Index Number:
Part 1

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Syllabus: Index Number

- **Syllabus:**
  - **Index Number:** Construction of index numbers, properties, some well-known index number formulae, problem of construction of index numbers, chain indices, cost of living indices, splicing of index numbers, different types of index numbers used in India.
  - **Price Statistics:** Consumer price index numbers.
  - **Statics of Production:** Index of industrial production

Introduction

- **Definition:** An ‘Index Number’ (IN) is a device for comparing the general level of magnitude of a group of distinct, but related, variables in two or more situations. The situations may be different time points or different locations or different groups of people. Since the treatment is similar, we shall henceforth talk in terms of period only.

- An index combines the individual information and produces an indicator which gives the relative position of the target period, say, with respect to the base period, say, for the variable concerned. E.g., while comparing output of, say, consumer durables in a country in 2020 with what it was in 2010, we consider a group of variables as consumer durables – radio, refrigerator, carpet, etc. If these outputs change in the same ratio, we can take that ratio for comparison. Otherwise, we may have to combine these ratios or device some formula.

Price Relative

- For a particular commodity, if p_0 and p_1 are the prices of the commodity in suitable unit in the two situations denoted by '0' and '1', then the (relative) change in the price of the commodity from '0' to '1' is expressed as p_1/p_0, which is called price relative. The problem is to combine these various price relatives to get an overall index.
- **Definition:** The period being compared is called the **current or target period** and the period with which it is compared is called the **base or reference period**.

Index as Price Relative

- **Price Relatives**
- One of the simplest examples of an index number is a price relative, which is the ratio of the price of a single commodity in the current period to its price in the base period.
- If p_0 and p_1 denote the commodity prices during the base period and the current period respectively, then by definition

$$\text{Price relative } = P_{01} = \frac{p_1}{p_0},$$

- and is generally expressed as a percentage by multiplying by 100. i.e., price relative = $p_1/p_0 \times 100$. More generally if p_i and p_j are prices of a commodity during periods i and j respectively, the price relative in period j with respect to period i is defined as p_j/p_i and is denoted by P_{ij}.

Properties of Price Relatives

- **Properties of price relatives**

  If p_i, p_j, p_k, … denote prices in periods i, j, k respectively, the following properties exist for the associated price relatives.

- **1) Identity Property P_{ii} = 1**

  It is also evident as $P_{ii} = p_i/p_i = 1$ or 100%

  This states that the price relative for a given period with respect to the same period is 1 or 100%.

- **2) Time Reversal Property $P_{ij} \times P_{ji} = 1$ or $P_{ij} = 1/P_{ji}$.**

  As $P_{ij} \times P_{ji} = \left(p_j/p_i\right) \times \left(p_i/p_j\right) = 1.$

  This states that if two periods are interchanged, the corresponding price relatives are reciprocals of each other. So, price relatives follow time reversal property.

- **3) Cyclical or Circular Property According to this $P_{ij} \times P_{jk} \times P_{ki} = 1.$**

  As $P_{ij} \times P_{jk} \times P_{ki} = \left(p_j/p_i\right) \times \left(p_k/p_j\right) \times (p_i/p_k) = 1.$

  Modified Cyclical or Circular Property $P_{ij} \times P_{jk} = P_{ik}.$

Quantity or Volume Relatives

- **Quantity or Volume Relatives**
- Instead of comparing prices of a commodity, we may be interested in comparing quantities or volumes of the commodity, such as quantity or volume of production, consumption, exports, etc. In such cases we speak of quantity relatives or volume relatives. For simplicity, as in the case of prices, we assume that quantities are constant for any period. If they are not, an appropriate average for the period can be taken to make this assumption valid.
- If q_0 denotes the quantity or volume of a commodity produced, consumed, exported, etc., during a base period, while q_1 denotes the corresponding quantity produced, consumed, etc., during the current period, then we define
- Quantity or volume relative = q_1/q_0, which is generally expressed as a percentage.
- As in the case of price relatives, we use the notation Q_{ij} to denote the quantity relative in period j with respect to period i. The same remarks and properties pertaining to price relatives are applicable to quantity relatives.

