噪声抑制

DENOISE

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Conference on Computer Vision, Bombay,
India

Bilateral Filtering for Gray and Color Images

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The Idea

Low-pass domain filter :

$$\mathbf{h}(\mathbf{x}) = k_d^{-1}(\mathbf{x}) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{f}(\xi) c(\xi, \mathbf{x})\, d\xi$$

$\boxed{c(\xi, \boldsymbol{x}) :\ \xi \to \boldsymbol{x}}$

距离优先

$$k_d(\mathbf{x}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi, \mathbf{x})\, d\xi$$

Range filter

$$\mathbf{h}(\mathbf{x}) = k_r^{-1}(\mathbf{x}) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{f}(\xi) s(\mathbf{f}(\xi), \mathbf{f}(\mathbf{x}))\, d\xi$$

$\boxed{s\big(f(\boldsymbol{\xi}), f(\boldsymbol{x})\big):\ f(\boldsymbol{\xi}) \to f(\boldsymbol{x})}$

值优先

$$k_r(\mathbf{x}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(\mathbf{f}(\xi), \mathbf{f}(\mathbf{x}))\, d\xi$$

Bilateral filter

$$\mathbf{h}(\mathbf{x}) = k^{-1}(\mathbf{x}) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{f}(\xi) c(\xi, \mathbf{x}) s(\mathbf{f}(\xi), \mathbf{f}(\mathbf{x}))\, d\xi$$

$$k(\mathbf{x}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\xi, \mathbf{x}) s(\mathbf{f}(\xi), \mathbf{f}(\mathbf{x}))\, d\xi\ .$$

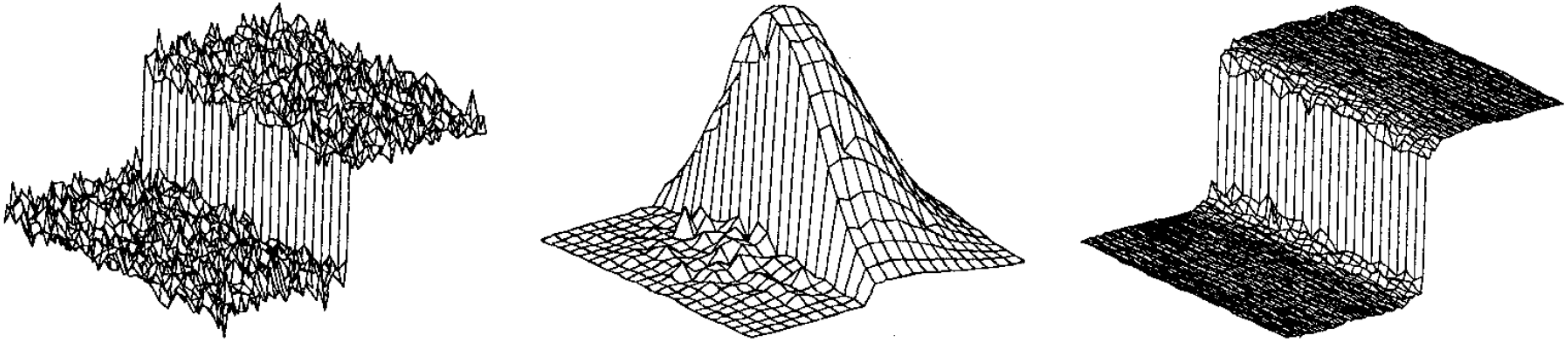
Example: the Gaussian Case

$$c(\xi, \mathbf{x}) = e^{-\frac{1}{2}\left(\frac{d(\xi,\mathbf{x})}{\sigma_d}\right)^2}$$

$$d(\xi, \mathbf{x}) = d(\xi - \mathbf{x}) = ||\xi - \mathbf{x}||$$

$$s(\xi, \mathbf{x}) = e^{-\frac{1}{2}\left(\frac{\delta(\mathbf{f}(\xi),\mathbf{f}(\mathbf{x}))}{\sigma_r}\right)^2}$$

