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Course: CSE330

Section: 10

Assignment no: 8

Ans: to the que no: 01

Given,

$$x_1 - x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x^3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

Here

$$\det = \begin{vmatrix} 1 & -1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{vmatrix}$$

$$= 40$$

As the value is not zero, we have an unique solution.

Ans: to the que no: 02

From 1

$$\begin{bmatrix} 1 & -1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$R_2 = R_2 - 4R_1$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 7 & -5 & 2 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$R_3 = R_3 - 3R_1$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 7 & -5 & 2 \\ 0 & 8 & 0 & 1 \end{array}\right]$$

$R_2 = R_3 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 5 & -1 \\ 0 & 8 & 0 & 1 \end{array}\right]$$

$R_3 = R_3 - 8R_2$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & -40 & 9 \end{array}\right]$$

Here

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & -40 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix}$$

$$x_3 = \frac{-9}{40}$$

$$x_2 = -1 - 5x_3$$

$$\Rightarrow x_2 = -1 - 5\left(\frac{-9}{40}\right) = \frac{1}{8}$$

$$x_1 - x_2 + x_3 = 1$$

$$\Rightarrow x_1 = +x_2 - x_3 + 1$$

$$= \frac{1}{8} - \frac{-9}{40}$$

$$= \frac{27}{20}$$

$$\therefore x_1 = \frac{27}{20}$$

$$x_2 = \frac{1}{8}$$

$$x_3 = \frac{-9}{40}$$

Ans: to the que no: 03

We got

$$\begin{bmatrix} 1 & -1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 3 & 5 & 3 \end{bmatrix} \quad R_2 = R_2 - 4R_1$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 8 & 0 \end{bmatrix} \quad R_3 = R_3 - 3R_1$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 40/7 \end{bmatrix} \quad R_3 = R_3 - \frac{8}{7}R_2$$

$\therefore$ The obtained Matrix, $U = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & \frac{40}{7} \end{bmatrix}$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 8/7 & 1 \end{bmatrix}$$

Ans: to the que no: 4

From 3

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 8/7 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 40/7 \end{bmatrix}$$

Considering

$$Ux = y$$
$$Ly = b$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 8/7 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

$$\therefore y_1 = 1$$

$$4y_1 + y_2 = 6$$

$$\Rightarrow 4 + y_2 = 6 \Rightarrow y_2 = 2$$

$$3y_1 + \frac{8}{7}y_2 + y_3 = 4$$

$$\Rightarrow y_3 = -9/7$$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -\frac{9}{7} \end{bmatrix}$$

Now for $Ux = y$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 40/7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -9/7 \end{bmatrix}$$

$$\therefore \quad x_3 \left(\frac{40}{7}\right) = \frac{-9}{7}$$

$$\Rightarrow x_3 = \frac{-9}{40}$$

$$7x_2 - 5x_3 = 2$$

$$\Rightarrow 7x_2 - 5\left(\frac{-9}{40}\right) = 2$$

$$\Rightarrow x_2 = \frac{1}{8}$$

$$\therefore x_1 - x_2 + x_3 = 1$$

$$\Rightarrow x_1 - \frac{1}{8} + \left(\frac{-9}{40}\right) = 1$$

$$\Rightarrow x_1 = \frac{27}{20}$$

$$\therefore \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{27}{20} \\ \frac{1}{8} \\ -9/40 \end{bmatrix}$$