Complex Numbers

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I. MCQs with One or More than One Correct

1) If ω is an imaginary cube root of unity, then $\left(1+\omega-\omega^2\right)^7$ equals (1998-2 Marks)
   a) 128ω
   b) 128ω
   c) $128\omega^2$
   d) $-128\omega^2$
2) The value of the sum $\sum_{n=1}^{13}\left(i^n+i^{n+1}\right)$ where $i=\sqrt{-1}$, equals: (1998 - 2 Marks)

   a) i b) $i-1$ c) $-i$ d) 0

3) If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x+iy$,then (1998 - 2 Marks)
   a) $x=3, y=4$
   b) $x=1, y=3$
   c) $x=0, y=4$
   d) $x=0, y=0$
4) Let z_1 and z_2 be two distinct complex numbers and let $z=(1-t)z_1+tz_2$ for some real number t with $0<t<1$. If $\arg(w)$ denotes the principal argument of a non-zero complex number w, then: (2010)
   a) $|z-z_1|+|z-z_2|=|z_1-z_2|$
   b) $\arg(z-z_1)=\arg(z-z_2)$
   c) $\begin{vmatrix} z-z_1 & \overline{z}-\overline{z_1} \\ z_2-z_1 & \overline{z_2}-\overline{z_1} \end{vmatrix}$
   d) $\arg(z-z_1)=\arg(z_2-z_1)$
5) Let $w=\frac{\sqrt{3}+i}{2}$ and $P=\{w^n : n=1,2,3,\ldots\}$. Further, let $H_1=\left\{z\in\mathbb{C}\middle|\mathrm{Re}(z)>\frac{1}{2}\right\}$ and $H_2=\left\{z\in\mathbb{C}\middle|\mathrm{Re}(z)<-\frac{1}{2}\right\}$, where $\mathbb{C}$ is the set of all complex numbers. If $z_1\in H_1$, $z_2\in H_2$, and O represents the origin, then the angle $\angle z_1Oz_2$ is: (JEE Adv. 2013)

   a) $\frac{p}{2}$ b) $\frac{p}{6}$ c) $\frac{2p}{3}$ d) $\frac{5p}{6}$

6) Let $a,b\in\mathbb{R}$ and $a^2+b^2\neq 0$. Suppose $S=\left\{z\in\mathbb{C}\middle|z=\frac{1}{a+ibt},\ t\neq 0\right\}$, where $i=\sqrt{-1}$. If $z=x+iy$ and $z\in S$, then (x,y) lies on: (JEE Adv. 2016)
   a) the circle with radius $\frac{1}{2a}$ and center $\left(\frac{1}{2a},0\right)$ for $a>0, b\neq 0$.
   b) the circle with radius $\frac{1}{2a}$ and center $\left(\frac{-1}{2a},0\right)$ for $a<0, b\neq 0$.
   c) the x-axis for $a\neq 0, b=0$.
   d) the y-axis for $a=0, b\neq 0$.
7) Let a,b,x, and y be real numbers such that $a-b=1$ and $y\neq 0$. If the complex number $z=x+iy$ satisfies $\mathrm{Im}\left(\frac{az+b}{z+1}\right)=y$, then which of the following is (are) possible value(s) of x? (JEE Adv. 2017)
   a) $-1+\sqrt{1-y^2}$
   b) $-1-\sqrt{1-y^2}$

c) $1+\sqrt{1+y^2}$
d) $1-\sqrt{1+y^2}$

8) For a non-zero complex number z , let $\arg(z)$ denote the principal argument with $-\pi < \arg(z) \le \pi$. Then,which of the following statement(s) is (are) FALSE? (JEE Adv. 2018)
 a) $\arg(-1-i) = \frac{\pi}{4}$,where $i = \sqrt{-1}$
 b) The function $f : \mathbb{R} \to (-\pi\ ,\ \pi]$, defined by $f(t) = \arg(-1+it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$
 c) For any two complex numbers z_1 and z_2 ,$\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π
 d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z-z_2)}{(z-z_3)(z_2-z_1)}\right) = \pi$,lies on a straight line

9) Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x+iy$ $\left(x, y, \in \mathbb{R}, i = \sqrt{-1}\right)$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$.Then, which of the following statements(s) is (are) TRUE? (JEE Adv. 2018)
 a) Let L has exactly one element, then $|s| \neq |t|$
 b) If $|s| = |t|$,then L has infinitely many elements
 c) The number of elements in $L \cap \{z : |z-1+i| = 5\}$ is at most 2
 d) If L has more than one element, then L has infinitely many elements

II. Subjective Problems

1) Express $\frac{1}{1-\cos\theta+2i\sin\theta}$ in the form $x+iy$. (1978)
2) If $x = a+b$, $y = a\gamma + b\beta$ and $z = a\beta + b\gamma$ where γ and β are the complex cube roots of unity, show that $xyz = a^3 + b^3$ (1978)
3) If $x+iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$ (1979)
4) Find real values of x and y for which the following equation is satisfied $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ (1980)
5) Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$. (1981 - 4 Marks)
6) Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2+z_2^2-z_1z_2 = 0$. (1983 - 3 Marks)