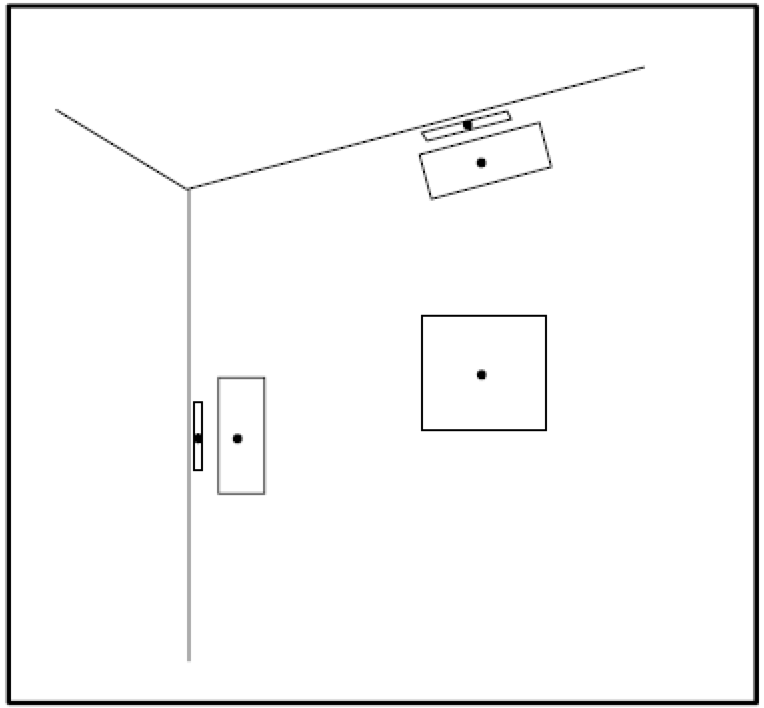
$$\delta(\phi, \mathbf{f}) = \delta(\phi - \mathbf{f}) = ||\phi - \mathbf{f}||$$



Computer Vision and Image Understanding 111 (2008) 155–169

An adaptive window mechanism for image smoothing

Ardeshir Goshtasby , Martin Satter

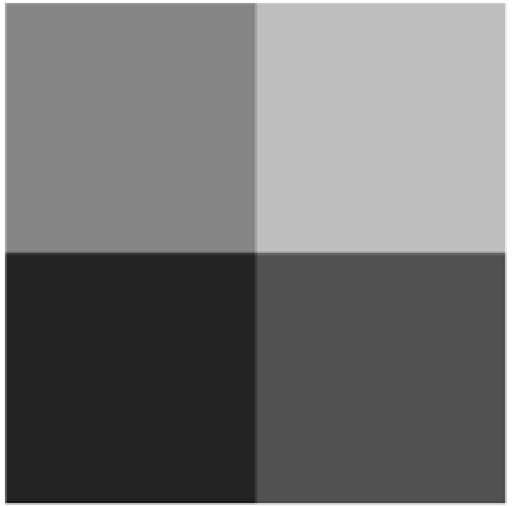


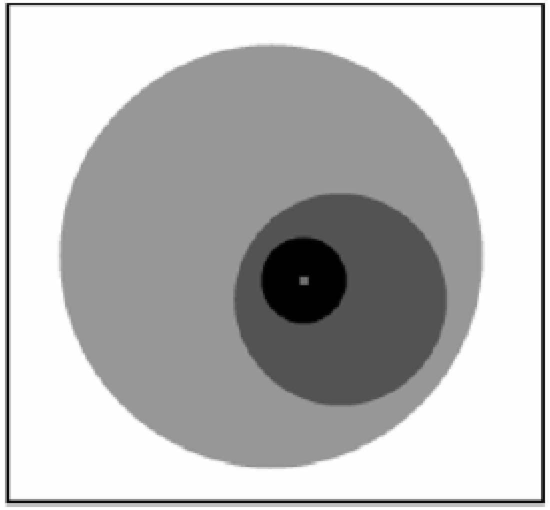
$$W = a/(g_n + 1)$$
$$H = a/(g_m + 1)$$

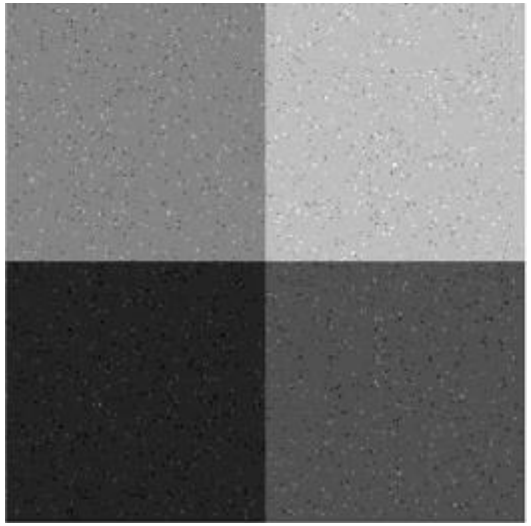
$$G(x, y) = G(x) \times G(y)$$

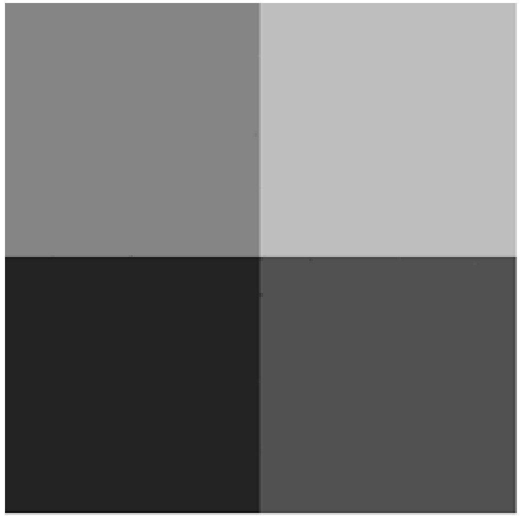
$$G(X, Y) = \exp\left\{-\frac{X^2}{2\sigma_X^2}\right\} \exp\left\{-\frac{Y^2}{2\sigma_Y^2}\right\}$$

