

# **Index Number:**

## **Part 2**

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# Chain Index

- **Chain Index**
- The index numbers, we have considered so far, are of the fixed base type; i.e., the base period with which we compare the other time periods remains fixed with the progress of time. If the base period is '0', then we considered only  $I_{01}$ ,  $I_{02}$ ,  $I_{03}$ , ... and so on. We have also noted that with the passage of time new commodities enter the market and old ones disappear; besides, the quality of the commodities may undergo a change. Also, the relative importance of various commodities, being dependent on the tastes and habits of the consumers, changes.
- Thus, fixed base IN becomes more and more inaccurate as the gap between base and current period increases. The weights become Out-of-date.
- For this reason, it is sometimes considered desirable to make comparison between closely situated periods, in which the  $q$ 's are not likely to change much. If indices are computed for the successive time periods it is not necessary to use a fixed base '0'. We use the previous period as base and construct what are called link indices.
- There is no change in the method of computation, only the base period changes for each comparison and in each case, it is the previous period.

# Chain Index

- Thus, for computing  $I_{0n}$ , we first construct the link indices  $I_{01}$ ,  $I_{12}$ ,  $I_{23}$ , ...,  $I_{n-1,n}$  and then compute  $I_{0n}$  as

$$I_{0n} = I_{01} \times I_{12} \times I_{23} \times \dots \times I_{n-1,n}.$$

- Such an index is called “Chain Index”.
- For example, let  $I_{01} = 110$ ,  $I_{12} = 120$  and  $I_{23} = 90$  then

$$I_{03} = I_{01} \times I_{12} \times I_{23} = 100 \times 1.1 \times 1.2 \times .9 = 129.$$

- Although the precise meaning of  $I_{03}$  is not simple in character, because it is based on a changing collection of goods, never the less there is a sense in which the weights are kept up to date in the chained index, because the  $q$ 's are unlikely to change radically between adjacent periods. These chain indices will not in general be equal to the corresponding fixed base indices unless the formula used meets the so-called Circular test.
- **Home Task:** Prove it. Also state some formula which does not satisfy Circular test but satisfy time reversal test.

# Chain Index

- The base period can be shifted to any convenient subsequent period if the formula satisfies the circular test.

$$I_{kn} = \frac{I_{0n}}{I_{0k}}.$$

- We have seen that the fixed-base index numbers become more and more inaccurate as the distance between the base period and the current period increases. As the chain-base index numbers are based on a number of link-indices, each of which is expected to be quite accurate, it is claimed that the chain-base index numbers are more accurate than the fixed-base ones, so far as long-term comparison is concerned. Also, a chain index fully utilizes the information regarding prices and quantities of all the intervening periods between the base period and the current period, whereas a fixed-base index requires data concerning the base period and the current only.
- Some authorities, on the other hand, hold that since a chain index is obtained by multiplying a number of link indices, it may involve a cumulative error, although none has put forward any convincing proof of the existence of such error.
- Fixed-base index numbers are generally easier to calculate and are more easily understood by users of index numbers than chain-base index numbers.

# Errors in Index Numbers:

## Formula Error

- **Errors in Index Numbers**
- The index numbers thus constructed will be subject to different types of errors. The errors are generally classified as (i) formula error, (ii) sampling error and (iii) homogeneity error.
- **Formula Error:** The formula error arises out of the choices of a particular formula in the construction of an index number. There cannot be any universally accepted formula which can measure the price changes with exactitude, and hence each formula is subject to some error inherent in the formula.
- One way of measuring formula error is to measure  $D = L_{01} - P_{01}$  (say), since  $L_{01}$  and  $P_{01}$  are trying to measure the same thing. Clearly if  $D$  is small, it must mean that the  $q_0$  and  $q_1$  are not too divergent. But if  $D$  is large, there is no way of identifying which one is more accurate.

# Errors in Index Numbers:

## Sampling Error

- **Sampling Error:** Sampling error arises from the fact that calculations are based on a set of  $n$  commodities which represents the whole list of  $N$  commodities. A random sample may not represent the population as accurately as we want.
- One way of reducing the error is to increase sample size. A random sample of  $n$  commodities may not have any representative from one or more of these commodity groups.
- A stratified sample ensures sample observations from each of the commodity groups. The sampling variance of a stratified sample is generally smaller than the variance from SRS if each stratum is homogenous i.e., between strata variance is high. So proper stratification reduces sampling variance.

