Session 10, Bangalore 27/02/25

Optimization with logistic model

Let Y takes values only 0 or 1, so that

$$P(Y=1) = p = \left(\frac{\exp(a^T x + b)}{1 + \exp(a^T x + b)} \right).$$

where $x \in \mathbb{R}^n$ is the vector of variables that effects the probability and 'a' and 'b' are parameters.

Example: $Y=1$, survival of a person who is insured by a insurance company.

x = age, BMI, Occupation etc.

The variable x, over which we are optimizing, also can be subject to constraints.

$F^T x \leq g$. (F and

Question: Formulate the problem as a convex optimization problem.

(1) Goal: Maximize $P(Y=1)$, that is the survival probability.

(2) Maximize expected profit. Let $c^T x + d$ be the profit from selling the product (if the person survive)

Assume that profit is always positive.

Goal is to maximize expected profit.

Solution to (a)

$$P(Y=1) = \frac{\exp(a^T x + b)}{1 + \exp(a^T x + b)} \quad \text{—(1)}$$

Define, $$f(u) = \frac{e^u}{1+e^u}$$

compute $$\frac{d}{du} f(u) = \frac{e^u}{(1+e^u)^2} > 0$$

$\Rightarrow$ $f(u)$ is monotonically increasing in u

$\Rightarrow$ maximizing $f(u)$ is same as maximizing u

Take $u = a^T x + b$ in equation (1)

Then we can maximize $\frac{\exp(a^T x + b)}{1 + \exp(a^T x + b)}$

by maximizing $a^T x + b$.

The problem become Linear programme

$$\left.\begin{array}{l} \max_x \; a^T x + b \\ \text{subject } F^T x \leq g \end{array}\right\}$$

(b) Maximize expected profit

Expected profit

$$E(c^T x + d) = \frac{\exp(a^T x + b)}{1 + \exp(a^T x + b)} \times (c^T x + d)$$

Goal $\max_x E(c^T x + d)$ — (2)

with constraint $F^T x \leq g$

H.W check if the term of R.H.S of (2) is convex or not?

Ref: 4.61 (exercise of textbook), page 210

○

○