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Course: CSE330

Section: 10

Assignment no: 09

Sub:

Answer to the que no: 01

Given

$f(0) = 3$

$f(4) = -2$

$f(-1) = 2$

$f(1) = 1$

We know

$P_2(x) = a_0x^2 + a_1x + a_2$

$\Rightarrow P_2(0) = 3$

$\Rightarrow P_2(4) = -2 = 16a_0 + 4a_1 + a_2$

$\Rightarrow P_2(-1) = a_0 - a_1 + a_2 = 2$

$\Rightarrow P_2(1) = a_0 + a_2 + a_2 = 1$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 16 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 3 \end{bmatrix}$$

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2.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 16 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 1 & 1 & 16 & 0 \\ 1 & -1 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A^T \cdot A = \begin{bmatrix} 258 & 64 & 18 \\ 64 & 18 & 4 \\ 18 & 4 & 4 \end{bmatrix}$$

$$A^T \cdot b = \begin{bmatrix} 1 & 1 & 16 & 0 \\ 1 & -1 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -29 \\ -9 \\ 4 \end{bmatrix}$$

3.

$$Ax = b \Rightarrow A^T A x = A^T b$$

$$\therefore x = A^T b \, (A^T A)^{-1}$$

Here

$$\det(A^T A) = \begin{vmatrix} 258 & 64 & 18 \\ 64 & 18 & 4 \\ 18 & 4 & 4 \end{vmatrix}$$

$$= 1448$$

$$\therefore \det(A^T A) \neq 0 \; \therefore (A^T A)^{-1} = \frac{\operatorname{adj}(A^T A)}{\det(A^T A)} = \frac{1}{1448} \begin{bmatrix} 56 & -184 & -68 \\ -189 & 708 & 120 \\ -68 & 120 & 548 \end{bmatrix}$$

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$$\therefore (A^T A)^{-1} = \frac{1}{342}\begin{bmatrix} 14 & -46 & -17 \\ -46 & 177 & 30 \\ -17 & 30 & 137 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} -29 \\ -9 \\ 4 \end{bmatrix} \times \frac{1}{342}\begin{bmatrix} 14 & -46 & -17 \\ -46 & 177 & 30 \\ -17 & 30 & 137 \end{bmatrix}$$

$$\therefore \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \frac{-60}{362} \\ \frac{-139}{362} \\ \frac{771}{362} \end{bmatrix}$$

Therefore

$$P_2(x) = \frac{771}{362} - \frac{139}{362}x - \frac{60}{362}x^2$$

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Ans: to the que no: 2

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 16 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 3 \end{bmatrix}$$

1.

Let

$u_1 = (1, 1, 16, 0)$

$u_2 = (1, -1, 4, 0)$

$u_3 = (1, 1, 1, 1)$

2.

Now

$$\beta_1 = u_1 = (1, 1, 16, 0)$$

$$\therefore \gamma_2 = u_2 - \frac{\langle u_2 \beta_1 \rangle}{\langle \beta_1 \beta_1 \rangle} \beta_1$$

$$= u_2 - \frac{32}{129}$$

$$= \frac{1}{129}\left[129(1, -1, 4, 0) - 32(1, 1, 16, 0)\right]$$

$$= \frac{1}{129}(97, -161, 4, 0)$$

$$\beta_2 = (97, -161, 4, 0)$$

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$$\beta_3 = U_3 - \frac{\langle U_3, \beta_1\rangle}{\langle \beta_1, \beta_1\rangle}\beta_1 - \frac{\langle U_3 \beta_2\rangle}{\langle \beta_2, \beta_2\rangle}\beta_2$$

$$= U_3 - \frac{1+1+16+0}{1^2+1^2+16^2}\beta_1 - \frac{97-161+4+0}{97^2+161^2+4^2+0}\beta^2$$

$$= U_3 - \frac{3}{43}\beta_1 + \frac{10}{5891}\beta_2$$

$$\therefore \beta_3 = \frac{43}{5891}(150, 90, -15, 137)$$

Orthogonal basis $= \{\beta_1, \beta_2, \beta_3\}$

Orthonormal basis $= \left\{ \frac{\beta_1}{\|\beta_1\|}, \frac{\beta_2}{\|\beta_2\|}, \frac{\beta_3}{\|\beta_3\|} \right\}$

$$q_1 = \frac{1}{\sqrt{258}}(1, 1, 16, 0)$$

$$q_2 = \frac{1}{\sqrt{35346}}(97, -161, 4, 0)$$

$$q_3 = \frac{1}{\sqrt{49594}}(150, 90, -15, 137)$$

$$\therefore Q = \begin{bmatrix} \frac{1}{\sqrt{258}} & \frac{97}{\sqrt{35346}} & \frac{150}{\sqrt{49594}} \\ \frac{1}{\sqrt{258}} & \frac{-161}{\sqrt{35346}} & \frac{90}{\sqrt{495494}} \\ \frac{16}{\sqrt{258}} & \frac{4}{\sqrt{35346}} & \frac{-15}{\sqrt{49594}} \\ 0 & 0 & \frac{137}{\sqrt{49394}} \end{bmatrix}$$

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3)

$$R = \begin{bmatrix} \frac{1}{\sqrt{258}} & \frac{1}{\sqrt{258}} & \frac{16}{\sqrt{258}} & 0 \\ \frac{97}{\sqrt{35346}} & \frac{-161}{\sqrt{35346}} & \frac{4}{\sqrt{35346}} & 0 \\ \frac{150}{\sqrt{49594}} & \frac{90}{\sqrt{49894}} & \frac{-15}{\sqrt{49894}} & \frac{137}{\sqrt{49594}} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 16 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{258}{\sqrt{258}} & \frac{64}{\sqrt{258}} & \frac{18}{\sqrt{258}} \\ 0 & \frac{274}{\sqrt{35346}} & \frac{-1860}{\sqrt{35346}} \\ 0 & 6 & \frac{362}{\sqrt{49594}} \end{bmatrix}$$