# Errors in Index Numbers:

## Homogeneity error

- **Homogeneity error:** Homogeneity error arises from the fact that index numbers are calculated from the data on binary commodities, i.e., the commodities which exist in both the base and the target periods, whereas they should be based on all the commodities marketed in the base period and the current period, including both binary and unique commodities. Since with the passage of time many old commodities disappear from the market and new commodities appear, the homogeneity error increases as the gap between the two periods increases.
- To get some idea about the magnitude of homogeneity error one can find R as

$$R = \frac{\text{number of unique commodities}}{\text{number of unique \& binary commodities}} = \frac{N_1 + N_0}{N_1 + N_0 + N_{01}},$$

where  $N_1$  = number of commodities in period 1 only;  $N_0$  = number of commodities in period 0 only and  $N_{01}$  = number of commodities in both periods (binary). Clearly, R lies in between 0 and 1 and  $R = 0$  when there is no unique commodity ( $N_1 = 0$  and  $N_0 = 0$ ), i.e., homogeneity error is nil or it is perfectly homogeneous.  $R = 1$  when there is no binary commodity ( $N_{01} = 0$ ), that is perfect heterogeneity.

# Errors in Index Numbers:

## Homogeneity error

- R may be defined in set theoretic term also. Let us define B and C to be the sets of base year and current year commodities respectively. Then BC is the set of binary commodities,  $B - BC$  is the set of unique commodities in base period and  $C - BC$  is the set of unique commodities in the current period. Assume that  $\mathcal{N}(A)$  is the number of commodities in the set A, say, then

$$\begin{aligned} R &= \frac{\mathcal{N}(B - BC) + \mathcal{N}(C - BC)}{\mathcal{N}(B - BC) + \mathcal{N}(C - BC) + \mathcal{N}(BC)} = \frac{\mathcal{N}(B) + \mathcal{N}(C) - 2\mathcal{N}(BC)}{\mathcal{N}(B) + \mathcal{N}(C) - \mathcal{N}(BC)} \\ &= 1 - \frac{\mathcal{N}(BC)}{\mathcal{N}(B) + \mathcal{N}(C) - \mathcal{N}(BC)}. \end{aligned}$$

- Notice that  $\mathcal{N}(B) + \mathcal{N}(C) - \mathcal{N}(BC) = \mathcal{N}(BUC)$ , where BUC is the union of B and C in the set theoretic term.



# Uses of Index Numbers:

## Purchasing power

- **Uses of Index Numbers**

- In addition to serving the basic purpose for which they are constructed, index numbers are also of use for following purposes.

- (a) **Purchasing power**

- The purchasing power of money (say rupee) is the quantity of goods that a given quantity of money will buy. The reciprocal of a price index number is used to show the purchasing power of money. A price index is the amount of money required to purchase a fixed basket of goods, and the reciprocal of the price index – the purchasing power – represents the quantity of goods that can be purchased with a fixed amount of money. The purchasing power will be relative to the base period of the price index.

# Uses of Index Numbers: Deflation

## **(b) Deflation:**

- Another use of index numbers is in adjusting a value series by dividing the series by a price index or by multiplying the series by the index of purchasing power. By this the unit of money is expressed in terms of the purchasing power in the base year. This process, which is known as deflation, is not limited to value series only. Wages are deflated by cost-of-living index and departmental store sales by retail price index, and so on.
- Since price of goods and services increases over time, the purchasing power of money decreases. Thus, the current income needs to be deflated by the price index number to get real income.

# Uses of Index Numbers: Deflation

- The amount of money you currently have is your nominal money. You buy a certain amount of goods and services with that money. Suppose one asks “how much money would one need to get the same goods and services (as one gets now) in the base period?” The answer is the real income. It gives us a platform to compare your purchasing power over time. Thus,

$$\text{Real income} = \frac{\text{Current income}}{\text{CPI}}.$$

- The process of dividing by the price index is called the deflation. GNP of a country needs to be deflated by the whole sale price index.

# **Uses of Index Numbers: Indicator of General Business Condition**

## **(c) Indicator of General Business Condition**

- Index numbers are also used in studying the general business conditions. A company may plan its activities by studying the wholesale price index numbers. The index of industrial production may be studied to follow changes in the volume of production, etc.
- Price adjustment in business & industrial contracts are often made by looking at price indexes.