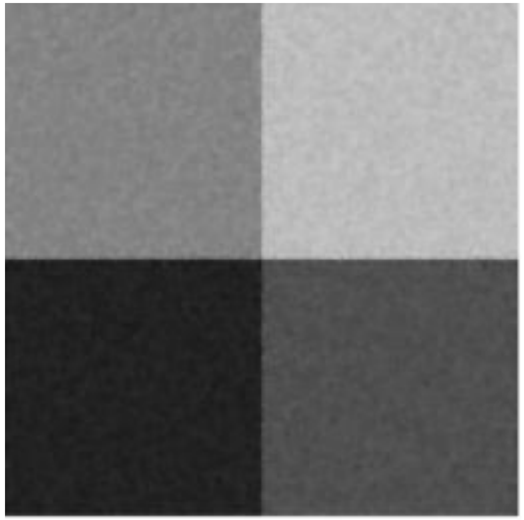
$$\sigma_x = a/2(g_n + 1)$$
$$\sigma_y = a/2(g_m + 1)$$

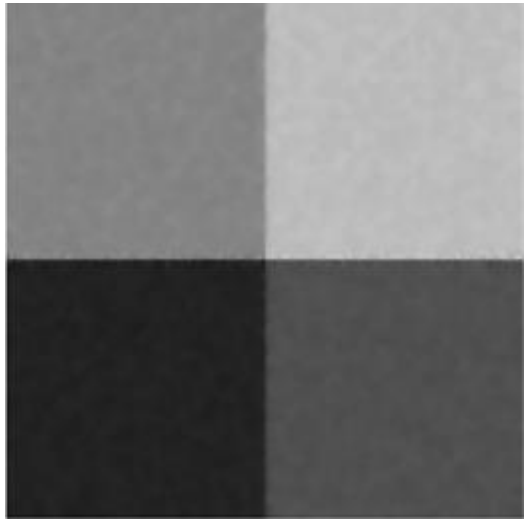








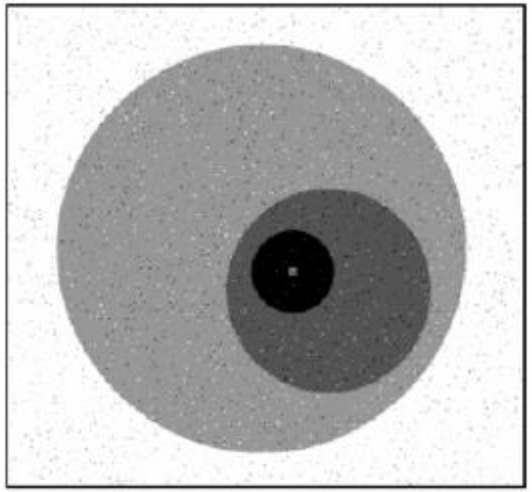


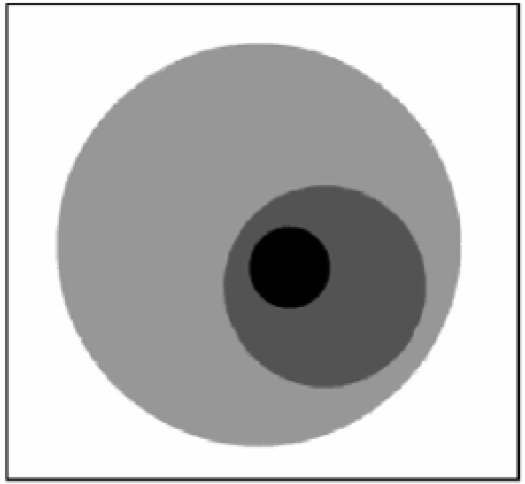


Median Filter

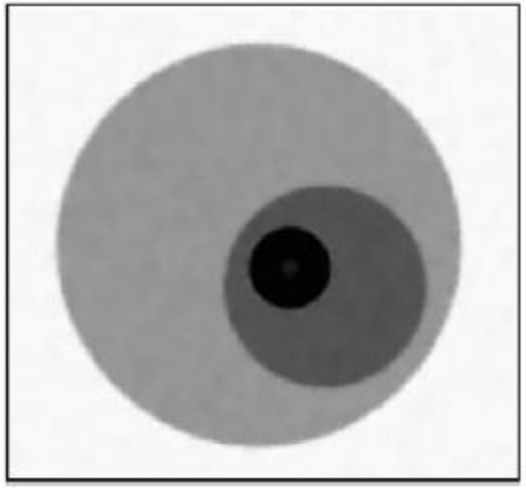
