1. a) $x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt.$

a) $\int_8^{20} f(t)\, dt$ — Simpson's $1/3^{rd}$, 4 intervals. $h = \frac{20-8}{4} = 3$

∴ points $x \rightarrow$ $x_0 = 8$, $x_1 = 11$, $x_2 = 14$, $x_3 = 17$, $x_4 = 20$.

∴ By M rule, $\frac{h}{3} \left[f(x_0) + 4 \sum_{\substack{i=1 \\ odd}}^{n-1} f(x_i) + 2 \sum_{\substack{i=2 \\ even}}^{n-2} f(x_i) + f(x_n) \right].$

$= \frac{3}{3} \left[f(8) + 4 \left[f(11) + f(17) \right] + 2 \left[f(14) \right] + f(20) \right]$

$= \left[177.266 + 4 \left[252.847 + 422.142 \right] + 2 \left[334.24 \right] + 517.349 \right]$

$= \boxed{4063.051}$

b) $\int_{20}^{30} f(t)\, dt$ using trapezoidal rule, (2 intervals).

$h = \frac{30-20}{2} = 5$ ∴ Points, $x_0 = 20$, $x_1 = 25$, $x_2 = 30$.

∴ $\int_{20}^{30} = \frac{h}{2} \left[f(x_0) + f(x_1) \right] + \frac{h}{2} \left[f(x_1) + f(x_2) \right]$

$= \frac{5}{2} \left[517.349 + 695.007 + 695.007 + 901.674 \right]$

$= 7022.5925$

c) Whole integral $\int_8^{30} f(t)\, dt = \int_8^{20} f(t)\, dt + \int_{20}^{30} f(t)\, dt$

$= 4063.051 + 7022.593$

$= 11085.643$ (approx)

x	$y=f(x)=\sin x$
0	0
$\pi/4$	$1/\sqrt{2}$
$\pi/2$	1.

2] No. of given points = 3,

$\therefore$ Polynomial order = 2

$\therefore f_2(x) = a_0 + a_1 x + a_2 x^2$

Newtons Divided Diff.

$f(0) = 0$ (b_0)

$f[\pi/4, 0] = \frac{1/\sqrt{2} - 0}{\pi/4} = 0.9003$ (b_1)

$f(\pi/4) = 1/\sqrt{2}$

$f[\pi/2, \pi/4] = \frac{1 - 1/\sqrt{2}}{\pi/2 - \pi/4} = 0.3729$

$f(\pi/2) = 1$

$f[\pi/2, \pi/4, 0] = \frac{0.3729 - 0.9003}{\pi/2 - 0} = -0.3357$ (b_2)

$\therefore$ Polynomial : $b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$

$= 0 + 0.9003(x-0) + -0.3357(x-0)(x-\pi/4)$

$f_2(x) = 1.164x + -0.3357x^2$ (Ans).

3. We cannot predict this using "intrapolation of Data" since

1. The ~~point~~ value D = 10 is greater than all given values of D.
2. The methodes of intrapolation, i.e Newton's Divided Method, & Lagrange's method are all "bracketing methods," i.e. the point must ~~be~~ have ~~included~~ within a data point on either side.

We can estimate a value using "extrapolation"

But this is out of scope of our syllabus.

Regression can also be used, but, the values obtained will not be accurate, since we are not using bracketing points.

 1805006

4.) Lagrange poly. of order 3 requires 4 distinct points,

for $x = 4$, we choose, $x = 2, 3, 5, 7$
$\quad x_0 \quad x_1 \quad x_2 \quad x_3$

$\therefore$ Lagrange poly:

$$f(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2(x) f(x_2) + L_3(x) f(x_3)$$

$$= \left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)\left(\frac{x-x_3}{x_0-x_3}\right) f(x_0) + \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)\left(\frac{x-x_3}{x_1-x_3}\right) f(x_1)$$

$$+ \left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right)\left(\frac{x-x_3}{x_2-x_3}\right) f(x_2) + \left(\frac{x-x_0}{x_3-x_0}\right)\left(\frac{x-x_1}{x_3-x_1}\right)\left(\frac{x-x_2}{x_3-x_2}\right) f(x_3)$$

$$= -0.2 \times 12 + 0.75 \times 19 + 0.5 \times 33 + (-0.05)(51)$$

$$\boxed{f(4) = 25.8}$$

5] Gauss Jordan requires

we rearrange the i^{st} row we with the maximum $|a_{ji}|$ from

$$\left(a_{jj},\ a_{(j+1)j},\ a_{(j+2)j},\ a_{(j+3)j},\ \ldots\ a_{nj}\right)$$

This requires ~~0~~ some extra operations.

In gaussian Elimination there is a chance (if pivot = 0) $a_{ii} = 0$ for any i of Division by 0. So, we can use Gauss Jordan to remove this drawback

 1805006

6] By Bisection $V = \sqrt{2gH} \tanh\left(\frac{\sqrt{2gH}}{2L} \times 4.5 \times 2\right)$

H_{low}	H_{hi}	H_m	V	e_r
0	10	5		

7) $y = axe^{bx}$

$\therefore \mathcal{E} = (y - axe^{bx})$ Minimize $\sum \mathcal{E}^2 = \sum (y - axe^{bx})^2$

$\frac{\partial}{\partial a}(\sum \mathcal{E}^2) = \frac{\partial}{\partial a} \sum (y_i - ax_i e^{bx_i})^2$

$= \sum -2 x_i e^{bx_i}(y_i - ax_i e^{bx_i}) = 0$

$\frac{\partial}{\partial b}(\sum \mathcal{E}^2) = \sum -2ax_i^2 [y_i - ax_i e^{bx_i}] = 0.$

$\therefore$ Solving,

$\sum x_i y_i e^{bx_i} - \sum a x_i^2 e^{2bx_i} = 0$

$\sum a x_i^2 y_i - \sum a^2 x_i^3 e^{bx_i} = 0$

from (i),

$$a = \frac{\sum x_i y_i e^{bx_i}}{\sum x_i^2 e^{2bx_i}} \quad \text{—(I)}$$

(I) in (II)

$\Rightarrow \sum x_i^2 y_i \frac{\sum x_i y_i e^{bx_i}}{\sum x_i^2 e^{2bx_i}} - \sum x_i^3 e^{bx_i} \left(\frac{\sum x_i y_i e^{bx_i}}{\sum x_i^2 e^{2bx_i}}\right)^2 = 0$

We can solve thus using bisection method, to get b, inputting value in (I) we can get a.

We can calculate the $\sum$ using given points

x	y
0.1	0.75
0.2	1.25
0.4	1.45.