# Uses of Index Numbers: Splicing

## (d) Splicing:

- Sometimes we have an index for a number of periods with a certain base and wish to change the base period. The way we change the base period to get a new index for the same current period is called splicing. Suppose we want to change the base year  $t_0$  to  $t_1$ . The current time is 't'. Then,

$$I_{t_1t} = \frac{I_{t_0t}}{I_{t_0t_1}} \times 100.$$

- It is easy to visualize the situation if we assume a single commodity in the market. Note that the change of base can be done accurately only if the index satisfies circularity, e.g., an aggregative index with fixed weights.
- (a) Regulation of dearness allowances in Government organizations are made using CPI.

# Wholesale Price Index

- **Wholesale Price Index (WPI):**
- It is a tool of measuring variations in exchange value or purchasing power of money. It has been utilized for other purposes, e.g., forecasting changes in business conditions, deflating aggregates such as national/domestic product and the like. In India, the aim of WPI has been to measure, as accurately as possible, changes in the general level of price of commodities in primary markets (i.e., mining, forestry, agriculture, fishing – relating to first important commercial transaction in large lots).
- Inflation rates are based on WPI which is released by the Office of Economic Advisor under the Ministry of Commerce and Industry. Since 1947 the index is being published regularly. Six revisions have taken place introducing the new base years, viz., 1952-53, 1961-62, 1970-71, 1981-82, 1993-94 and 2004-05. The new series of WPI with base year of 2011-12 is the seventh revision of WPI.

# Wholesale Price Index

- Following the recommendation of the Working Group for the revision of the WPI (Base 2004-05) series, a representative commodity basket comprising 697 items has been selected.
- The weighting structure derived for the new series is consistent with the structure of the economy for the year 2011-12. The number of quotations selected for collecting price data for the above items are 8331.
- A comparative statement of weights, number of items and number of quotations between the old series (Base 2004-05 = 100) and new series (Base: 2011-12 =100) is given for the major groups in Table 1.1.

**Table 1.1: Comparative Weights, No. of Items and  
No. of Quotations in Old and New WPI series**

<b>Major Groups/Groups</b>	<b>Weight</b>		<b>No. of Items</b>		<b>No. of Quotations</b>	
	2011-12	2004-05	2011-12	2004-05	2011-12	2004-05
<b>All Commodities</b>	100.00	100.00	697	676	8331	5482
<b>I. Primary Articles</b>	22.61756	20.11815	117	102	983	579
<b>II. Fuel &amp; Power</b>	13.15190	14.91021	16	19	442	72
<b>III. Manufactured Products</b>	64.23054	64.97164	564	555	6906	4831



# Wholesale Price Index:

## Product Groups in Major Groups

- In each of the major groups there are some product groups.
- **Primary Articles** consists of
  - (i) Food Articles, (ii) Non-Food Articles, (iii) Minerals (iv) Crude Petroleum & Natural Gas.
- **Fuel & Power** consists of
  - (i) Coal, (ii) Mineral Oils and (iii) Electricity.
- **Manufactured Products** consists of
  - (i) Food Products, (ii) Beverages, ... (xxi) Furniture and (xxii) Other manufacturing.
- Each product group in the item basket has been represented by such number of items. Two new groups “Manufacture of furniture”, and “Other Manufacturing” have been added in the new WPI (Base 2011-12) series. In each product groups there are a number of items and price quotations.

# Method of Calculation of WPI

- **Method of Calculation of WPI**

- The Compilation of new WPI series broadly consists of two stages –
  - (i) First, the item level indices (i.e. elementary price index) are calculated using “**Jevons Index formula**”, which uses the **Geometric Mean (GM)** of Price Relatives (i.e. the price change). Price relatives are calculated as the percentage ratios, i.e. by dividing the current price by the base period price and multiplying the quotient by 100. These elementary indices are the lowest level of aggregation where prices are combined into price indices.
  - (ii) In the second stage, these elementary price indices are aggregated using weighted arithmetic mean to obtain higher level (at sub-group/ group/ major group/ All Commodities level) indices, which has a fixed base-year weighting diagram operative through the entire life span of the series.

# Method of Calculation of WPI

- The formula used is:

$$I = \frac{\sum I_i \times W_i}{\sum W_i}$$

- where,

$I$  = Index Number of wholesale prices of a sub-group/group/major group/All commodities

$W_i$  = weight assigned to the  $i$ th item/sub-group/group/major group

$I_i$  = Index of the  $i$ th item/sub-group/group/major group

- Observe that, in the new WPI series, elementary price index (i.e. at the item level) has been computed using the geometric mean of the price relatives (Jevons' Index) as opposed to the practice of taking arithmetic mean of price relatives (Carli Index) as was the case in the previous WPI series.
- The revision of the Index of Wholesale Prices was discussed in the meeting of Committee of Secretaries (CoS) held on 14th March 2017 which approved the release of the new series.

