the different activities is different.
The assignment problem can be stated in the form of mxn matrix cij called a Cost Matrix (or) Effectiveness Matrix where cij is the cost of assigning i th machine to j th job.

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}$$

Mathematical Formulation
Consider an assignment problem of assigning n jobs to n machines (one job to one machine). Let cij be the unit cost of assigning ith machine to the jth job and,i th machine to j th job. Let xij = 1 , if j th job is assigned to i th machine. xij = 0 , if j th job is not assigned to i th machine.

Minimize Z = Xn i=1Xn j=1cijxij
subject to the constraints
Xn i=1 xij = 1, j = 1, 2, ..., n
Xn j=1 xij = 1, i = 1, 2, ..., n
xij = 0 or 1.

Difference between transportation and assignment problem

S no.	Transportation Problem	Assignment Problem
1	Supply at any source may be any positive quantity ai.	Supply at any source (machine)will be 1. i.e.,ai = 1.
2	Demand at any destination may be any positive quantity bj.	Demand at any destination (job) will be 1. i.e.,bj = 1
3	One or more sources to any number of destinations.	One source (machine) to only one destination (job).

Aim

The aim of the study is to improve the Hungarian method for linear assignment problems so that it becomes more efficient. To achieve the aim, the following objectives have been set: – to increase the number of the smallest uncovered elements; – to create more than one zero elements at each iteration; – to illustrate the proposed algorithm by an example.

METHODS

Hungarian Method

The Hungarian method for solving the assignment problem was developed and published in 1955 [1]. It was named the Hungarian method because two theorems by two Hungarian mathematicians [2, 3] were used. In 1957 [4], it was noticed that this algorithm was strongly polynomial and has a complexity of order O(n4). This is the reason why the Hungarian method is also known as the Kuhn-Munkres algorithm. Later on, in 1971 [5] and 1972 [6], it was improved to a complexity of order O(n3). A lot of work has been done on this algorithm after that but up to now this algorithm still has an obvious weakness. A smallest uncovered element is selected to create a single zero at every iteration. In this paper, this weakness is alleviated by selecting more than one smallest uncovered element thus creating more than one zero at every iteration to come up with what we now call the

Accelerating Hungarian (AH) method. Assignment model and the Hungarian method have application in addressing the Weapon Target Assignment (WTA) problem. This is the problem of assigning weapons to targets while considering the maximum probability of kill. The assignment problem is also used in the scheduling problem of physicians and medical staff in the outpatient department of large hospitals with multi-branches. The mathematical modelling of these assignment problems results in complex problems. A hybrid meta-heuristic algorithm SCA-VNS combining a Sine Cosine Algorithm (SCA) and Variable Neighbourhood Search (VNS) based on the Iterated Hungarian algorithm is normally used.

New Method

In our project we have studied the method applied in the paper "Optimal Solution of an Assignment problem as a special case of Transportation Problem"", International Journal of Emerging Technologies and Innovative Research, ISSN:2349-5162, Vol.5, Issue 1, page no.198-203, January-2018.

Our mathematical model of an Assignment problem is a specific instance of the transportation problem with the following features:
1) the cost matrix is a square lattice and 2) the optimal solution matrix for the problem would have just a single Assignment in a given line (row) or a segment (column).

In the method to find optimal assignments as studied in this project, the following steps have been followed:
Step 1: Construction of cost matrix with dummy rows or columns if needed
Step 2: Subtract the smallest element found in the first row and first column from those respective row/column only.
Step 3: For each row and each column, identify the smallest element and subtract it in the row/column except the 'L-shape' of matrix
Step 4: Making assignments row-wise top to bottom first and then performing the same column-wise from left to right. We cross off all the other zeros in the corresponding row/column. In case there are multiple unmarked zeros, we choose a zero element arbitrarily.

Step 5: Repeat the above steps until optimality arises

Test for optimality

Step 1:

a) If all zero elements have been marked and there exists exactly one assignment for each row/column, then it is an optimal solution.

b) If a zero element was chosen arbitrarily, it means there exists alternative optimal solutions.

c) If there is no assignment in a particular row/column, then follow Step 2

Step 2:

Make a set of horizontal and vertical lines to cover all the zeros in the revised cost matrix and develop the revised cost matrix by:

a) Identify the minimum element ‘t’ such that it is not covered by any line
b) Subtracting ‘t’ from every element in matrix that is not covered by a line
c) Adding ‘t’ to every element at intersection of two lines
d) Other elements remain unchanged.

Repeat the above steps till optimality has been achieved.

Final solution: Add the original costs for elements in the occupied cells.

