MATH 1530 Problem Set 3

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Problem 1. Please complete the mid-semester survey. Write "I have completed the mid-semester survey" and sign your name.

Problem 2. Let a be an element of a group G. Prove that $\langle a^m \rangle \cap \langle a^n \rangle$ is cyclic, where n, m are integers. What is its generator?

Proof. Let $a^k \in \langle a^m \rangle \cap \langle a^n \rangle$. We have that $a^k \in \langle a^m \rangle \implies a^k = a^{ms}$ where $s \in \mathbb{Z}$. We also have that $a^k \in \langle a^n \rangle \implies a^k = a^{nt}$ where $t \in \mathbb{Z}$. Together, we have

$$a^k = a^{ms} = a^{nt} \implies k = ms = nt$$

In other words, k must be a common multiple of both m and n. Since every common multiple of m and n is itself a multiple of $\mathrm{lcm}(m, n)$, we have that $\langle a^m \rangle \cap \langle a^n \rangle$ is equal to $\langle a^{\mathrm{lcm}(m,n)} \rangle$. $\square$

Problem 3. Let a and b belong to a group. If $|a|$ and $|b|$ are relatively prime, prove that $\langle a\rangle \cap \langle b\rangle = \{e\}$.