# Limitations of Wholesale Price Index

- **Limitations of Wholesale Price Index**
- Being an economy-wide index, it is natural for WPI to capture the most important items. Selection of items also depends on the availability of the monthly information (price quotations).
- Provisional WPI is released with a time lag of two weeks of the reference month and is initially provisional. After eight weeks, the index is finalized and final figures are released and then frozen thereafter. Even if any response to the queries comes after eight weeks, it cannot be incorporated in the final figures.

# Limitations of Wholesale Price Index

- There has normally been a difference between provisional and final figures and this is not unique to WPI. Every other series including GDP, IIP, CPI etc. also have a mechanism of revising the initial estimates during a well-defined time interval.
- WPI series has been revised by shifting base year from 2004-05 to 2011-12 and the basket of commodities/items has been reselected afresh in order to capture the recent structure of the economy. As such, there may not be one-to-one correspondence for all commodities in the two series.

# True Cost of Living Index Number

- **True Cost of Living Index Number:**
- The cost of living is the minimum expenditure needed to attain a certain level of utility. In simpler terms, the true cost-of-living index is the cost of achieving a certain level of utility (or standard of living) in one year relative to the cost of achieving the same level of utility in the target year.
- The utility remains same if the same commodity bundle  $q_0$  is used in the target period and this gives the Laspeyres' index, i.e.,

$$L_{01} = \frac{\sum_{i=1}^n p_1^i q_0^i}{\sum_{i=1}^n p_0^i q_0^i}.$$

- But there are many other bundles giving the same level of utility. Consider that set to be ' $S_L$ '. The same level of utility may be achieved by some other bundle from  $S_L$ , say,  $q_*$  that costs less than that of  $q_0$ .

# True Cost of Living Index Number

- The index

$$L_{01} = \frac{\sum_{i=1}^n p_1^i q_*^i}{\sum_{i=1}^n p_0^i q_0^i}$$

- is, in fact true CLI corresponding to the base year utility. As argued above, Laspeyres' index overestimates the corresponding true CLI.
- In a similar manner, if the true CLI is based on the target year quantities then the Paasche's index underestimates the true CLI. This is the reason why it is said that the true CLI lies between Paasche's index and Laspeyres' index.
- But for convenience, we use Laspeyres method.

# Consumer Price Index (CPI)

- **Consumer Price Index No. (CPI):**
- The National Statistical Office (NSO), Ministry of Statistics and Programme Implementation (MoSPI) compiles All India as well as state-wise Consumer Price Index (CPI) for Rural, Urban, Combined sectors and releases the CPI numbers on 12th day of every month for current month and Final Index of previous month.
- In India, the Consumer Price Index (CPI) is calculated and released by the Ministry of Statistics and Programme Implementation. The CPI is calculated using the Laspeyres formula, which involves selecting a fixed basket of goods and services that are commonly consumed by households and tracking the change in the prices of these items over time.



# Consumer Price Index (CPI)

- The basket of goods and services used for the calculation of CPI in India is based on the consumption pattern of households in both rural and urban areas. The basket is divided into various categories, such as food and beverages, housing, clothing, transportation, and healthcare, with each category assigned a specific weightage based on its importance in the overall consumption pattern.
- The prices of each item in the basket are collected from a sample of retail outlets and markets across the country on a monthly basis. The data is then compiled and analyzed to calculate the CPI for each category and for the overall basket. The CPI is expressed as an index number, with a base year of 2012. The base year is typically updated periodically to ensure that the basket of goods and services used to calculate the CPI remains relevant and representative of the consumption patterns of households.

# Consumer Price Index (CPI)

- CPI is constructed through 3 stages, viz., (1) the subgroup index is compiled (2) the subgroup indices are combined into group indices (3) group indices are combined into the general index of the CPI, separate weights are derived at three levels, i.e., commodity, subgroups and groups. The weights at each stage are derived in terms of proportion of expenditure.
- The CPI is an important indicator of inflation in the economy, and it is used by policymakers, economists, and market analysts to track price changes and make informed decisions regarding monetary and fiscal policies.

