# MATH2204 Time Series and Forecasting

# Final Project Report

Declaration of contributions:

| No | Name of Team Member | Contribution to the project |
|---|---|---|
| 1 | Tanjeena Tahrin Islam | 1/3 |
| 2 | Priyanka Tiwari | 1/3 |
| 3 | Syeda Shabnam Khan | 1/3 |
| | Sum: | 1 |

RMIT University,

School of Science

2025

## Introduction

This project aims to analyse and forecast the number of department stores in Australian retail sector over time using advanced time series modelling techniques. The dataset comprises monthly observations of department store count from April 1982 onwards, with no missing time point, enabling a comprehensive and continuous temporal analysis.

By examining this data, the study seeks to uncover underlying patterns such as ling-term trends and seasonal fluctuations that influence retail dynamics. Understanding these patterns is crucial for stakeholders to make informed decision related to inventory management, marketing strategies, and business expansion.

The central research question guiding this project is: How can Seasonal Autoregressive Integrated Moving Average models be effectively applied to model and forecast the number of department stores in Australia's retail sector, and which model specifications yield the best forecasting performance?

To answer this, this project involves the following steps:

- Conducting exploratory data analysis to identify initial trends, seasonality, and any anomalies.
- Performing stationarity testing and applying variance-stabilizing transformations as needed.
- Fitting a range of SARIMA models with different parameter configurations.
- Evaluating model performance using accuracy metrics such as AIC, BIC, RSME, MAE; and
- Generating forecasts for the next 10 months of department store counts based on the selected optimal SARIMA model.

## Methodology

### Dataset

The dataset used in this project contains monthly retail turnover figures (in millions of Australian dollars) specifically for department stores across Australia, spanning from April 1982 to March 2025. It provides a continuous, complete time series with no missing values, making it well-suited for time series analysis and forecasting. The data was sourced from the Australian Bureau of Statistic Website. Table 1. Retail turnover, by industry group, identified by the reference code A3348618 [1]. For this analysis, the focus is on the department store turnover values to explore trends and seasonal patterns in the Australian retail sector.

### Data Preprocessing and Exploration

The dataset was reviewed for the presence of missing values and missing timestamps. Basic descriptive statistics were generated to get an idea on the overall distribution of the data. The data was then transformed into a time series format. The time series plot was observed for key insights about the series. Additionally, ACF and PACF plots, as well as scatter plots for lags 1 and 2, were generated to provide further insights about the series.

### Normality and Stationarity Assessment

To assess the normality of the time series data, a Q-Q plot was generated, and the Shapiro-Wilk test was conducted. If the series deviated from normality, appropriate transformations were applied to the data to approximate a normal distribution.

For stationarity, the Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test, and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test were performed. If the series was found to be non-stationary, differencing was applied to ensure stationarity.

If the series was found with seasonality, seasonal difference has been applied to remove seasonal effect before further analysis.

### Model Specification and Selection

Residual approach was applied to determine the seasonal components of the model. Autocorrelation (ACF) and Partial Autocorrelation (PACF) plots, along with the Extended Autocorrelation Function (EACF) and Bayesian Information Criterion (BIC), were used to identify the various parameters for potential models. These analyses led to the identification of several plausible models.

Goodness-of-fit indicators, including the Akaike Information Criterion (AIC) and BIC, were then examined to select the best-fitting model. Additionally, error metrics were explored to ensure the chosen model provided the most consistent and accurate forecasts.

### Diagnostic Checking

Residuals from the finally fitted SARIMA model were examined for autocorrelation, normality, and constant variance.

### Forecasting

Best fitted model was used to forecast department store turnover for the next 10 months.

## Descriptive Analysis

The lowest monthly department store turnover recorded was 378.0 million AUD, while the highest record was 3,216.1 million AUD. In this data, 25% of the monthly turnover values are below 829.7 million AUD, median monthly turnover is 1,198.0 million AUD and 75% of the monthly turnover values are below 1,472.1 million AUD. The average monthly turnover across the entire time series is 1,221.8 million AUD, slightly higher than the median.

A check confirmed that the dataset contains no missing values or missing time points, ensuring a continuous and complete monthly time series.

**Figure 1.1: Time Series plot of total number of department stores in retail**

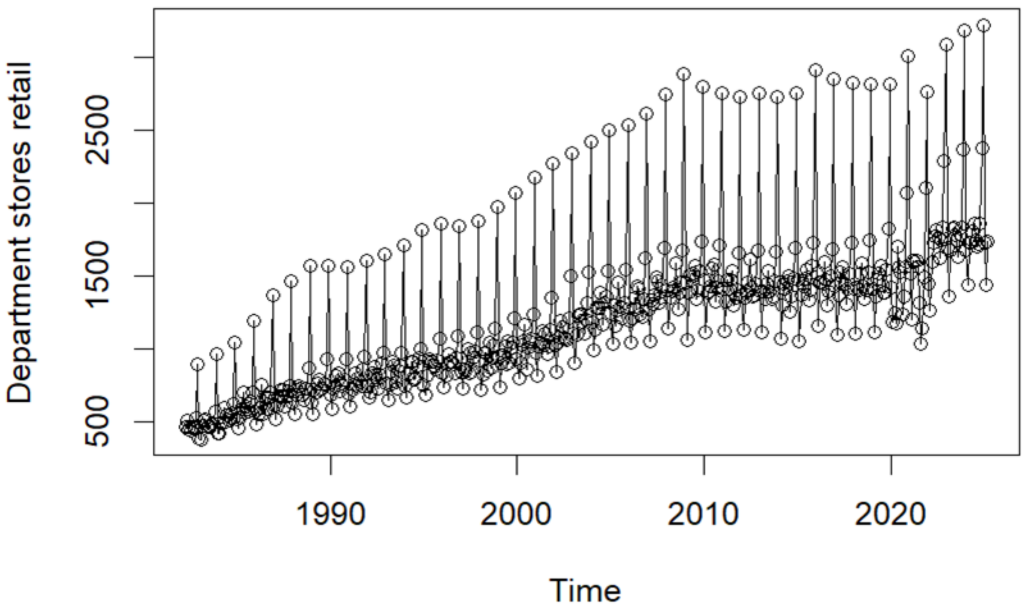
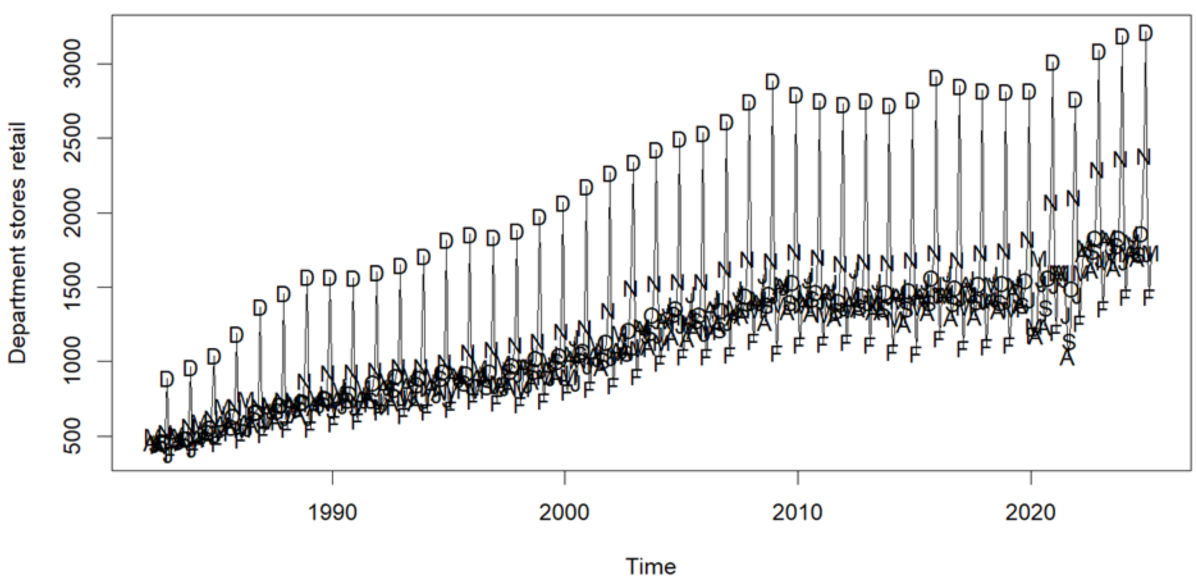
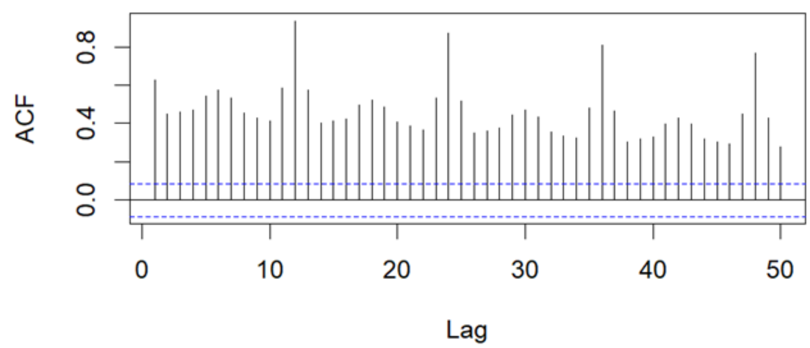



**Figure 1.2: Time series plot of total number of department stores in retail.**



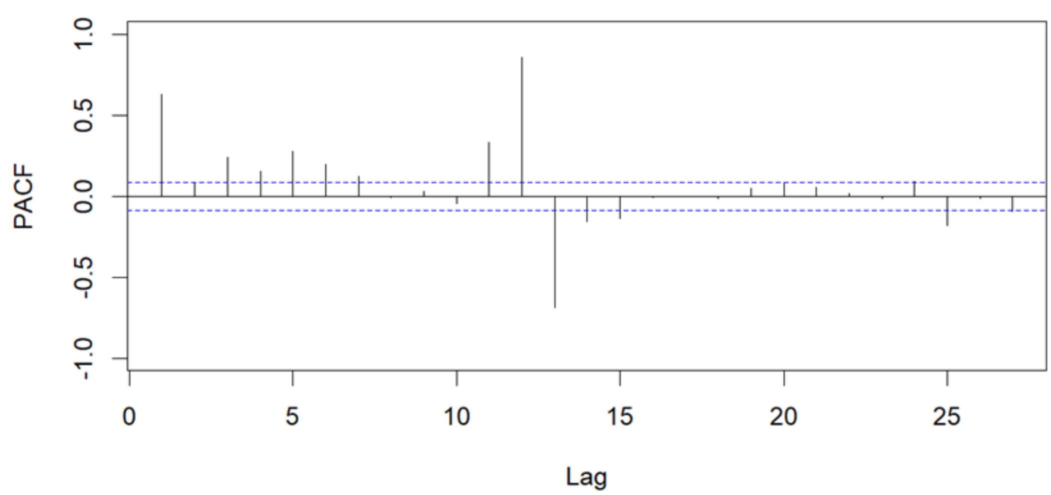

The time series plot (figure 1.1 & 1.2) reveals several key features of the department store retail turnover data. It displays a clear linear increasing trend over time, with a strong seasonal pattern and a consistent and prominent peaks occurring every December, likely driven by heightened consumer spending during the Christmas season. The plot also exhibits changing variance, with fluctuations becoming more pronounced in recent years. The underlying behavior appears to be largely autoregressive (AR) in nature but can be influenced by seasonal components. Finally, there is no visible change point.

**Figure 1.3: ACF of number of department stores in retail**



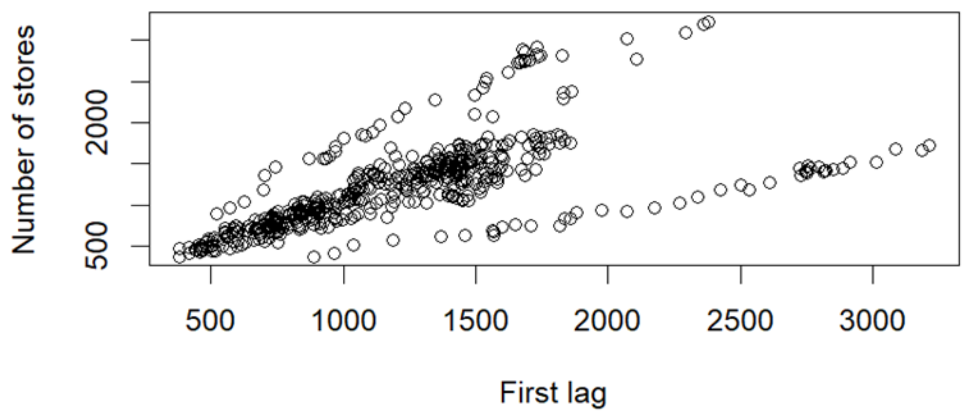

The ACF plot (figure 1.3) a monthly frequency of 12, showing a repeating seasonal pattern and slowly decaying lags, suggesting an AR process. A high autocorrelation at lag 1 also supports this. The PACF plot (figure 1.4) shows no clear cutoff, but several lags exceed the confidence bounds, indicating a complex structure and possibly multiple AR terms. These findings point to non-stationarity with strong seasonal and autoregressive behavior.

