MuS8

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Problem 8.1: Birth-death processes

Given is a queuing system with two Markov service units and two waiting slots. The service rate is μ. The arrival rate is λ if no customer is waiting, $\frac{\lambda}{2}$ otherwise. In the following, the offered load is denoted by a $= \frac{\lambda}{\mu}$ and the relative offered load is $p = \frac{a}{n}$.

1. Give the states and brie???y describe their meaning!

2. Give the state transition diagram including transition rates!

3. Formally derive the stationary state probabilities! De???ne appropriate macro states for this purpose!

4. Plot diagrams with states on the x-axis and corresponding probabilities on the y-axis! Connect the points belonging to the same $p \in$ {0.3, 0.7, 0.9} with an interpolating line and scale the diagram appropriately!

5. Calculate the time-average probabilities that new customers need to wait or are blocked!

6. Calculate the waiting and blocking probability for new customers! Take into account that the arrival rate depends on the system state!

7. Plot the waiting and blocking probability for new customers in the interval $p \in$ (0, 1)!

8. Calculate the mean server utilization and the mean waiting queue length!

9. Plot the mean utilization and the mean waiting queue length (as multiple of $\frac{1}{\mu}$) in the interval $p \in$(0, 1)!

10. Formally derive the distribution function of the waiting time for all waiting and for all non-rejected customers, respectively!

11. Plot the complementary distribution function of the waiting time for all waiting and for all non-rejected customers for $p = 0.9$ in one diagram! Plot the waiting time as a multiple of $\frac{1}{\mu}$ on the x-axis and the corresponding probabilities on the y-axis!