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Parameter Estimation

(1) $$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$X_1, X_2, X_3 \ldots X_n$ sample of size n

$$L(X_1, X_2, X_3 \ldots X_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) f(x_n)$$

taking ln on both side

$$\ln(L) = \frac{-n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left[\frac{(x_i-\mu)^2}{2\sigma^2}\right] \quad (1)$$

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i-\mu)}{2\sigma^2}\right) = 0$$

$$\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\cancel{n\bar{x} - n\mu = 0}$$

$$n\bar{X} - n\mu = 0$$

$$\boxed{\bar{X} = \mu}$$

$\boxed{\theta_1 = \bar{X}}$ is therefore sample mean

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$$

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2}$$

(2) Binomial distribution $^{n}C_{x_i}\, \theta^{x_i}(1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i}(1-\theta)^{n-x_i}$$

$$\log L = \sum_{i=1}^{n} \left(\log({}^{n}C_{x_i}) + \log \theta^{x_i} + \log(1-\theta)^{n-x_i}\right)$$

$$\log L = \sum_{i=1}^{n} \log({}^{n}C_{x_i}) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n-x_i)$$

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{1}{1-\theta} \Sigma(n-x_i) = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \Sigma x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\Sigma x_i}{n^2}}$$