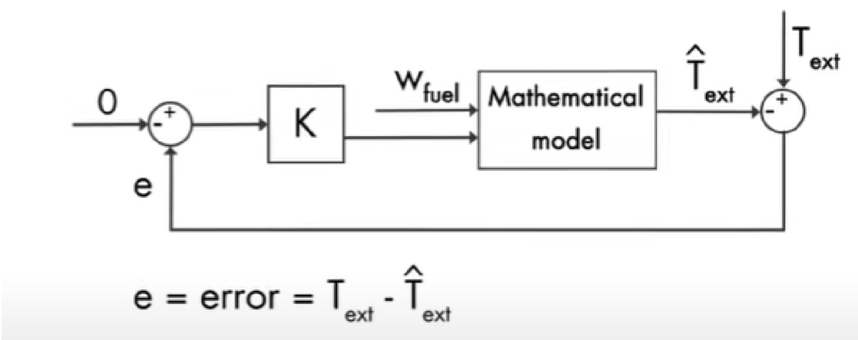
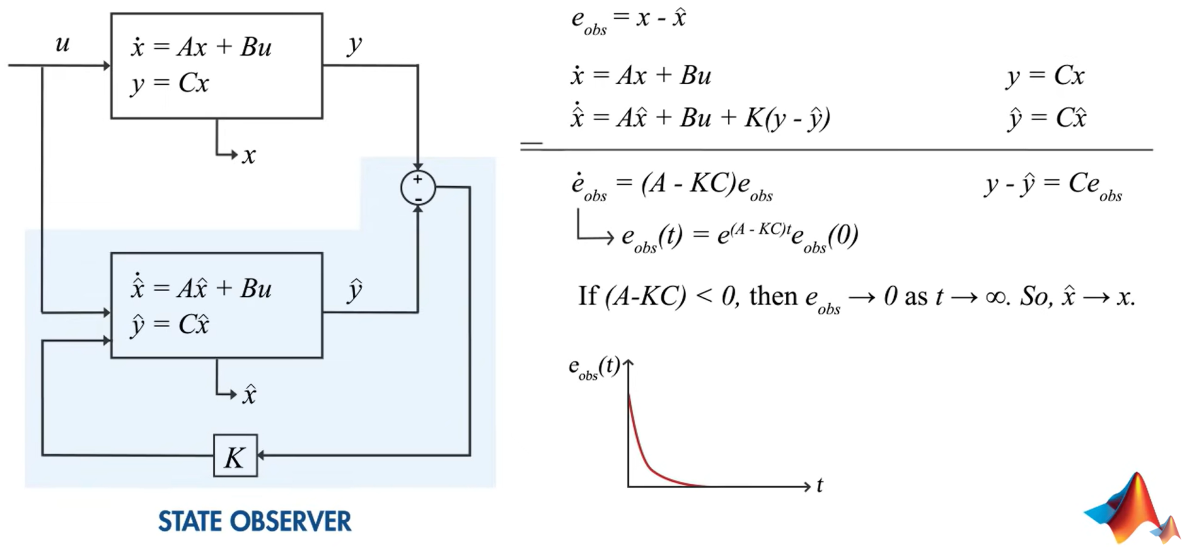
Kalman Filters

- Generally used to obtain the optimal estimate measurement after fusion of sensors when some sensors give uncertain or no value
- Algorithm generally used in applications like determining location speed etc
- State estimator- used to find quantities that are difficult or unable to find using mathematical model
- The methodology used in kalman filter is similar to feedback control mechanism where the difference between estimated and measured is minimised by using a controller K
- e here is the error of output.



Here is a typical example of how Kalman filters work



State estimator refers to the filter responsible for estimating the current state of a dynamic system

Kalman algorithm generally follows steps as follows

- Prediction- expected measurement
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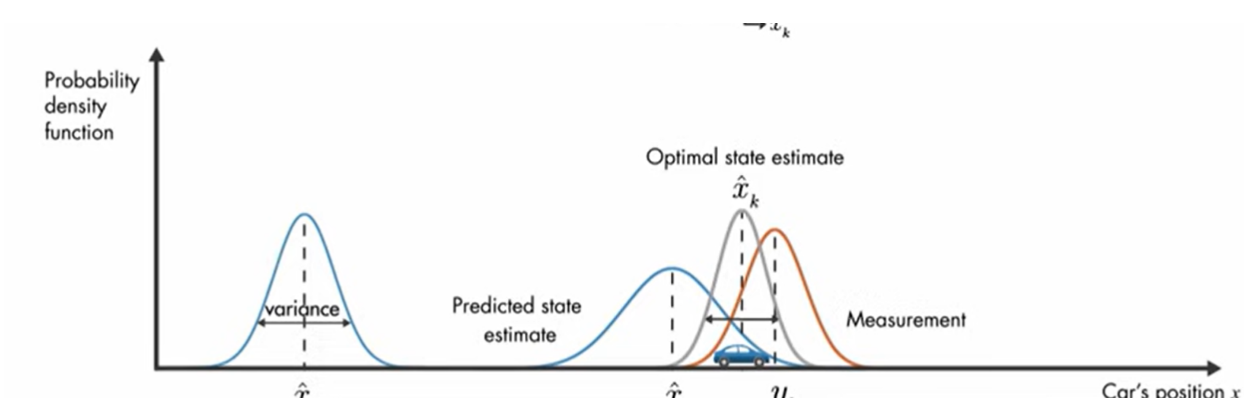


