Complex Numbers

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- I. MCQs with One or More than One Correct
- 1) If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)'$ equals (1998-2 Marks)
 - a) 128ω
 - b) 128ω
 - c) $128\omega^2$
 - d) $-128\omega^2$
- 2) The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$ where i = $\sqrt{-1}$, equals: (1998 - 2 Marks)
 - b) i 1 c) -i
- 3) If $\begin{vmatrix} 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then (1998 2 Marks)
 - a) x = 3, y = 4
 - b) x = 1, y = 3
 - c) x = 0, y = 4
 - d) x = 0, y = 0
- 4) Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with 0 < t < 1. If arg (w) denotes the principal argument of a non-zero complex number w, then: (2010)
 - a) $|z z_1| + |z z_2| = |z_1 z_2|$

 - b) $\arg(z z_1) = \arg(z z_2)$ c) $\begin{vmatrix} z z_1 & \overline{z} \overline{z_1} \\ z_2 z_1 & \overline{z_2} \overline{z_1} \end{vmatrix}$
 - d) $\arg(z z_1) = \arg(z_2 z_1)$
- 5) Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further, let $H_1 = \left\{ z \in \mathbb{C} \middle| \operatorname{Re}(z) > \frac{1}{2} \right\}$ and $H_2 =$ $\{z \in \mathbb{C} | \operatorname{Re}(z) < -\frac{1}{2} \}$, where \mathbb{C} is the set of all complex numbers. If $z_1 \in H_1$, $z_2 \in H_2$, and O represents the origin, then the angle $\angle z_1Oz_2$ is: (JEE Adv. 2013)
 - a) $\frac{p}{2}$ b) $\frac{p}{6}$ c) $\frac{2p}{3}$ d) $\frac{5p}{6}$
- 6) Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Suppose S = $\left\{z \in \mathbb{C} \middle| z = \frac{1}{a+ibt}, \ t \neq 0\right\}$, where $i = \sqrt{-1}$. If z = x+iy and $z \in S$, then (x,y) lies on:

(JEE Adv. 2016)

- a) the circle with radius $\frac{1}{2a}$ and center $(\frac{1}{2a}, 0)$ for $a > 0, b \neq 0$.
- b) the circle with radius $\frac{1}{2a}$ and center $\left(\frac{-1}{2a},0\right)$ for $a < 0, b \neq 0$.
- c) the x-axis for $a \neq 0$, b = 0.
- d) the y-axis for a = 0, $b \ne 0$.
- 7) Let a, b, x, and y be real numbers such that a - b = 1 and $y \ne 0$. If the complex number z = x + iy satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is (are) possible value(s) of x? (JEE Adv. 2017)
 - a) $-1 + \sqrt{1 y^2}$ b) $-1 \sqrt{1 y^2}$ c) $1 + \sqrt{1 + y^2}$ d) $1 \sqrt{1 + y^2}$
- 8) For a non-zero complex number z, let arg(z)denote the principal argument with $-\pi$ < $\leq \pi$. Then, which of the following statement(s) is (are) FALSE? (JEE Adv. 2018)
 - a) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$
 - b) The function $f: \mathbb{R} \to (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of \mathbb{R} , where $i = \sqrt{-1}$
 - c) For any two complex numbers z_1 and z_2 $\operatorname{arg}\left(\frac{z_1}{z_2}\right) - \operatorname{arg}\left(z_1\right) + \operatorname{arg}\left(z_2\right)$ is an integer multiple of 2π
 - d) For any three given distinct complex numbers z_1, z_2 and z_3 , the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z-z_2)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line
- 9) Let s, t, r be non-zero complex numbers and L be the set of solutions z = x + iy $(x, y, \in \mathbb{R}, i = \sqrt{-1})$ of the equation $sz + t\overline{z} + r =$ 0, where $\bar{z} = x - iy$. Then, which of the following statements(s) is (are) TRUE? (JEE Adv. 2018)
 - a) Let L has exactly one element, then $|s| \neq |t|$
 - b) If |s| = |t|, then L has infinitely many elements
 - c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2
 - d) If L has more than one element, then L has

infinitely many elements

II. Subjective Problems

- 1) Express $\frac{1}{1-\cos\theta+2i\sin\theta}$ in the form x+iy. (1978) 2) If x=a+b, $y=a\gamma+b\beta$ and $z=a\beta+b\gamma$ where γ and β are the complex cube roots of
- unity, show that $xyz = a^3 + b^3$ (1978) 3) If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$
- 4) Find real values of x and y for which the following equation is satisfied $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ (1980)
- 5) Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$. (1981 - 4 Marks)
- 6) Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if z_1^2 + $z_2^2 - z_1 z_2 = 0.$ (1983 - 3 Marks)