**Figure 1.4: PACF of total number of department stores in retail**



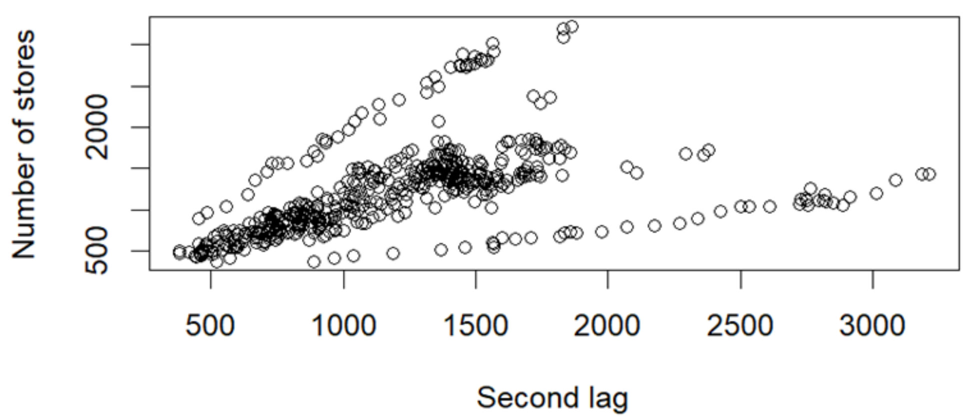
## Lag Scatter Plots and Autocorrelation

Figure 1.5: Scatter plot of neighboring number of stores




The scatter plot for lag 1 (figure 1.5) shows a moderate to strong positive autocorrelation, with a correlation coefficient of 0.6305, indicating a noticeable relationship with the previous time point.

Figure 1.6: Scatter plot of number of stores and its second lag values

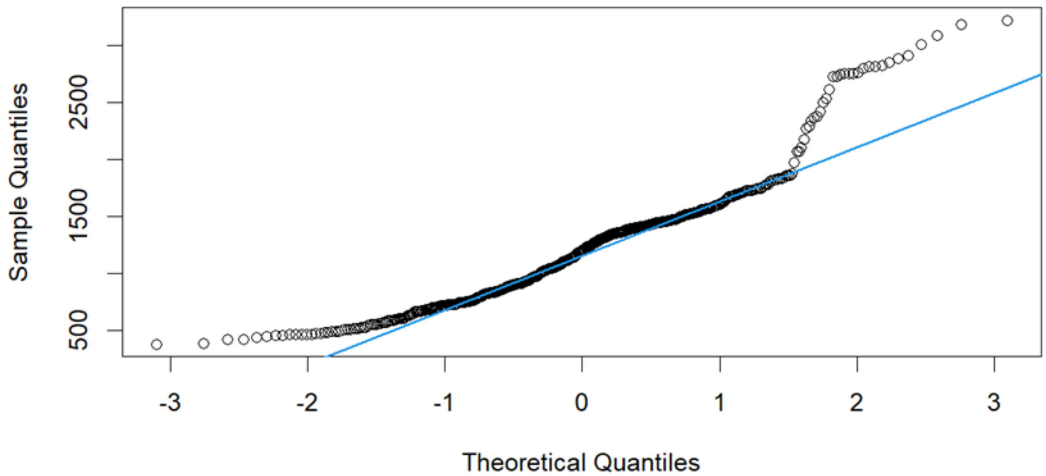



The scatter plot for lag 2 (figure 1.6) displays a weaker but still positive autocorrelation, with a correlation coefficient of 0.4513. These results are consistent with the patterns observed in the PACF plot, further supporting the presence of autoregressive behavior in the time series.

## Transformation and Stationarity

The time series plot showed changing variance, prompting normality checks.

Figure 2.1: QQ plot of number of department stores in retail

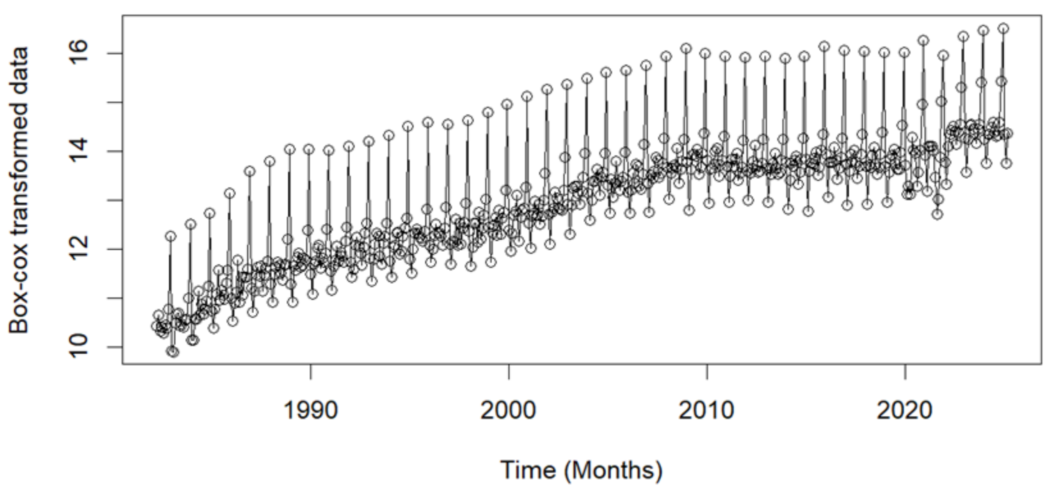



The Q-Q plot (figure 2.1) revealed significant deviation from normality. In Shapiro-Wilk test (W = 0.92163, p-value = 9.723e-16), we reject the null hypothesis of normality. Therefore, the presence of changing variance is confirmed, and a transformation has been applied to stabilize the variance.

## Transformation

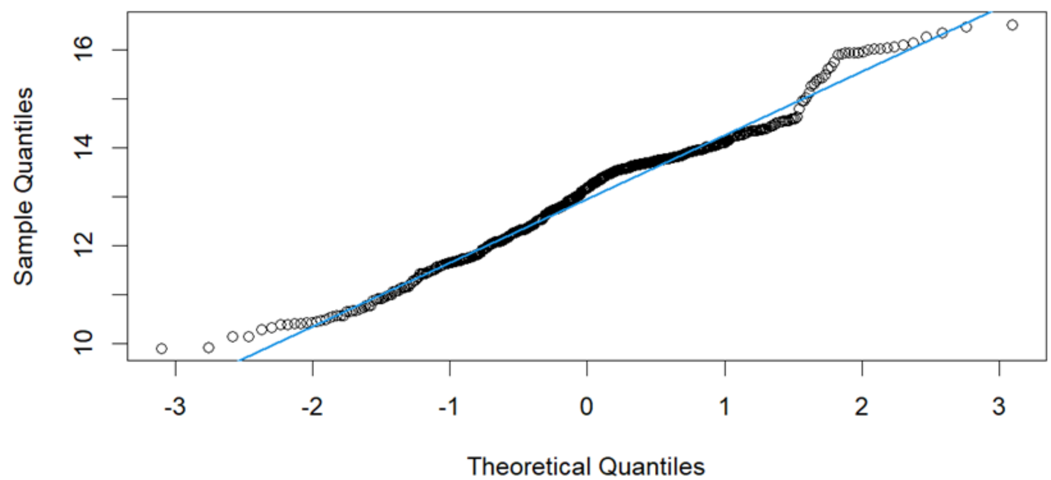
The Box-Cox transformation was attempted with various lambda ranges, finally with $\lambda = 0.16$ the Box-Cox transformation was applied. The transformation improved the stationarity of the model (figure 2.2).

Figure 2.2: Box-Cox transformed data of number of department stores in retail



Following the Box-Cox transformation, the Q-Q plot (figure 2.3) demonstrated an improved alignment with normality compared to the original series.

Figure 2.3: QQ plot of Box-cox transformed data




However, in Shapiro-Wilk test (W = 0.9825, p-value = 7.14e-06), we still reject the null hypothesis of normality. This deviation from normality may be due to underlying seasonality or trend in the data. Nonetheless, since the transformation successfully stabilized the variance, the transformed series was retained for subsequent analysis.

## Differencing

To assess stationarity, three formal tests were conducted on the transformed series.

| Test | Test Statistic | p-value | Conclusion |
|---|---|---|---|
| **Augmented Dickey-Fuller (ADF)** | -5.7922 | 0.01 | Stationary (reject $H_0$) |
| **Phillips-Perron (PP)** | -325.22 | 0.01 | Stationary (reject $H_0$) |
| **KPSS (Level Stationarity)** | 6.8217 | 0.01 | Non-stationary (reject $H_0$) |

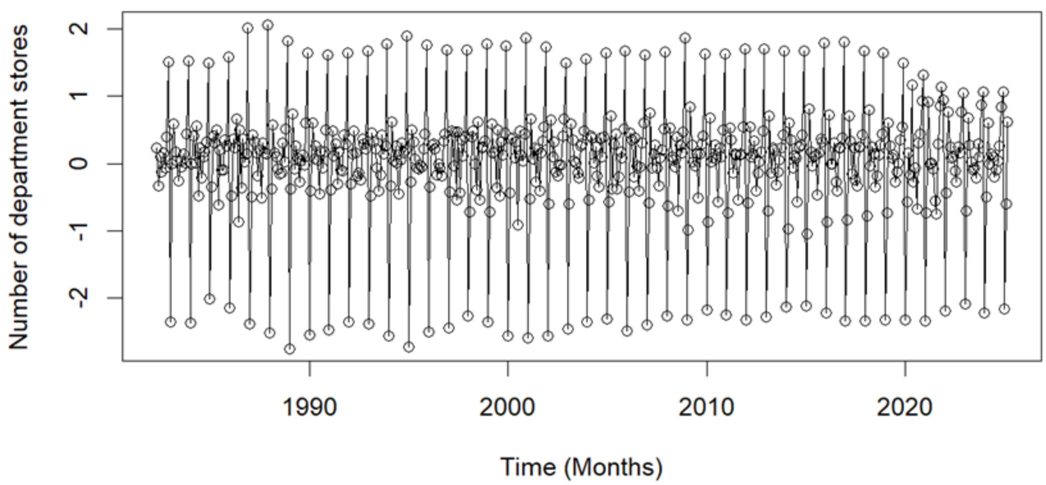
Table 2.1: Stationarity tests

The Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test both returned p-values of 0.01, suggesting that the series is stationary. However, the KPSS test with a p-value of 0.01 indicates non-stationarity. Given the test outcomes and visual evidence, first differencing was applied to proceed with stationarity.

### First lag Differencing

After first differencing, the series became stationary.

Figure 2.4: First Difference of Transformed data of number of department stores.




Key features of this time series plot (figure 2.4):

1. No trend
2. Seasonality present
3. Reduced changing variance compared to the original series
4. Behaviour is mostly autoregressive (AR)
5. No changing point observed

### Stationarity Check after Differencing

ACF & PACF plots, the Dickey-Fuller Test, Phillips-Perron Unit Root Test, and the KPSS test were used to confirm non-stationarity after differencing.

