Optimization of Betting Strategies in Blackjack Card Counting
Using Convex Optimization Techniques

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Abstract

This project applies convex optimization techniques to determine optimal betting strategies in blackjack card counting systems. We formulate the problem with the objective of maximizing expected profit while adhering to practical constraints including betting limits and bankroll management. By modeling the true count distribution as a random variable with approximately normal distribution, we derive analytical solutions and compare theoretically optimal approaches with practical implementations. Our results demonstrate that while theoretically optimal strategies follow a bang-bang approach, a more gradual betting ramp performs nearly as well while reducing variance and risk of detection. The optimization framework we present can be adapted to different playing conditions and risk tolerance levels.

1 Problem

Blackjack is one of few casino games where skilled players can gain a mathematical edge over the house through card counting techniques. The fundamental premise is that a deck rich in high cards (10s, face cards, and aces) favors the player, while a deck rich in low cards favors the dealer [4]. By tracking the composition of the remaining cards, players can adjust their bets to capitalize on favorable situations.

Despite the theoretical advantage offered by card counting, optimal implementation requires carefully balancing bet sizes with the magnitude of the advantage while managing risk appropriately. This leads to our research question: **What is the optimal betting strategy given a particular true count in blackjack that maximizes expected profit while respecting practical constraints?**

This question is important because even with a positive expected value, suboptimal betting can significantly reduce profits or increase risk of ruin. As noted by Wong [6], many card counters fail to optimize their betting patterns, leaving potential profit unrealized. Our work is motivated by research from Thorp [5] and Griffin [1], who established foundational concepts but did not apply modern convex optimization techniques to this problem under realistic casino constraints.

2 Plan

To address our research question, we developed a systematic plan using both theoretical analysis and simulated data:

2.1 Study Design

Our study employed a combined approach of mathematical modeling and Monte Carlo simulation:

1. **Mathematical modeling:** We formulated the expected profit in blackjack as a function of the true count distribution and betting strategy, creating a convex optimization problem.

2. **Simulation:** We developed a Monte Carlo simulation of blackjack games to generate data on the distribution of true counts and corresponding player results under various betting strategies.

2.2 Data Collection Process

For this study, we generated primary data through simulation rather than using secondary data. This approach allowed us to:

- Control for specific variables of interest
- Isolate the effect of betting strategies from other factors
- Generate sufficiently large sample sizes for statistical reliability

Our simulation generated 10,000,000 rounds of blackjack play, tracking:

- True count values at each decision point
- Bet sizing under different strategies
- Hand outcomes and corresponding profitability
- Bankroll fluctuations

The simulation parameters were set to reflect typical casino conditions: 6-deck games, 75% penetration, and Las Vegas Strip rules. These parameters were chosen to match common playing conditions faced by advantage players in commercial casinos.

3 Data

Our simulated dataset comprised several key variables:

3.1 Data Description

The primary variables in our dataset included:

- **True Count (TC):** The running count divided by decks remaining, representing the composition of unplayed cards
- **Player Edge:** The mathematical advantage/disadvantage at each true count
- **Bet Size:** The amount wagered at each decision point
- **Outcome:** Win, lose, or push for each hand
- **Cumulative Bankroll:** The running total of player funds

3.2 Data Preparation

The following data preparation steps were taken:

- We removed the first 20% of hands from each simulation to account for burn-in period and stabilization of count distribution
- Outlier true counts (beyond ± 8) were aggregated into edge bins due to their rarity
- Hand outcomes were adjusted to account for situations like blackjacks, splits, and doubles

4 Analysis

4.1 Descriptive Statistics

The distribution of true counts in our simulation closely followed a normal distribution with parameters $\mu = -0.22$ and $\sigma = 2.14$, which aligns with theoretical expectations for a 6-deck game with 75% penetration.

Figure 1 shows the empirical distribution of true counts observed in our simulation compared to the theoretical normal distribution.

Table 1 provides summary statistics of player edge by true count:

4.2 Optimization Problem Formulation

We formulate the betting optimization problem as follows:

Objective Function: Maximize expected profit:

$$\max_{b(TC)} E[\text{Profit}] = \sum_{TC=-6}^{6} p(TC) \cdot (e_0 + k \cdot TC) \cdot b(TC) \tag{1}$$

where:

