

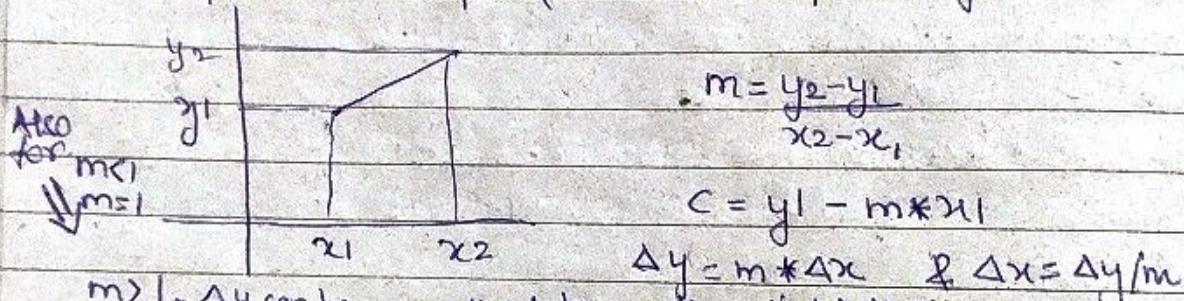
UNIT-2 Graphics Primitives

Points - Point plotting is done by converting a single coordinate posⁿ furnished by an application program into appropriate operations for the output device in use.

Line drawing is done by calculating intermediate positions along the line path b/w two specified endpoint position
 → For raster video display; the line intensity is loaded in frame buffer at corresponding pixel position.

LINE DRAWING ALGORITHMS:-

→ Cartesian slope intercept eq. for straight line is $y = mx + c$
 m represents slope & c intercept at y



$m > 1$, Δy can be proportional to small vertical deflection voltage &
DDA Algorithm horizontal is set proportional to Δx which is calculated from above eqn.
 Digital differential Analyzer is scan conversion line drawing algorithm based on calculating either Δy or Δx using above eqn.

* Ex:- $m < 1$

$$(5, 4) (12, 7)$$

$$\Delta x = 12 - 5 = 7$$

$$\Delta y = 7 - 4 = 3$$

Steps 7

$$x_{\text{incr}} = \frac{7}{7} = 1$$

$$y_{\text{incr}} = \frac{3}{7} = 0.4$$

x	y	.	.	
5	4	m	↑	Not
6	4.4	4		Smooth
7	4.8	5		line
8	5.2	5		
9	5.6	6		
10	6	6		
11	6.4	6		
12	6.8	7		

Procedure for DDA :-

```

Void lineDDA ( int xa, int ya, int xb, int yb )
{
    int dx = xb - xa, dy = yb - ya, steps, k;
    float xincrement, yincrement, x = xa, y = ya;
    if (abs(dx) > abs(dy))
        Steps = abs(dx);
    else
        Steps = abs(dy);
    xincrement = dx / (float)Steps;
    yincrement = dy / (float)Steps;
    set pixel (ROUND(x), ROUND(y));
    for (k=0; k<steps; k++)
    {
        x += xincrement;
        y += yincrement;
        set pixel (ROUND(x), ROUND(y));
    }
}

```

Advantage :-

- It is faster Algorithm
- It is simple Algorithm

Disadvantage :-

- floating point arithmetic is time consuming.
- Poor end point accuracy.

BRESENHAM'S LINE ALGORITHM:-

An accurate & efficient Raster line-generating algorithm, developed by bresenham which scan convert line using only incremental integer calculation that can be modified to display circles & other curves.

Desired

Slope < 1



$$y = m(x_k + 1) + c$$

$$\text{if } P_k < 0 \quad P_{k+1} = P_k + 2\Delta y$$

$$\text{if } P_k \geq 0 \quad P_{k+1} = P_k + 2\Delta y - 2\Delta x$$

$$P_i = 2\Delta y - 4x$$

Algorithm Bresenham (x_1, y_1, x_2, y_2)

$$\{ x = x_1 ;$$

$$y = y_1 ;$$

$$\Delta x = x_2 - x_1 ; \quad \Delta y = y_2 - y_1 ;$$

$$P = 2\Delta y - \Delta x$$

while ($x \leq x_2$)

{ put pixel (x, y) ;

$x++$;

if ($P < 0$)

$$\{ P = P + 2\Delta y$$

}

else

$$\{ P = P + 2\Delta y - 2\Delta x ;$$

y++;