**Figure 2.5: ACF of first difference of transformed data**

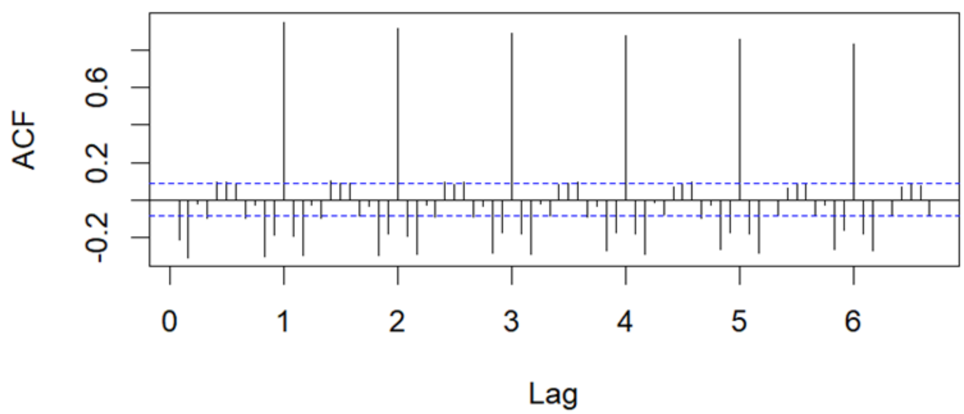
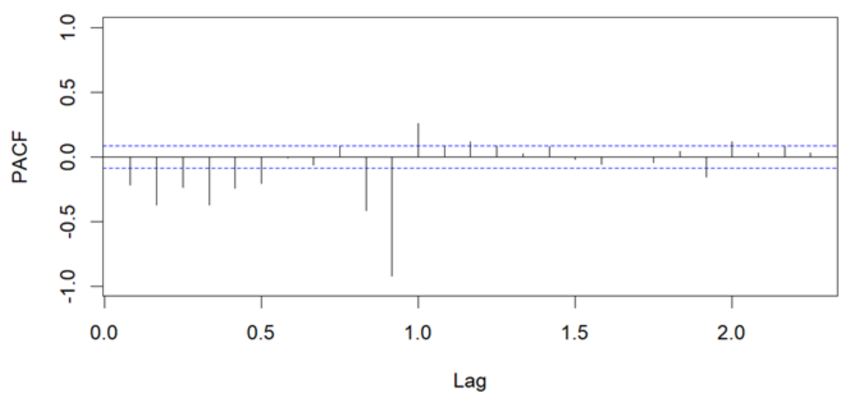



**Figure 2.6: PACF of first difference of transformed data**




The ACF plot (figure 2.5) shows no decreasing pattern but has high seasonal lags, and the PACF plot (figure 2.6) has several lags outside the borders, indicating that the series is non-stationary after differencing.

| Test | **Test Statistic** | **p-value** | **Conclusion** |
|---|---|---|---|
| Augmented Dickey-Fuller (ADF) | -10.173 | 0.01 | Stationary (reject $H_0$) |
| Phillips-Perron (PP) | -427.85 | 0.01 | Stationary (reject $H_0$) |
| KPSS (Level Stationarity) | 0.013852 | 0.1 | Stationary (fail to reject $H_0$) |

Table 2.2: Stationarity tests of first differenced series

The agreement between ADF, PP, and KPSS tests indicates that after applying the Box-Cox transformation and first differencing, the time series has achieved stationarity (table 2.2).

## Seasonal differencing

Finally, seasonal differencing has been applied to the transformed and first differenced series to control seasonal patterns.

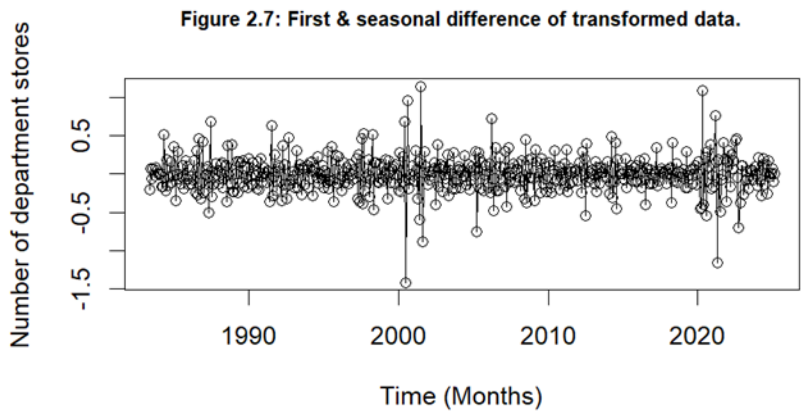


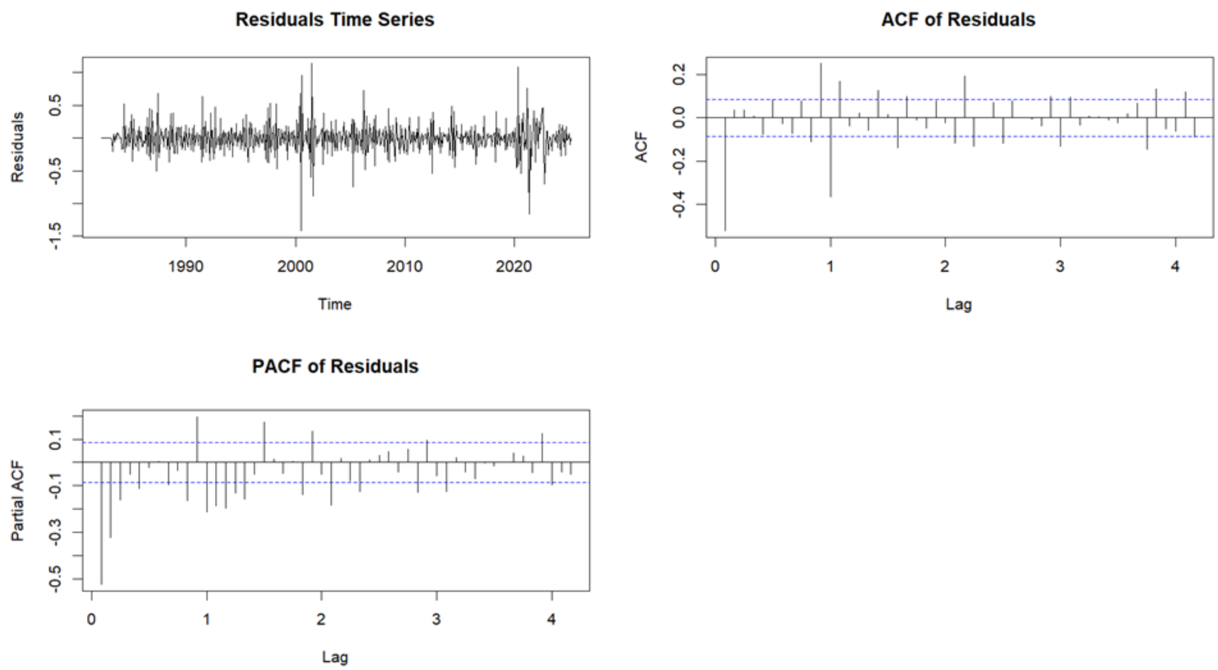

Figure 2.7 shows the first and seasonal differenced of the transformed series with no seasonal pattern.

## Model Specification

We have determined the model's components d=1 and D=1 with s=12. Now, we implied residual approach to determine the other seasonal components P and Q.

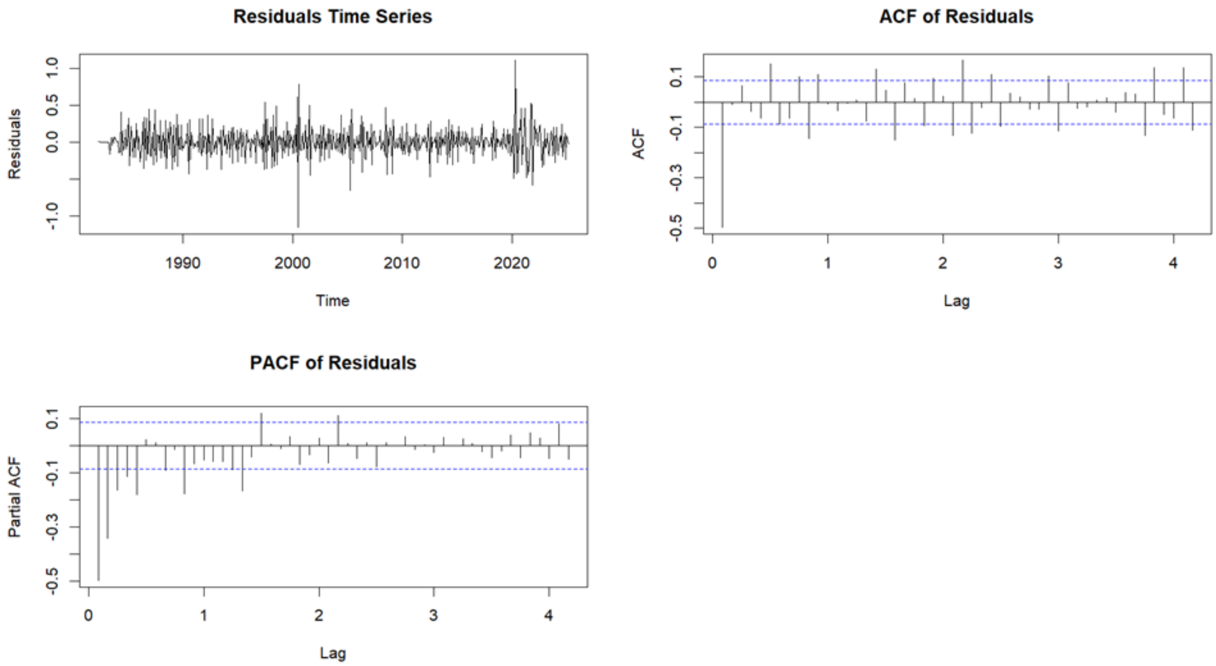
### Residual approach to confirm model's seasonal components (P & Q)

#### Considering SARIMA(0,1,0)x (0,1,0)_[12 ]model




The residual diagnostics for the SARIMA(0,1,0)(0,1,0)_12 model shows no trend or seasonality in the residual time series plot. However, the ACF and PACF plots reveal significant autocorrelations at various lags. In the ACF plot, spikes are observed at lags 1, 10, 11, and 13, along with seasonal spikes at lags 12 and 36. The PACF plot shows significant lags at 1, 2, 3, 5, 8, and 10, and a weak seasonal spike at lag 48. So, this model fails to adequately capture both seasonal and non-seasonal structures in the data.

## Considering SARIMA(0,1,0)x(1,1,2)_[12] model




Although this model also shows residuals that are not white noise, the lack of strong seasonal spikes in the ACF and PACF plots suggests that the seasonal components (P=1, D=1, Q=2) are appropriate.

Now, we determine the other components (p and q) of the model.

## ACF and PACF Plots

**Figure 3.1: ACF of first & seasonal difference of transformed data**

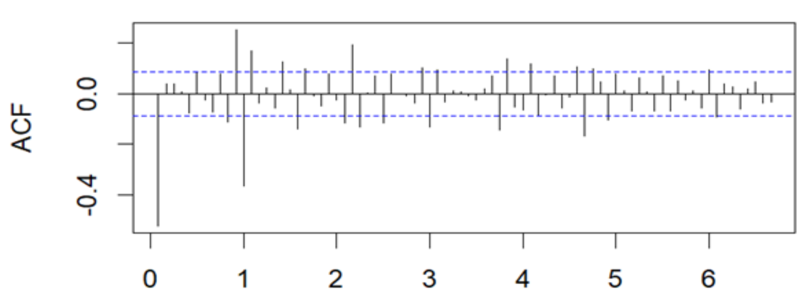
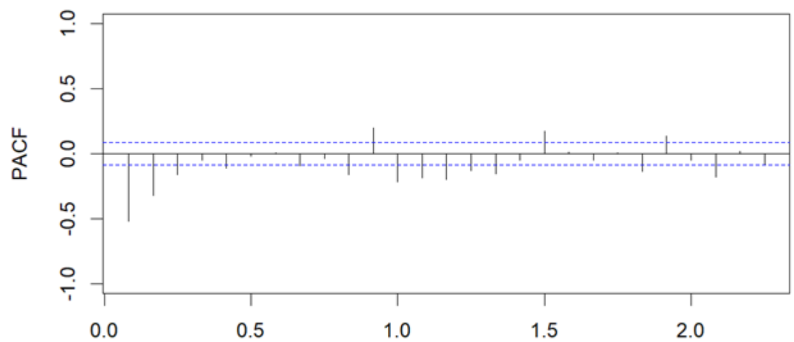


**Figure 3.2: PACF of first & seasonal difference of transformed data**




Based on the ACF plot (figure 3.1), significant spikes are observed at lags 3 and 4, suggesting that values of q = 3 and 4 should be considered. In the PACF plot (figure 3.2), lags 5, 6, and 7 cross the significance boundary, indicating potential values of p = 5, 6, and 7. Therefore, we explore the following SARIMA models:

- SARIMA(5,1,3)x(1,1,2)_[12]
- SARIMA(5,1,4)x(1,1,2)_[12]
- SARIMA(6,1,3)x(1,1,2)_[12]
- SARIMA(6,1,4)x(1,1,2)_[12]
- SARIMA(7,1,3)x(1,1,2)_[12]
- SARIMA(7,1,4)x(1,1,2)_[12]