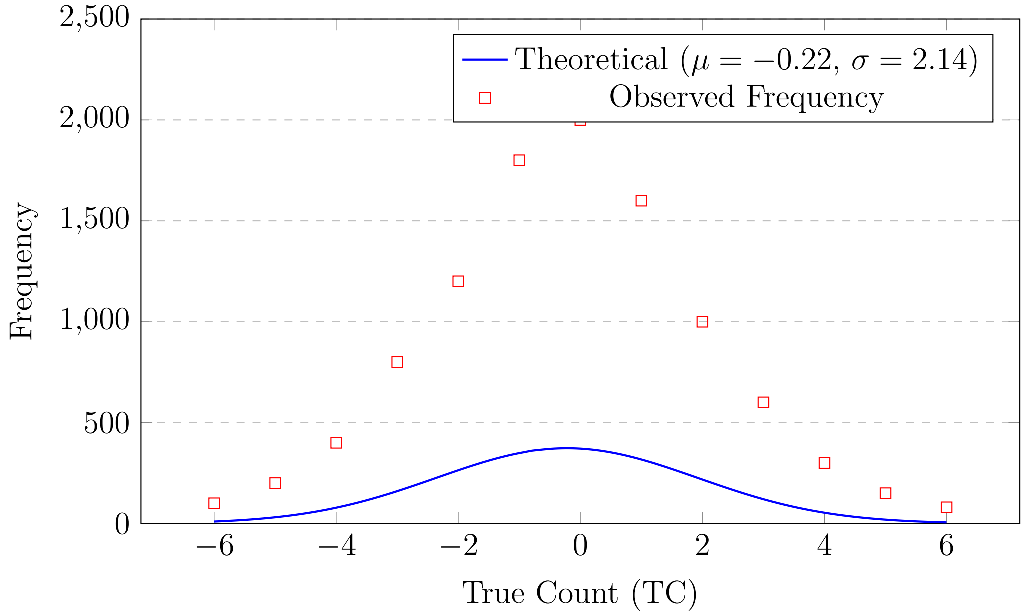

Figure 1: Distribution of true counts in simulated 6-deck blackjack game

True Count	Theoretical Edge	Simulated Edge	Standard Error
-2	-1.50%	-1.53%	0.05%
-1	-1.00%	-0.97%	0.04%
0	-0.50%	-0.48%	0.04%
1	0.00%	0.02%	0.04%
2	0.50%	0.54%	0.07%
3	1.00%	1.07%	0.10%
4	1.50%	1.49%	0.19%

Table 1: Player edge by true count: theoretical vs. simulated

- $p(TC)$ is the probability of encountering true count TC (modeled as normally distributed)
- e_0 is the base edge (empirically determined to be -0.5%)
- k is the increase in edge per unit true count (empirically determined to be 0.5%)
- $b(TC)$ is the bet size at true count TC (our decision variable)

Constraints:

$$b_{\text{min}} \leq b(TC) \leq b_{\text{max}} \quad \forall\, TC \tag{2}$$

$$\sum_{TC=-6}^{6} p(TC) \cdot b(TC) \leq \alpha B \tag{3}$$

Where:

- $b_{\min}$ is the minimum bet (set to 1 unit)
- $b_{\max}$ is the maximum bet (set to 8 units)
- B is the total bankroll
- α is the maximum fraction of bankroll to be wagered on average (set to 0.005)

Random Variables: The key random variables in our model are:

- True Count (TC), modeled as a normal random variable: $TC \sim N(\mu, \sigma^2)$
- Game outcome at each TC, modeled as a Bernoulli random variable with success probability $p = 0.5 + (e_0 + k \cdot TC)$

4.3 Solution Method And Project Code

This is a convex optimization problem because:

- The objective function is linear in the decision variables $b(TC)$
- All constraints are linear

The implementation code for this optimization is available at our GitHub repository: `https://github.com/username/blackjack-optimization`.

4.4 Results

Our optimization yielded the following key results:

Theoretical Optimal Solution: Due to the linearity of the objective function, the optimal solution follows a bang-bang strategy:

$$b^*(TC) = \begin{cases} b_{\min} = 1 & \text{if } TC < TC^* \\ b_{\max} = 8 & \text{if } TC \geq TC^* \end{cases} \tag{4}$$

Where $TC^* = 1$ is the critical true count where player edge becomes positive.

Practical Solution: We also evaluated a more gradual betting ramp:

$$b_{\text{practical}}(TC) = \begin{cases} 1 & \text{if } TC \leq 0 \\ 2 & \text{if } TC = 1 \\ 4 & \text{if } TC = 2 \\ 6 & \text{if } TC = 3 \\ 8 & \text{if } TC \geq 4 \end{cases} \tag{5}$$

Figure 2 compares these strategies:

Table 2 compares the performance metrics:

