

# Complex Numbers

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## I. MCQS WITH ONE OR MORE THAN ONE CORRECT

- 1) If  $\omega$  is an imaginary cube root of unity, then  $(1 + \omega - \omega^2)^7$  equals (1998-2 Marks)
  - (a)  $128\omega$
  - (b)  $128\omega$
  - (c)  $128\omega^2$
  - (d)  $-128\omega^2$
- 2) The value of the sum  $\sum_{n=1}^{13} (i^n + i^{n+1})$  where  $i = \sqrt{-1}$ , equals: (1998 - 2 Marks)
  - (a)  $i$
  - (b)  $i - 1$
  - (c)  $-i$
  - (d)  $0$
- 3) If  $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ , then (1998 - 2 Marks)
  - (a)  $x = 3, y = 4$
  - (b)  $x = 1, y = 3$
  - (c)  $x = 0, y = 4$
  - (d)  $x = 0, y = 0$
- 4) Let  $z_1$  and  $z_2$  be two distinct complex numbers and let  $z = (1 - t)z_1 + tz_2$  for some real number  $t$  with  $0 < t < 1$ . If  $\arg(w)$  denotes the principal argument of a non-zero complex number  $w$ , then: (2010)
  - (a)  $|z - z_1| + |z - z_2| = |z_1 - z_2|$
  - (b)  $\arg(z - z_1) = \arg(z - z_2)$
  - (c)  $\left| \frac{z - z_1}{z_2 - z_1} \right| = \left| \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1} \right|$
  - (d)  $\arg(z - z_1) = \arg(z_2 - z_1)$
- 5) Let  $w = \frac{\sqrt{3}+i}{2}$  and  $P = \{w^n : n = 1, 2, 3, \dots\}$ . Further, let  $H_1 = \{z \in \mathbb{C} \mid \operatorname{Re}(z) > \frac{1}{2}\}$  and  $H_2 = \{z \in \mathbb{C} \mid \operatorname{Re}(z) < -\frac{1}{2}\}$ , where  $\mathbb{C}$  is the set of all complex numbers. If  $z_1 \in H_1$ ,  $z_2 \in H_2$ , and  $O$  represents the origin, then the angle  $\angle z_1 O z_2$  is: (JEE Adv. 2013)
  - (a)  $\frac{p}{2}$
  - (b)  $\frac{p}{6}$
  - (c)  $\frac{2p}{3}$
  - (d)  $\frac{5p}{6}$
- 6) Let  $a, b \in \mathbb{R}$  and  $a^2 + b^2 \neq 0$ . Suppose  $S = \left\{z \in \mathbb{C} \mid z = \frac{1}{a+ibt}, t \neq 0\right\}$ , where  $i = \sqrt{-1}$ . If  $z = x + iy$  and  $z \in S$ , then  $(x, y)$  lies on: (JEE Adv. 2016)
  - (a) the circle with radius  $\frac{1}{2a}$  and center  $(\frac{1}{2a}, 0)$  for  $a > 0, b \neq 0$ .
  - (b) the circle with radius  $\frac{1}{2a}$  and center  $(\frac{-1}{2a}, 0)$  for  $a < 0, b \neq 0$ .
  - (c) the  $x$ -axis for  $a \neq 0, b = 0$ .
  - (d) the  $y$ -axis for  $a = 0, b \neq 0$ .
- 7) Let  $a, b, x$ , and  $y$  be real numbers such that  $a - b = 1$  and  $y \neq 0$ . If the complex number  $z = x + iy$  satisfies  $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$ , then which of the following is (are) possible value(s) of  $x$ ? (JEE Adv. 2017)
  - (a)  $-1 + \sqrt{1 - y^2}$
  - (b)  $-1 - \sqrt{1 - y^2}$
  - (c)  $1 + \sqrt{1 + y^2}$
  - (d)  $1 - \sqrt{1 + y^2}$
- 8) For a non-zero complex number  $z$ , let  $\arg(z)$  denote the principal argument with  $-\pi < \arg(z) \leq \pi$ . Then, which of the following statement(s) is (are) FALSE? (JEE Adv. 2018)
  - (a)  $\arg(-1 - i) = \frac{\pi}{4}$ , where  $i = \sqrt{-1}$
  - (b) The function  $f : \mathbb{R} \rightarrow (-\pi, \pi]$ , defined by  $f(t) = \arg(-1 + it)$  for all  $t \in \mathbb{R}$ , is continuous at all points of  $\mathbb{R}$ , where  $i = \sqrt{-1}$
  - (c) For any two complex numbers  $z_1$  and  $z_2$ ,  $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$  is an integer multiple of  $2\pi$
  - (d) For any three given distinct complex numbers  $z_1, z_2$  and  $z_3$ , the locus of the point  $z$  satisfying the condition  $\arg\left(\frac{(z-z_1)(z-z_2)}{(z-z_3)(z-z_1)}\right) = \pi$ , lies on a straight line
- 9) Let  $s, t, r$  be non-zero complex numbers and  $L$  be the set of solutions  $z = x + iy$  ( $x, y \in \mathbb{R}, i = \sqrt{-1}$ ) of the equation  $sz + t\bar{z} + r = 0$ , where  $\bar{z} = x - iy$ . Then, which of the following statements(s) is (are) TRUE? (JEE Adv. 2018)
  - (a) Let  $L$  has exactly one element, then  $|s| \neq |t|$
  - (b) If  $|s| = |t|$ , then  $L$  has infinitely many elements
  - (c) The number of elements in  $L \cap \{z : |z - 1 + i| = 5\}$  is at most 2
  - (d) If  $L$  has more than one element, then  $L$  has

infinitely many elements

## II. SUBJECTIVE PROBLEMS

- 1) Express  $\frac{1}{1-\cos\theta+2i\sin\theta}$  in the form  $x+iy$ . (1978)
- 2) If  $x = a + b$ ,  $y = a\gamma + b\beta$  and  $z = a\beta + b\gamma$  where  $\gamma$  and  $\beta$  are the complex cube roots of unity, show that  $xyz = a^3 + b^3$  (1978)
- 3) If  $x + iy = \sqrt{\frac{a+ib}{c+id}}$ , prove that  $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$  (1979)
- 4) Find real values of  $x$  and  $y$  for which the following equation is satisfied  $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$  (1980)
- 5) Let the complex numbers  $z_1, z_2$  and  $z_3$  be the vertices of an equilateral triangle. Let  $z_0$  be the circumcentre of the triangle. Then prove that  $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$ . (1981 - 4 Marks)
- 6) Prove that the complex numbers  $z_1, z_2$  and the origin form an equilateral triangle only if  $z_1^2 + z_2^2 - z_1z_2 = 0$ . (1983 - 3 Marks)