Value Relatives

- **Value Relatives**
- If p is the price of a commodity during a period and q is the quantity or volume produced, sold, etc., during the period, then pq is called the total value. Thus if 1000 items are sold at 30 Rs. each the total value is Rs 30 x 1000 = Rs 30,000.
- If p_0 and q_0 denote the price and quantity of a commodity during the base period while p_1 and q_1 denote the corresponding price and quantity during the current period, then the total values during these periods are given by V_0 and V_1 respectively and we define
- Value relative = $V_1/V_0 = p_1q_1/p_0q_0 = (p_1/p_0)\,(q_1/q_0)$ = price relative × quantity relative.
- The same remarks, notation and properties pertaining to price and quantity relatives can be applied to value relatives.
- In particular if P_{ij}, Q_{ij} and V_{ij} denote the price, quantity and value relatives of period j with respect to period i then we have

$$V_{ij} = P_{ij} \times Q_{ij}.$$

Different Types of Index Number

- **Different Types of Index Number:**
- We may compare prices, quantities, or values leading to the respective indices: price index, quantity index or value index.

(1) **Price Index Numbers:** There may be different types of price index numbers.

(i) **Wholesale price index (WPI):** WPI measures the general change in the wholesale price level of a group of commodities. It measures variation in exchange value or purchasing power of money. It is needed in forecasting business conditions, deflating aggregates such as national/domestic product etc.

(ii) **Consumer price index (CPI)**: It measures variation in the retail prices of goods and services which enter into consumers' expenditure.

Different Types of Index Number

(iii) True Cost of living index or simply **cost of living index (CLI)**: CLI measures the relative change in the amount of money required to produce equivalent satisfaction under two different situations. We measure change in money income which will be necessary for a person to maintain his original standard of living – no more, no less. However, satisfaction and standard of living is an abstract thing which cannot be measured. We can only approximate it by taking some assumptions and thus it reduces to some CPI formula. Thus, CPI is sometimes referred to as CLI.

(2) Quantity index number (QIN): QIN is the index to measure the change in the quantity of goods produced.

Different Types of Index Number

- Besides, one can think of other indices like Index of Industrial Production.

(3) Index of Industrial Production (IIP): It measures movements in the quantum of production of individual firms and industries which contribute to the national aggregate. It helps in comparing the rates of change of production in the various industries of an economy and to compare these rates with changes in employments etc.

- Most widely used index number is the price index number.

Aggregative and Average Type Formulae

- Initially economists discussed only two types of formulae for IN. (1) **Aggregative type** and (2) **Average type**. The convention is that we write index numbers in percentage form.

  (1) Aggregative type index numbers may be simple aggregate of prices or weighted aggregate of prices. The most popular indices fall in the group of weighted aggregate of prices. The examples are Laspeyres's index, Paasche's index, Edgeworth-Marshall index etc.

  (2) Similarly average type may be simple average or weighted average. One can think of simple arithmetic mean (AM) of price relatives, simple geometric mean (GM) of price relatives, weighted AM of price relatives, weighted GM of price relatives, and so on.

- Depending on the weights average type becomes aggregative type and vice versa.
- There is a third type of index number which is the weighted average of two or more indices which may be termed as **Mixed type**.

Notations

- Consider the vectors p_0, q_0, p_1, q_1 as

$$p_0 = (p_0^1, p_0^2 \ldots, p_0^n), q_0 = (q_0^1, q_0^2 \ldots, q_0^n),$$

$$p_1 = (p_1^1, p_1^2 \ldots, p_1^n), q_1 = (q_1^1, q_1^2 \ldots, q_1^n).$$

- where p_0^i and q_0^i are price and quantity of ith commodity in the base period and p_1^i and q_1^i are price and quantity of ith commodity in the current period.

Steps Followed in the Construction of IN

- Steps followed in the construction of IN.

  (1) Defining the purpose of index number

  (2) Choice of the base period

  (3) Collection of data

  (4) Choice of commodities

  (5) Method of combining the data

  (6) Choice of weights/formula

  (7) Interpretation of the index

- We shall explain these steps assuming that we are constructing a price index number.