}

$$\Delta x = 7$$

$$\Delta y = 4$$

$$P = 2 \times 4 - 7$$

= 1

$$2\Delta y = 2 \times 4 = 8$$

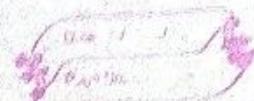
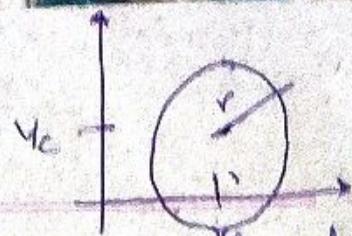
$$2 \times \Delta y - 2 \times \Delta x$$

$$2 \times 4 - 2 \times 7 = -6$$

Y
Y
EXAMPLE :- Point $(1,1)$ to $(8,5)$

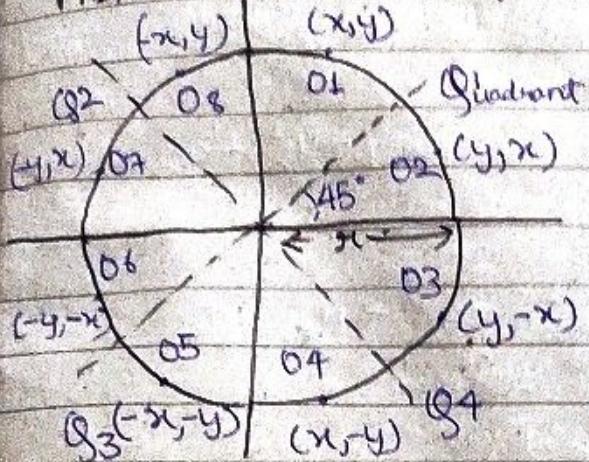
x	y	P
1	1	1
2	2	-5
3	2	3
4	3	-3
5	3	5
6	4	-1
7	4	7
8	5	1

CIRCLE :-



A circle is defined as the set of points that are all at a given distance r from a center position say (x_c, y_c)

MIDPOINT CIRCLE ALGORITHM :-



8 Octant - 8 way symmetry

Circle Equation

$$x^2 + y^2 = r^2 \quad (C: (0,0))$$

Brute force

$$x \rightarrow 0 - \infty$$

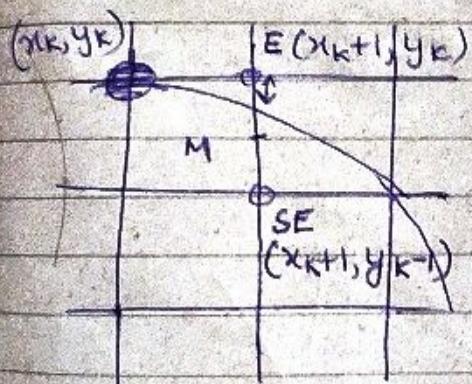
$$+ \infty, y ?$$

$$y = \sqrt{r^2 - x^2}$$

\Rightarrow time consuming

to overcome that

Midpoint Algo is used



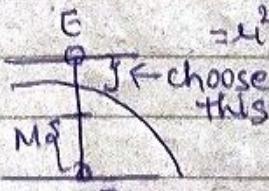
$$x^2 + y^2 = r^2 \quad P(0,0)$$

$$x^2 + y^2 - r^2 = 0$$

$\leq r^2 < 0$ Inside circle

$> r^2 > 0$ Outside

$= r^2$ On Circle



If Midpoint lie
inside circle then
take E Point ~~other~~

If Outside then take SE

$$\text{MidPoint} \left(\frac{x_{k+1} + x_{k+1}}{2}, \frac{y_k + y_{k-1}}{2} \right) = \left(x_{k+1}, y_{k-\frac{1}{2}} \right)$$

$$P_K = x_M^2 + y_M^2 - r^2 = (x_K + 1)^2 + (y_K - \frac{1}{2})^2 - r^2$$

$$P_{KH} = (x_{KH} + 1)^2 + (y_{KH} - \frac{1}{2})^2 - r^2$$

$$P_{KH} - P_K = (x_K)^2 + 4 + 4x_K - x_K^2 - 1 - 2x_K + y_{KH}^2 + y_K^2 - y_{KH}^2 - y_K^2$$

