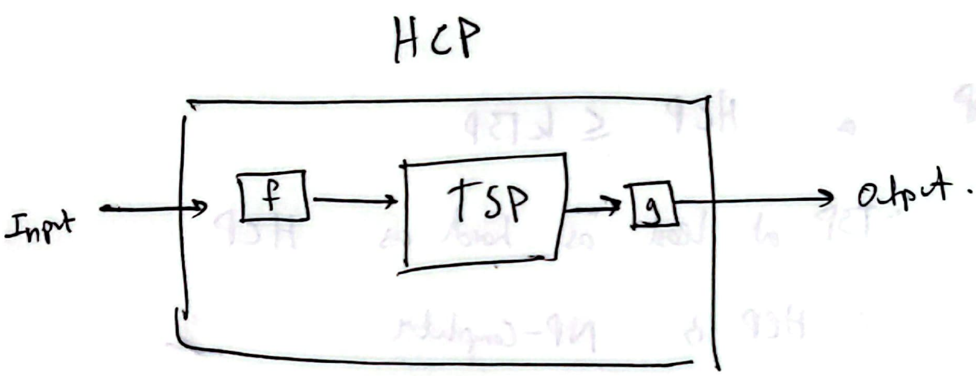
HCP (Hamiltonian Cycle Problem)

TSP (Travelling Salesman Problem)

1805006

$HCP <= K.TSP$



HCP input → $G(V, E)$.

$f: G \to G'(V, E')$

s.t. for $u, v \in V$, if ~~E(u,v)~~ $(u,v) \in E$

then $(u,v) = 1$ in E''

~~for u,v ∉ E~~ if $(u,v) \notin E$

then $(u,v) = \alpha$ in E'

$|E'| = \binom{V}{2}$

Solve TSP problem — see later for solⁿ.

define y: TSP ~~out~~ out $\rightarrow \{0, 1\}$

if TSP out $= n \rightarrow$ return 1

else $\longrightarrow$ return 0

(α)

Pr TSP : HCP $\leq_k$ TSP

$\because$ TSP at least as hard as HCP

$\because$ HCP is NP-Complete

$\therefore$ TSP is NP-Hard

TSP is NP:

Given sequence of nodes, promised yes ans for k size.

Check $\rightarrow$ sequence of corresponding edges.

$\rightarrow$ if sum $\leq k \rightarrow$ ok. ✓

Given TSP of (length n) & ($k < \alpha$) $\rightarrow$ then ~~exists~~ there is HCP (that cycle by defn).

if No TSP of (length n) $\rightarrow$ No HCP, since could not form cycle of length n.

n $k = \alpha$