

Complex Numbers

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I. MCQS WITH ONE OR MORE THAN ONE CORRECT

- 1) If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals (1998-2 Marks)
 - a) 128ω
 - b) 128ω
 - c) $128\omega^2$
 - d) $-128\omega^2$
- 2) The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$ where $i = \sqrt{-1}$, equals: (1998 - 2 Marks)
 - a) i
 - b) $i - 1$
 - c) $-i$
 - d) 0
- 3) If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then (1998 - 2 Marks)
 - a) $x = 3, y = 4$
 - b) $x = 1, y = 3$
 - c) $x = 0, y = 4$
 - d) $x = 0, y = 0$
- 4) Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If $\arg(w)$ denotes the principal argument of a non-zero complex number w , then: (2010)
 - a) $|z - z_1| + |z - z_2| = |z_1 - z_2|$
 - b) $\arg(z - z_1) = \arg(z - z_2)$
 - c) $\left| \frac{z - z_1}{z_2 - z_1} \right| = \left| \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1} \right|$
 - d) $\arg(z - z_1) = \arg(z_2 - z_1)$
- 5) Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \dots\}$. Further, let $H_1 = \{z \in \mathbb{C} | \operatorname{Re}(z) > \frac{1}{2}\}$ and $H_2 = \{z \in \mathbb{C} | \operatorname{Re}(z) < -\frac{1}{2}\}$, where \mathbb{C} is the set of all complex numbers. If $z_1 \in H_1$, $z_2 \in H_2$, and O represents the origin, then the angle $\angle z_1 O z_2$ is: (JEE Adv. 2013)
 - a) $\frac{p}{2}$
 - b) $\frac{p}{6}$
 - c) $\frac{2p}{3}$
 - d) $\frac{5p}{6}$
- 6) Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Suppose $S = \{z \in \mathbb{C} | z = \frac{1}{a+ibt}, t \neq 0\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on: (JEE Adv. 2016)
 - a) the circle with radius $\frac{1}{2a}$ and center $(\frac{1}{2a}, 0)$ for $a > 0, b \neq 0$.
 - b) the circle with radius $\frac{1}{2a}$ and center $(\frac{-1}{2a}, 0)$ for $a < 0, b \neq 0$.
 - c) the x -axis for $a \neq 0, b = 0$.
 - d) the y -axis for $a = 0, b \neq 0$.
- 7) Let a, b, x , and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is (are) possible value(s) of x ? (JEE Adv. 2017)
 - a) $-1 + \sqrt{1 - y^2}$
 - b) $-1 - \sqrt{1 - y^2}$
 - c) $1 + \sqrt{1 + y^2}$
 - d) $1 - \sqrt{1 + y^2}$
- 8) For a non-zero complex number z , let $\arg(z)$ denote the principal argument with $-\pi < \arg(z) \leq \pi$. Then, which of the following statement(s) is (are) FALSE? (JEE Adv. 2018)
 - a) $\arg(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$
 - b) The function $f : \mathbb{R} \rightarrow (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in \mathbb{R}$, is continuous at all points of \mathbb{R} , where $i = \sqrt{-1}$
 - c) For any two complex numbers z_1 and z_2 , $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π
 - d) For any three given distinct complex numbers z_1, z_2 and z_3 , the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z-z_2)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line
- 9) Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ ($x, y \in \mathbb{R}, i = \sqrt{-1}$) of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statements(s) is (are) TRUE? (JEE Adv. 2018)
 - a) Let L has exactly one element, then $|s| \neq |t|$
 - b) If $|s| = |t|$, then L has infinitely many elements
 - c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2
 - d) If L has more than one element, then L has

infinitely many elements

II. SUBJECTIVE PROBLEMS

- 1) Express $\frac{1}{1-\cos\theta+2i\sin\theta}$ in the form $x+iy$. (1978)
- 2) If $x = a + b$, $y = a\gamma + b\beta$ and $z = a\beta + b\gamma$ where γ and β are the complex cube roots of unity, show that $xyz = a^3 + b^3$ (1978)
- 3) If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$ (1979)
- 4) Find real values of x and y for which the following equation is satisfied $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ (1980)
- 5) Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$. (1981 - 4 Marks)
- 6) Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1z_2 = 0$. (1983 - 3 Marks)