Average Filter

Gaussian Filter

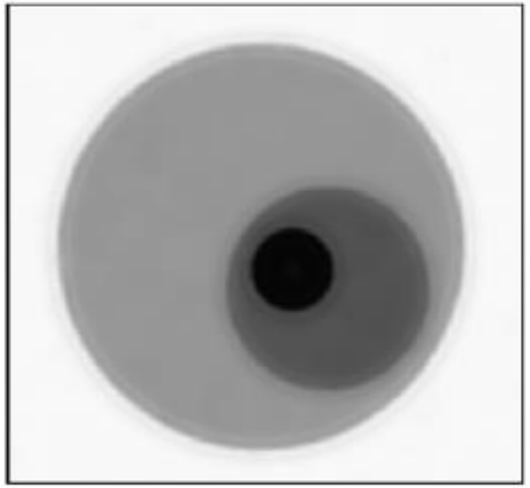




Median Filter

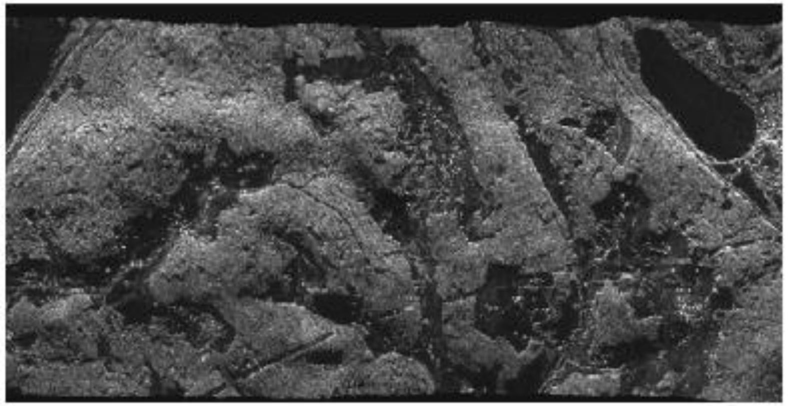
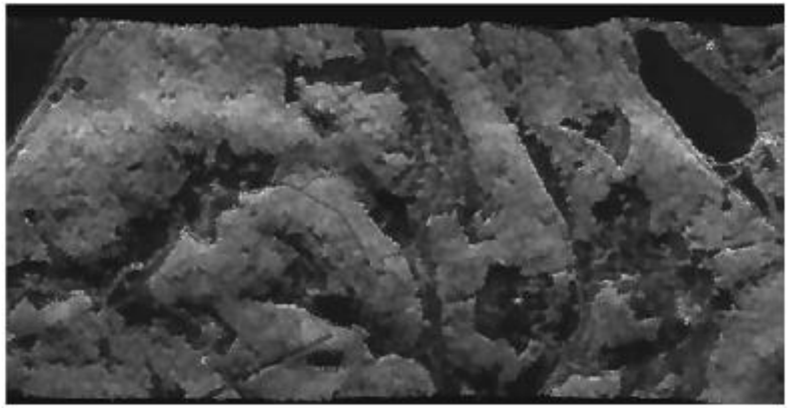
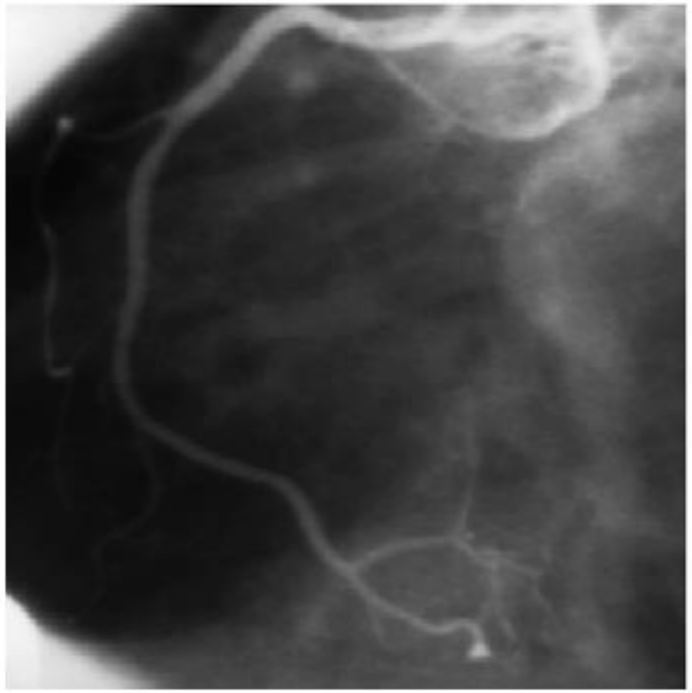
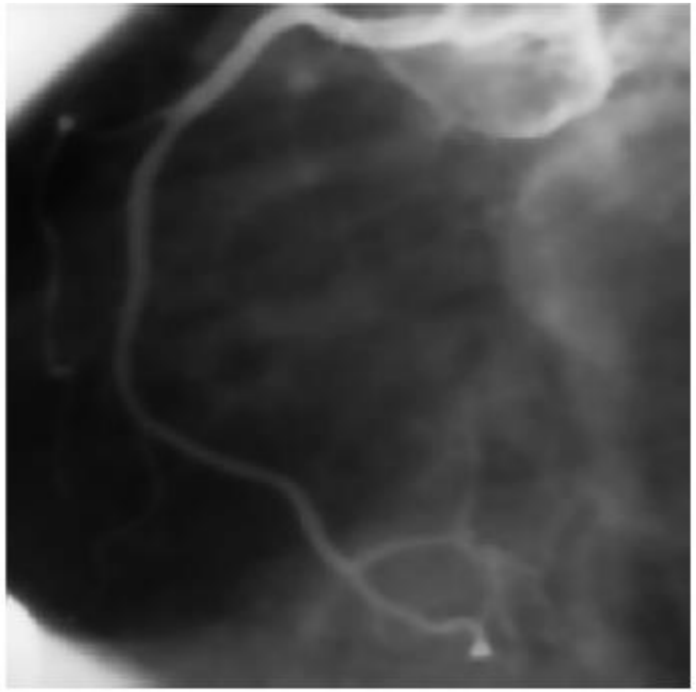