## EACF Matrix

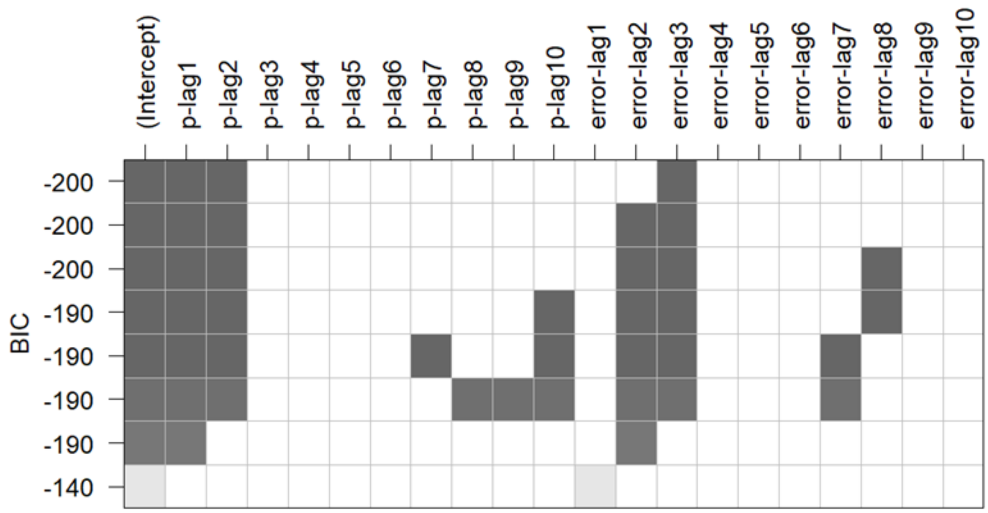
| | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | x | o | o | o | o | o | o | o | o | x | x |
| 1 | x | x | o | o | o | o | o | x | o | o | o |
| 2 | x | x | o | o | o | o | o | o | o | o | o |
| 3 | x | x | x | o | o | o | o | o | o | o | o |
| 4 | x | x | x | x | o | o | o | o | o | o | o |
| 5 | x | x | x | x | o | o | o | o | o | o | o |
| 6 | x | o | x | x | o | o | x | o | o | o | o |
| 7 | o | x | x | x | x | x | o | o | o | o | o |
| 8 | x | x | x | o | x | x | o | o | o | o | o |
| 9 | x | x | x | o | x | x | o | o | o | o | o |
| 10 | x | o | x | x | x | x | o | o | o | o | x |

The top-left 'o' indicates p = 0 and q = 1, and the neighboring 'o's are also considered when deciding on the models. From the EACF matrix, we identified the following models:

- SARIMA(0,1,1)(1,1,2)_12
- SARIMA(0,1,2)(1,1,2)_12
- SARIMA(1,1,2)(1,1,2)_12

## BIC plot

**Figure 3.3: BIC selection plot for models**




In the BIC plot (figure 3.3), the SARIMA(1,1,3)x(1,1,2)_[12] and SARIMA(2,1,3) x(1,1,2)_[12] models are identified as the best models due to their lowest BIC values. The second-best models are SARIMA(1,1,2)x(1,1,2)_[12] and SARIMA(2,1,2)x(1,1,2)_[12]. Models with q = 8 were not considered because there wasn't enough evidence or support.

## Final suggested models:

1. SARIMA(0,1,1)x(1,1,2)$_{12}$
2. SARIMA(0,1,2)x(1,1,2)$_{12}$
3. SARIMA(1,1,2)x(1,1,2)$_{12}$
4. SARIMA(1,1,3)x(1,1,2)$_{12}$
5. SARIMA(2,1,2)x(1,1,2)$_{12}$
6. SARIMA(2,1,3)x(1,1,2)$_{12}$
7. SARIMA(5,1,3)x(1,1,2)$_{12}$
8. SARIMA(5,1,4)x(1,1,2)$_{12}$
9. SARIMA(6,1,3)x(1,1,2)$_{12}$
10. SARIMA(6,1,4)x(1,1,2)$_{12}$
11. SARIMA(7,1,3)x(1,1,2)$_{12}$
12. SARIMA(7,1,4)x(1,1,2)$_{12}$

## Model Fitting & Selection

To identify the most appropriate SARIMA model for the transformed time series data, a comprehensive model fitting process was conducted using three estimation methods:

- Maximum Likelihood (ML)
- Conditional Sum of Squares (CSS)
- Hybrid approach, (CSS-ML)

All models were fitted with a **fixed seasonal component of (1,1,2)[12]** and candidate non-seasonal parameters (p,d,q) were derived based on:

- ACF/PACF analysis
- Extended ACF (EACF)
- Bayesian Information Criterion (BIC)

As the series was already differenced and transformed, the differencing order was fixed at d = 1. A range of simple to complex models were explored capture the underlying structure of the data.

### Model Evaluation Criteria:

Each model was evaluated using the following criteria:

- Coefficient significance tests (Z-tests via coeftest)
- Information criteria:
    - Akaike Information Criterion (AIC)
    - Bayesian Information Criterion (BIC)
- Forecast accuracy metrics:
    - Mean Absolute Percentage Error (MAPE)
    - Root Mean Square Error (RMSE)
    - Mean Absolute Scaled Error (MASE)

### Exclusion of Unstable or Inadequate Models

Many of the higher-order models estimated using the CSS method exhibited convergence issues or returned NaN standard errors, indicating unreliable parameter estimates. These were excluded from final consideration.

Several simpler models, including SARIMA(0,1,1), (0,1,2), (1,1,2), (1,1,3) and (2,1,2) featured fewer parameters and were computationally efficient. However, they were not selected as the best models due to the following reasons:

- They consistently showed higher error metrics and lower forecast accuracy.
- AIC and BIC values were higher, indicating poorer model fit.
- These models underfit the data, missing key seasonal or autocorrelation structures and thus limiting forecasting capability.

## 4.1 Coefficient Significance Analysis:

The coeftest results provided detailed statistical summaries of the autoregressive (AR), moving average (MA), and seasonal components:

- In well-performing models such as SARIMA(5,1,3)x(1,1,2)_[12] and SARIMA(6,1,3)x(1,1,2)_[12] under both ML and CSS-ML methods, majority of estimated parameters were statistically significant($p<0.05$),indicating reliable model specificantion.
- These significant coefficients confirm that included lag terms contributed meaningfully to explaining variance in the time series.
- In more complex models like SARIMA(6,1,4)x(1,1,2)_[12] , SARIMA(7,1,3)x(1,1,2)_[12] , SARIMA(7,1,4)(1,1,2)[12] some coefficients had p-values > 0.05, indicating potential overfitting or redundancy, also had NaN standard errors which indicated non-convergence.

### Co-Efficient Test ML Method:

| Model | Significant Coefficient | Comments |
|---|---|---|
| SARIMA(0,0,1)x(1,1,2)_[12] | ma1: -0.783, sma1: -0.818 | MA(1) and seasonal MA(1) strongly significant |
| SARIMA(0,0,2)x(1,1,2)_[12] | ma1: -0.783, sma1: -0.819 | Additional MA(2) which is insignificant. |
| SARIMA(1,0,2)x(1,1,2)_[12] | Sma1: -0.861 | Only Seasonal MA(1) significant. |
| SARIMA(1,0,3)x(1,1,2)_[12] | sma1: -0.822 | Only Seasonal MA(1) significant. |
| SARIMA(2,0,2)x(1,1,2)_[12] | sma1: -0.817 | Only Seasonal MA(1) significant. |
| SARIMA(2,0,3)x(1,1,2)_[12] | sma1: -0.819 | Only Seasonal MA(1) significant. |
| SARIMA(5,0,3)x(1,1,2)_[12] | ar1: -1.057, ar2: -0.739 ,ar3: 0.291,ma1: 0.312, ma3: -0.861, sma1: -0.959 | Multiple AR and MA terms significant; seasonal MA(1) significant |
| SARIMA(5,0,4)x(1,1,2)_[12] | ar2: 0.269, ar3: 0.969 ,ma1: -0.642,ma2: -0.335, ma3: -0.822 , ma4: 0.818 , sma1: -0.968 | Most AR/MA terms significant; seasonal MA(1) significant |
| SARIMA(6,0,3)x(1,1,2)_[12] | ar1: -1.025, ar2: -0.693, ar3: 0.340, ar4: 0.267, ma1: 0.277 , ma3: -0.882 , sma1: -1.014 | Several AR and MA terms significant; seasonal MA(1) significant |

Table 4.1: Coeftest ML Method

**Co-Efficient Test CSS-ML Method:**

| | | |
|---|---|---|
| SARIMA(0,0,1)x(1,1,2)_[12] | ma1: -0.783, sma1: -0.818 | MA(1) and seasonal MA(1) significant |
| SARIMA(0,0,2)x(1,1,2)_[12] | ma1: -0.783, sma1: -0.817 | Ma(1) and sma(1) are siginifcant. Rest all are not significant. |
| SARIMA(1,0,2)x(1,1,2)_[12] | sma1: -0.819 | Only seasonal MA(1) significant |
| SARIMA(1,0,3)x(1,1,2)_[12] | sma1: -0.819 | Only seasonal MA(1) significant |
| SARIMA(2,0,2)x(1,1,2)_[12] | sma1: -0.819 | Only seasonal MA(1) significant |
| SARIMA(2,0,3)x(1,1,2)_[12] | ar1: -1.156, ar2: -0.992, ma1: 0.413, ma2: 0.081, ma3: -0.762, sma1: -0.930 | Strong AR and MA terms; seasonal MA(1) also significant |
| SARIMA(5,0,3)x(1,1,2)_[12] | ar1: -1.057, ar2: -0.738, ar3: 0.292, ma1: 0.311, ma3: -0.862, sma1: -0.966 | Several AR and MA terms; seasonal MA(1) significant |
| SARIMA(5,0,4)x(1,1,2)_[12] | sma1: -0.926 | Only seasonal MA(1) significant; others had NaN |
| SARIMA(6,0,3)x(1,1,2)_[12] | ar1: -1.025, ar2: -0.693, ar3: 0.340, ar4: 0.267, ma1: 0.277, ma3: -0.882, sma1: -1.014 | Several AR and MA terms significant; seasonal MA(1) significant |

Table 4.2: Coeftest (CSS-ML) Method

## 4.2 Parameter Estimation Comparison: ML vs. CSS-ML

Both ML and CSS-ML methods identified similar model structures, but key distinctions were observed:

- Coefficient Stability: ML typically produced more precise estimates with lower standard errors and narrower confidence intervals.
- Significance Patterns: Parameters that were significant under ML were generally also significant under CSS-ML, indicating robustness of model dynamics across methods.
- Efficiency: ML is theoretically more efficient under correct model specification, which was evident in tighter bounds and better convergence behaviour.

### 4.3.Model Ranking and Selection

| Model | AIC | BIC | RSME | MAE | MAPE | MASE | ACF1 | Notes |
|---|---|---|---|---|---|---|---|---|
| SARIMA(6,1,3)x (1,1,2)_[12] | **-411.98** | -351.11 | **0.1527** | **0.1075** | 207.42 | **0.6259** | -0.004 | Best overall model with lowest RMSE and MAE. |
| SARIMA(5,1,4)x (1,1,2)_[12] | -411.96 | **-354.78** | 0.1530 | 0.1078 | **203.15** | 0.6278 | -0.004 | Good model with slightly higher error. |
| SARIMA(5,1,3)x (1,1,2)_[12] | -411.96 | -361.31 | 0.1529 | 0.1077 | 200.58 | 0.6272 | -0.003 | This model is a very close competitor with best BIC. |
| SARIMA(2,1,3)x (1,1,2)_[12] | -381.42 | -343.43 | 0.1589 | 0.1147 | 268.55 | 0.6678 | -0.032 | Moderate, decent AIC, but higher error |
| SARIMA(2,1,2)x (1,1,2)_[12] | -379.77 | -346.01 | 0.1595 | 0.1144 | 266.38 | 0.6661 | -0.0008 | Moderate, less efficient. |
| SARIMA(1,1,3)x (1,1,2)_[12] | -379.73 | -345.97 | 0.1595 | 0.1146 | 267.18 | 0.6671 | -0.0015 | Moderate like above. |
| SARIMA(1,1,2)x (1,1,2)_[12] | -381.73 | -352.19 | 0.1595 | 0.1146 | 266.95 | 0.6671 | -0.0015 | Moderate, with minor variation |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| SARIMA(0,1,2)x(1,1,2)_[12] | -383.73 | -358.41 | 0.1595 | 0.1146 | 267.08 | 0.6671 | -0.0015 | Moderate less accurate. |
| SARIMA(0,0,1)x(1,1,2)_[12] | -385.73 | **-364.63** | 0.1595 | 0.1146 | 266.95 | 0.6671 | -0.00115 | Worst model among all with highest MAE and RMSE. |