  - Optimal state function(gaussian function) is obtained by multiplying the predicted and measured Probability Density Function and the mean of this gives the optimal estimate

- More the variance less the prediction is so our main aim is to reduce it so the probability density function value is high and we derive the optimal state estimate

- Variance and standard deviation

$$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2$$

Kalman filter

$$\hat{x}_k = \underbrace{\hat{x}_k^-}_{\text{Predict}} + \underbrace{K_k(y_k - C\hat{x}_k^-)}_{\text{Update}}$$

- Posterori estimate is the xk- which refers to the previous value

Prediction

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k$$

$$P_k^- = AP_{k-1}A^T + Q$$

Initial estimates for $\hat{x}_{k-1}$ and P_{k-1}

Update

$$K_k = \frac{P_k^- C^T}{CP_k^- C^T + R}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(y_k - C\hat{x}_k^-)$$

$$P_k = (I - K_kC)P_k^-$$

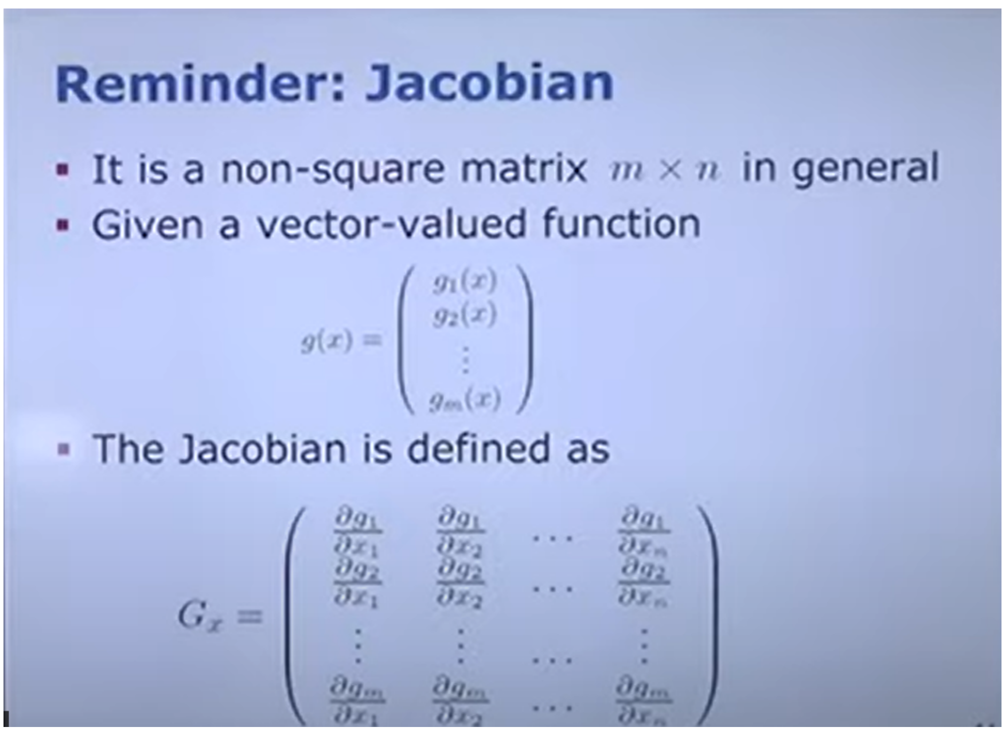
- Distribution in graph

- Precision

Extended Kalman Filters

The Extended Kalman Filter (EKF) is an extension of the traditional Kalman Filter (KF) that allows it to handle nonlinear systems . But how’s it different from extended kalman filters?

- Kalman filters assume that both the system's dynamics (state transition model) and measurement model (relationship between the state and measurements) are linear. But EKF is non linear
- Accuracy wise, KF is more accurate than the EKF in linear systems but in non linear terms EKF is very effective
- EKF uses Jacobian matrices and KF uses gaussian distributions for state and measurement noise.



EKF Linearization: First Order Taylor Expansion

- Prediction:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \underbrace{\frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}}_{=:\, G_t} (x_{t-1} - \mu_{t-1})$$

- Correction:

$$h(x_t) \approx h(\bar{\mu}_t) + \underbrace{\frac{\partial h(\bar{\mu}_t)}{\partial x_t}}_{=:\, H_t} (x_t - \bar{\mu}_t)$$

Jacobians