$$P_{KH} = P_K + 2x_K + 3 + y_{KH}^2 - y_{KH} - y_K^2 + y_K$$

$$P_K < 0 \quad y_{KH} = y_K$$

$$P_{KH} = P_K + 2x_K + 3 + (y_K^2) - y_K^2 - y_K + y_K$$

$$\text{if } P_K > 0 \Rightarrow y_{KH} = y_K - 1$$

$$P_{KH} = P_K + 2x_K + 3 + (y_K - 1)^2 - y_K^2 - (y_K - 1) + y_K$$

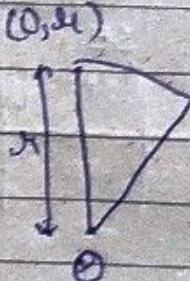
$$P_{KH} = P_K + 2x_K - 2y_K + 5$$

$P_0 \rightarrow$ Initial Decision Parameter

$$x_K = 0 \quad y_K = 0$$

$$P_0 = \frac{5}{4} - r$$

$$r = 1 - r$$



NUMERICAL :-

Plot 1st Octant of Circle centre at origin having radius 10 units

1) Plot $(0, r) \rightarrow (0, 10) \rightarrow (x_0, y_0)$

$$p_0 = 1 - x_0 = 1 - 10 = -9$$

$$\therefore p_0 < 0 \rightarrow p_1 = p_0 + 2x_0 + 3 = -9 + 3 = -6 \boxed{(x_1, y_1) = (1, 10)}$$

$$2) p_1 < 0 \rightarrow p_2 = p_1 + 2x_1 + 3$$

$$= -6 + 2 \times 1 = -4 \boxed{(x_2, y_2) = (2, 10)}$$

$$3) p_2 < 0 \rightarrow p_3 = p_2 + 2x_2 + 3 = -4 + 2 \times 2 + 3 = 6$$

$$\boxed{(x_3, y_3) = (3, 10)}$$

$$4) p_3 > 0 \quad P_4 = P_3 + 2(x_3 - y_3) + 5 = -3$$

$$\boxed{(x_4, y_4) = (4, 9)}$$

$$5) p_4 < 0 \quad P_5 = P_4 - 3 + 8 + 3 = 8$$

$$\boxed{(x_5, y_5) = (5, 9)}$$

$$6) P_5 > 0 \quad P_6 = 8 + 2(5 - 9) + 5 = 5$$

$$\boxed{(x_6, y_6) = (6, 8)}$$

$$7) P_6 > 0 \rightarrow P_7 = 5 + 2(6 - 8) + 5 = 6$$

$$\boxed{(x_7, y_7) = (7, 7)}$$

this or if $x > y$
stop one
step before

Bresenham's Circle Drawing ALGO:-

1. Determine the radius (r) of circle from given information
2. Ensure the centre of circle lies at origin $(0,0)$
3. Plot first pixel of the first Octant as $(0,r)$ [i.e. $y=r$]
4. Calculate initial decision parameter as $d = 3 - 2r$

5. Repeat till $x \leq y$:

(i) If $d_k < 0$ then:

$$\hookrightarrow d_{k+1} = d_k + 4x_k + 6$$

$$\hookrightarrow x_{k+1} = x_k + 1 \quad \rightarrow y_{k+1} < y_k$$

ii) Else if $(d_k \geq 0)$ then:

$$\hookrightarrow d_{k+1} = d_k + 4(x_k - y_k) + 10$$

$$\hookrightarrow x_{k+1} = x_k + 1$$

$$\hookrightarrow y_{k+1} = y_k - 1$$

6. Plot (x_{k+1}, y_{k+1})

7. Determine & Plot other octant as well

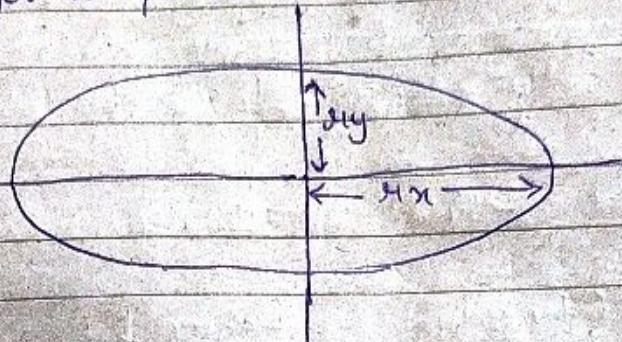
Q1 Centre is at (X_c, Y_c)

then

$$(x_{k+1}, y_{k+1}) \rightarrow (x_{k+1} + X_c, y_{k+1} + Y_c)$$

ELLIPSE :-

An ellipse is defined as the set of points such that the sum of the distances from two fixed positions (foci) is same for all points.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{centre, at } (0, 0)$$

Major axis = $2a = 24x$ Semi-major axis $a = 4x$
 Minor axis = $2b = 24y$, Semi-minor axis $b = 4y$

