Detailed Explanation of the 2DCentreOfMass Program

The 2DCentreOfMass program is designed to interactively create and manipulate polygons in a two-dimensional space, allowing the user to calculate geometric properties such as the center of mass and the area. It also enables the user to rotate the polygon so that the center of mass aligns directly below a chosen pinning point. This document explains the implementation details and the mathematics behind these calculations.

Overview of the Program

The program is structured into three main components:

1. **Geometry Class:** Handles geometric calculations.
2. **Shape Class:** Manages the vertices of the shape and uses the Geometry class for calculations.
3. **2DCentreOfMass Class:** Manages user interactions and visual representation of the shape using Matplotlib.

Geometry Class

Calculating the Center of Mass and Area

The center of mass (COM) and the area of a simple polygon can be calculated using the formulas derived from Green's theorem. Given a list of vertices $(x_0, y_0), (x_1, y_1), \ldots, (x_{n-1}, y_{n-1})$:

$$A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i)$$

$$C_x = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

$$C_y = \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

where $(x_n, y_n) = (x_0, y_0)$ to close the polygon.

Rotating a Point

To rotate a point (x, y) about another point (ox, oy) by an angle θ, the coordinates (x', y') after rotation are given by:

$$x' = ox + (x - ox)\cos(\theta) - (y - oy)\sin(\theta)$$

$$y' = oy + (x - ox)\sin(\theta) + (y - oy)\cos(\theta)$$

Shape Class

This class maintains a list of vertices defining the polygon and uses the Geometry class to compute the center of mass and area when the shape is closed.

2DCentreOfMass Class

User Interaction

This class uses Matplotlib for graphical display and handles user input through mouse events:

- Left-click to add vertices.
- Right-click to close the shape and compute properties.
- Another left-click to choose the pinning point and perform rotation.

Rotation Mechanism

Upon selecting a pinning point, the shape is rotated so that the center of mass is directly underneath this point, aligning vertically with gravity. The rotation angle θ is computed based on the current position of the center of mass relative to the pinning point.

Mathematical Insight

The approach to rotate the shape to align the center of mass with the pinning point involves calculating an angle θ that brings the center of mass directly under the pinning point in the simplest manner. This involves understanding of basic trigonometry and coordinate geometry transformations.