1. Defining the Purpose of Index Number

1. **Defining the Purpose of Index Number:**

   (i) The purpose for which the index number is being constructed should be clearly and unambiguously stated, since most of the later problems will depend upon the purpose. For instance, if we want to construct an index number for measuring the change in the consumer price index number, we have to take the retail prices of consumer goods and the costs of services like electricity charges.

   (ii) When the purpose is to measure the changes in general price level of goods, we are to consider the wholesale prices of finished product, intermediate products, agricultural products, mineral products, etc.

2. Choice of the Base Period

2. Choice of the Base Period:

- Suppose we want to compare the price levels of two time periods, say price level of 2020 with that of 2010. The base period, i.e., 2010, constitutes the basis of comparison. The price level of the base period is taken as 100 and the price level of the current period, 2020, is expressed relative to that.

(i) The base period must be a normal period. The prices of the base period should not be subject to boom or depression or effects of catastrophes like wars, floods, famines, etc.

(ii) it is also desirable to select a base period which is not too far in the past, for then we may not get comparable figures. Market conditions, i.e., tastes and habits of people, may undergo some change, resulting in the replacement of old goods by new ones.

Thus, we find that when a base, on being used for a number of years, becomes a period in the remote past, it is to be shifted to a period in the recent past for all subsequent comparisons.

Choice of the Base Period

(iii) The base period should not be too short or too long. It should not be too short, e.g., a single day, as in the case of share prices per unit of share, because the prices for too short a period are highly unstable and unreliable. Again, it should not be too long, e.g., five years, for then the average price for that period may smooth out some important fluctuations.

3. Collection of Data

3. Collection of data:

- Makers of price index numbers take great pains to collect the necessary data in each period for all the commodities included in the index number.
- The price of a commodity at a particular period of time will vary from one market to another and also for different grades. So, we are to collect prices of a commodity from a number of representative markets for a few important grades of the commodity.
- Each of these prices is referred to as a **price quotation**. An utmost care should be taken to get accurate data.

4. Choice of Commodities

4. Choice of commodities:

- It is practically impossible to include the prices of all commodities of an economy in constructing a price index number. The reason is that it involves too much time, money and labour. We are to take a suitable sample of commodities. But selection of commodities should be by judgment sampling, not by random sampling.
- All the important commodities (consumed by most of the people) should be taken in the sample and a subset from the rest of the commodities. The commodities may have to be classified into groups showing similar patterns of price fluctuations, and a number of commodities, representative of each of these groups are to be selected.
- The quality of the selected commodities should not vary much from period to period, and no commodities should disappear from the market.
- No rigid rule can be laid down for the number of commodities to be included. But it may be stated that the number should not be too large or too small.

5. Method of Combining Data

5. Method of combining data:

- The price fluctuations of different commodities are reflected in the price relatives. We want to represent the changes by means of a single number. So, we are to consider some means of combining these individual price fluctuations. It has been found that the distribution of price relatives is bell shaped with a marked central tendency. Amongst the various measures of central tendency, the arithmetic mean and the geometric mean may be used.

$$I_{01} = \frac{1}{n}\sum_{i=1}^{n} \frac{p_1^i}{p_0^i},$$

assuming that there are n commodities. This a simple or unweighted index using the arithmetic mean of price relatives. Similarly, the simple geometric mean of price relatives is

$$I_{01} = \left(\prod_{i=1}^{n} \frac{p_1^i}{p_0^i}\right)^{1/n}.$$

Method of Combining Data

- In the same way, the simple harmonic mean, median or mode of price relatives may be used.
- So far, we have considered some kind of average price relative to get the index number. We can also get a simple index number by comparing the simple aggregate of actual prices for the current period relative to that of the base period. Symbolically,

$$I_{01} = \frac{\sum_{i=1}^{n} p_1^i}{\sum_{i=1}^{n} p_0^i}.$$

- This is called a **simple aggregative index**. This formula has serious drawback. It depends too much on the units of prices. The average of price relatives is independent of unit of measurement.
- We can also get **weighted average of price relatives**, or weighted aggregative index. This will be discussed elsewhere.
- We are to multiply each formula by 100 to express the index in the percentage form.