Average Filter



Gaussian Filter

Gaussian Filter

The 22nd British Machine Vision Conference
University of Dundee, 29 August – 2 September 2011

Image denoising with patch-based PCA: local versus global

Charles-Alban Deledalle[1], Joseph Salmon[2], Arnak Dalalyan[3]

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August 31, 2011

Noisy image (with additive white Gaussian noise)

$$y_i = f_i + w_i \text{ for } i = 1, \cdots, M$$

patch model

$$Y_i = F_i + W_i \text{ for } i = 1, \cdots, M$$

Patch based Global PCA

patches of size $n = W_{\mathsf{P}} \times W_{\mathsf{P}}$

$n \times n$ empirical covariance matrix

$$\Sigma = \frac{1}{M}\sum_{k=1}^{M} Y_k Y_k' - \bar{Y}\bar{Y}', \qquad \bar{Y} = \frac{1}{M}\sum_{k=1}^{M} Y_k$$

Principal Component Analysis

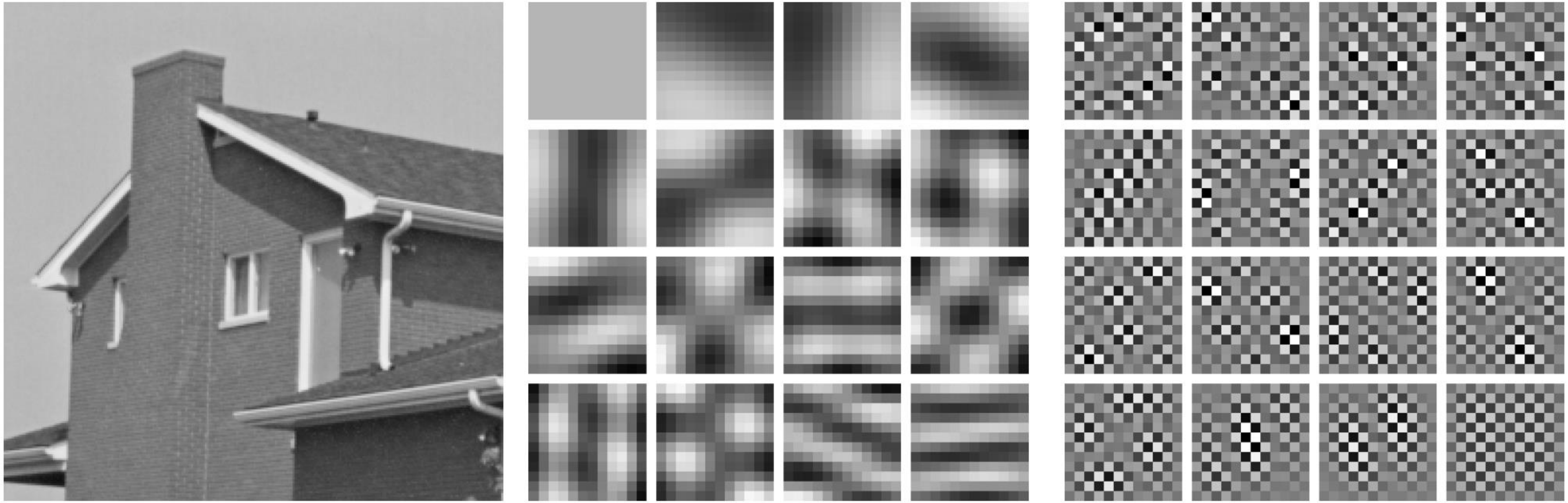
eigenvalues of Σ $\quad \lambda_1 \geq \ldots \geq \lambda_n \geq 0$