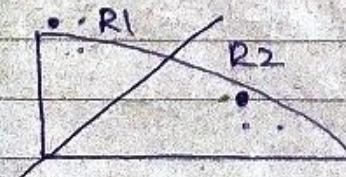
$$\Rightarrow b^2x^2 + a^2y^2 - a^2b^2 = 0 \quad \text{OR}$$

$$4y^2x^2 + 16x^2y^2 - 16x^2y^2 = 0$$

$= 0$ < 0 > 0
 ON Ellipse Inside Outside

Quadrant Symmetry 4 way Symmetry

2 different Octant of a Quadrant to Plot



Algorithm :-

- Read radii R_x & R_y
- Initialise starting point of Region 1 as $x=0$ $y=R_y$

- Calculate

$$P_1 = R_y^2 - R_x^2 R_y^2 + \frac{1}{4} R_x^2$$

- Calculate $dx = 2R_y^2 x$ $dy = 2R_x^2 y$

- Repeat while ($dx < dy$) (Region 1)

→ Plot (x, y)

→ if ($P_1 < 0$)

$$\left\{ \begin{array}{l} x = x + 1 \\ \text{Update } dx \end{array} \right.$$

$$2R_y^2 x$$

y same

$$P_1 = P_1 + \frac{2R_y^2 x + R_y^2}{dx}$$

y

else → $P_1 \geq 0$

$$\left\{ \begin{array}{l} x = x + 1, y = y - 1 \\ \text{Update } dx \rightarrow 2R_y^2 x \\ \text{Update } dy \rightarrow 2R_x^2 y \end{array} \right.$$

$$P_1 = P_1 + dx - dy + R_x^2 y$$

- when ($dx \geq dy$) Plot Region 2 at :

→ Find $\boxed{P_2 = R_y^2 \left(x + \frac{1}{2} \right)^2 + R_x^2 (y-1)^2 - R_x^2 R_y^2}$

→ Repeat till ($y > 0$)

→ Plot (x, y)

→ if ($P_2 > 0$)

$$\left\{ \begin{array}{l} x = x \\ y = y - 1 \end{array} \right.$$

Update $dy : 2R_x^2 y$

$$\left\{ \begin{array}{l} P_2 = P_2 - dy + R_x^2 y \end{array} \right.$$

else { $x = x + 1$

$$y = y - 1$$

$$\begin{aligned} \text{new } dy &= dy - 2x^2 \quad \text{for } 2x^2 \\ \frac{dy}{dx} &= dx + 2y^2 \quad \text{for } 2y^2 \\ p_2 &= p_2 + dx - dy + 4x^2 \end{aligned}$$

$$\begin{array}{l} dx = 4 \\ dy = 6 \end{array}$$

$$\begin{array}{ll} ex & dx = 4 \quad dy = 6 \\ & (x_k, y_k) \\ P_1 & P_1 = -332 \end{array}$$

$$\therefore (0, 6)$$

$$P_{1,1} = -332 + 72 + 36 = -224$$

(x_{k+1}, y_{k+1})	$2x_{k+1} y_{k+1}^2$	$2y_{k+1}^2 x_{k+1}^2$
$(1, 6)$	$(2(1)(36)) = 72$	$2(6)(64) = 768$
$(2, 6)$	$4(36) = 144$	$= 768$

$$x^2(2, 6) \quad P_{1,2} = -224 + 144 + 36 = -44 \quad (3, 6) \quad 6(36) = 216 \quad 768$$

$$x^3(3, 6) \quad P_{1,3} = -44 + 216 + 36 = 208 \quad (4, 5) \quad 8(36) = 288 \quad 640$$

$$x^4(4, 5) \quad P_{1,4} = 208 + 288 + 36 - 640 = -108 \quad (5, 5) \quad 10(36) = 360 \quad 640$$

$$x^5(5, 5) \quad P_{1,5} = -108 + 360 + 36 = 288 \quad (6, 4) \quad 12(36) = 432 \quad 8(64) = 512$$

$$x^6(6, 4) \quad P_{1,6} = 288 \quad (7, 3) \quad 504 \quad 384$$

$$\begin{array}{ll} P_2 & P_2 = 36\left(7+\frac{1}{2}\right)^2 + 64(2)^2 - \\ & 36(64) = 23 \end{array} \quad (8, 2) \quad 16(36) \\ = 576 \quad 256$$

$$\begin{array}{ll} P_{2,1} & P_{2,1} = -23 + 576 + 64 - 256 \\ & = 361 \end{array} \quad (8, 1) \quad 576 \quad 2(64) \\ = 128$$

$$\begin{array}{ll} P_{2,2} & P_{2,2} = 361 - 128 + 64 \\ & = 297 \end{array} \quad (8, 0) \quad - \quad -$$