Table 4.3: SARIMA Model Comparison –(ML) Method

| Model | AIC | BIC | RSME | MAE | MAPE | MASE | ACF1 | Notes |
|---|---|---|---|---|---|---|---|---|
| SARIMA(6,1,3)x(1,1,2)_[12] | -411.98 | -357.11 | **0.1527** | **0.1075** | 207.42 | **0.6259** | -0.0043 | Strong performer with best RSME and MASE |
| SARIMA(5,1,4)x(1,1,2)_[12] | -409.62 | -354.75 | 0.1530 | 0.1078 | 202.79 | 0.6276 | -0.0045 | Moderate Model, but with no best AIC/BIC |
| SARIMA(5,1,3)x(1,1,2)_[12] | -411.96 | -361.31 | 0.1529 | 0.1077 | **200.77** | 0.6272 | -0.0034 | Compared to above model but with slightly higher errors. |
| SARIMA(2,1,3)x(1,1,2)_[12] | **-413.29** | **-375.31** | 0.1536 | 0.1077 | 211.16 | 0.6271 | 0.0017 | Best AIC and BIC among all the models |
| SARIMA(2,1,2)x(1,1,2)_[12] | -379.77 | -346.00 | 0.1595 | 0.1144 | 266.63 | 0.6662 | -0.0010 | Weak AIC/BIC and errors. |
| SARIMA(1,1,3)x(1,1,2)_[12] | -379.76 | -345.99 | 0.1595 | 0.1145 | 266.91 | 0.6665 | -0.0019 | Among the least effective. |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| SARIMA(1,1,2) x(1,1,2)_[12] | -381.74 | -352.20 | 0.1595 | 0.1146 | 267.44 | 0.6671 | -0.003 | Poor fit, no gain from added complexity. |
| SARIMA(0,1,2) x(1,1,2)_[12] | -383.73 | -358.41 | 0.1595 | 0.1146 | 267 | 0.6671 | -.0015 | High errors |
| SARIMA(0,0,1) x(1,1,2)_[12] | -385.73 | -364.63 | 0.1595 | 0.1146 | 266.94 | 0.6671 | -0.0012 | High errors, simple model. |

Table 4.4: SARIMA Model Comparison –(CSS-ML) Method

## 4.4 Final Model Recommendations

Based on the combination of AIC/BIC scores, parameter significance, and forecast accuracy, the top models are:

- Best-Overall Models:
- SARIMA(6,1,3)x(1,1,2)_[12] (CSS-ML),(ML)
    - Demonstrated the lowest RMSE and MASE, indicating excellent short-term predictive accuracy.
    - Balanced model complexity with strong forecast performance.
    - Since SARIMA(6,1,3)x(1,1,2)_[12] (ML) convergence we are going to consider method for diagnostic checking.
- SARIMA(5,1,3)x(1,1,2)_[12](ML)
    - Slightly higher AIC/BIC than the top model.
    - Comparable accuracy with statistically significant coefficients.
    - Benefited from ML's precision and stable parameter estimation.
- SARIMA(2,1,3)x(1,1,2)_[12](CSS-ML)
    - Best lowest AIC and lowest BIC among all the models, suggesting the best overall fit under information criteria.
    - Slightly higher RMSE and MASE than SARIMA(6,1,3)x(1,1,2)_[12], but still within acceptable accuracy limits.

While exploring more complex variants of the final model SARIMA(6,1,3)×(1,1,2)121212, such as SARIMA(6,1,4)×(1,1,2)121212 and SARIMA(7,1,3)×(1,1,2)121212, estimation instability was observed. Specifically, coefficient significance tests returned NaN values, indicating non-identifiable parameters and suggesting overfitting due to excessive model complexity. Comparisons between SARIMA(5,1,3)×(1,1,2)121212, SARIMA(6,1,3)×(1,1,2)121212, and SARIMA(5,1,4)×(1,1,2)121212 were also conducted, with SARIMA(5,1,4)×(1,1,2)121212 eliminated due to poor forecast accuracy. Additionally, adjacent models for SARIMA(2,1,3)×(1,1,2)121212 were excluded as they failed to converge under both estimation methods.

## Diagnostic Checking

We considered the top 3 models for diagnostic checking. For each model, we analysed the residuals to understand whether there were any important information that the models were unable to capture.

### SARIMA(2,1,3)x(1,1,2)_[12] – CSS-ML

The smallest and best performing model in coefficient test is SARIMA(2,1,3)x(1,1,2)[12]. Figure 5.1 shows the histogram and QQ plot of the residuals from the model. The histogram appears symmetric, with few large outliers. The tails of the QQ plot deviate from the reference line. This suggests that the residuals are not normally distributed, which is further confirmed by Shapiro-wilk test ($p<0.05$) in table 5.1.

Fig 5.1 (a) Residuals from the SARIMA(2,1,3)x(1,1,2)_12 Model

Fig 5.1 (b): Q-Q plot for Residuals: SARIMA(2,1,3)x(1,1,2)_12 Model.

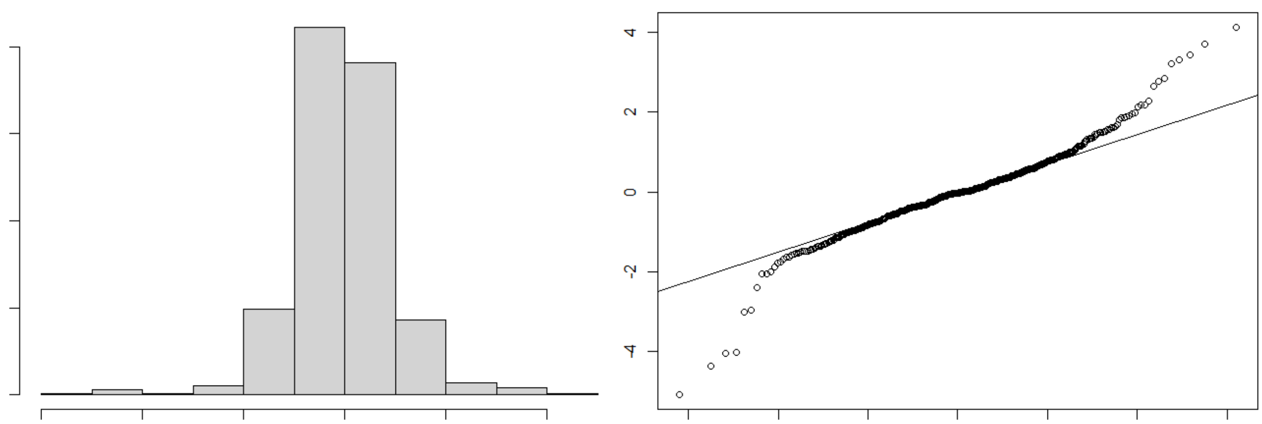



**Table 5.1: Shapiro-Wilk normality test**

```
data:  res.213
W = 0.94404, p-value = 4.868e-13
```

The standardized residuals plotted in fig 5.2 (a) appear random, without any obvious trend. We can infer that this model captures trend sufficiently. The ACF of residuals do not have any significant seasonal lag, suggesting that the seasonality was captured by the model. The pacf of residuals in fig 5.3 confirms this. The Ljung-box test shows several p-values below the threshold of 0.05, meaning that the model fails to capture autocorrelation.

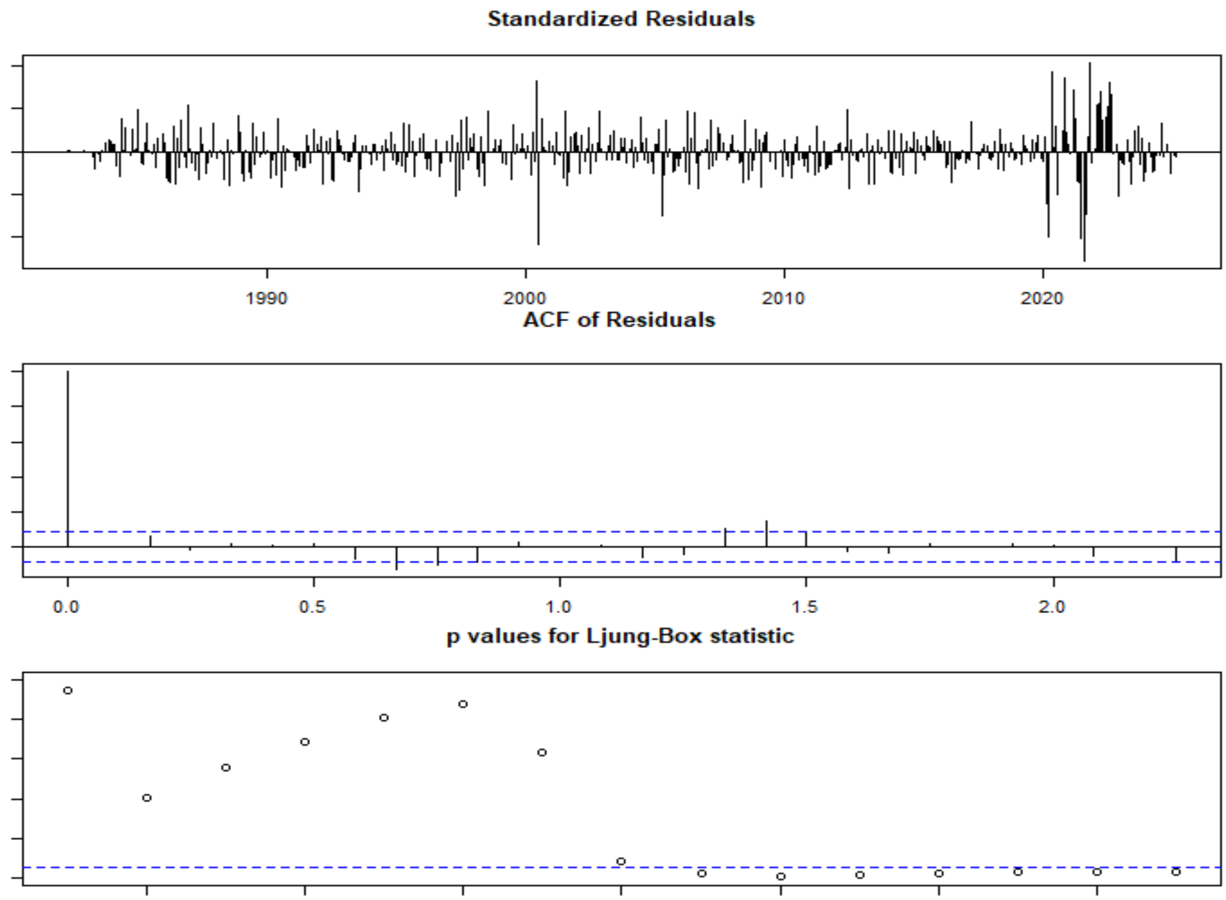
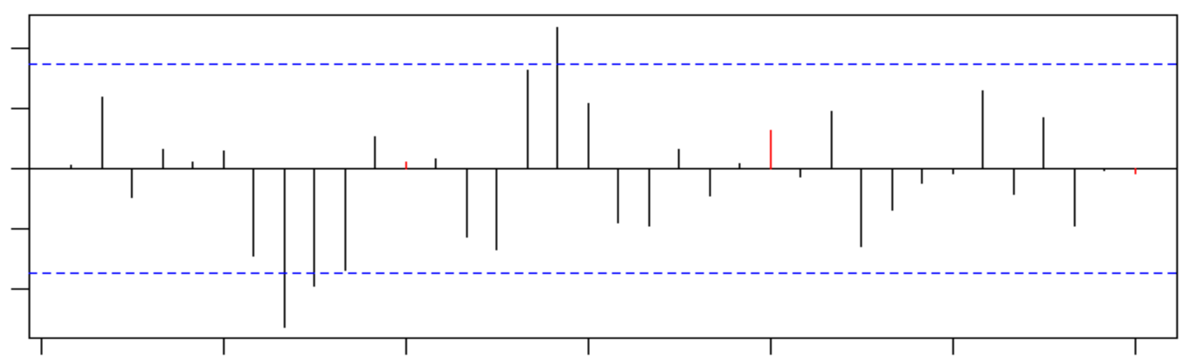



Fig 5.2: (a)Standardized Residuals (b)ACF of Residuals (c) Ljung-Box statistic: SARIMA(2,1,3)x(1,1,2)_[12]

**Fig 5.3: PACF of Residuals from the SARIMA(2,1,3)x(1,1,2)_12 Model.**



## SARIMA(5,1,3)x(1,1,2)_[12] –ML

The histogram of SARIMA(5,1,3)x(1,1,2)[12] is symmetric, with some extreme values. The QQ plot suggests that that the distribution of the residuals is not normal. The tail at the further end of the plot does not align with the QQ line. Shapiro test confirms that the distribution is not normal ($p<0.05$).

Fig 5.4 (a) Residuals from the SARIMA(5,1,3)x(1,1,2)_12 Model

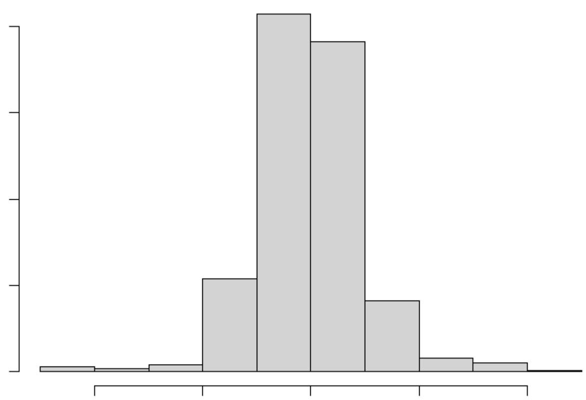


Fig5.4 (b): Q-Q plot for Residuals: SARIMA(5,1,3)x(1,1,2)_12 Model.