eigenvectors $\quad X_1, \ldots, X_n$

any patch Y_i $\quad Y_i = \sum_{k=1}^{n} \langle Y_i | X_k \rangle X_k$

patch denoising $\quad \hat{F}_{KOK,i} = \bar{Y} + \sum_{k=1}^{n'} \langle Y_i - \bar{Y} | X_k \rangle X_k$

$n' < n$



(a) Input image (b) 16 first axes (c) 16 last axes

Figure 2: An image (House), its 16 first axes and 16 last axes obtained by a PCA over all the patches of the image.

$$\hat{F}_i = \bar{Y} + \sum_{k=1}^{n} \eta(\langle Y_i - \bar{Y} | X_k \rangle) X_k$$

threshold parameter λ

$$\eta_{\mathrm{ST}}(x) = \mathrm{sign}(x) \cdot (|x| - \lambda)_+$$

$$(t)_+ = \max(0, t)$$

$$\eta_{\mathrm{HT}}(x) = x \cdot \mathbb{1}(\lambda < |x|)$$

非局部均值算法
Non-local averaging for image denoising

A.Buades, B.Coll, *et al*, A non-local algorithm for image denoising, Proc. IEEE Computer Society Conf Computer Vision and Pattern Recognition (CVPR'05), Vol 2, 2005 :60-65

$$v(i) = u(i) + n(i)$$

$$0 \le \omega(i,j) \le 1$$

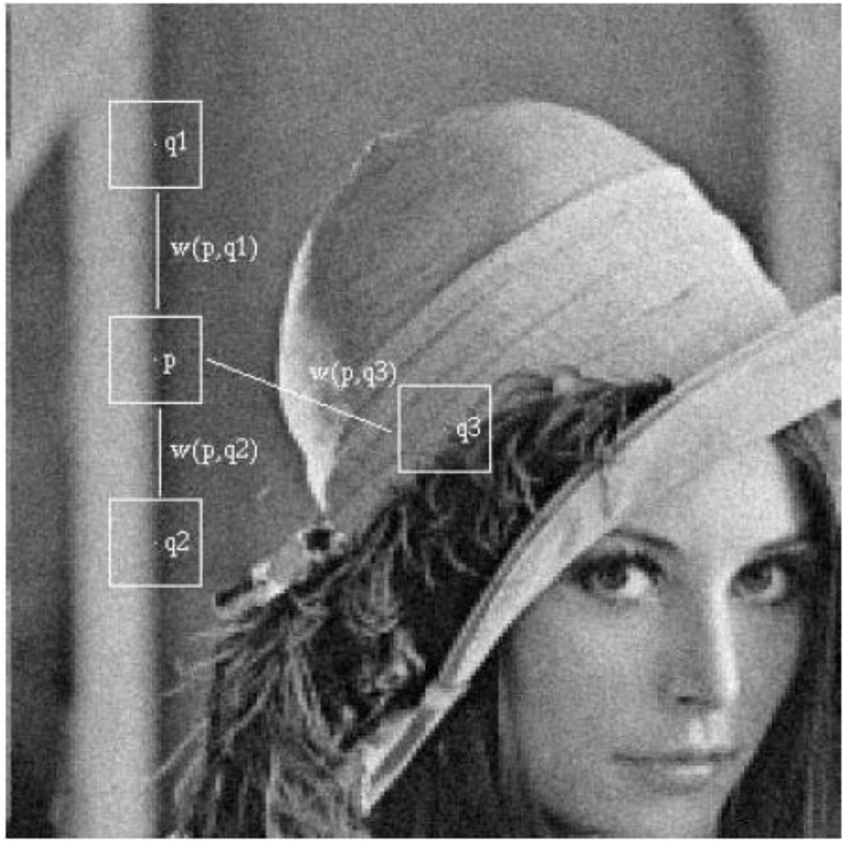
$$\sum_j \omega(i,j) = 1$$

$$NL[v](i) = \sum_{j \in I} \omega(i,j) v(j)$$

Similarity between pixel i and j

Weighted Euclidean distance

$$\left\| v(\Omega_i) - v(\Omega_j) \right\|_{2,\sigma}^2$$



$$\omega(i,j) = \frac{1}{Z(i)} e^{-\frac{\left\| v(\Omega_i) - v(\Omega_j) \right\|_{2,\sigma}^2}{h^2}}$$