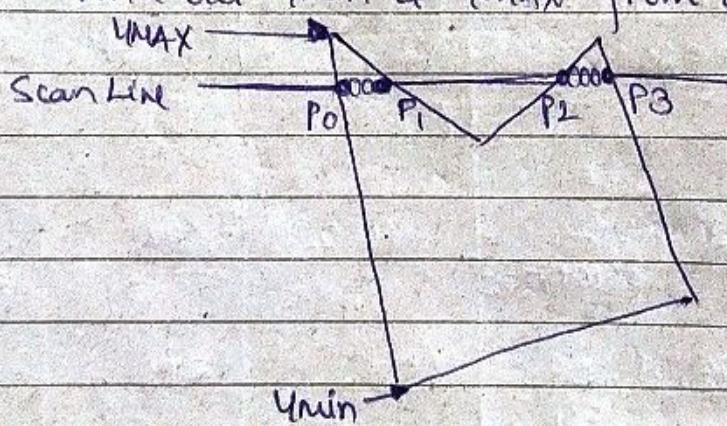
FILLED AREA PRIMITIVES -

- In practical we often use polygon which are filled with some color or pattern inside it.
- Two basic approach to area filling on raster system
 - One way is to determine the overlap intervals for scan line that cross the area.
- Another way is to fill the area is to start from given interior position & paint outwards from this point until we encounter boundary.

SCAN-LINE POLYGON FILL ALGORITHM -

this algo works by intersecting scanline ~~scanning~~ with polygon edges & fills the polygon between pairs of intersections.

Step 1 - Find out y_{min} & y_{max} from given polygon.



Step 2 - Scanline intersects with each edge of polygon from y_{min} to y_{max} . Name each intersection point of the polygon. As per fig they are name P_0, P_1, P_2, P_3 .

Step 3 - Sort the intersection point in the increasing order of X coordinate i.e. $P_0, P_1, P_3, P_2, \& P_1, P_3$.

Step 4 - fill all those pair of coordinates that are inside polygons & ignore the alternate pairs.

INSIDE OUTSIDE TEST

this method is also known as counting no. method. While filling an object, we often need to identify whether particular point is inside the object or outside it. There are two methods by which we can identify whether particular point is inside or outside an object.

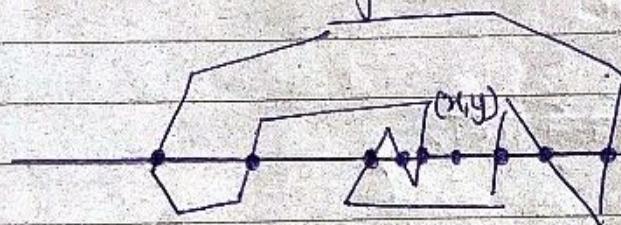
- Odd - Even Rule
- Non zero winding number Rule

Odd Even Rule - Also known as odd parity rule or even

In this technique,

we count the edge crossing along the line from any point x,y to infinity.

If no. of intersection is odd, then point x,y is an interior point
if no. is even, then x,y is an exterior point

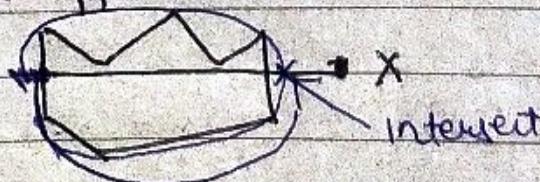


No of intersections point on left side is 5 & on right is 3 from both end the no. of intersection point is odd, so the point is considered within object.

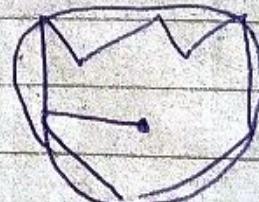
NON ZERO WINDING RULE :-

this method is also used with simple polygons to test given point is interior or not..

