Indian Statistical Institute
BSDS: 2024-26
First Year: Semester – II
Economics-II
Home Task 1

Home Task 1.1: Prove that the general form of the weighted arithmetic mean of price relatives can be written as the general form of the weighted aggregative index and vice versa.

Answer: The weighted arithmetic mean of the price relatives is given by

$$I_{01} = \frac{\sum_{i=1}^{n} \frac{p_1^i}{p_0^i} w_i}{\sum_{i=1}^{n} w_i}.$$

$$\text{Put } w_i = \frac{p_0^i v_i}{\sum p_0^j v_j} \text{ to get } I_{01} = \frac{\sum_{i=1}^{n} \frac{p_1^i}{p_0^i} \frac{p_0^i v_i}{\sum p_0^j v_j}}{\sum_{i=1}^{n} \frac{p_0^i v_i}{\sum p_0^j v_j}} = \frac{\sum_{i=1}^{n} p_1^i v_i}{\sum_{i=1}^{n} p_0^i v_i}.$$

$$\text{Again take } I_{01} = \frac{\sum_{i=1}^{n} p_1^i v_i}{\sum_{i=1}^{n} p_0^i v_i}.$$

$$\text{Put } v_i = \frac{w_i}{\sum w_j} \text{ to get } I_{01} = \frac{\sum_{i=1}^{n} p_1^i \frac{w_i}{\sum w_j}}{\sum_{i=1}^{n} p_0^i \frac{w_i}{\sum w_j}} = \frac{\sum_{i=1}^{n} p_1^i w_i}{\sum_{i=1}^{n} p_0^i w_i}.$$

Home Task 1.2: Prove that if an index satisfies circular test then the chain index will be equal to the corresponding fixed base index. Also state some formula which does not satisfy Circular test but satisfy time reversal test.

Answer: Suppose the chain base index is C_{0n} and the corresponding fixed base index is I_{0n}.

The circular test says that

$$I_{01} \times I_{12} \times I_{23} \times \ldots I_{n-1,n} \times I_{n0} = 1.$$

$$I_{01} \times I_{12} \times I_{23} \times \ldots I_{n-1,n} \times I_{n0} = 1 \Rightarrow I_{01} \times I_{12} \times I_{23} \times \ldots I_{n-1,n} = \frac{1}{I_{n0}}. \qquad \ldots \text{(I)}$$

If an index obeys the circular test, then putting n = 1 we get $I_{01} \times I_{10} = 1$. This implies that $I_{10} = 1/I_{01}$. By the same principle we have $I_{12} \times I_{21} = 1 \Rightarrow I_{21} = 1/I_{12}$, and so on. The chain base index C_{0n} is defined as

$$C_{0n} = I_{01} \times I_{12} \times I_{23} \times \ldots \times I_{n-1,n}.$$

$$C_{0n} = I_{01} \times I_{12} \times I_{23} \times \ldots \times I_{n-1,n} = \frac{1}{I_{10}} \frac{1}{I_{21}} \cdots \frac{1}{I_{n,n-1}} = \frac{1}{I_{10} I_{21} \ldots I_{n,n-1}}$$

$$= \frac{1}{I_{n,n-1}I_{n-1,n-2}\cdots I_{21}I_{10}} = I_{0n}, \text{since } I_{n,n-1}I_{n-1,n-2}\cdots I_{21}I_{10}I_{0n} = 1 \text{ by circular test.}$$

Now, consider Fisher's index

$$F_{01} = \sqrt{\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}}$$

$$F_{01} \times F_{10} = \sqrt{\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}} \times \sqrt{\frac{\sum_{i=1}^{n} p_0^i q_1^i}{\sum_{i=1}^{n} p_1^i q_1^i} \times \frac{\sum_{i=1}^{n} p_0^i q_0^i}{\sum_{i=1}^{n} p_1^i q_0^i}} = 1.$$

$$F_{01} \times F_{12} \times F_{20}$$

$$= \sqrt{\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}} \times \sqrt{\frac{\sum_{i=1}^{n} p_2^i q_1^i}{\sum_{i=1}^{n} p_1^i q_1^i} \times \frac{\sum_{i=1}^{n} p_2^i q_2^i}{\sum_{i=1}^{n} p_1^i q_2^i}} \times \sqrt{\frac{\sum_{i=1}^{n} p_0^i q_2^i}{\sum_{i=1}^{n} p_2^i q_2^i} \times \frac{\sum_{i=1}^{n} p_0^i q_0^i}{\sum_{i=1}^{n} p_2^i q_0^i}} \neq 1.$$

(QED)

Home Task 1.3: Prove that Fisher's ideal index number formula satisfies time reversal test and factor reversal test but does not satisfy circular test.

Answer: In Home Task 1.2, we have already proved that Fisher's ideal index number formula satisfies time reversal test but does not satisfy circular test. Now it remains to show that Fisher's ideal index number formula satisfies factor reversal test.

$$\sqrt{\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_1^i}} \times \sqrt{\frac{\sum_{i=1}^{n} q_1^i p_0^i}{\sum_{i=1}^{n} q_0^i p_0^i} \times \frac{\sum_{i=1}^{n} q_1^i p_1^i}{\sum_{i=1}^{n} q_0^i p_1^i}} = \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_0^i}. \qquad \text{Proved.}$$

Home Task 1.4: Prove that None of the indices discussed so far (other than Fisher's ideal index) satisfy the factor reversal test.

Answer:

We shall prove it for L_{01} only. You should prove for other indices.

$$\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} q_1^i p_0^i}{\sum_{i=1}^{n} q_0^i p_0^i} \neq \frac{\sum_{i=1}^{n} p_1^i q_1^i}{\sum_{i=1}^{n} p_0^i q_0^i}. \qquad \text{Proved.}$$

Home Task 1.5: Investigate whether the proposed indices (other than Fisher's ideal index) satisfy time reversal test.

Answer:

We shall show it for L_{01} only. You should show for other indices.

$$\frac{\sum_{i=1}^{n} p_1^i q_0^i}{\sum_{i=1}^{n} p_0^i q_0^i} \times \frac{\sum_{i=1}^{n} p_0^i q_1^i}{\sum_{i=1}^{n} p_1^i q_1^i} \neq 1. \qquad \text{Proved.}$$