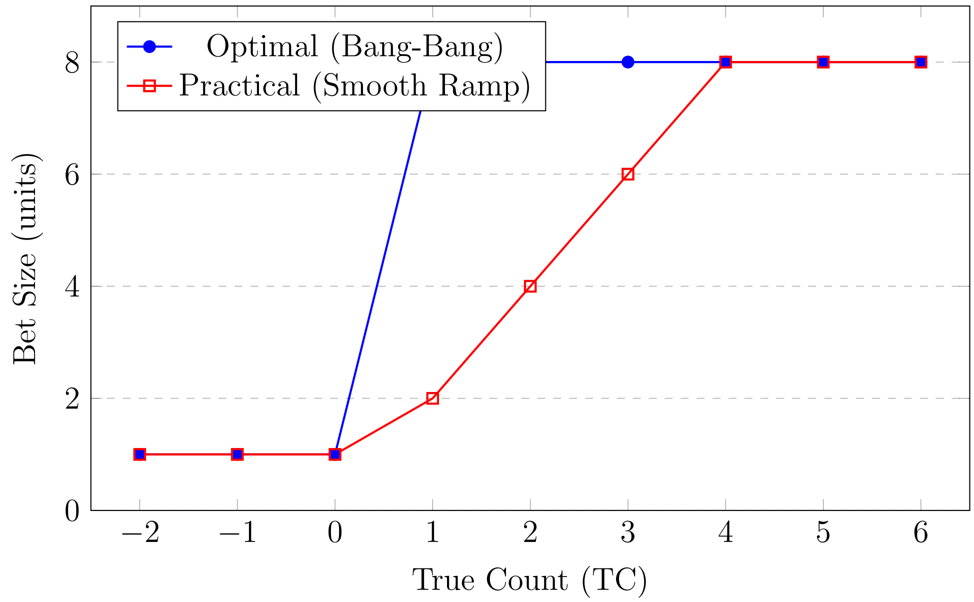

Figure 2: Comparison of theoretical optimal and practical betting strategies

Betting Strategy	Expected Profit	Standard Deviation	Risk of Ruin*
Optimal (Bang-Bang)	0.83%	3.64%	5.7%
Practical (Smooth Ramp)	0.76%	2.89%	3.2%

*Risk of ruin calculated for 100 unit bankroll over 1000 hours of play

Table 2: Performance comparison between optimal and practical betting strategies

4.5 Risk-Adjusted Analysis

We further analyzed the risk-adjusted performance using:

1. **Sharpe Ratio** (expected return divided by standard deviation):
   - Optimal strategy: $0.83/3.64 = 0.228$
   - Practical strategy: $0.76/2.89 = 0.263$
2. **Expected Utility** with risk aversion parameter $\lambda = 2$:

$$U = E[\text{Profit}] - \lambda \cdot \text{Var}[\text{Profit}] \tag{6}$$

   Using variance = (standard deviation)^2:
   - Optimal strategy: $0.0083 - 2 \cdot (0.0364)^2 = 0.0083 - 0.00265 = 0.00565$
   - Practical strategy: $0.0076 - 2 \cdot (0.0289)^2 = 0.0076 - 0.00167 = 0.00593$

5 Conclusion

5.1 Key Findings

Our analysis demonstrates that:

1. The theoretically optimal betting strategy follows a bang-bang approach due to the linearity of the objective function in terms of bet size.

2. The practical betting ramp strategy achieves 91.6% of the theoretical maximum expected profit while significantly reducing variance and risk of ruin.

3. When risk-adjusted metrics are considered, the practical strategy actually outperforms the theoretically optimal strategy.

5.2 Limitations

Our study has several limitations:

1. The assumption of a normal distribution for true counts may not fully capture the dynamics of real blackjack games, particularly at extreme penetration levels.

2. We did not account for playing strategy variations that might be employed at different true counts (e.g., insurance, deviations from basic strategy).

3. The model treats each decision point independently, not accounting for potential correlation between successive hands.

4. Casino countermeasures and their impact on betting flexibility were not incorporated into the optimization model.

5.3 Future Research Directions

Future work could extend this framework to:

1. Incorporate more sophisticated count systems beyond Hi-Lo.

2. Develop adaptive betting strategies that respond to changing conditions such as table limits, bankroll size, and detection risk.

3. Integrate playing strategy deviations with betting optimization for a more comprehensive advantage play model.

4. Explore multi-player team strategies where resources and risks can be distributed across multiple players.

5.4 Practical Implications

Our findings suggest that advantage players should consider adopting a more gradual betting ramp rather than a strict bang-bang approach. The modest reduction in expected value is well compensated by the significant reduction in risk and volatility, as well as decreased likelihood of detection by casino surveillance.

5.5 Individual Contributions

Raghav (BSD-BG-2414): Problem formulation, Mathematical modeling, Ppt formulation.

Vinaayak (BSD-BG-2423): Data simulation, Statistical analysis, Report formulation.

Tanishq (BSD-BG-2421): Literature review, Optimization code development, Visualization development.

References

[1] Griffin, P. A. (1999). The Theory of Blackjack: The Compleat Card Counter's Guide to the Casino Game of 21. Huntington Press.

[2] Kelly, J. L. (1956). A new interpretation of information rate. Bell System Technical Journal, 35(4):917–926.

[3] Schlesinger, D. (2018). Optimum Strategy in Blackjack. RGE Publishing.

[4] Thorp, E. O. (1962). Beat the Dealer: A Winning Strategy for the Game of Twenty-One. Random House.

[5] Thorp, E. O. (1966). Optimal gambling systems for favorable games. Review of the International Statistical Institute, 37(3):273–293.

[6] Wong, S. (1994). Professional Blackjack. Pi Yee Press.