Ω_i：以i为中心的矩形窗

$$Z(i) = \sum_j e^{-\frac{\left\| v(\Omega_i) - v(\Omega_j) \right\|_{2,a}^2}{h^2}}$$

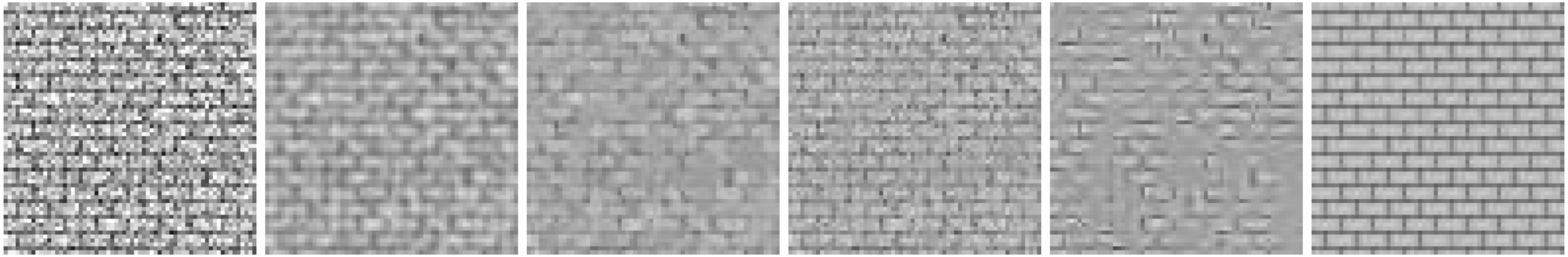


Fig. 4. Denoising experience on a periodic image. From left to right: noisy image (standard deviation 35), Gauss filtering, Total variation, Neighborhood filter, translation invariant wavelet thresholding and NL-means algorithm. It seems that a non-local algorithm is necessary to reconstruct the periodic and fine structure of the wall pattern.

基于PDE的图像增强算法
Image Enhancement based on PDE

P.Perona, J.Marlk, Scale-space and edge detection using anisotropic diffusion, IEEE Trans Pattern Analysis and Machine Intelligence, 12(7), 1990 : 629-639

热扩散模型（Thermal diffusion model）

各向同性介质（For isotropic media）

热通量密度（Heat Flux Density)

热场

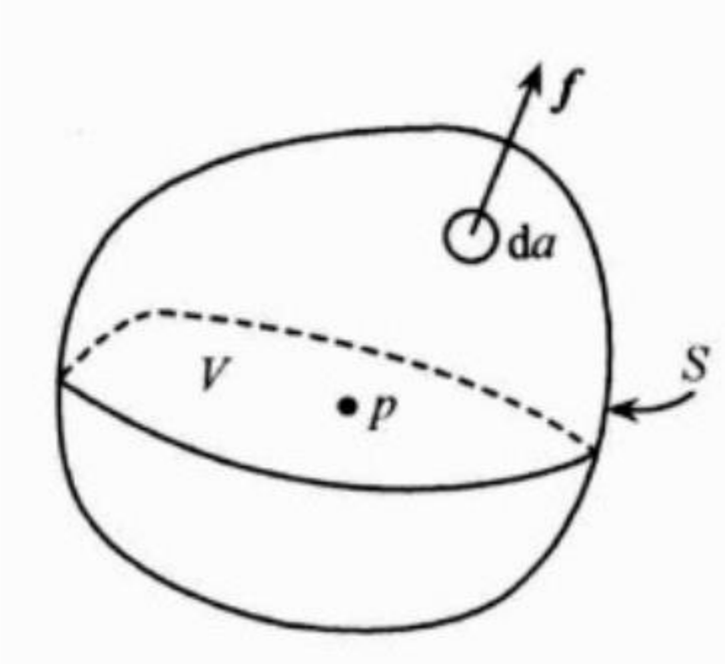
$$\vec{f} = -a\,\nabla \underline{u}$$

$$a: constant, a(x,y,z), a(x,y,z,u)$$

各向异性介质（For anisotropic media）

$$\vec{f} = -D\,\nabla u$$

$$D: 2 \times 2, 3 \times 3\ tensor$$



$$F = \oiint_S \vec{f} \cdot \overrightarrow{da}$$

$$F = \iiint_V div(\vec{f})\, dv$$

$$\frac{\partial}{\partial t} \iiint_V u dv = -F = -\iiint_V div\left(\vec{f}\right) dv$$

$$\boxed{\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} = -\boldsymbol{div}(\vec{\boldsymbol{f}})} \qquad \boxed{\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} = -\boldsymbol{\nabla} \cdot \vec{\boldsymbol{f}}}$$

