GATE 2

AI24BTECH11033 - Tanishq

1) Let R be a ring. If $R[x]$ is a principal ideal domain,then R is necessarily a
   a) Unique Factorization Domain
   b) Principal Ideal Domain
   c) Euclidean Domain
   d) Field
2) Consider the group homomorphism $\varphi : M_2(\mathbb{R}) \to \mathbb{R}$ given by $\varphi(A) = \text{trace}(A)$. The kernel of φ is isomorphic to which of the following groups?
   a) $M_2(\mathbb{R}) / \{A \in M_2(\mathbb{R}) : \varphi(A) = 0\}$
   b) $\mathbb{R}^2$
   c) $\mathbb{R}^3$
   d) $GL_2(\mathbb{R})$
3) Let X be a set with at least two elements. Let τ and τ' be two topologies on X such that $\tau' \neq \{\phi, X\}$. Which of the following conditions is necessary for the identity function id : $(X, \tau) \to (X, \tau')$ to be continuous?
   a) $\tau \subseteq \tau'$
   b) $\tau' \subseteq \tau$
   c) no conditions on τ and τ'
   d) $\tau \cap \tau' = \{\phi, X\}$
4) Let $A \in M_3(\mathbb{R})$ be such that $\det(A - I) = 0$, where I denotes the $3x3$ identity matrix. If the trace$(A) = 13$ and $\det(A) = 32$,then the sum of squares of the eigenvalues of A is ___________
5) Let V denote the vector space $C^5[a, b]$ over $\mathbb{R}$ and $W = \left\{f \in V : \frac{d^4 f}{dt^4} + 2\frac{d^2 f}{dt^2} - f = 0\right\}$. Then
   a) $\dim(V) = \infty$ and $\dim(W) = \infty$
   b) $\dim(V) = \infty$ and $\dim(W) = 4$
   c) $\dim(V) = 6$ and $\dim(W) = 5$
   d) $\dim(V) = 5$ and $\dim(W) = 4$
6) Let V be a real inner product space of dimension 10. Let $x, y \in V$ be non-zero vectors such that $\langle x, y\rangle = 0$. Then the dimension of $\{x\}^\perp \cap \{y\}^\perp$ *is*___________
7) Consider the following linear programming problem: Minimize $x_1 + x_2$ Subject to:

$$2x_1 + x_2 \geq 8 \tag{7.1}$$

$$2x_1 + 5x_2 \geq 10 \tag{7.2}$$

$$x_1, x_2 \geq 0 \tag{7.3}$$

   The optimal value to this problem is ___________.

8) Let

$$f(x) := \begin{cases} -3\pi & \text{if } -\pi < x \le 0 \\ 3\pi & \text{if } 0 < x < \pi \end{cases} \tag{8.1}$$

be a periodic function of period 2π. The coefficient of $\sin 3x$ in the Fourier series expansion of $f(x)$ on the interval $[-\pi, \pi]$ is ___________.

9) For the sequence of functions

$$f_n(x) = \frac{1}{x^2}\left(\sin\frac{1}{nx}\right), \quad x \in [1\ ,\infty), \tag{9.1}$$

consider the following quantities expressed in terms of Lebesgue integrals:

I. $\lim_{n\to\infty} \int_1^\infty f_x(x)\,dx.$

II. $\int_1^\infty \lim_{n\to\infty} f_n(x)\,dx.$

Which of the following is **TRUE**?

a) The limit in I does not exist.
b) The integrand in II is not integrable on $[1\ ,\infty)$.
c) Quantities I and II are well-defined, but I $\neq$ II.
d) Quantities I and II are well-defined and I = II.

10) Which of the following statements about the spaces l^p and $L^p[0,1]$ is **TRUE**?

a) $l^3 \subset l^7$ and $L^6[0,1] \subset L^9[0,1]$
b) $l^3 \subset l^7$ and $L^9[0,1] \subset L^6[0,1]$
c) $l^7 \subset l^3$ and $L^6[0,1] \subset L^9[0,1]$
d) $l^7 \subset l^3$ and $L^9[0,1] \subset L^6[0,1]$

11) The maximun modulus of e^{z^2} on the set S=$\{z \in \mathbb{C} : 0 \le \mathrm{Re}(z) \le 1, 0 \le \mathrm{Im}(z) \le 1\}$ is

a) $\frac{2}{e}$
b) e
c) $e+1$
d) e^2

12) Let d_1, d_2 and d_3 be matrices on a set X with at least two elements. Which of the following is **NOT** a metric on X?

a) $\min\{d_1, 2\}$
b) $\max\{d_2, 2\}$
c) $\frac{d_3}{1+d_3}$
d) $\frac{d_1+d_2+d_3}{3}$

13) Let $\Omega = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$ and let C be a smooth curve lying in Ω with initial point $-1+2i$ and final point $1+2i$. The value of $\int_C \frac{1+2z}{1+z}\,dz$ is

a) $4 - \frac{1}{2}\ln 2 + i\frac{\pi}{4}$
b) $-4 + \frac{1}{2}\ln 2 + i\frac{\pi}{4}$
c) $4 + \frac{1}{2}\ln 2 - i\frac{\pi}{4}$
d) $4 - \frac{1}{2}\ln 2 + i\frac{\pi}{2}$