TYPES OF ASSIGNMENT PROBLEMS

1. Maximization
2. Minimisation
3. Restricted
4. Unbalanced

Maximization Problem

Excel Implementation

			Jobs		
Men	I	II	III	IV	V
1	20	15	18	20	25
2	18	20	12	14	15
3	21	23	25	27	25
4	17	18	21	23	20
5	18	18	16	19	20

			Jobs						
Men	I	II	III	IV	v	Assigned	Constraint		
1	0	1	0	0	0	1	=	1	
2	0	0	0	1	0	1	=	1	
3	1	0	0	0	0	1	=	1	
4	0	0	0	0	1	1	=	1	
5	0	0	1	0	0	1	=	1	
Men Assig	1	1	1	1	1				
Constraint	=	=	=	=	=				
	1	1	1	1	1		Total hours=		86

Datasource:
https://ijisrt.com/wp-content/uploads/2017/10/New-Approach-to-Solve-Assignment-Problem.pdf

Result Analysis:

When manually solving the assignment problem with the new method the Z(max) value is 87 hours while the excel implementation provided us with a value of 86 hours. The new method can be said to be a nearly accurate optimal solution when used for maximization problems.

Restricted Problem
Excel Implementation

	Projects				
Crew	P	Q	R	S	T
A	9	7000	6	5	4
B	6	5	8	6	4
C	3	5	2	5	5
D	4	4	4	3	4
E	6	5	8	7	6

	Project					Project		Project
Crew	P	Q	R	S	T	Assigned	Constraint	Required
A	0	0	0	0	1	1	=	1
B	0	1	0	0	0	1	=	1
C	0	0	1	0	0	1	=	1
D	0	0	0	1	0	1	=	1
E	1	0	0	0	0	1	=	1
Crews Assigned	1	1	1	1	1			
Constraint	=	=	=	=	=		Minimum days =	20
Crews Required	1	1	1	1	1			

Dataset source: Jaskowski, Piotr & Tomczak, Michał. (2014). Assignment problem and its extensions for construction project scheduling. Technical Transactions. Civil Engineering. 111. 241-248.

Result Analysis:

When manually solving the assignment problem with the new method the Z(min) value is 21 days while the excel implementation provided us with a value of 20 days. The new method can be said to be nearly accurate when used for restricted problems.

Minimization problem using the Hungarian Method

→Forming an assignment matrix

Operator	I	II	III	IV	V	VI
A	10	8	3	9	24	13
B	14	24	2	32	18	12
C	44	16	2	22	15	19
D	2	2	3	1	1	1
E	31	32	4	43	28	41
F	25	62	2	29	46	22

Step 1 → Subtract minimum value of each row from corresponding row values.

=

7	5	0	6	21	10
12	22	0	30	16	10
42	14	0	20	13	17
1	1	2	0	0	0
27	28	0	39	24	37
23	60	0	27	44	20

Step 2 → Subtract minimum value of each column from corresponding column values.

6	4	0	6	21	10
11	21	0	30	16	10
41	13	0	20	13	17
0	0	2	0	0	0
26	27	0	39	24	37
22	59	0	27	44	20

Step 3 → Draw minimum numbers of lines through rows & columns in which it covers all zero

6	4	0	6	21	10
11	21	0	30	16	10
41	13	0	20	13	17
0	0	2	0	0	0
26	27	0	39	24	37
22	59	0	27	44	20

The number of lines $h_1 = 2$. This implies that the solution is not optimal. The smallest uncovered element is $e_s = 4$
We then add 4 to the elements covered by two lines and subtract 4 from all elements that are uncovered.

2	0	0	3	17	6
7	17	0	26	12	6
37	9	0	16	9	13
0	0	6	0	0	0
22	23	0	35	20	33
28	55	0	23	40	16

→ Adding 4 to elements covered by 2 lines

→ subtracting 4 from the uncovered elements

Covering zeros with min. no. of lines

The number of lines $h_2 = 3$. This implies that the soln. is not optimal. The smallest uncovered element is $e_s = 6$.
We then add 6 to the elements covered by two lines and subtract 6 from all elements that are uncovered

2	0	6	3	17	6
1	11	0	20	6	0
31	3	0	10	3	7
0	0	12	0	0	0
16	17	0	29	14	27
22	49	0	17	34	10

Covering zeros with min. no. of lines

The min. no. of lines $h_3 = 4$.
This implies that the solution obtained is not optimal.
The smallest uncovered element is $e_s = 1$.
We add 1 to all elements covered by two lines and subtract 1 from all elements that are uncovered.