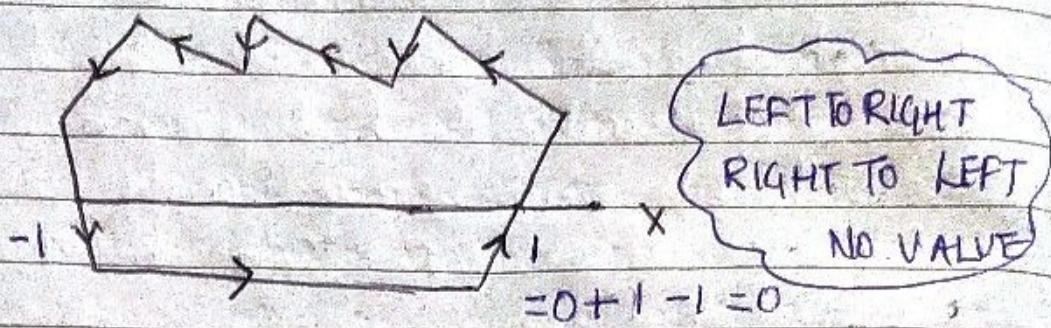
Pin elastic approach :-



Point of entry if none
otherwise inside



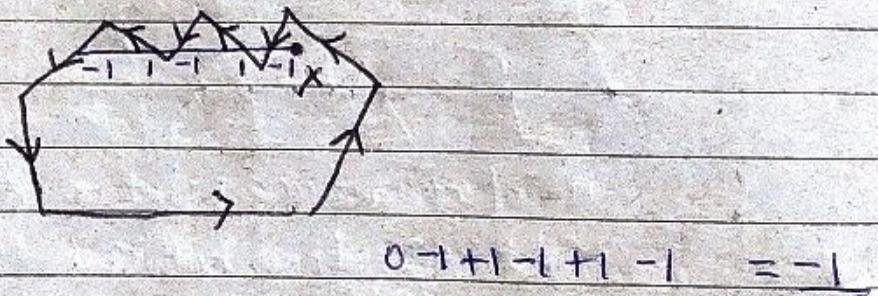
No line cut



Initial Winding No = 0

- If line crosses an edge directed bottom to top add 1 to winding no
- TOP - BOTTOM ADD -1 to W.no.

→ POINT OUTSIDE IF WNO. IS Zero
→ Otherwise Inside



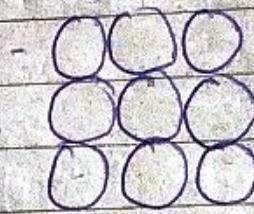
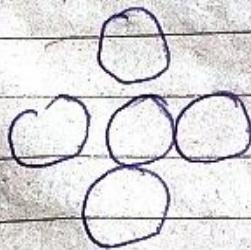
Comparison B/w odd even & Non zero Winding :-
for simple object gives same result
but for complicated shape both rule gives different result

BOUNDARY FILLED ALGO :-

This Algo uses recursive method. First of all, a starting pixel called as the seed is considered.

→ The Algo checks boundary pixel or adjacent pixels are colored or not. If pixel is already filled then leave it, otherwise fill it.

Filling is done using 4 connect. or 8 connected approaches.



Color specified in parameter fill color (f-color)

Boundary specified with parameter boundary color (b-color)

Procedure :-

boundary-fill4(x, y, f-color, b-color)

{ if(getpixel(x,y) != b-color && getpixel(x,y) != f-color)

 putpixel(x, y, f-color)

 boundary-fill4(x+1, y, f-color, b-color);

 " " (x, y+1, " ", ");

 " " (x-1, y, " ", ");

 " " (x, y-1, " ", ");

}

}

Same procedure is used for 8-connected by including 4 additional statements to test diagonal positions, such as (x+1, y+1)

FLOOD FILL ALGO :-

In this method, a point or seed which is inside region is selected. The four connected or eight connected approach are used to fill with specific color.

In this, we start from a specific interior point (x, y) & means all pixel values are currently set to a given interior color with desired color.

Disadvantage :-

Slow, May be fail for large polygon, Initial pixel requires more knowledge about surrounding pixels

Algo :-

Procedure :-

flood-fill4($x, y, \text{new-color}, \text{old-color}$)

{

if (getpixel(x, y) == old-color)

{

putpixel($x, y, \text{new-color}$)

flood-fill4($x+1, y, \text{new-color}, \text{old-color}$);

" ($x, y+1, \text{new-color}, \text{old-color}$);

" ($x-1, y, \text{new-color}, \text{old-color}$);

" ($x, y-1, \text{new-color}, \text{old-color}$);

}

}

CHARACTER GENERATION:-

- We can display letter & no in variety of size & ~~font~~ style.
- Typeface - The overall design style for the set of character.
- Originally font was referred to a set of cast metal character forms in particular size & format.
But now, the term font & typeface are often used inter-changeably.
- Two different representations are used for storing computer fonts :-

Bitmap Font

Simple Method for representing character shapes in particular type face is to use rectangular grid pattern.