# Consumer Price Index (CPI)

- Here's a table with the different types of Consumer Price Index in India, along with their importance:

S. No.	Type	Importance
1	CPI (Combined)	Covers both rural and urban areas
2	CPI (Urban)	Covers only urban areas
3	CPI (Rural)	Covers only rural areas
4	CPI (Industrial)	Covers workers in the organized industrial sector
5	CPI (Agricultural)	Covers agricultural labourers and rural workers

# Consumer Price Index (CPI)

- Each type of CPI is based on a different basket of goods and services and has a different weightage assigned to various categories, depending on the consumption patterns of the target population. The CPI (Combined) is the most widely used CPI in India and is considered to be the most representative measure of inflation in the economy.
- **Difference Between Consumer Price Index and Wholesale Price Index**
- Here's a table highlighting the key differences between Consumer Price Index (CPI) and Wholesale Price Index (WPI).

# Key Differences Between CPI and WPI

Feature	Consumer Price Index (CPI)	Wholesale Price Index (WPI)
<b>Definition</b>	Measures the change in prices of goods and services consumed by households.	Measures the change in prices of goods sold in bulk by producers and wholesalers.
<b>Coverage</b>	Covers both urban and rural areas.	Covers only goods sold in bulk in the primary and wholesale markets.
<b>Computation</b>	Based on a fixed basket of goods and services consumed by households.	Based on a fixed basket of goods sold in bulk by producers and wholesalers.
<b>Weightage</b>	The weightage is assigned to various categories based on the consumption pattern of households.	The weightage is assigned to various categories based on the value of production.
<b>Inflation</b>	Reflects the price changes at the retail level.	Reflects the price changes at the wholesale level.
<b>Purpose</b>	Used to measure inflation, calculate the Cost-of-Living Index, and adjust salaries, wages, pensions, and government subsidies.	Used to measure inflation in the production sector, calculate Producer Price Index, and adjust prices of goods and services sold by producers and wholesalers.

# **Index of Industrial Production (IIP)**

- **Index of Industrial Production (IIP)**
- Index of Industrial Production represents the status of production in the industrial sector for a given period of time as compared to a reference period of time. It is a statistical device which enables us to arrive at a single representative figure to measure the general level of industrial activity in the economy. It is calculated and published by the Central Statistical Office (CSO) every month. It has recently been merged with NSSO to assume a new name as National Statistical Office (NSO).
- **Official Definition of IIP**
- “It is a composite indicator that measures the short-term changes in the volume of production of a basket of industrial products during a given period with respect to that in a chosen base period.”

# **Index of Industrial Production (IIP)**

- The Central Statistical Organisation (CSO) under the “Ministry of Statistics and Programme Implementation” was responsible for the compilation and publication of the Index of Industrial Production (IIP) since 1950. Now it is done by NSO. IIP is published monthly, six weeks after the reference month ends.
- This indicator is of paramount importance and is being used by various organisations including Ministries/Departments of Government of India, Industrial Associations, Research Institutes and Academicians.

# Index of Industrial Production (IIP)

- The general scope of the index of industrial production as recommended by the United Nations Statistical Office (UNSO), is defined to include **mining, manufacturing, construction, electricity and gas sectors**. But due to constraints of the data availability, the present general index of industrial production compiled in India has in its scope **mining, manufacturing and electricity only**.
- Though UNSO recommends that the base year of the IIP may be revised quinquennially, the base year 1937 was revised successively to 1946, 1951, 1956, 1960, 1970, 1980-81, 1993-94, 2004-05 and 2011-12.



# Formulae for the Index of Industrial Production

- **Formulae for the Index of Industrial Production**
- Index of quantum of production for a given firm is given by

$$I_L = \frac{\sum P_0 Q_1 - \sum p_0 q_1}{\sum P_0 Q_0 - \sum p_0 q_0} \text{ (Laspeyres' form),}$$

$$I_P = \frac{\sum P_1 Q_1 - \sum p_1 q_1}{\sum P_1 Q_0 - \sum p_1 q_0} \text{ (Paasche's form),}$$

- where P and Q are prices and quantities of output and p and q are the prices and quantities of input. This is basically a quantity index.

