1-1.5-32

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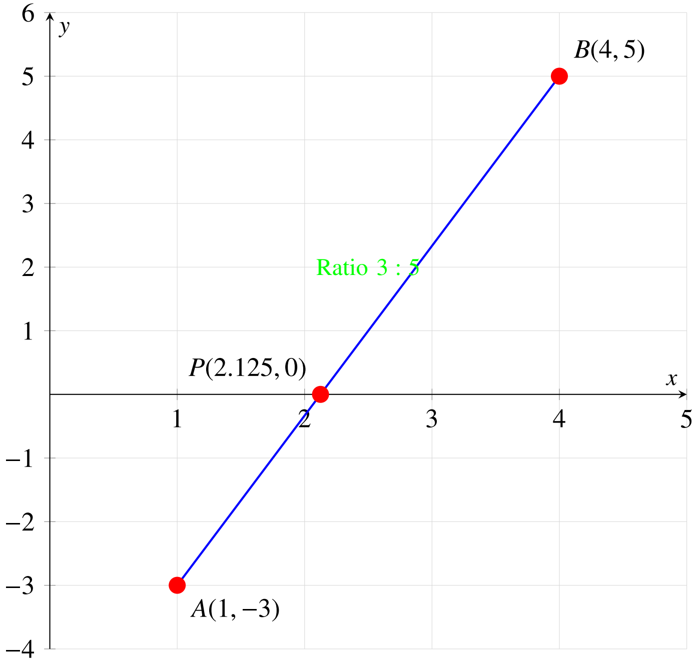
Question:
Find the ratio in which the line segment joining the points $(1, -3)$ and $(4, 5)$ is divided by X axis.
Solution:



1. **Equation of the Line Segment:** The line segment joining $A = (1, -3)$ and $B = (4, 5)$ can be expressed parametrically as:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

where $(x_1, y_1) = (1, -3)$ and $(x_2, y_2) = (4, 5)$.
So,

$$\frac{x - 1}{4 - 1} = \frac{y + 3}{5 + 3}$$

Simplify to:

$$\frac{x - 1}{3} = \frac{y + 3}{8}$$

2. **Finding Intersection with the x-Axis:**

The intersection with the x-axis occurs when $y = 0$. Substitute $y = 0$ into the parametric equation:

$$\frac{x-1}{3} = \frac{0+3}{8}$$

Simplify:

$$\frac{x-1}{3} = \frac{3}{8}$$

Solving for x:

$$x - 1 = \frac{3 \cdot 3}{8} = \frac{9}{8}$$

$$x = 1 + \frac{9}{8} = \frac{8+9}{8} = \frac{17}{8}$$

Therefore, the point of intersection with the x-axis is $\left(\frac{17}{8}, 0\right)$.

3. **Using Section Formula to Find Ratio:**

Let this point $\left(\frac{17}{8}, 0\right)$ divide the segment AB in the ratio $k : 1$. The section formula for a point dividing a line segment in the ratio $k : 1$ is given by:

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$

Here, substituting the coordinates $A = (1, -3)$ and $B = (4, 5)$:

$$\left(\frac{k \cdot 4 + 1}{k+1}, \frac{k \cdot 5 - 3}{k+1}\right) = \left(\frac{17}{8}, 0\right)$$

Equate the y-coordinates:

$$\frac{k \cdot 5 - 3}{k+1} = 0$$

$$k \cdot 5 - 3 = 0$$

$$k = \frac{3}{5}$$

Now, we need to verify the x-coordinate:

$$\frac{\frac{3}{5} \cdot 4 + 1}{\frac{3}{5} + 1} = \frac{\frac{12}{5} + 1}{\frac{8}{5}} = \frac{\frac{12+5}{5}}{\frac{8}{5}} = \frac{\frac{17}{5}}{\frac{8}{5}} = \frac{17}{8}$$

This confirms that our ratio $k = \frac{3}{5}$ is correct.

Hence, the ratio in which the line segment joining the points $(1, -3)$ and $(4, 5)$ is divided by the x-axis is $3 : 5$.

$$\text{Ratio} = \frac{3}{5} : 1 = 3 : 5$$