Name : Tanjim Reza

Student ID : 20101065

Course : CSE330

Section : 10

Assignment no: 03

Answer to the que no:01

Given,

$$f(x) = \sin(x)$$

$$x_0 = 0,$$

$$x_1 = \frac{\pi}{2}$$

$$x_2 = \pi$$

$$\therefore f(x_0) = \sin(0) = 0$$

$$f(x_1) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$f(x_2) = \sin(\pi) = 0$$

Now

Newton's divided difference method:

$$P_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$$

$$= f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2]$$

$$(x-x_0)(x-x_1)$$

Now

here,

$x_0 = 0$. $f[x_0] = 0$. $f[x_0 x_1] = \dfrac{1-0}{\frac{\pi}{2} - 0}$

$= \dfrac{2}{\pi}$

$x_1 = \dfrac{\pi}{2}$. $f[x_1] = 1$. $f[x_1, x_2] = \dfrac{0-1}{\pi - \frac{\pi}{2}}$

$= -\dfrac{2}{\pi}$

$x_2 = \pi$ $f[x_2] = 0$ $f[x_0, x_1, x_2] = \dfrac{-\frac{2}{\pi} - \frac{2}{\pi}}{\pi - 0}$

$= -\dfrac{4}{\pi^2}$

So,

$a_0 = f[x_0] = 0$

$a_1 = f[x_0, x_1] = \dfrac{2}{\pi}$

$a_2 = f[x_0, x_1, x_2] = -\dfrac{4}{\pi^2}$

Ans

Ans: to the que no: 02

The interpolating polynomial will be:

$\therefore P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$

$f[x_0] = 0$

$f[x_0, x_1] = \frac{2}{\pi}$

$f[x_0, x_1, x_2] = \frac{-4}{\pi^2}$

$= 0 + \frac{2}{\pi}(x - 0) + \left(-\frac{4}{\pi^2}\right)(x - 0)\left(x - \frac{\pi}{2}\right)$

$= \frac{2x}{\pi} - \frac{4x}{\pi^2}\left(x - \frac{\pi}{2}\right)$

$\therefore P_2(x) = +\frac{2x}{\pi} - \frac{4x}{\pi^2}\left(x - \frac{\pi}{2}\right)$

Ans: to the que no: 03

Given, New node $= \frac{3\pi}{2}$

$$\therefore f(x_3) = \sin\left(\frac{3\pi}{2}\right)$$

$$= -1$$

From (1)

$$f[x_0, x_1] = \frac{2}{\pi}$$

$$f[x_0, x_1, x_2] = \frac{-4}{\pi^2}$$

$$f[x_1, x_2] = -\frac{2}{\pi}$$

$$f[x_1, x_2, x_3] = 0$$

$$f[x_2, x_3] = -\frac{2}{\pi}$$

$$\therefore f[x_0, x_1, x_2, x_3] = \frac{0 + \frac{4}{\pi^2}}{\frac{3\pi}{2} - 0}$$

$$= \frac{8}{3\pi^3}$$

$\therefore$ New Interpolating Polynomial

$$P_3(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

$$= \frac{2x}{\pi} - \frac{4x}{\pi^2}\left(x - \frac{\pi}{2}\right) + \frac{8}{3\pi^3} x\left(x - \frac{\pi}{2}\right)(x - \pi)$$

Ans: to the que no: 04

We know

Error term:

$$|f(x) - P_3(x)| = \frac{f^4(\xi)}{4!}(x - 0)\left(x - \frac{\pi}{2}\right)(x - \pi)\left(x - \frac{3\pi}{2}\right)$$

$$= \frac{\sin(\xi)}{24} \cdot W(x)$$

Here, $\xi \in [-1, 1]$

$$\therefore W(x) = x\left(x - \frac{\pi}{2}\right)(x - \pi)\left(x - \frac{3\pi}{2}\right)$$

$$= \left(x^2 - \frac{\pi x}{2}\right)\left(x^2 - \frac{3\pi x}{2} - x\pi + \frac{3\pi^2}{2}\right)$$

$$= x^4 - \frac{3x^3\pi}{2} - \pi x^3 + \frac{3x^2\pi^2}{2} - \frac{x^3\pi}{2} + \frac{3\pi^2 x^2}{4}$$

$$+ \frac{\pi^2 x^2}{2} - \frac{3x\pi^3}{4}$$

$$\boxed{\begin{aligned} &= x^4 - \frac{3x^3\pi}{2} - \frac{3x\pi^3}{4} - \pi x^3 + \frac{3x^2\pi^2}{2} \\ &- \frac{x^3\pi}{2} + \frac{3\pi^2 x^2}{4} - \frac{\pi^2 x^2}{2} \end{aligned}}$$

$$= x^4 - \frac{3x^3\pi}{2} - \frac{\pi x^3}{1} - \frac{x^3\pi}{2} + \frac{3x^2\pi^2}{2} +$$

$$\frac{3x^2\pi^2}{4} + \frac{x^2\pi^2}{2} - \frac{3x\pi^3}{4}$$

$$= x^4 - \left(\frac{3x^3\pi + 2\pi x^3 + x^3\pi}{2}\right) + \cancel{\frac{3x^2\pi^2 + 2x^2\pi^2}{4}}$$

$$+ \frac{6x^2\pi^2 + 3x^2\pi^2 + 2x^2\pi^2}{4} - \frac{3x\pi^3}{4}$$

$$= x^4 - 3x^3\pi + \frac{11x^2\pi^2}{4} - \frac{3x\pi^3}{4}$$

Ans

Ans: to the que: no: 05

Now

for, $\sin(\xi)$, max error $= \sin(1)$

Now

$$W(x) = x^4 - 3x^3\pi + \frac{11x^2\pi^2}{4} - \frac{3x\pi^3}{4}$$

$$W'(x) = 4x^3 + \frac{11x\pi^2}{2} - \frac{3\pi^3}{4} - 9\pi x^2$$

So, when, $(W'(x)) = 0$,

$$4x^3 + \frac{11x\pi^2}{2} - \frac{3\pi^3}{4} - 9\pi x^2 = 0$$

$$4x^3 + 54{\cdot}28x - 9\pi x^2 - 23{\cdot}25 = 0$$

$\therefore \quad x = 0.599,\ 2.356,\ 4.112$

$\therefore$ Interpolation error (max)

$$|f(x) - P_3(x)| = \frac{\sin(1)}{24} \times 60.821$$

$$= 0.0442 \quad \left[\text{when } x = -1 \rightarrow 60.821 \text{ (max)}\right)$$

Ans