1	1	1	1	0
0	1	0	1	1
0	1	1	0	0
0	1	0	1	1
1	1	1	1	0

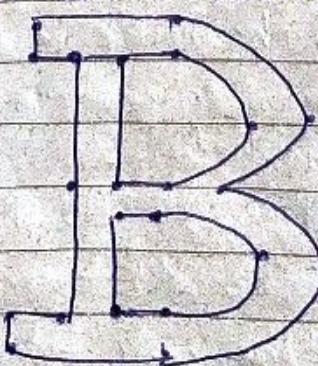
B

→ Simplest to design & display as character grid only need to maintain frame Buffer Position.

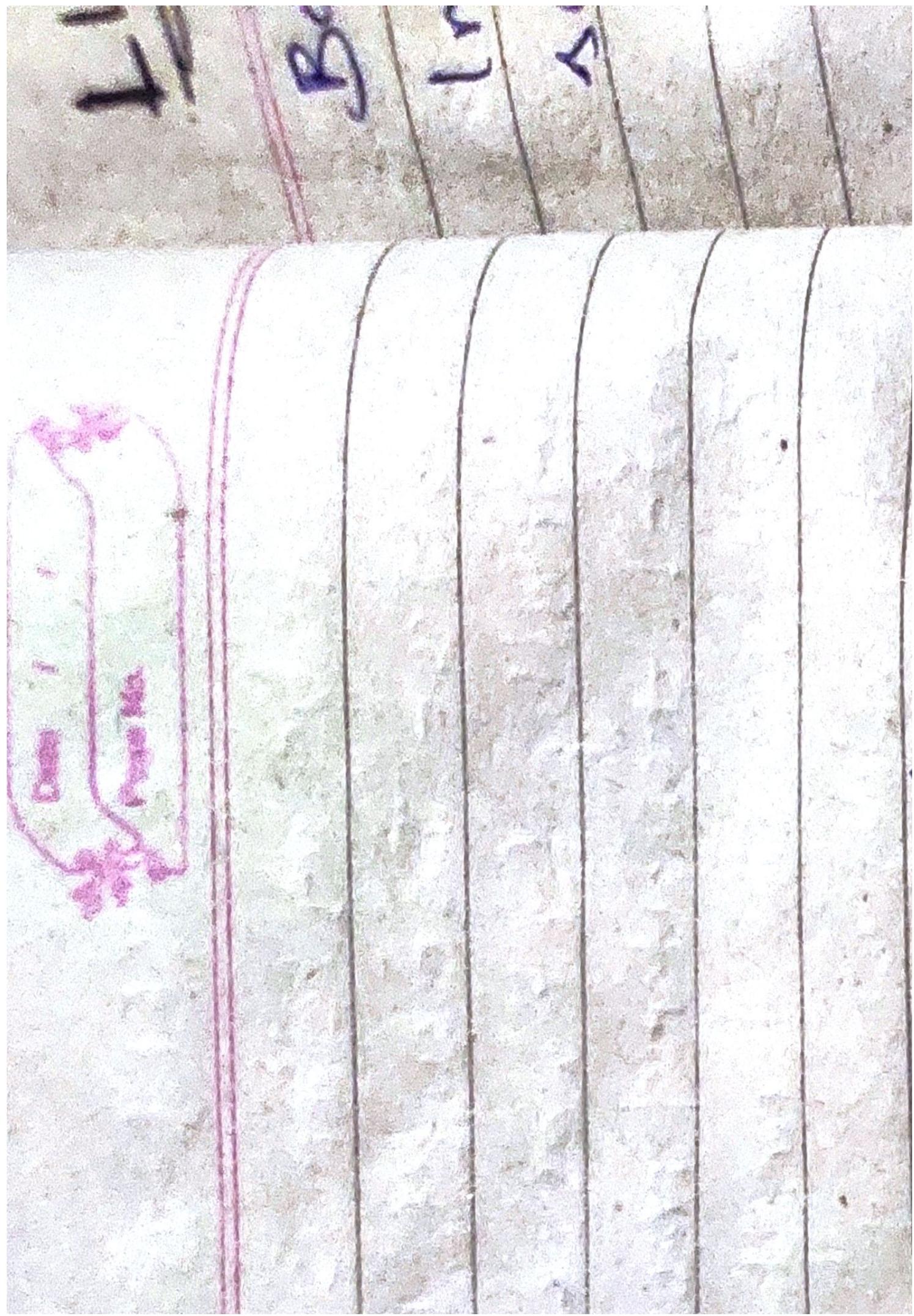
→ It requires more space as each variation must be stored in a font cache.

