The University of Hong Kong
Department of Electrical and Electronic Engineering

ELEC6026 Digital Signal Processing

Assignment: Design of Linear Phase FIR Filters

1. Objectives:

The purpose of this assignment is to study the design of one dimensional finite-duration impulse response (FIR) linear phase filters using MATLAB. The window method and parks-McClellan method will be studied.

2. Equipment Required

MATLAB software.

If you do not have the MATLAB software, you may use the PC workstations located at the CYC103 Laboratory.

Reference: A.V. Oppenheim, Discrete-time Signal Processing, Prentice-Hall, 1991.

3. Suggested Duration

Three hours.

4. Theory

4.1 Window method

Window method generally begins with an ideal frequency response:

$$H_d(e^{jw}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n} \tag{1}$$

where $h_d(n)$ is the corresponding impulse response sequence. Many idealized systems are defined by piecewise-constant or piecewise-functional frequency responses with discontinuities at the boundaries between bands. As a result, they have impulse response that are noncausal and infinitely long. Window method truncates the infinitely long impulse response by multiplying it with a finite sequence called the window functions:

$$h(n) = h_d(n)\omega(n) \tag{2}$$

The Fourier transform of the FIR filter is then given by:

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})W(e^{j(\omega-\theta)})d\omega \tag{3}$$

which is the periodic convolution of the desired ideal frequency response with the Fourier transform of the windows. Hence, $H(e^{j\omega})$ will be a smeared version of $H_d(e^{j\omega})$. The most popular window functions being the Kaiser window:

$$w(n) = \begin{cases} \dfrac{I_0[\beta(1-[(n-\alpha)/\alpha]^2)^{1/2}]}{I_0(\beta)} & 0 \le n \le M, \\ 0 & otherwise, \end{cases} \tag{4}$$

where $\alpha = M/2$, and I_0 represents the zeroth-order modified Bessel function of the first kind. The design procedure involves the determination of the length M and the shape parameter β from the given specification. Let

$\delta = \min\{\delta_1, \delta_2\}$

$\Delta\omega = \omega_s - \omega_p$ (for lowpass approximation)

where δ_1, δ_2 are respectively the pass and stop band ripples in the specification.

ω_s, ω_p are respectively the stop and pass band edge frequencies.

Define

$$A = -20\log_{10}\delta \tag{5}$$

Kaiser determined empirically that the value of β and M needed to meet the specification is given by:

$$\beta = \begin{cases} 0.1102(A-8.7), A > 50, \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), 21 \le A < 50, \\ 0, A < 21, \end{cases} \tag{6}$$

$$M = \frac{A-8}{2.285\Delta\omega}. \tag{7}$$

Similar procedure can be applied to design multiband filters.

4.2 Parks-McClellan method

The Parks-McClellan algorithm uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with optimal ∞ error norm, i.e. the maximum error between the desired frequency response and the actual frequency response is minimized. Filter designed in this way exhibit an equiripple behavior in their frequency response.

Unlike the Kaiser window method, the pass and stop band ripples need not be the same. Kaiser has obtained the following simplified formula for the estimation of filter order for a given specification:

$$M = \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.324\Delta\omega} \tag{8}$$

4.3 PC-MATLAB

PC-MATLAB is an interactive environment for high level simulation of general numerical applications such as numerical linear algebra, signal processing, etc.

MATLAB works with essentially only one kind of object, a rectangular numerical matrix with possibly complex elements. Refer to PC-MATLAB Reference and Tutorial for more details of matrix declaration and manipulation.

User defined function or command files can be added to MATLAB's vocabulary if they are expressed in terms of other existing functions. The commands and functions that comprise the new function are put in a file whose name defines the name of the new function, with a file type of .m appended. See the PC-MATLAB reference pp.3-84-3.85 for details. M-files can be created using ordinary text-editor and placed in the same directory of MATLAB or other defined search path in MATLAB. Type the name of the M-file in MATLAB environment will invoke the commands or functions defined in the corresponding M-file.

In this experiment, we shall make use of two functions in the Signal Processing Toolbox in the PC-MATLAB for designing linear phase FIR filters.

fir1 implements the classical method of windowed linear-phase FIR digital filter design. It is formulated to design filters in standard lowpass, bandpass, highpass and bandstop configurations. For example

```
b=fir1(n,Wn,Kaiser(n+1,β))
```

Returns row vector b containing the n+1 coefficients of the order n Kaiser windowed lowpass linear-phase FIR filter with normalized cutoff frequency Wn. Refer to MATLAB Signal Processing Toolbox Tutorial pp. 1-32 - 1-33 and 2-61 - 2-62 for more details.

remez is a function to design linear phase FIR filters using the Parks-McClellan algorithm.

```
b=remez(n,f,m,w)
```

returns row vector b containing the n+1 coefficients of the order n FIR filter whose frequency-magnitude characteristics match those given by vector f and m. For later versions of the MATLAB, the **remez** function is replaced by **firpm**. The usage of firpm is similar to that of the **remez** function

```
b=firpm(n,f,m,w)
```

f is a vector of frequency points, specified in the range between 0 and 1, where 1.0 corresponds to half the sample frequency.

m is a vector containing the desired magnitude response at the points specified in f. The elements of m must appear in equal-valued pairs.

w is the weight used in each frequency band.

Refer to pp. 1-34 - 1-35 and 2-91 - 2-93 of the Signal Processing Toolbox tutorial or the online help manual of the PC-MATLAB for more details.

5. Procedure

The assignment package 'ass1.zip' contains the following files:

- `ass1_main.m`: The script is used to calculate and display the spectrogram of the synthetic signal which is stored in `Signal.mat`. The script will load a variable named '`x`', which is a signal. The total duration of the signal is 2 seconds and the sampling rate is 200 Hz.
- `stft.m`: In-house MATLAB scripts to perform STFT.

(i) Run `ass1_main.m` to display the synthetic signal and the STFT-based spectrogram using the default setting (window length = 0.3s). From the signal and its spectrogram, comment on the properties of the signal and identify the frequency of the signal at $t = 0{,}1$ and 2 s.

(ii) Re-calculate the spectrogram using the in-house program `stft` with different window sizes: 0.2 and 0.5. Modify the MATLAB code in `ass1_main.m` to plot the two spectrograms. Describe the difference between these spectrograms. Discuss how the window size affects the time and frequency resolutions of the spectrograms.

(iii) Use the function **fir1** in MATLAB to design an FIR lowpass filter using **Kasier Window** method with the following specifications:

$$\omega_p = 0.6\pi,\ \omega_s = 0.7\pi,\ \delta_1 = 0.01, \text{and } \delta_2 = 0.001, \qquad (9)$$

Obtain the estimate of the filter from eqn. (5) – (7).

(iv) Use the **Parks-McClellan** method **(**function **firpm** in MATLAB) to design a lowpass filter with the specification as in (9). Obtain the estimate of the filter from eqn. (8).

(v) Plot and discuss the frequency response of filters designed in (ii) and (iii).

(vi) Comment on the two approaches in designing linear phase FIR filters.

(vii) $^{(*)}$Use the filter designed by **Parks-McClellan** method to filter signal 'x' by linear convolution.

> **Remarks** $^{(*)}$ You are **not allowed** to use the function conv in MATLAB for your implementation. However, you may use it to verify the accuracy of your own code.

(viii) Calculate the spectrogram of filtered signal with default window size. Plot the filtered signal and its spectrogram.

(ix) Comment and explain the difference between the spectrogram of signal ‘x’ and the filtered signal.

END