Complex Numbers

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I. MCQs with One or More than One Correct

1) If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals   (1998-2 Marks)
   (a) 128ω
   (b) 128ω
   (c) $128\omega^2$
   (d) $-128\omega^2$

2) The value of the sum $\sum_{n=1}^{13} \left(i^n + i^{n+1}\right)$ where $i = \sqrt{-1}$, equals: (1998 - 2 Marks)

   (a) i (b) $i - 1$ (c) $-i$ (d) 0

3) If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy,$
   then   (1998 - 2 Marks)
   (a) $x = 3, y = 4$
   (b) $x = 1, y = 3$
   (c) $x = 0, y = 4$
   (d) $x = 0, y = 0$

4) Let z_1 and z_2 be two distinct complex numbers and let $z = (1-t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If $\arg(w)$ denotes the principal argument of a non-zero complex number w, then:
   (2010)
   (a) $|z - z_1| + |z - z_2| = |z_1 - z_2|$
   (b) $\arg(z - z_1) = \arg(z - z_2)$
   (c) $\begin{vmatrix} z - z_1 & \overline{z} - \overline{z_1} \\ z_2 - z_1 & \overline{z_2} - \overline{z_1} \end{vmatrix}$
   (d) $\arg(z - z_1) = \arg(z_2 - z_1)$

5) Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further, let $H_1 = \left\{z \in \mathbb{C} \mid \text{Re}(z) > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in \mathbb{C} \mid \text{Re}(z) < -\frac{1}{2}\right\}$, where $\mathbb{C}$ is the set of all complex numbers. If $z_1 \in H_1$, $z_2 \in H_2$, and O represents the origin, then the angle $\angle z_1 O z_2$ is:
   (JEE Adv. 2013)

   (a) $\frac{p}{2}$ (b) $\frac{p}{6}$ (c) $\frac{2p}{3}$ (d) $\frac{5p}{6}$

6) Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Suppose $S = \left\{z \in \mathbb{C} \mid z = \frac{1}{a+ibt}, t \neq 0\right\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on:
   (JEE Adv. 2016)
   (a) the circle with radius $\frac{1}{2a}$ and center $\left(\frac{1}{2a}, 0\right)$ for $a > 0$, $b \neq 0$.
   (b) the circle with radius $\frac{1}{2a}$ and center $\left(\frac{-1}{2a}, 0\right)$ for $a < 0$, $b \neq 0$.
   (c) the x-axis for $a \neq 0$, $b = 0$.
   (d) the y-axis for $a = 0$, $b \neq 0$.

7) Let $a, b, x,$ and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is (are) possible value(s) of x?
   (JEE Adv. 2017)
   (a) $-1 + \sqrt{1 - y^2}$
   (b) $-1 - \sqrt{1 - y^2}$
   (c) $1 + \sqrt{1 + y^2}$
   (d) $1 - \sqrt{1 + y^2}$

8) For a non-zero complex number z , let $\arg(z)$ denote the principal argument with $-\pi < \arg(z) \leq \pi$. Then,which of the following statement(s) is (are) FALSE?
   (JEE Adv. 2018)
   (a) $\arg(-1 - i) = \frac{\pi}{4}, where\ i = \sqrt{-1}$
   (b) The function $f : \mathbb{R} \rightarrow (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$
   (c) For any two complex numbers z_1 and z_2 , $\arg(\frac{z_1}{z_2}) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π
   (d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z_2-z_2)}{(z-z_3)(z_2-z_1)}\right) = \pi$,lies on a straight line

9) Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ $\left(x, y, \in \mathbb{R}, i = \sqrt{-1}\right)$ of the equation $sz + t\overline{z} + r = 0$, where $\overline{z} = x - iy$.Then, which of the following statements(s) is (are) TRUE?
   (JEE Adv. 2018)
   (a) Let L has exactly one element, then $|s| \neq |t|$
   (b) If $|s| = |t|$,then L has infinitely many elements

(c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(d) If L has more than one element, then L has infinitely many elements

II. Subjective Problems

1) Express $\frac{1}{1 - \cos\theta + 2i\sin\theta}$ in the form $x + iy$.

(1978)

2) If $x = a + b$, $y = a\gamma + b\beta$ and $z = a\beta + b\gamma$ where γ and β are the complex cube roots of unity, show that $xyz = a^3 + b^3$

(1978)

3) If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $\left(x^2 + y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$

(1979)

4) Find real values of x and y for which the following equation is satisfied $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$

(1980)

5) Let the complex numbers $z_1, z_2 \text{ and } z_3$ be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$.

(1981 - 4 Marks)

6) Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1 z_2 = 0$.

(1983 - 3 Marks)