Introduction to Computing

Mathematics for Computing

Malay Bhattacharyya

Associate Professor

MIU, CAIML, TIH
Indian Statistical Institute, Kolkata

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Decimal, Binary, Octal, Hexadecimal

Given the number $x_k x_{k-1} \cdots x_1 x_0$ represented in base b, its decimal value is:

$$\sum_{i=0}^{k} x_i * b^i.$$

Number conversions

	To decimal	**Multiplier**
Binary [0-1]	$b_k \ \cdots \ b_7 \ b_6 \ b_5 \ b_4 \ b_3 \ b_2 \ b_1 \ b_0$	
	$\cdots \ 2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$	Power of 2
Octal [0-7]	$b_k \ \cdots \ \overline{b_7 \ b_6} \ \overline{b_5 \ b_4 \ b_3} \ \overline{b_2 \ b_1 \ b_0}$	
	$\cdots \ o_1 * 8^1 \quad o_0 * 8^0$	Power of 8
Hexadecimal [0-F]	$b_k \ \cdots \ \overline{b_7 \ b_6 \ b_5 \ b_4} \ \overline{b_3 \ b_2 \ b_1 \ b_0}$	
	$\cdots \ h_1 * 16^1 \quad h_0 * 16^0$	Power of 16

Number conversions

Decimal 6.625 equals to 110.101 in binary.

Converting the integer part 6 to binary:

Integer	Operation	Quotient (Integer)	Remainder
6	6 / 2	3	0
3	3 / 2	1	1
1	1 / 2	0 [STOP]	1

↑

Converting the fractional part 0.625 to binary:

Fraction	Operation	Fraction	Integer
0.625	0.625 * 2	0.25	1
0.25	0.25 * 2	0.5	0
0.5	0.5 * 2	0 [STOP]	1

↓

Arithmetic operations in binary

Addition	Subtraction	Multiplication	Division
$0 + 0 = 0$	$0 - 0 = 0$	$0 \times 0 = 0$	$0 / 0 = \text{NA}$
$0 + 1 = 1$	$0 - 1 = 1$ (borrow 1)	$0 \times 1 = 0$	$0 / 1 = 0$
$1 + 0 = 1$	$1 - 0 = 1$	$1 \times 0 = 0$	$1 / 0 = \text{NA}$
$1 + 1 = 0$ (carry 1)	$1 - 1 = 0$	$1 \times 1 = 1$	$1 / 1 = 1$

Note: The carry bit and borrow bit are required whenever an overflow and underflow happen, respectively.

Representation of unsigned integers

Binary	Decimal
00000000 00000000	0
00000000 00000001	+1
00000000 00000010	+2
00000000 00000011	+3
...	...
01111111 11111111	+32767
10000000 00000000	+32768
10000000 00000001	+32769
...	...
11111111 11111111	+65535

Note: The number of bits is fixed (depends upon the architecture).

Representation of signed integers

Binary	Decimal
1 0000000 00000000	-32768
...	...
1 1111111 11111101	-3
1 1111111 11111110	-2
1 1111111 11111111	-1
0 0000000 00000000	0
0 0000000 00000001	+1
0 0000000 00000010	+2
0 0000000 00000011	+3
...	...
0 1111111 11111111	+32767

Note: 2's complement of $x = -x$.

Bitwise shift

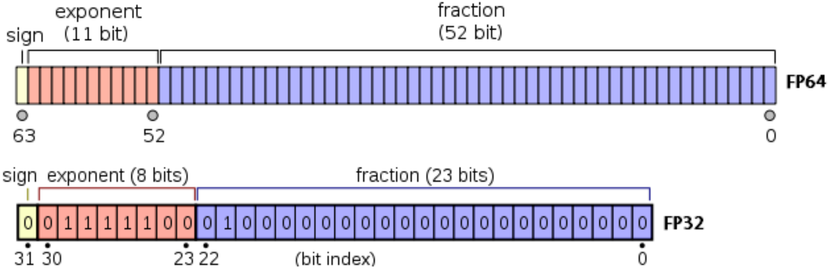
```
i = 11
i >> 1 = 5
i <<2 = 44
```

Bitwise shift

```
i = 11
i >> 1 = 5
i <<2 = 44
```

Decimal	**Binary**							
11	0	0	0	0	1	0	1	1
5	0	0	0	0	0	1	0	1
44	0	0	1	0	1	1	0	0

Floating point representation



Basics

Boolean algebra is a mathematical area that deals with operations on logical values with binary variables.

Boolean variables are represented as binary numbers to represent truthfulness, i.e., 1 denotes TRUE and 0 denotes FALSE.

De Morgan's theorems

- not (B_0 and B_1 and ... and B_n) = (not B_0) or (not B_1) or ... or (not B_n), where each B_i is a Boolean expression.
- not (B_0 or B_1 or ... or B_n) = (not B_0) and (not B_1) and ... and (not B_n), where each B_i is a Boolean expression.

Note: The original theorems are based on the union and intersection of sets.

Homework

- Prove that $(n << r) \mid (n >> (32 - r))$ will perform left circular shift (rotation) of the bits in n (32-bit representation) by r positions for integers n. Note that $<<$ and & denote bitwise left shift and bitwise and operation, respectively.
- Prove that $(n << 3) + (n << 1)$ will perform left rotation of the decimal digits in n by one position with zero padding for integers n. Note that $<<$ denotes bitwise left shift operation.
- Prove that if n & (n–1) is 0, then n is a power of 2. Note that & denotes bitwise and operation.
- Prove that for any arbitrary bits A, B and C, the following will hold.
  not (A and B) $=$ (not A) or (not B)
- Prove that for any arbitrary bits A, B and C, the following will hold.
  not (A or B) $=$ (not A) and (not B)