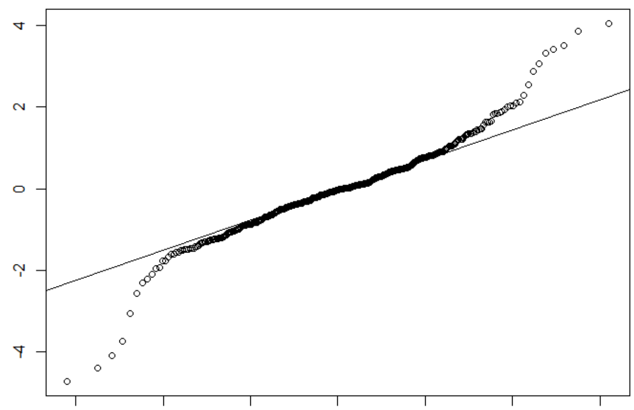



**Table 5.2 Shapiro-Wilk normality test**

```
data:  res.513
W = 0.94668, p-value = 1.127e-12
```

The residual plot in figure 5.5 appears a random with no strong trend, which means that the model captures trend quite well. The ACF plot in 5.5 (b) has no significant seasonal lag. PACF plot in figure 5.6 also shows no significant seasonal lag, meaning that this model captures all seasonality. The Ljung-box test shows all p-values above the significant level. This suggests that the model is able to capture autocorrelation well.

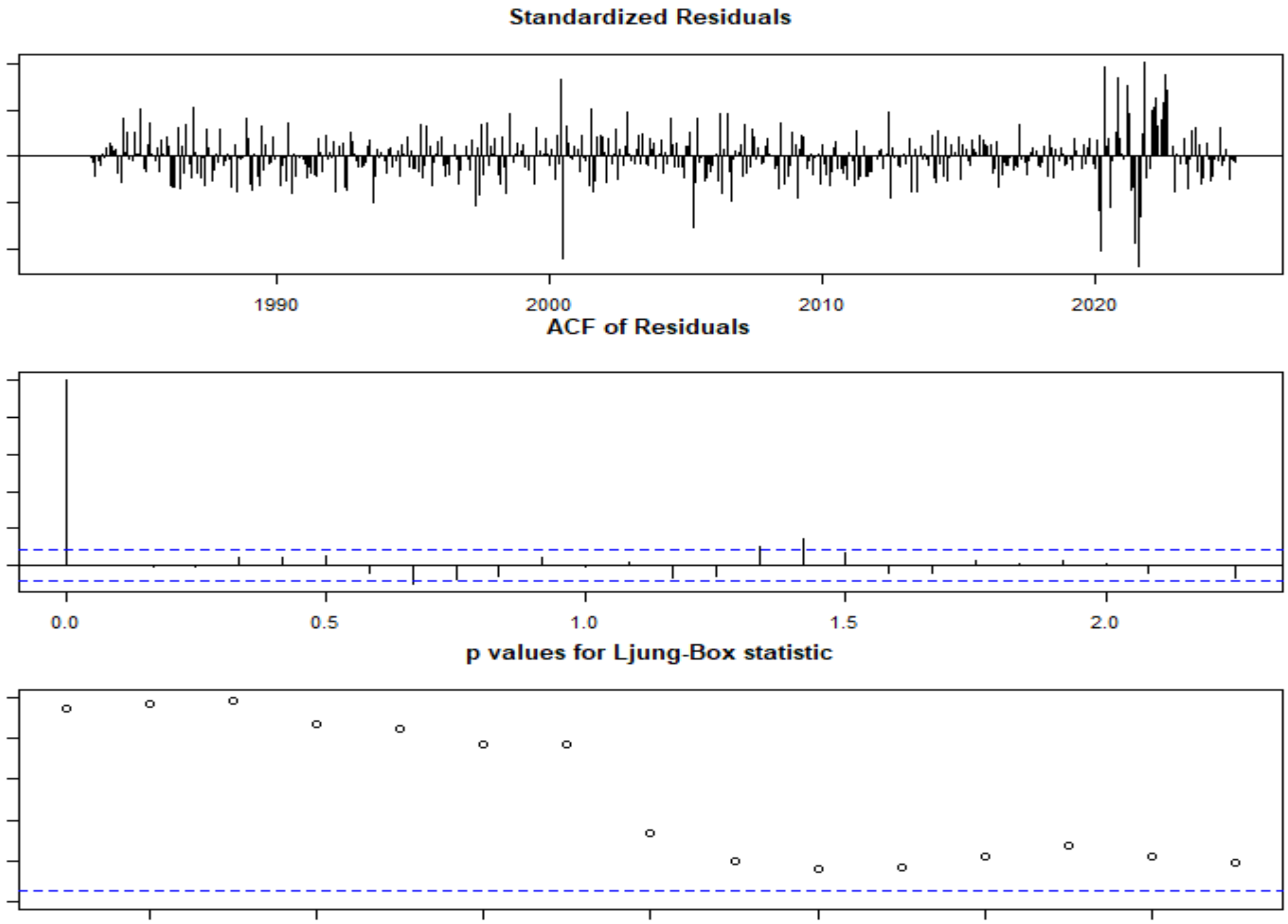
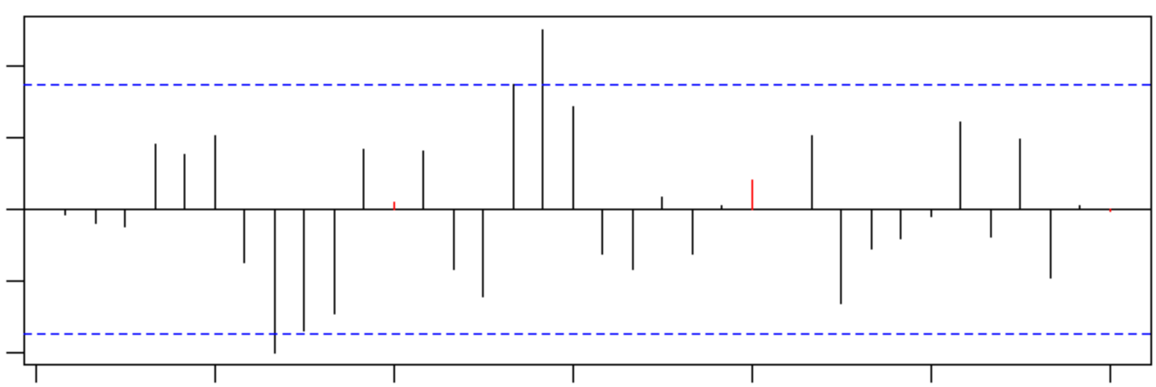



Fig 5.5: (a)Standardized Residuals (b)ACF of Residuals (c) Ljung-Box statistic: SARIMA(2,1,3)x(1,1,2)_[12]

**Fig 5.6: PACF of Residuals from the SARIMA(5,1,3)x(1,1,2)_12 Model.**



### SARIMA(6,1,3)x(1,1,2)_[12] –ML

SARIMA(6,1,3)x(1,1,2)[12] is the largest model amongst the candidates. The histogram of its residuals in figure 5.7 (a) once again shows a symmetric plot with some extreme values. The dots in the QQ plot align with QQ line, except at the tails. This shows the absence of normality in the residuals. This is confirmed by Shapiro Wilk test which has p-value < 0.05.

Fig 5.7 (a) Residuals from the SARIMA(6,1,3)x(1,1,2)_12 Model

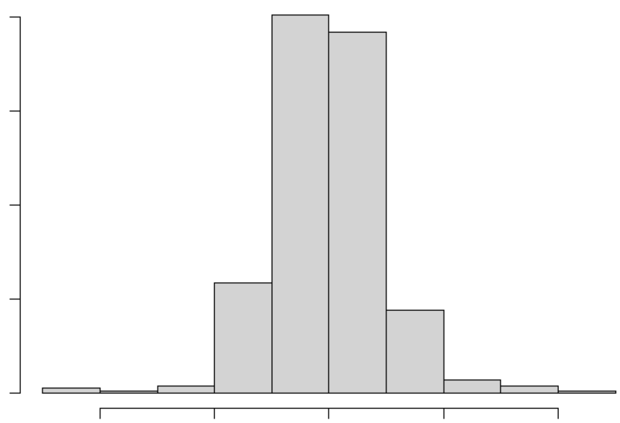


Fig 5.7 (b): Q-Q plot for Residuals: SARIMA(6,1,3)x(1,1,2)_12 Model.

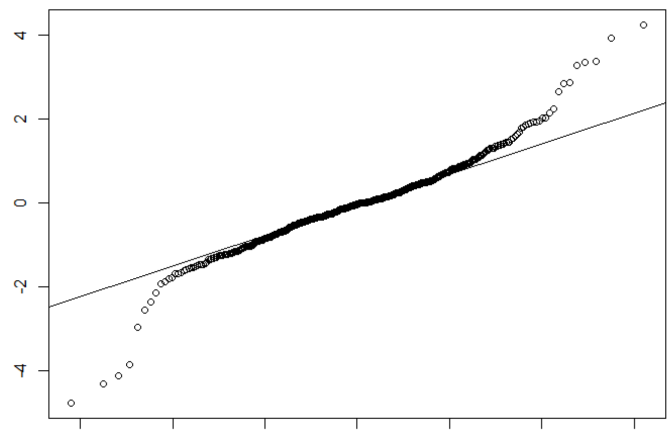



**Table 5.3: Shapiro-Wilk normality test**

```
data:  res.613
W = 0.94547, p-value = 7.655e-13
```

Figure 5.8 gives the standardized residuals, ACF and Ljung-box test. Fig 5.9 shows the PACF of the model. The standardized residuals do not have any apparent trend. The data appears random. This means that the model captures trend well. The ACF and PACF do not have any significant seasonal lag, meaning that the model is able to capture seasonality. Finally, the results of the Ljung-box test show all p-value above the threshold, suggesting that the autocorrelation was also captured well by this model.

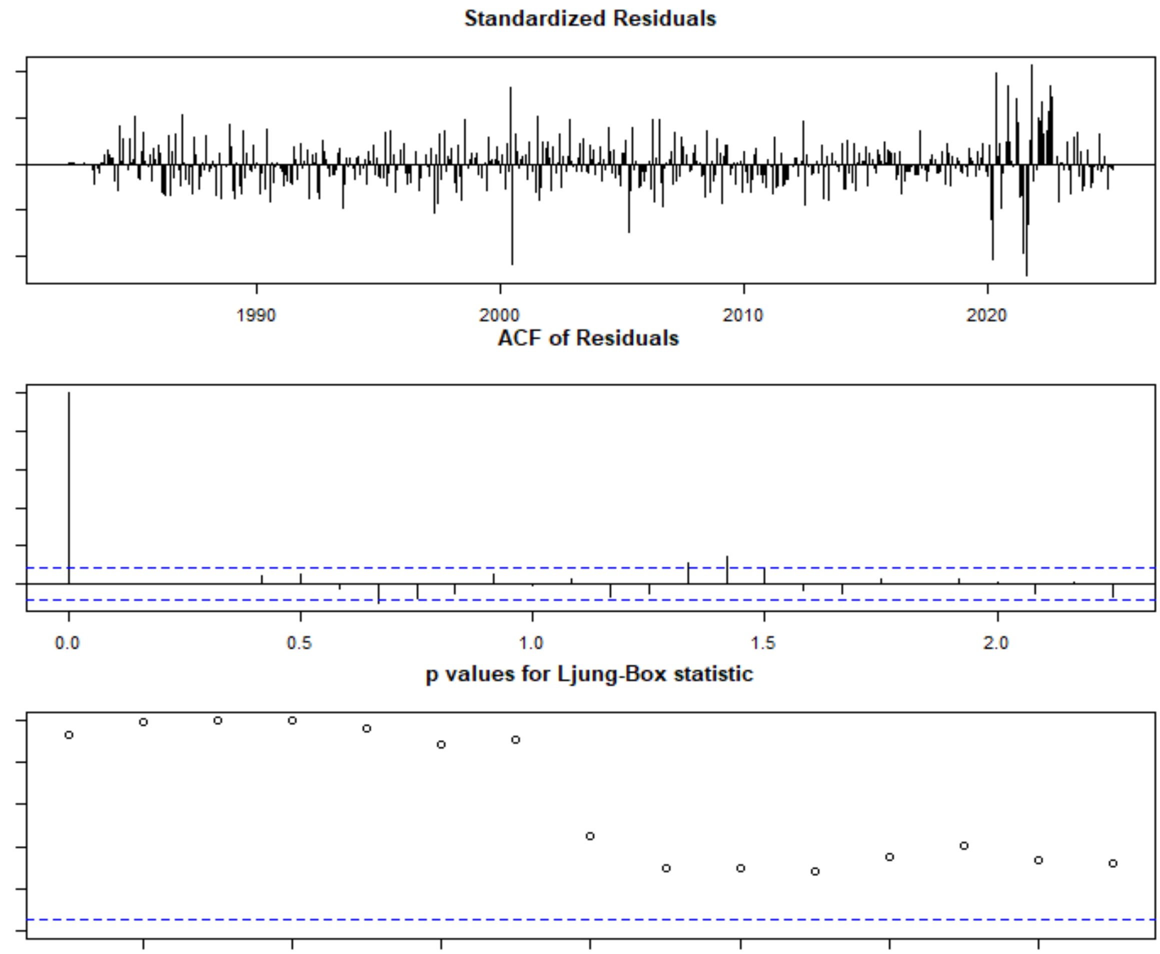
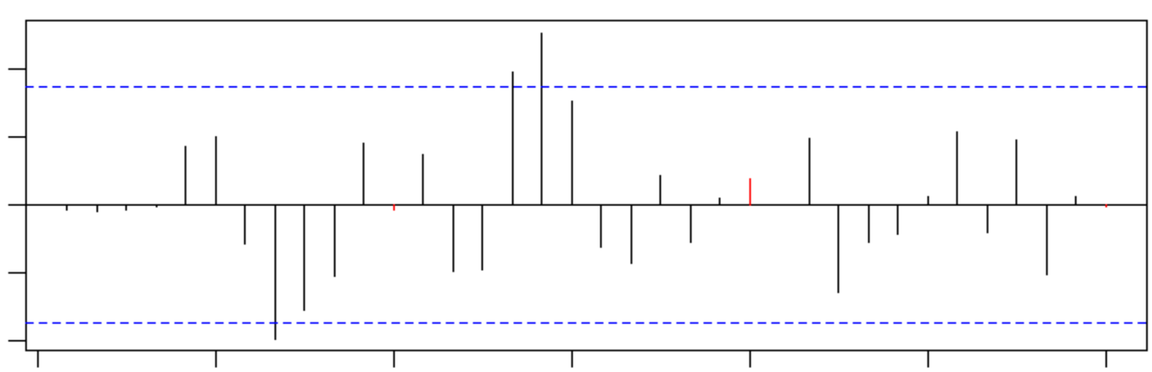