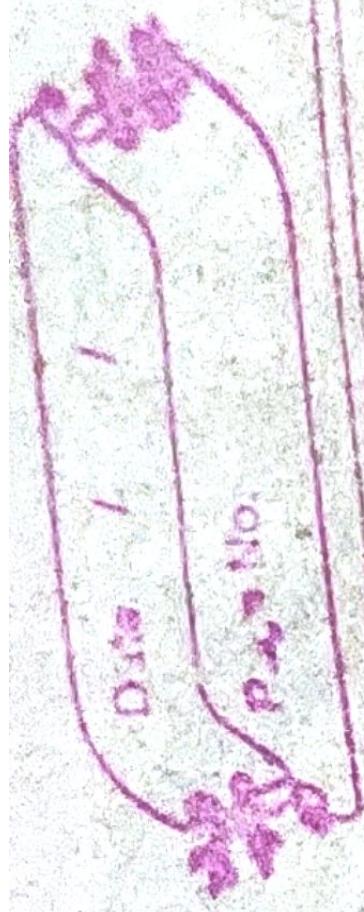
Outline Font -

→ In this method character is generated using curve section & straight line as combine assembly



→ To display character we need to fill interior region of character.
→ This method requires less storage as each variation does not require a distinct





• its dimension & its color
• its shape, it also be displayed using

Solid
cashed

Line is displayed

or Constantly changing

or Index (Ic)

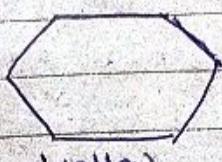
AREA FILL ATTRIBUTES

- for filling area we have choice between solid color or pattern.
- Area can be painted by various brushes & styles.

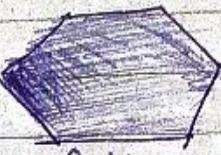
Fill Style :

→ In PHIGS set InteriorStyle(f₁)

→ Solid, hollow, pattern etc



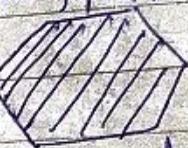
Hollow



Solid



Pattern



Diagonal
hatchfill



Diagonal
Cross hatch
fill

→ for setInteriorColorIndex(fc)

Pattern fill :-

setInteriorStyleIndex(pi)

Index(pi)	Pattern(cp)
1	$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$
2	$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

SOFTFILL :- (it is modified Boundary & flood fill algo.)

it is used to recolor or repaint so that we can obtain layer of multiple color & get new color combination.

→ One use of this algo is soften the fill boundary

so that blurred effect will reduce the aliasing effect.

for ex:-

$$\text{Pixel} \rightarrow p = tF + (1-t)B$$

where t amount of foreground color
F is foreground color

B is background color

If we use RGB component then:

$$p = (p_r, p_g, p_b) \quad f = (f_r, f_g, f_b) \quad b = (b_r, b_g, b_b)$$

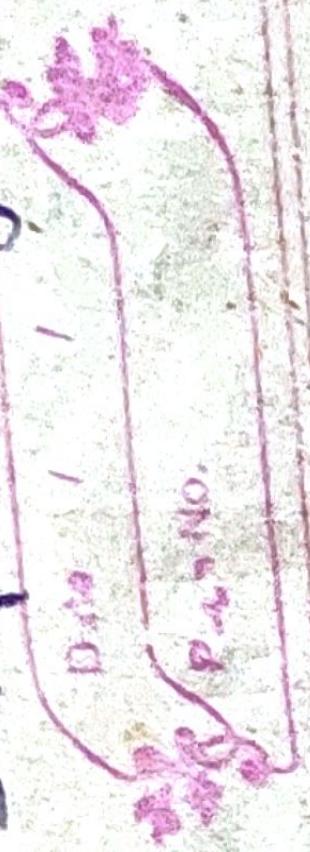
then we calculate, $t = \frac{p_k - b_k}{f_k - b_k}$



is controlled by attributes
information

Audio Signals in time

Any Sampled Signal



presence of jagged
or zig-zag edges
in the image can

unusually occurs due
to noise in the

considering
region:

→ & compute pixel
such objects

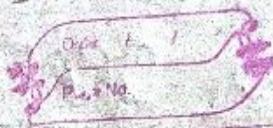
shades of gray based

Tue

Thu

...

of



Two drawbacks of this method :-

- There is a technical & economic limit for increasing the resolution of virtual image.
- ∵ the frequency of image can extend to infinity, it just reduces aliasing by raising the Nyquist limit - Shift (the effect of the frequency spectrum).