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Course: CSE 330

Section: 10

Assignment no: 02

Ans: to the que no: 01

(1) Given,

$$f(x) = e^{x} + e^{-x}$$

Now, from Taylor Expansion,

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$\therefore f(x) = e^{x} + e^{-x}$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) + \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)$$

$$= 2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\right)$$

Ans

(2)

$$f(x) = 2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right) \text{ [from 1]}$$

$$= 2 + x^2 + \frac{2x^4}{4!} + \cdots$$

We know

$$P_n(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n$$

Now, If the function is interpolated by degree four polynomial $P_4(x)$,

$$P_4(x) = 2 + x^2 + \frac{2x^4}{4!}$$

Now

Comparing $P_4(x)$ with $P_n(x)$

$a_0 = 2$, $a_1 = 0$, $a_2 = 1$,

$a_3 = 0$, $a_4 = \frac{2}{4!} = \frac{1}{12}$.

Ar

(3) Given,

$$f(x) = e^{x} + e^{-x}$$

$$f(0.1) = e^{(0.1)} + e^{-(0.1)}$$

$$= 1.105170918 + 0.904837418$$

$$= 2.010008336$$

Taking 7 significant digits

$$= 2.010008$$

Now,

$$P_4(x) = 2 + x^2 + \frac{2x^4}{4!} \quad \text{[got before)}$$

$$\therefore P_4(0.1) = 2 + (0.1)^2 + \frac{2(0.1)^4}{24}$$

$$= 2.010008$$ [taking 7 significant digits]

(4) We know from (3),

$$f(0.1) = 2.010008$$

$$P_4(0.1) = 2.010008$$

Now, Percent Error,

$$\frac{|f(0.1) - P_4(0.1)|}{f(0.1)} \times 100\%$$

$$= \frac{|2.010008 - 2.010008|}{2.010008} \times 100\%$$

$$= \frac{0}{2.010008} \times 100\%$$

$$= 0\%$$

Ans

[Even if we take all the digits the ans is same. Following the previous instruction]

Ans: to the que no: 02

(1) Given function,

$$f(x) = e^{x} + e^{-x}$$

Now, Constructing a Vandermonde matrix V if $f(x)$ passes through $-1, 0$ and 1 nodes

We know

$$V = \begin{bmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ Vandermonde Matrix = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(2)

Computing det(V)

$$\det |V| = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1(0-0) + 1(1-0) + 1(1-0)$$

$$= 0 + 1 + 1$$

$$= 2$$

Ans

(3)

Firstly

Inverse matrix of v,

$$v^{-1} = \frac{adj(v)}{det(v)} \qquad [det(v) = 2]$$

Given v (from 1)

$$V = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

We know,

$$C_{ij} = (-1)^{i+j} M_{ij}$$

So,

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = -1$$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$C_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = 2$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -2$

$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1$

$\therefore$ Cofactor Matrix $= \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$\therefore$ Adjoint Matrix $= \begin{bmatrix} 0 & 2 & 0 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$

$$\therefore\ V^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 2 & 0 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$ [det(v) = 2]

$$= \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix}$$

An

(4) Given,

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = V^{-1} \begin{bmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} e^{-1} + e^{1} \\ 2 \\ e^{1} + e^{-1} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1.086161 \end{bmatrix}$$ (Calculator)

(5) $P_2(x) = 2 + 1.086161x^2$

$P_2(0.1) = 2 + 1.086161\,(0.1)^2$

$= 2.01086161$

Taking 5 decimal

$= 2.01086$

Now

$f(x) = e^{x} + e^{-x}$

$\therefore f(0.1) = e^{(0.1)} + e^{(-0.1)}$

$= 2.0100083336$

Taking 5 decimals

$= 2.01001$

(6) Percent Error,

$$\frac{|f(0.1) - P_2(0.1)|}{f(0.1)} \times 100\%$$

$$= \frac{|2.01001 - 2.01086|}{2.01001} \times 100\%$$

$$= \frac{8.5 \times 10^{-4}}{2.01001} \times 100\%$$

$$= 4.228834682 \times 10^{-4} \times 100\%$$

$$\cancel{= 4.2288 \times 10^{-4}}$$

$$= 0.042288\ \%$$

If we consider all the values/digits

$$= \frac{|-2.01086161 + 2.010008336|}{2.010008336} \times 100\%$$

$$= 4.24512 \times 10^{-4} \times 100\%$$

$$= 0.0424512\ \%$$