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Course : CSE330

Section : 10

Assignment : 05

Ans: to the que no: 01

1

Given, $f(x) = 5e^{-2x}$

$x = 0.4$

$h = 0.32$

Now

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$\Rightarrow f'(0.4) = \frac{f(0.4+0.32) - f(0.4-0.32)}{2(0.32)}$$

$$= \frac{f(0.72) - f(0.08)}{0.64}$$

$$= \frac{1.1846 - 4.2607}{0.64}$$

$$= -4.8064$$

Ans

2

$x = 0{\cdot}4$, $h = 0{\cdot}16$

$$f'(0{\cdot}4) = \frac{f(0{\cdot}4+0{\cdot}16) - f(0{\cdot}4-0{\cdot}16)}{(2 \times 0{\cdot}16)}$$

$$= \frac{f(0{\cdot}56) - f(0{\cdot}24)}{0{\cdot}32}$$

$$= \frac{1{\cdot}6314 - 3{\cdot}0939}{0{\cdot}32}$$

$$= -4{\cdot}5703$$

An

3

Here

$$D^{(1)}_{0{\cdot}32} = \frac{2^2 (D_{\frac{h}{2}}) - D_h}{2^2 - 1}$$

$D_h = -4{\cdot}8064$
$D_{h/2} = -4{\cdot}5703$
$n = 2$

$$= \frac{4(-4{\cdot}5703) - (-4{\cdot}8064)}{3}$$

$$= -4{\cdot}4916$$

4

The exact value of derivative, $f'(0.4) = -4.4933$

$\therefore$ Percentage error $= \left| \dfrac{-4.4933-(-4.4916)}{-4.4916} \right| \times 100\%$

$= 0.037848492\%$

Ans: to the que no: 02

1

Expanding Taylor's Series:

$$D_h^{(1)} = f'(x_1) + \frac{f^3(x_1)}{3!} h^2 + \frac{f^5(x_1)}{5!} h^4 + \frac{f^7(x_1)}{7!} h^6 + o(h^8)$$

$$\therefore D_{\frac{h}{2}}^{(1)} = f'(x_1) + \frac{f^3(x_1)}{3!} \left(\frac{h}{2}\right)^2 + \frac{f^5(x_1)}{5!} \left(\frac{h}{2}\right)^4 + \frac{f^7(x_1)}{7!} \left(\frac{h}{2}\right)^6 + o(h^8)$$

2

$$D_h^{(2)} = \frac{2^4 D_{\frac{h}{2}}^{(1)} - D_h^{(1)}}{2^4 - 1}$$

Here

$$2^2 D^{(1)}_{\frac{h}{2}} - D^{(1)}_h = f'(x_1)(2^2-1) + \left(\frac{1}{2^2}-1\right)\left(\frac{f^5(x_1)h^4}{5!}\right) + \left(\frac{1}{2^4}-1\right)\frac{f^7(x_1)h^6}{7!} + o(h^8) \cdots (i)$$

Now dividing by (2^2-1)

$$\frac{2^2 D^{(1)}_{\frac{h}{2}} - D^{(1)}_h}{2^2-1} = f'(x_1) + \left(-\frac{1}{4}\right)\frac{f^5(x_1)h^4}{5!} + \left(-\frac{5}{16}\right)\frac{f^7(x_1)}{7!}h^6 + o(h^8)$$

Now

$$D^{(1)}_h = f'(x_1) + \left(-\frac{1}{4}\right)\frac{f^5(x_1)}{5!}h^4 + \left(-\frac{5}{16}\right)\frac{f^7(x_1)}{7!}h^6 + o(h^8)$$

$$D^{(1)}_{\frac{h}{2}} = f'(x_1) + \left(-\frac{1}{4}\right)\frac{f^5(x_1)}{5!}\left(\frac{h}{2}\right)^4 + \left(-\frac{5}{16}\right)\frac{f^7(x_1)}{7!}\left(\frac{h}{2}\right)^6 + o(h^8)$$

$$2^4 D_{\frac{h}{2}} - D_h = (2^4-1)f(x_1) + \left(\frac{15}{64} \times \frac{f^7(x_1)}{7!}h^6\right) + o(h^8) - (ii)$$

Now Dividing this (2^4-1)

$$\frac{2^4 D\frac{h}{2} - Dh}{2^4-1} = f'(x_1) + \frac{15}{64(2^4-1)} \times \frac{f^7(x_1)}{7!} h^6 + O(h^8)$$