Fig 5.8: (a)Standardized Residuals (b)ACF of Residuals (c) Ljung-Box statistic: SARIMA(2,1,3)x(1,1,2)_[12]

**Fig 5.9: PACF of Residuals from the SARIMA(6,1,3)x(1,1,2)_12 Model.**



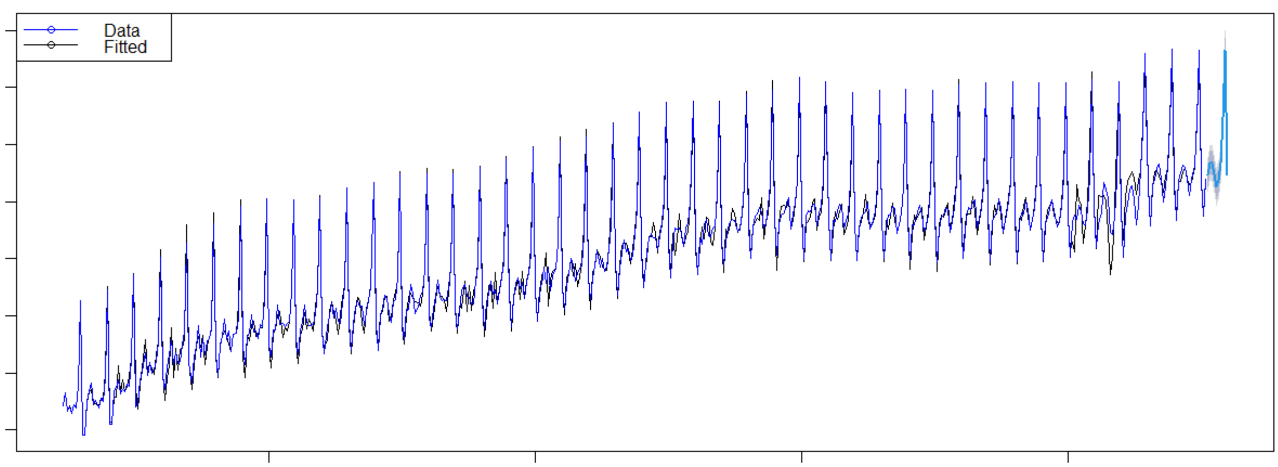
## Forecasting

Based on the outcome of the diagnostic tests, SARIMA(5,1,3)x(1,1,2)[12] and SARIMA(6,1,3)x(1,1,2)[12] are strong candidates for forecasting. However, adhering to parsimony principle, we will use SARIMA(5,1,3)x(1,1,2)[12]. It is the smaller and less complicated model among the two. Table 6.1 shows the forecasted values. It provides predicted values for the next 10 months, along with the corresponding high and low bounds within the 80% and 95% confidence intervals for each month. Figure 6.1 shows the plot of the predicted value, along with the data and the fitted model. We can observe a rising trend peaking, followed by a drop. The widening of the boundary values reflects the model's decreasing certainty over longer horizons.

| Point | Forecast | Lo 80 | Hi 80 | Lo 95 | Hi 95 |
|---|---|---|---|---|---|
| Apr 2025 | 14.46479 | 14.26423 | 14.66534 | 14.15807 | 14.77151 |
| May 2025 | 14.67045 | 14.46349 | 14.87740 | 14.35393 | 14.98696 |
| Jun 2025 | 14.68009 | 14.46647 | 14.89372 | 14.35338 | 15.00681 |
| Jul 2025 | 14.49174 | 14.27299 | 14.71048 | 14.15719 | 14.82628 |
| Aug 2025 | 14.26385 | 14.04078 | 14.48691 | 13.92270 | 14.60499 |
| Sep 2025 | 14.37542 | 14.15002 | 14.60081 | 14.03070 | 14.72013 |
| Oct 2025 | 14.67378 | 14.44474 | 14.90282 | 14.32350 | 15.02407 |
| Nov 2025 | 15.46841 | 15.23619 | 15.70062 | 15.11326 | 15.82355 |
| Dec 2025 | 16.65034 | 16.41612 | 16.88456 | 16.29213 | 17.00855 |
| Jan 2026 | 14.46304 | 14.22529 | 14.70079 | 14.09943 | 14.82665 |

**Table 6.1: Forecast for the next 10 intervals**

**Fig 6.1: Forecasts from SARIMA(5,1,3)(1,1,2)[12]**



## Conclusions

In this project we analysed the time series data of retail turnover in Australia between April 1982 to March 2025. Close inspection of the data revealed a series with trend, seasonality, changing variance and autoregressive behaviour. Exploratory data analysis confirmed clear seasonal patterns and long-term trend. We tested for stationarity and applied appropriate transformations to mitigate changing variance. Next multiple SARIMA models were fitted and evaluated. Model selection was guided by performance metrics including AIC, BIC, RMSE, and MAE, ensuring a balance between model complexity and predictive accuracy. The selected SARIMA model, SARIMA(5,1,3)x(1,1,2)[12] was then used to predict 10 months ahead with 80% and 95% confidence intervals.

### Limitations

We used differencing to remove linear trend in the data. While this ensures a stationary data for modelling, it also leads to loss of some information. Despite applying appropriate transformation, changing variance was not completely removed form the data. This may influence the reliability of the forecasting. Additionally, the model we used is relatively large and complex. Therefore, there is a risk of overfitting, leading to poor generalization of future values. Finaly, as the forecasting horizon extends further into the future, the model's accuracy tends to decline due to the accumulation of uncertainty and potential divergence from historical patterns.

### Recommendations

Since changing variance was not fully corrected, alternative approaches such as ARCH/GARCH models or hybrid models could be considered. Furthermore, additional diagnostic testing could be performed on all the selected models to ensure robust model selection, potentially avoiding complex, over-fitted models.

## References


[1] Australian Bureau of Statistics. (2024). *Retail Trade, Australia, Table 1. Retail turnover, by industry group.* [Online]. Available: *https://www.abs.gov.au/statistics/industry/retail-and-wholesale-trade/retail-trade-australia/latest-release#data-downloads* [Accessed: Jun. 14, 2025].

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[3]MATH13182204_Module3_pres_annotated_2025.pdf. RMIT Canvas: https://rmit.instructure.com/courses/140832/files/44536644?module_item_id=7224915 [Accessed: May. 10, 2025].

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[11] solutionModule7Tasks_2024.R. RMIT Canvas: https://rmit.instructure.com/courses/140832/files/45375451?module_item_id=7307940 [ Accessed: May. 22, 2025].