# Index of Industrial Production: The Ratio of Value Added

- Since input is nothing but the output of some other firms, if we do not subtract input then there will be a problem of double counting. Thus, quantum of production should be defined as the quantity component of the value added of the firm. Gross value added provides a value for the amount of goods and services that have been produced, less the cost of all inputs and raw materials that are directly attributable to that production.

$$\begin{aligned}\text{The Ratio of values added} &= V_{01} = \frac{\sum P_1 Q_1 - \sum p_1 q_1}{\sum P_0 Q_0 - \sum p_0 q_0} \\ &= \frac{\sum P_1 Q_1 - \sum p_1 q_1}{\sum P_0 Q_1 - \sum p_0 q_1} \times \frac{\sum P_0 Q_1 - \sum p_0 q_1}{\sum P_0 Q_0 - \sum p_0 q_0}.\end{aligned}$$

# **Index of Industrial Production: The Ratio of Value Added**

- Thus, ratio of value added has two multiplicative components. The first component is the price index using Paasche's formula and the second component is the quantity index using Laspeyres' formula. Also

$$V_{01} = \frac{\sum P_1 Q_1 - \sum p_1 q_1}{\sum P_0 Q_0 - \sum p_0 q_0} = \frac{\sum P_1 Q_0 - \sum p_1 q_0}{\sum P_0 Q_0 - \sum p_0 q_0} \times \frac{\sum P_1 Q_1 - \sum p_1 q_1}{\sum P_1 Q_0 - \sum p_1 q_0}.$$

- Thus,  $V_{01}$  can also be written as the product of price index using Laspeyres' formula and quantity index using Paasche's formula. Since we are interested in the index of quantum of production, we may take one of these two quantity indices defined above, i.e.,  $I_L$  and  $I_P$ .

# Index of Industrial Production: The Final Formula

- Suppose we have computed  $I_L$  indices for individual firms and we want to combine these into an index of industrial production as a whole. We use the following formula

$$\begin{aligned} IIP_L &= \frac{\sum (\sum P_0 Q_1 - \sum p_0 q_1)}{\sum \sum P_0 Q_0 - \sum p_0 q_0} = \frac{\sum I_L (\sum P_0 Q_0 - \sum p_0 q_0)}{\sum \sum P_0 Q_0 - \sum p_0 q_0} \\ &= \sum I_L \frac{(\sum P_0 Q_0 - \sum p_0 q_0)}{\sum \sum P_0 Q_0 - \sum p_0 q_0}. \end{aligned}$$

# Computation of IIP: Method of Deflation

- **Computation:**

**(1) Method of Deflation:** Consider the formula

$$I_L = \frac{\sum P_0 Q_1 - \sum p_0 q_1}{\sum P_0 Q_0 - \sum p_0 q_0}.$$

Estimate  $\sum P_0 Q_1$  by  $\sum P_1 Q_1 / P_{01}$ , where  $P_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1}$ .

Estimate  $\sum p_0 q_1$  by  $\sum p_1 q_1 / p_{01}$ , where  $p_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1}$ .

# Computation of IIP: Method of Indicators

## (2) Method of Indicators:

- The basis of method of indicators is the breaking up of the industrial area under consideration into small sectors for which the values added in the base period are known.
- If  $I_{01}$  is an indicator of quantum of production from period 0 to 1 in the small sector, the index for the larger field is given by

$$N_{01} = \frac{\sum I_{01} V_0}{\sum V_0},$$

- where  $V_0$  is the value added in the period 0.

# Computation of IIP: Method of Indicators

$$V_0 = \sum P_0 Q_0 - \sum p_0 q_0 ,$$

- where  $\Sigma$  is over items in the small sector. Since  $N_{01}$  is an estimate of

$$\frac{\Sigma(\Sigma P_0 Q_1 - \Sigma p_0 q_1)}{\Sigma(\Sigma P_0 Q_0 - \Sigma p_0 q_0)} = \frac{\Sigma \frac{\Sigma P_0 Q_1 - \Sigma p_0 q_1}{\Sigma P_0 Q_0 - \Sigma p_0 q_0} (\Sigma P_0 Q_0 - \Sigma p_0 q_0)}{\Sigma(\Sigma P_0 Q_0 - \Sigma p_0 q_0)}$$

- $I_{01}$ 's are approximation to

$$\frac{\Sigma P_0 Q_1 - \Sigma p_0 q_1}{\Sigma P_0 Q_0 - \Sigma p_0 q_0} .$$

- Accordingly, the principal difference between the method of deflation and the method of indicators is that in the former,  $\frac{\Sigma P_0 Q_1 - \Sigma p_0 q_1}{\Sigma P_0 Q_0 - \Sigma p_0 q_0}$  is estimated directly in detail and in the latter it is approximated by an indicator series. The method of indicators is more practicable and is the commonest method found in practice.

# Output and Input Indicators

- Output Indicator:

$$I_{01} = \frac{\sum P_0 Q_1}{\sum P_0 Q_0}.$$

- Input Indicator:

$$I_{01} = \frac{\sum p_0 q_1}{\sum p_0 q_0}.$$