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Course : CSE330

Section : 10

Assignment no: 04

Ans: to the que no: 1

1. Given,

$$x = [-1, 1]$$

$\therefore$ $x_0 = -1$, $f(x_0) = 0$, $f'(x_0) = 1$

$x_1 = 0$, $f(x_1) = 1$, $f'(x_1) = 0$

$x_2 = 1$, $f(x_2) = 0$, $f'(x_2) = 1$

Langrange Basis:

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$= \frac{x(x-1)}{(-1)(-2)}$$

$$= \frac{x(x-1)}{2}$$

$$\cancel{l_2(x) = (x-x_0)(x-x_2)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$= \frac{(x+1)(x-1)}{(1)(-1)}$$

$$= \frac{x^2-1}{(-1)}$$

$$= -(x^2-1)$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{(x+1)x}{(2)(1)}$$

$$= \frac{x(x+1)}{2}$$

2

According to hermite polynomial interpolation

$$P_5(x) = f(x_0)h_0(x) + f(x_1)h_1(x) + f(x_2)h_2(x)$$

$$+ f'(x_0)\hat{h}_0(x) + f'(x_1)\hat{h}_1(x) + f'(x_2)\hat{h}_2(x)$$

$$= h_1(x) + \hat{h}_0(x) + \hat{h}_2(x)$$

Now

$$l_1(x) = -(x^2 - 1)$$

$$= 1 - x^2$$

$$l_1'(x) = -2x = -2\cdot 0 = 0$$

$$h_1(x) = \{1 - 2(x - x_1)\, l_1'(x_1)\}\, l_1^2(x)$$

$$= 1 \cdot (1 - x^2)^2$$

$$= (1 - x^2)^2$$

$$\hat{h}_0(x) = (x - x_0)\, l_0^2(x)$$

$$= \frac{(x+1)(x^2 - x)^2}{4}$$

$$\hat{h}_2(x) = (x - x_2)\, l_2^2(x)$$

$$= \frac{(x-1)(x^2 + x)^2}{4}$$

3

Hermite Interpolating Polynomial.

$$P_5(x) = \hat{h}_1(x) + \hat{h}_0(x) + \hat{h}_2(x)$$

$$= (1-x^2)^2 + \frac{(x+1)(x^2-x)^2}{4} + \frac{(x-1)(x^2+x)^2}{4}$$

4

$x = 0.5$

$$P_5(0.5) = (1-(0.5)^2)^2 + \frac{(0.5+1)((0.5)^2-0.5)^2}{4} + \frac{(0.5-1)((0.5)^2+0.5)^2}{4}$$

$$= \frac{33}{64}$$

$$= 0.515625$$