APMA 1650 Problem Set 1

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1. (a) $S = \{\text{turn left, turn right, continue straight}\}$
   (b) $2/3$
   (c) $(2/3)^2 = 4/9$

2. (a) i. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.12 - 0.12 = 0.4$
   ii. $P(A \cap B) = P(B) = 0.12$
   iii. $P((A \cap B)^C) = 1 - P(A \cap B) = 1 - 0.12 = 0.88$
   (b) i. $P(A \cup B) = P(A) + P(B) = 0.4 + 0.12 = 0.52$
   ii. $P(A \cap B) = 0$ (disjointness implies $A \cap B = \emptyset,\ P(\emptyset) = 0$)
   iii. $P((A \cap B)^C) = 1 - P(A \cap B) = 1$

3. (a) $31/366$
   (b) $12/366 = 2/61$
   (c) $1/366$

4. (a) $S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), 4, 5, 6\}$
   (b) 21 elements
   (c) $A = \{(1,5), (2,4), (2,6), (3,3), (3,5), 6\}$
   (d) 6 elements
   (e) $P(A) = 5(\frac{1}{36}) + \frac{1}{6} = \frac{11}{36}$
   (f) $3/36 = 1/12$

5. (a) *Proof.* Let $A \subseteq S$. By the definition of a complement, we have that $A \cup A^C = S$ and $A \cap A^C = \emptyset$. Thus,

$$\begin{aligned} 1 &= P(S) && (P(S) = 1) \\ &= P(A \cup A^C) && \text{(definition of complement)} \\ &= P(A) + P(A^C) && (A \text{ and } A^C \text{ disjoint}) \end{aligned}$$

Therefore, we can conclude that $P(A) = 1 - P(A^C)$ □

6. (a) $P(E_1) + \cdots + P(E_5) = 1$
$P(E_4) + P(E_5) = 1 - (P(E_1) + \cdots + P(E_3)) = 1 - 0.7 = 0.3$
Thus, $P(E_4) = 0.2$ and $P(E_5) = 0.1$.

(b) Let $p = P(E_3) = P(E_4) = P(E_5)$.
We have that $P(E_1) = 3P(E_2) = 0.3$, which implies that $P(E_2) = 0.1$.
$P(E_1) + \cdots + P(E_5) = 1$. So, $3p = 1 - P(E_1) - P(E_2) = 0.6$.
Therefore, $p = 0.6/3 = 0.2$.