# Appendix

## Appendix 1: R code

```
# Clear workspace
rm(list = ls())
# Set working directory
setwd("D:/Git/time_series_project")

# Load necessary libraries
library(readr)
library(tseries)
library(lmtest)
library(forecast)
library(TSA)

## Necessary functions ##
# function to check if a series follows normal distribution
check_normality <- function(series, plot_title) {
  par(mfrow = c(1, 1))
  # QQ plot
  qqnorm(series, main = plot_title)
  qqline(series, col = 4, lwd = 1.5, lty = 1)

  # Shapiro-Wilk test
  SWresult <- shapiro.test(series)
  print(SWresult)
}

# function for stationarity check using Dickey-Fuller and Phillips-Perron Unit Root test
test_stationarity <- function(ts_data) {
```

```
  adf_result <- adf.test(ts_data)
  print(adf_result)
  pp_result <- pp.test(ts_data)
  print(pp_result)
  kpss_result <- kpss.test(ts_data)
  print(kpss_result)
}
#function for sort AIC and BIC
sort.score <- function(x, score = c("bic", "aic")){
  if (score == "aic"){
    x[with(x, order(AIC)),]
  } else if (score == "bic") {
    x[with(x, order(BIC)),]
  } else {
    warning('score = "x" only accepts valid arguments ("aic","bic")')
  }
}

# Check the data
check_data <- function(data) {
  print(class(data))
  print(any(is.na(data)))
  print(summary(data))
  print(head(data))
}

# function to plot different SARIMA models (p=0, d=1, q=0)(P=0, D=1, Q=0)_s
## purpose: (1) Check residuals of series (ts plot, acf plot, pacf plot)
```

```
## purpose: (2) Returns model
sarima_res_test <- function(ts_data, p = 0, d = 1, q = 0, P = 0, D = 1, Q = 0, s = 12) {
  model <- arima(ts_data, order = c(p, d, q),
                 seasonal = list(order = c(P, D, Q), period = s))
  res <- residuals(model)
  # ts plot, acf and pacf plots of residuals
  par(mfrow = c(3, 1))
  plot(res, xlab = 'Time', ylab = 'Residuals', main = 'Residuals Time Series', type = 'l')
  acf(res, lag.max = 50, main = 'ACF of Residuals')
  pacf(res, lag.max = 50, main = 'PACF of Residuals')
  par(mfrow = c(1, 1))
  return(model) # returns SARIMA model
}

#getwd()
# Set working directory
#setwd("/Users/priyankatiwari/Documents/RMIT/Sem4/time_series/Assignment3")



data =read_csv("DepartmentStoreretailing.csv")
check_data(data)
# Result: All data available
# checking for missing timepoints
time_str <- data$Time  # e.g., "Apr-1982"
months <- substr(time_str, 1, 3)
years <- as.numeric(sub(".*-", "", time_str))

time_numeric <- years * 12 + match(months, month.abb)
```

```
full_range <- min(time_numeric):max(time_numeric)
full_months <- (full_range - 1) %% 12 + 1
full_years <- (full_range - 1) %/% 12
full_labels <- paste0(month.abb[full_months], "-", full_years)
actual_labels <- paste0(months, "-", years)

missing_months <- setdiff(full_labels, actual_labels)
if (length(missing_months) == 0) {
  cat("None, all timepoints available\n")
} else {
  cat("Missing timepoint(s):\n")
  print(missing_months)
}
# Result: All timepoints available

retail_data= ts(data$`Department store retailing`, start = c(1982,4), frequency = 12)
class(retail_data)
head(retail_data)
summary(retail_data)

plot(retail_data,cex.main = 0.8, type='o',ylab="Department stores retail", main="Figure 1.1: Time Series plot of total number of department stores in retail ")
plot(retail_data,cex.main = 0.8,,type='l', ylab = "Department stores retail", main="Figure 1.2: Time series plot of total number of department stores in retail.")
points(y=retail_data,x=time(retail_data), pch=as.vector(season(retail_data)))

# Sample every 6 months for plot clarity
idx <- seq(1, length(retail_data), by = 6)
plot(time(retail_data)[idx], retail_data[idx], type = "o",
```

```
    xlab = "Year", ylab = "Retail Sales (sampled every 6 months)",
    main = "Sampled Department Store Retail Sales",
    col = "black", pch = 16, lwd = 2)

# ACF & PACF plot to confirm frequency
acf(as.vector(retail_data), main = "Figure 1.3: ACF of number of department stores in retail", xlab = "Lag", ylab = "ACF", lag.max=50)
pacf(as.vector(retail_data), main = "Figure 1.4: PACF of total number of department stores in retail", xlab = "Lag", ylab = "PACF",ylim = c(-1, 1))

# first lag scatter plot
lag1_retail_data <- zlag(retail_data)
index = 2:length(retail_data)
plot(y=retail_data,x=lag1_retail_data, ylab='Number of stores', xlab='First lag', main= "Figure 1.5: Scatter plot of neighboring number of stores",cex.main = 0.8)
cor(retail_data[index], lag1_retail_data[index])

# second lag scatter plot
lag2_retail_data <- zlag(lag1_retail_data)
index2 = 3:length(lag2_retail_data)
plot(y=retail_data,x=lag2_retail_data, ylab='Number of stores', xlab='Second lag', main= "Figure 1.6: Scatter plot of number of stores and its second lag values",cex.main = 0.8)
cor(retail_data[index2], lag2_retail_data[index2])

## Changing variance, seasonality, non-stationary
# checking if the original series follows normal distribution
check_normality(retail_data, "Figure 2.1: QQ plot of number of department stores in retail")
# Transformation
## trying different seq for BoxCox transformation
```

```
# BC1 <- BoxCox.ar(retail_data,lambda = seq(-1, 0, 0.01)) # Error

# BC2 <- BoxCox.ar(retail_data,lambda = seq(-1, 1, 0.01)) # Error

# BC3 <- BoxCox.ar(retail_data,lambda = seq(0, 1, 0.01))  # seq worked

BC4 <- BoxCox.ar(retail_data,lambda = seq(0, 2, 0.01))  # seq worked

# BC5 <- BoxCox.ar(retail_data,lambda = seq(0, 3, 0.01))  # Error

BC4$ci

lambda <- BC4$lambda[which(max(BC4$loglike) == BC4$loglike)]

lambda

rt_dataBC = (retail_data^lambda-1)/lambda  #Box-cox transformed data

# check again --> plot and normality

plot(rt_dataBC, type = "o", ylab = "Box-cox transformed data", xlab = "Time (Months)",
main = "Figure 2.2: Box-Cox transformed data of number of department stores in retail",
cex.lab = 0.5, cex.main = 0.8)

# check noramality of transformed data

check_normality(rt_dataBC, "Figure 2.3: QQ plot of Box-cox transformed data")


# Trend & Seasonality

# checking for stationarity of transformed series

test_stationarity(rt_dataBC)


# 1st difference on transformed series

rt_dataBCDiff <- diff(rt_dataBC, differences = 1)


# time series plot of 1st difference on transformed series

plot(rt_dataBCDiff, type = "o", ylab = "Number of department stores", xlab = "Time
(Months)", main = "Figure 2.4: First Difference of Transformed data of number of
department stores.", cex.main = 0.8)


# ACF & PACF plot of 1st difference on transformed series
```

```
acf(rt_dataBCDiff, main = "Figure 2.5: ACF of first difference of transformed data", xlab = "Lag", ylab = "ACF", lag.max = 80, cex.main = 0.8)

pacf(rt_dataBCDiff, main = "Figure 2.6: PACF of first difference of transformed data", xlab = "Lag", ylab = "PACF",ylim = c(-1, 1), cex.main = 0.8)


# checking for stationarity in 1st difference of transformed series

test_stationarity(rt_dataBCDiff)


# seasonal difference on 1st difference on transformed series

rt_dataBCDiffSD <- diff(rt_dataBCDiff, lag = 12)


# time series plot of 1st & seasonal difference on transformed series

plot(rt_dataBCDiffSD, type = "o", ylab = "Number of department stores", xlab = "Time (Months)", main = "Figure 2.7: First & seasonal difference of transformed data.", cex.main = 0.8)



# Residuals approach to finalize seasonal components of model

# --> applied on transformed data


## SARIMA (0,1,0)x(0,1,0)12 returns residual model

m1 = sarima_res_test(rt_dataBC)

# SARIMA(0,1,0) (1,1,2)_12 returns residual model

m2 = sarima_res_test(rt_dataBC, P=1, Q=2)


# nonseasonal parameter determinations

# --> applied on 1st and seasonal difference on transformed data

# ACF & PACF plot of 1st & seasonal difference on transformed series

acf(rt_dataBCDiffSD, main = "Figure 3.1: ACF of first & seasonal difference of transformed data", xlab = "Lag", ylab = "ACF", lag.max = 80, cex.main = 0.8)
```

```
pacf(rt_dataBCDiffSD, main = "Figure 3.2: PACF of first & seasonal difference of transformed data", xlab = "Lag", ylab = "PACF",ylim = c(-1, 1), cex.main = 0.8)


# EACF plot
eacf(rt_dataBCDiffSD, ar.max = 10, ma.max = 10)
# BIC based model selection
set.seed(92397)
res = armasubsets(y=rt_dataBCDiffSD, nar=10, nma=10, y.name='p',ar.method='ols')
plot(res, main = "")
title(main = "Figure 3.3: BIC selection plot for models", line = 6)


###p = 5, 6, 7
##q = 3, 4


###### MODEL_FITTING USING ML, CSS_ML, CSS(CSS some models failed and were assigned NA)
library(forecast)
library(lmtest)


# Define seasonal component for SARIMA
seasonal_order <- list(order = c(1, 1, 2), period = 12)


# List of (p,d,q) orders to fit
orders_to_fit <- list(
  c(0, 0, 1), c(0, 0, 2), c(1, 0, 2), c(1, 0, 3), c(2, 0, 2),
  c(2, 0, 3), c(5, 0, 3), c(5, 0, 4), c(6, 0, 3)  #c(6, 0, 4), c(7,0,3)c(7, 0, 4) not considered due to non convergence.
```

```
# Function to fit SARIMA models with specified estimation method and print coeftest
fit_sarima_models <- function(series, method) {
  model_list <- list()

  for (ord in orders_to_fit) {
    label <- paste0("SARIMA(", paste(ord, collapse = ","), ")x(1,1,2)[12]_Method-", method)
    cat("\nFitting:", label, "\n")

    fit <- Arima(series, order = ord, seasonal = seasonal_order, method = method)

    # Print coeftest output for the fitted model
    cat("Coefficient test for", label, ":\n")
    print(coeftest(fit))
    cat("\n------------------------------\n")

    model_list[[label]] <- fit
  }

  return(model_list)
}

# The rest of your code remains unchanged:
# compare_models(), extract_accuracy_metrics(), and the loop running all three methods

# Fit models with all three methods
methods <- c("ML", "CSS-ML", "CSS")
all_results <- list()
```

```
for (m in methods) {
  cat("\n==================== Fitting models with method:", m,
"====================\n")
  all_results[[m]] <- fit_sarima_models(rt_dataBCDiff, method = m)
}

# Compare and display results for each method
for (m in methods) {
  cat("\n==== Model comparison for method:", m, "====\n")
  scores <- compare_models(all_results[[m]])

  cat("\nSorted by AIC:\n")
  print(scores[order(scores$AIC), ])

  cat("\nSorted by BIC:\n")
  print(scores[order(scores$BIC), ])

  cat("\nAccuracy metrics:\n")
  acc <- extract_accuracy_metrics(all_results[[m]])
  print(acc)
}



#------------------------------------------------------------------------
# Diagnostic Checking
#------------------------------------------------------------------------

helper <- function(class = c("acf", "pacf"), ...) {
```

```
# Capture additional arguments
params <- match.call(expand.dots = TRUE)
params <- as.list(params)[-1]

# Calculate ACF/PACF values
if (class == "acf") {
  acf_values <- do.call(acf, c(params, list(plot = FALSE)))
} else if (class == "pacf") {
  acf_values <- do.call(pacf, c(params, list(plot = FALSE)))
}

# Extract values and lags
acf_data <- data.frame(
  Lag = as.numeric(acf_values$lag),
  ACF = as.numeric(acf_values$acf)
)

# Identify seasonal lags to be highlighted
seasonal_lags <- acf_data$Lag %% 1 == 0

# Plot ACF/PACF values
if (class == "acf") {
  do.call(acf, c(params, list(plot = TRUE)))
} else if (class == "pacf") {
  do.call(pacf, c(params, list(plot = TRUE)))
}
```

```
  # Add colored segments for seasonal lags
  for (i in which(seasonal_lags)) {
    segments(x0 = acf_data$Lag[i], y0 = 0, x1 = acf_data$Lag[i], y1 = acf_data$ACF[i], col = "red")
  }
}


# seasonal_acf -----------------------------------------------------------

seasonal_acf <- function(...) {
  helper(class = "acf", ...)
}


# seasonal_pacf ----------------------------------------------------------

seasonal_pacf <- function(...) {
  helper(class = "pacf", ...)
}
###########################################


#-----------ML513---------------------
fit_ml_5_1_3 <- Arima(rt_dataBC, order = c(5,1,3), seasonal = c(1,1,2), method = "ML")

res.513 <- rstandard(fit_ml_5_1_3)

par(mfrow = c(1, 2))
```

```
# Histogram

hist(res.513,xlab='Standardized Residuals',main="Fig 5.4 (a) Residuals from the SARIMA(5,1,3)x(1,1,2)_12 Model")


#QQ plot

qqnorm(res.513,main="Fig5.4 (b): Q-Q plot for Residuals: SARIMA(5,1,3)x(1,1,2)_12 Model.")

qqline(res.513)


#Shapiro wilk

shapiro.test(res.513)



par(mar=c(1,1,4,1))

tsdiag(fit_ml_5_1_3,gof=15,omit.initial=F)


par(mfrow = c(2, 1))




# Seasonal PACF

seasonal_pacf(res.513,

            lag.max=36,

            main="Fig 5.6: PACF of Residuals from the SARIMA(5,1,3)x(1,1,2)_12 Model.")



#-----------ML613---------------------

fit_ml_6_1_3 <- Arima(rt_dataBC, order = c(6,1,3), seasonal = c(1,1,2), method = "ML")
```

```
res.613 <- rstandard(fit_ml_6_1_3)


par(mfrow = c(1, 2))
# Histogram
hist(res.613,xlab='Standardized Residuals',main="Fig 5.7 (a) Residuals from the SARIMA(6,1,3)x(1,1,2)_12 Model")


#QQ plot
qqnorm(res.613,main="Fig 5.7 (b): Q-Q plot for Residuals: SARIMA(6,1,3)x(1,1,2)_12 Model.")
qqline(res.613)


#Shapiro wilk
shapiro.test(res.613)



par(mar=c(1,1,4,1))
tsdiag(fit_ml_6_1_3,gof=15,omit.initial=F)


par(mfrow = c(2, 1))



# Seasonal PACF
seasonal_pacf(res.613,
        lag.max=36,
        main="Fig 5.9: PACF of Residuals from the SARIMA(6,1,3)x(1,1,2)_12 Model.")



#-----------CSS213--------------------
```

```
fit_css_ml_2_1_3 <- Arima(rt_dataBC, order = c(2,1,3), seasonal = c(1,1,2), method = "CSS-ML")

res.213 <- rstandard(fit_css_ml_2_1_3)

par(mfrow = c(1, 2))
# Histogram
hist(res.213,xlab='Standardized Residuals',main="Fig 5.1 (a) Residuals from the SARIMA(2,1,3)x(1,1,2)_12 Model")

#QQ plot
qqnorm(res.213,main="Fig 5.1 (b): Q-Q plot for Residuals: SARIMA(2,1,3)x(1,1,2)_12 Model.")
qqline(res.213)

#Shapiro wilk
shapiro.test(res.213)

par(mar=c(1,1,4,1))
tsdiag(fit_css_ml_2_1_3,gof=15,omit.initial=F)

par(mfrow = c(1, 1))

# Seasonal PACF
seasonal_pacf(res.213,
        lag.max=36,
        main="Fig 5.3: PACF of Residuals from the SARIMA(2,1,3)x(1,1,2)_12 Model.")
```

```
#-------------------------------------
# Forecasting
#---------------------------------------
frc.513.ml <- forecast(fit_ml_5_1_3, h=10)
print(frc.513.ml)

plot(frc.513.ml)
lines(fitted(frc.513.ml), col= "blue")
legend("topleft", lty=1, pch=1, col=c("blue","black"), text.width = 2, c("Data", "Fitted "))

#-------------------------------------
# Forecasting
#---------------------------------------
frc.513.ml <- forecast(fit_ml_5_1_3, h=10)
print(frc.513.ml)

plot(frc.513.ml, main = "Fig 6.1: Forecasts from SARIMA(5,1,3)(1,1,2)[12]")
lines(fitted(frc.513.ml), col= "blue")
legend("topleft", lty=1, pch=1, col=c("blue","black"), text.width = 2, c("Data", "Fitted "))
```

## Appendix 2: Acknowledgement on Generative AI Tool

Generative AI tools, including ChatGPT, Microsoft Copilot and Val were used in this assignment. These tools assisted with sentence rewriting, table formatting, English grammar checking, and improving sentence structure.