1	0	6	3	17	6
0	11	0	19	5	0
30	3	0	9	2	7
0	1	13	0	0	1
15	17	0	28	13	27
21	49	0	16	33	10

Covering zeros with min. no. of lines

$h_4 = 4$

Implies not optimal

$e_5 = 2$

→ add 2 to all elements covered by 2 lines

→ subtract 2 from others

1	0	8	3	17	6
0	11	2	19	5	0
28	1	0	7	0	5
0	1	15	0	0	1
13	15	0	26	11	25
19	47	0	14	31	8

Covering zeros with min. no. of lines

no. of lines $h_5 = 5$

soln. not optimal

$e_s = 8$

→ add 8 to covered

→ subtract 8 from others

1	0	16	3	17	6
0	11	10	19	5	0
28	1	8	7	0	5
0	1	23	0	0	1
5	7	0	8	3	17
11	39	0	6	23	0

Covering zeros with min. no. of lines

the number of lines $n_6 = 6$

This implies that soln is optimal

Now finding Zmin:

For A, theres only one 0 in the row
the original cost for II box = 8

For B, there are 2 0s in the row
the original cost for I box = 14

For C, there's one 0 in the row
the original cost for V box = 15

For D, there are 3 0's in the row
the original cost for IV box = 1

For E, theres only one 0 in the row
the original cost for III box = 4

For F, there are 2 0's in the row
the original cost for VI box = 22

thus, adding the costs of A, B, C, D, E, F we get:

$\Rightarrow 8 + 14 + 15 + 1 + 4 + 22$

$Z_{min} = 64$

which is equal to the solution obtained in the excel sheet.

EXCEL:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
2	operator	I	II	III	IV	V	VI			Worker			operator	I	II	III	IV	V	VI
3	A	0	1	0	0	0	0	1	=	1			A	10	8	3	9	24	13
4	B	1	0	0	0	0	0	1	=	1			B	14	24	2	32	18	12
5	C	0	0	0	0	1	0	1	=	1			C	44	16	2	22	15	19
6	D	0	0	0	1	0	0	1	=	1			D	2	2	3	1	1	1
7	E	0	0	1	0	0	0	1	=	1			E	31	32	4	43	28	41
8	F	0	0	0	0	0	1	1	=	1			F	25	62	2	29	46	22
9		1	1	1	1	1	1												
10		=	=	=	=	=	=												
11	Demand	1	1	1	1	1	1												
12									Zmin	64									

Unbalanced Problem
Manual Solving

Machines

Jobs	A	B	C	D	E
1	5	7	11	6	7
2	8	5	5	6	5
3	6	7	10	7	3
4	10	4	8	2	4

No. of rows $\neq$ no. of columns, (no. of jobs $\neq$ no. of machines) hence this is an unbalanced assignment problem.

$\therefore$ Add Dummy Row with cost $= 0$.
and,
calculate min element from first row and subtract it from each element of the first row.

Formulation of the problem:

$$Z_{min} = 5x_{11} + 7x_{12} + 11x_{13} + 6x_{14} + 7x_{15} + 8x_{21} + 5x_{22} + 5x_{23} + 6x_{24} + 5x_{25} + 6x_{31} + 7x_{32} + 10x_{33} + 7x_{34} + 3x_{35} + 10x_{41} + 4x_{42} + 8x_{43} + 2x_{44} + 4x_{45} + 0$$

Subject to

$c_1 : x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1$

$c_2 : x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 1$

$c_3 : x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 1$

$c_4 : x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 1$

$c_5 : x_{11} + x_{21} + x_{31} + x_{41} = 1$

$c_6 : x_{12} + x_{22} + x_{32} + x_{42} = 1$

$c_7 : x_{13} + x_{23} + x_{33} + x_{43} + \cancel{x_{44}} = 1$

$c_8 : x_{14} + x_{24} + x_{34} + x_{44} = 1$

$c_9 : x_{15} + x_{25} + x_{35} + x_{45} = 1$

Jobs \ Machines	A	B	C	D	E	min
1	5	7	11	6	7	5
2	8	5	5	6	5	
3	6	7	10	7	8	
4	10	4	8	2	4	
(Dummy) 5	0	0	0	0	0	

	A	B	C	D	E	min
1	0	2	6	1	2	
2	8	5	5	6	5	5
3	6	7	10	7	3	3
4	10	4	8	2	4	2
5	0	0	0	0	0	0

min element in column 1 = 0.
∴ Same table as above.
find min element from each row ~~and~~ except first row and subtract from each element in that row.

	A	B	C	D	E
1	$\boxed{0}$	2	6	1	2
2	8	$\boxed{0}$	$\cancel{0}$	1	$\cancel{0}$
3	6	4	7	4	$\boxed{0}$
4	10	2	6	$\boxed{0}$	2
5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
min		0	0	0	0

∴ same matrix as above for column reduction
since all rows have assignment, this is an optimal solution.