6. Choice of Weights/Formula

6. Choice of Weights/Formula: We have already discussed some indices without taking any weights. But the commodities included in the index number are not of equal importance. For instance, in constructing a consumer price index number for India, rice and wheat should have greater importance than tobacco. So, we must consider the problem of weighting the different commodities included in the index number according to their importance. If we ignore weights, we may end up with an inappropriate index.

The simple arithmetic mean of price relatives may be written in the following form.

$$I_{01} = \frac{1}{n}\sum_{i=1}^{n} \frac{p_1^i}{p_0^i} = \frac{\sum_{i=1}^{n} p_1^i \times \frac{1}{p_0^i}}{\sum_{i=1}^{n} p_0^i \times \frac{1}{p_0^i}},$$

which is a **weighted aggregative index of prices**, each weight being the reciprocal of the base year price or the number of units of the commodity that can be purchased by one unit of money in the base period.

Choice of Weights/Formula

- Thus, we must adopt a system of weighting for the price relatives or prices that will truly reflect the importance of each commodity.
- Since our index should not depend on the units in which the prices or quantities are reported, we shall weight the price relatives by values and prices by quantities. The quantity used for determining the weight may be the quantity of the commodity consumed, produced, marketed or sold. The prices and quantities required for the weights may relate either to the base period or to the current period.
- If w_i is the weight attached to the price relative for the ith commodity, then the **weighted arithmetic mean of the price relatives** is given by

$$I_{01} = \frac{\sum_{i=1}^{n} \frac{p_1^i}{p_0^i} w_i}{\sum_{i=1}^{n} w_i}.$$

- This may be regarded as the general form of weighted arithmetic mean of price relatives.

Choice of Weights/Formula

- The general form of the weighted geometric mean of price relatives is given by

$$I_{01} = \left\{\prod_{i=1}^{n} \left(\frac{p_1^i}{p_0^i}\right)^{w_i}\right\}^{1/\sum w_i},$$

and the weighted harmonic mean by

$$I_{01} = \frac{\sum_{i=1}^{n} w_i}{\sum_{i=1}^{n} \frac{p_0^i}{p_1^i} w_i}.$$

- Similarly, if w_i is the weight attached to the price of the ith commodity, then **the general form of the weighted aggregative index** is given by

$$I_{01} = \frac{\sum_{i=1}^{n} p_1^i w_i}{\sum_{i=1}^{n} p_0^i w_i}.$$

Choice of Weights/Formula:
Laspeyres' and Paasche's Index

- Now, let us consider some particular weighted index numbers of prices.
- If in the general form of the weighted aggregative price index w_i be taken as q_0^i, the base-period quantities, then we get

$$L_{01} = \frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i},$$

- which is known as **Laspeyres' index**. Again, taking q_1^i, the current period quantities, for w_i in the general form of the weighted aggregative index, we get

$$P_{01} = \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i},$$

- which is known as **Paasche's index**.

Choice of Weights/Formula:
Edgeworth-Marshall Index

- One can see that Laspeyres' and Paasche's indices can also be written as weighted average of price relatives taking weights as $p_0^i q_0^i$ and $p_1^i q_1^i$ respectively. $p_0^i q_0^i$ and $p_1^i q_1^i$ are nothing but the base and current period values respectively. In fact, any weighted aggregative price index can also be viewed as weighted average of price relatives and vice versa.

- Taking w_i as $\left(q_1^i + q_0^i\right)/2$, the average of current period and base period quantities, in the general form of aggregative price index, we get

$$P_{01} = \frac{\sum_{i=1}^{n} p_1^i \left(q_1^i + q_0^i\right)}{\sum_{i=1}^{n} p_0^i \left(q_1^i + q_0^i\right)},$$

- which is known as the **Edgeworth-Marshall index**.

