Index Number:

Part 3

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Axiomatic Approach to Price Index Number Formulation

- **Axiomatic Approach to Price Index Number Formulation:**
- In this approach, price index is viewed as a positive valued function depending on the prices and quantities of n commodities of a base year and the current year.

$$P:\ R^{4n}_{++} \rightarrow R_{++.}$$

- A price index number should satisfy the following axioms:

1. **Monotonicity Axiom:** The function is strictly increasing with respect to p_1 and strictly decreasing w.r.t. p_0.

$$P(p_0, q_0, p_1, q_1) > P(p_0, q_0, \bar{p}_1, q_1), \forall\ p_0, q_0, p_1, q_1 \text{ and } \bar{p}_1 \text{ such that } \bar{p}_1 < p_1.$$

$$P(p_0, q_0, p_1, q_1) < P(\bar{p}_0, q_0, p_1, q_1), \forall\ p_0, q_0, p_1, q_1 \text{ and } \bar{p}_0 \text{ such that } \bar{p}_0 < p_0.$$

2. **(Linear) Homogeneity Axiom:** The function is linear homogeneous in p_1.

$$P(p_0, q_0, \lambda p_1, q_1) = \lambda P(p_0, q_0, p_1, q_1)$$

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3. **Identity Axiom**: If prices remain unchanged then P = 1.

$$P(p_0, q_0, p_0, q_1) = 1$$

4. **Dimensionality Axiom**: P is invariant under change of unit of money.

$$(\lambda p_0, q_0, \lambda p_1, q_1) = P(p_0, q_0, p_1, q_1)$$

5. **Commensurability Axiom**: A price index P must be independent of the units of measurements of the quantities.

$$P\left(\lambda_1 p_0^1, \ldots, \lambda_n p_0^n, \frac{q_0^1}{\lambda_1}, \ldots, \frac{q_0^n}{\lambda_n}, \lambda_1 p_1^1, \ldots, \lambda_n p_1^n, \frac{q_1^1}{\lambda_1}, \ldots, \frac{q_1^n}{\lambda_n}\right)$$
$$= P(p_0, q_0, p_1, q_1).$$

Examples of Popular Price Indices Satisfying 1-5 Axioms

- **Examples of Popular Price Indices Satisfying 1-5 Axioms** are
- **E1. Laspeyre's index:**

$$L_{01} = \frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i}.$$

- **E2. Paasche's index:**

$$P_{01} = \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}.$$

- **E3. Fisher's ideal index:**

$$F_{01} = \sqrt{L_{01} \times P_{01}} = \sqrt{\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}}.$$

Other Examples of Price Indices Satisfying 1-5 Axioms

- **Other Examples of Price Indices Satisfying 1-5 Axioms** are

- E4. $I_{01} = \prod_{i=1}^{n} \left(\frac{p_1^i}{p_0^i}\right)^{\alpha_i}, \alpha_i > 0 \,\&\, \sum \alpha_i = 1.$

- E5. $I_{01} = \frac{\sum_{i=1}^{n}\left[\left(p_1^i q_0^i\right)^{-\rho}\right]^{-1/\rho}}{\sum_{i=1}^{n}\left[\left(p_0^i q_0^i\right)^{-\rho}\right]^{-1/\rho}}, \rho \neq 0.$

- E6. $I_{01} = \sum_{i=1}^{n} \beta_i \left(\frac{p_1^i}{p_0^i}\right)^{-\rho}, \beta_i > 0, \sum \beta_i = 1 \,\&\, \rho \neq 0.$

Theorem: None of These Five Axioms are Superfluous

- **Theorem:** Axioms 1 – 5 are independent.
- **Proof:** We shall prove that any four of them can be satisfied by a function that does not fulfill the remaining axiom.

For axiom (1)consider the function $I_{01} = \frac{p_1^1}{p_0^1}$.

For axiom (2)consider the function $I_{01} = \sqrt{\frac{p_1 q_0}{p_0 q_0}}$.

For axiom (3)consider the function $I_{01} = \frac{p_1 q_1}{p_0 q_0}$.

For axiom (4)consider the function $I_{01} = \frac{1}{\sum \ln\left(p_0^j q_0^j\right)} \sum \frac{p_1^i}{p_0^i} \ln\left(p_0^i q_0^i\right)$.

For axiom (5) consider the function $I_{01} = \frac{ap_1}{ap_0}$, where $a = (a_1, a_2, \dots, a_n)$.

Home Tasks

- **Home Task:** Prove the above theorem.
- **Home Task:** Prove the following.
- Every price index satisfying the axioms 1-5 satisfies proportionality test, i.e., if all prices change λ fold, then the index value is λ.

$$P(p_0, q_0, \lambda p_0, q_0) = \lambda.$$

Examples

- **Example 1:**
- Suppose, p_0 = base year price; p_1 = current year price; q_0 = base year quantity and q_1 = current year quantity. Prices and quantities for six commodities during the years 2000 and 2008 are given in the following table. The computations towards finding the different indices are also shown.

Commodity	2000		2008		$\frac{q_0 + q_1}{2} = q'$
	Price (p_0)	Quantity (q_0)	Price (p_1)	Quantity (q_1)	
(1)	(2)	(3)	(4)	(5)	(6)
Rice	277	10	367	62	36
Wheat	176	106	187	117	111.5
Jowar	151	42	183	55	48.5
Barley	122	24	181	10	17
Bajra	157	13	156	6	9.5
Gram	273	10	500	6	8

Calculation of Products

	p_0q_0	p_1q_0	p_0q_1	p_1q_1	p_0q'	p_1q'
Rice	2770	3670	17174	22754	9972	13212
Wheat	18656	19822	20592	21879	19624	20850.5
Jowar	6342	7686	8305	10065	7323.5	8875.5
Barley	2928	4344	1220	1810	2074	3077
Bajra	2041	2028	942	936	1491.5	1482
Gram	2730	5000	1638	3000	2184	4000
Sum	**35467**	**42550**	**49871**	**60444**	**42669**	**51497**

Calculation of Indices

- Then, **Laspeyres' Index:**

$$L_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{42550}{35467} \times 100 = 119.97.$$

- **Paasche's Index:**

$$L_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{60444}{49871} \times 100 = 121.20.$$

- **Edgeworth-Marshall's Index:**

$$EM_{01} = 100 \times \frac{\sum p_1 q'}{\sum p_0 q'} = \frac{51497}{42669} \times 100 = 120.69.$$

- **Fisher's Ideal Index:**

$$F_{01} = \sqrt{PIN_{01}^{L} \times PIN_{01}^{Pa}} = \sqrt{119.97 \times 121.20} = 120.58.$$

Combining Indices of Different Groups of Commodities

- **Example 2:**
- **Combining Indices of Different Groups of Commodities:**
- In calculating Index, all commodities are categorized into different commodity groups. Following an appropriate method, index is calculated for each of the commodity groups. These indices are treated equivalent to the price relatives. In the next step, we use weighted average of price relatives (i.e., group indices), where weight is the percent expenditure of the group to the total expenditure.

Calculations Towards Combining Indices of Different Groups of Commodities

- **Calculations Towards Combining Indices of Different Groups of Commodities**

Group	Index (2000-01=100)	Weight (% of total expenditure)
Primary articles	163.6	32.30
Fuel, power etc.	156.6	10.66
Manufactured products	168.6	57.04

- **Overall Index**:

$$\mathrm{I}_{01} = \frac{\sum \text{group index} \times \text{weight of the group}}{\sum \text{weight of the group}} = \frac{16570.569}{100} = 165.71.$$