Thus,
machine A will do Job 1
machine B will do Job 2
Machine D will do Job 4
Machine E will do Job 3
machine C is assigned the dummy row,
thus no job shall be performed by
machine C.

Total cost = 5+5+2+3
$= \boxed{15}$

Total cost is 15.

Excel Solution

	A	B	C	D	E	F	G	H	I	J
1										
2					*Machines*					
3		*Jobs*	A	B	C	D	E			
4		1	1	0	0	0	0	1	=	1
5		2	0	1	0	0	0	1	=	1
6		3	0	0	0	0	1	1	=	1
7		4	0	0	0	1	0	1	=	1
8		5 (Dummy)	0	0	1	0	0	1	=	1
9			1	1	1	1	1			
10			=	=	=	=	=		Zmin	15
11			1	1	1	1	1			
12										
13										

K	L	M	N	O	P	Q	R
				Machines			
	Jobs	A	B	C	D	E	
	1	5	7	11	6	7	
	2	8	5	5	6	5	
	3	6	7	10	7	3	
	4	10	4	8	2	4	
	5 (Dummy)	0	0	0	0	0	

REAL LIFE APPLICATIONS OF THE ASSIGNMENT PROBLEM

The assignment problem is a fundamental optimization problem that aims to assign a set of tasks to a set of resources in the most efficient way possible. This problem is commonly encountered in many real-life situations, and its solutions have practical applications in various fields. In this report, we will discuss some of the most common real-life applications of the assignment problem in optimization techniques.

Workforce Management:
One of the most common applications of the assignment problem is in workforce management. In this context, the problem is to assign employees to various tasks or projects in the most efficient way possible. The goal is to minimize the total time required to complete all the tasks while ensuring that each task is assigned to an appropriate employee based on their skills and availability.

Manufacturing:
In manufacturing, the assignment problem is used to optimize production by assigning workers to various production lines and machines. The goal is to minimize the overall production time and maximize the efficiency of the production process.

Transportation:
The assignment problem is also used in transportation planning, where the problem is to assign vehicles to delivery routes. The goal is to minimize the total distance traveled by the vehicles while ensuring that each delivery is made on time and at the lowest possible cost.

Sports Scheduling:
The assignment problem is commonly used in sports scheduling to assign teams to venues and timeslots. The goal is to minimize the overall travel distance for the teams while ensuring that each team plays an appropriate number of games against opponents of similar skill level.

Resource Allocation:
The assignment problem is used in resource allocation, where the problem is to allocate resources to various tasks or projects. The goal is to minimize the total cost of the resources while ensuring that each task or project is completed on time and to the required quality.

Medical Applications:
The assignment problem is also used in medical applications, such as assigning patients to hospital beds, assigning nurses to patients, and assigning doctors to medical procedures. The goal is to minimize the overall waiting time for patients and optimize the use of medical resources.

Data Association:
The assignment problem is used in data association, where the problem is to match observations with tracks in a surveillance system. The goal is to minimize the overall tracking error and optimize the use of resources.

Education:
The assignment problem is used in education to assign students to classes or courses based on their academic performance and interests. The goal is to maximize the overall academic performance of the students while ensuring that each student is assigned to a suitable class or course.

In conclusion, the assignment problem is a fundamental optimization problem that has practical applications in various fields, including workforce management, manufacturing, transportation, sports scheduling, resource allocation, medical applications, data association, and education. By applying optimization techniques to solve the assignment problem, we can minimize costs, maximize efficiency, and improve the overall performance of many real-life systems.

ADVANTAGES:

1.Ensures that each row and column of whole cost matrix has at least one zero element

2.The aggregate time taken by this strategy for finding optimal solution is less

3.Less iterations than Hungarian Method

DISADVANTAGES:

1.if is it an unbalanced problem some tasks are left not assigned

2.it assigns tasks in 1-1 mapping only

3.It requires a lot of iterations

CONCLUSION

In conclusion, the assignment problem is a fundamental topic in optimization techniques with numerous real-life applications in various fields. The problem involves assigning tasks or resources to individuals or objects in a way that optimizes a given objective function, such as minimizing cost or maximizing profit.

To solve the assignment problem, different methods can be used, including manual solving techniques such as maximization, minimization, balanced and unbalanced methods, and the Hungarian method, which is considered the most efficient algorithm for solving the problem.

Furthermore, advancements in technology have made it possible to solve the assignment problem using Excel, which provides a more efficient and accurate approach to solving the problem.

Overall, understanding the assignment problem and its various solving techniques is critical in making optimal decisions in real-life situations. By applying the appropriate method, individuals or organizations can effectively assign resources and achieve their desired objectives.

REFERENCES:

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