Choice of Weights/Formula:
Fisher's Ideal Index

- Irving Fisher tested a large number of formulae and selected the following formula which he obtained by taking geometric mean of Laspeyres' and Paasche's price indices.

$$F_{01} = \sqrt{\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}} = \sqrt{L_{01} \times P_{01}}.$$

- This is known as **Fisher's ideal index number**, because it satisfies certain tests of consistency which Irving Fisher considered appropriate.
- Dorbish and Bowley's Formula is the Arithmetic mean of L_{01} and P_{01}. The Lowe price index is a type of index in which the quantities are fixed and predetermined.
- In the majority of countries, the index numbers are computed using Laspeyres' formula or its equivalent, the weighted mean of price relatives, the weights being the base-year values. This formula is simple to calculate and the necessary data may be easily obtained. The other most commonly used formula, is the constant-weight aggregative or the constant-weight arithmetic mean of price relatives.

7. Interpretation of the Index

7. Interpretation of the index: The interpretation will depend on the purpose of the index number. The wholesale price index number measures the change in the general price level from the base period to the current period, while the cost-of-living index number compares the amounts of money required to purchase the same basket of goods and services for the two periods.

- Generally, the index numbers are expressed in percentage form and I_{00}, the index number for the base period compared with the base period itself, is taken as 100.
- Thus, the statement, “The wholesale price index number for India in the period 2020 compared with the base year 2010 is 175.8”, means that, as compared with the price level of the period 2010, the price level in the year 2020 increased by 1.758 times.

Fisher's Tests of Adequacy of Index Number

- **Fisher's Tests of Adequacy of Index Number:**
- From the historical point of view the axiomatic theory of price indices started not with axioms 1-5 but with some Tests due to Fisher (1922). [Fisher, I. 1922. The Making of Index Numbers. Boston: Houghton Mifflin]. Irving Fisher considered two tests of consistency which a price index number should satisfy, viz., the time reversal test and the factor reversal test.

  1. **Time Reversal Test**: $\mathrm{I}_{01} \times \mathrm{I}_{10} = 1$. I.e., we should get the same picture of the change in the price level if the base period and the current period are interchanged.
  2. **Factor Reversal Test**: $\mathrm{P}_{01} \times \mathrm{Q}_{01} = \mathrm{V}_{01}$

- where P_{01} is the price index, Q_{01} is the quantity index and V_{01} is the value index. It implies that the product of price index and quantity index should be value index.

Fisher's Tests of Adequacy of Index Number

- This needs further clarification. P_{01} is the usual price index. The value of P_{01} is found by putting the amounts of prices and quantities in the price index formula. We use the same formula for Q_{01} with prices and quantities interchanged. I.e., in the price index formula, we find the weighted average/aggregative/mixed of prices taking quantities as weights, whereas in the quantity index formula we find the weighted average/aggregative/mixed of quantities taking prices as weights, formula being the same. The value index is

$$V_{01} = \frac{p_1 q_1}{p_0 q_0} = \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_0^i}.$$

- This is the ratio of values for the two periods.

Fisher's Tests of Adequacy of Index Number: Circular Test

- It should be remembered that there is a test which is a generalized version of Time Reversal Test known as **Circular Test**.

  Circular Test: $\mathrm{I}_{01} \times \mathrm{I}_{12} \times \mathrm{I}_{23} \times \cdots \times \mathrm{I}_{n0} = 1.$

- Clearly if n = 1 then it reduces to Time Reversal Test. It just says that after some years, if the prices and quantities become same as that of base year then the product of the consecutive indices becomes one. It is easy to visualize the situation if we assume that there is only one commodity in the market.

- Laspeyres' and Paasche's formulae satisfy none of these two tests – time reversal test and factor reversal test. Fisher's formula is called ideal index number formula because it satisfies both these tests. At the time of Fisher, the circular test was not known and that is why Fisher's index number formula was regarded as ideal.

- Evidently, Fisher's ideal index number formula does not satisfy circular test.

Home Tasks

- **Home Task**: Prove that Fisher's ideal index number formula satisfies time reversal test and factor reversal test but does not satisfy circular test
- **Home Task**: Prove that None of the indices discussed so far (other than Fisher's ideal index) satisfy the factor reversal test.
- **Home Task**: Investigate whether the above proposed indices (other than Fisher's ideal index) satisfy time reversal test.