For isotropic media

$$\vec{f} = -a\nabla u \quad \Rightarrow \quad \frac{\partial u}{\partial t} = \nabla \cdot (a\nabla u)$$

For 2D :

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(a\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(a\frac{\partial u}{\partial y}\right)$$

$$\boxed{\boldsymbol{u_t = a(u_{xx}+u_{yy})}}$$

For anisotropic media:

$$\vec{f} = -D\,\nabla u \quad \Rightarrow \quad \frac{\partial u}{\partial t} = \nabla \cdot (D\nabla u)$$

For 2D: $$D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\overrightarrow{v_1}, \overrightarrow{v_2}: eigenvector\ of\ D \quad \Rightarrow \quad D = \mu_1 \vec{v}_1 \vec{v}_1^T + \mu_2 \vec{v}_2 \vec{v}_2^T$$

$$\nabla u = \frac{\partial u}{\partial v_1}\vec{v}_1 + \frac{\partial u}{\partial v_2}\vec{v}_2$$

$$D\nabla u = \left[\mu_1 \vec{v}_1 \vec{v}_1^T + \mu_2 \vec{v}_2 \vec{v}_2^T\right][\frac{\partial u}{\partial v_1}\vec{v}_1 + \frac{\partial u}{\partial v_2}\vec{v}_2]$$

$$= \mu_1 \frac{\partial u}{\partial v_1}\vec{v}_1 + \mu_2 \frac{\partial u}{\partial v_2}\vec{v}_2$$

$$\nabla \cdot (D\nabla u) = \frac{\partial}{\partial v_1}\left(\mu_1 \frac{\partial u}{\partial v_1}\right) + \frac{\partial}{\partial v_2}\left(\mu_2 \frac{\partial u}{\partial v_2}\right)$$

$$\boxed{\boldsymbol{u_t = \mu_1 u_{v_1 v_1} + \mu_2 u_{v_2 v_2}}}$$

Linear filter and isotropic diffusion

$$\boxed{\boldsymbol{u_t = a(u_{xx}+u_{yy})}}$$

$Let\ a = 1$

$$\boldsymbol{u_t = u_{xx} + u_{yy}}$$

$$\mathcal{F}\{u_t\} = \mathcal{F}\{u_{xx} + u_{yy}\}$$

$$\frac{\partial U}{\partial t} = [(j2\pi u)^2 + (\mathrm{j}2\pi v)^2]U$$

$$U = U_0 e^{(-(2\pi u)^2-(2\pi v)^2)t} \qquad U_0 = U(u, v, 0)$$

$$\boxed{\boldsymbol{u(x, y, t) = G_\sigma * u(x, y, 0)}}$$

P-M (Perona, Malik) Equation isotropic nonlinear filter

$$\begin{cases} u_t = \nabla \cdot [g(|\nabla u|)\nabla u] \\ u(x,y,0) = u_0(x,y) \end{cases}$$

$$\boxed{\frac{\partial u}{\partial t} = \nabla \cdot (a\nabla u)}$$

1D:

$$u_t = \frac{\partial}{\partial x}[g(|u_x|)u_x]$$

$$= g'(|u_x|)\frac{u_x u_{xx}}{\sqrt{u_x^2}}u_x + g(|u_x|)u_{xx}$$

$$= [g'(|u_x|)|u_x| + g(|u_x|)]u_{xx}$$

$$= \phi'(|u_x|)u_{xx}$$

Influence function

$$\phi(r) = rg(r)$$

$$g(r) = \frac{1}{1 + \left(\frac{r}{K}\right)^p}, \qquad p = 1,2$$

$$\phi(r) = \frac{r}{1 + \left(\frac{r}{K}\right)^p}$$

$$\boxed{\boldsymbol{u_t = \phi'(|u_x|)u_{xx}}}$$

$$\phi'(r) = \frac{1 - (p-1)\left(\frac{r}{K}\right)^p}{\left[1 + \left(\frac{r}{K}\right)^p\right]^2}$$

$$p = 1 : smoothing$$

$$p = 2 : \begin{cases} 0 \leq |\nabla u| < K : smoothing \\ |\nabla u| > K : sharpening \end{cases}$$

2D:

$$\eta = \frac{\nabla u}{|\nabla u|} = (\cos\theta, \sin\theta)$$

$$\xi = (-\sin\theta, \cos\theta)$$

$$\frac{\partial u}{\partial \xi} = 0, \frac{\partial u}{\partial \eta} \geq 0$$

$$|\nabla u| = \sqrt{\left(\frac{\partial u}{\partial \eta}\right)^2} = \frac{\partial u}{\partial \eta}$$

$$\frac{\partial |\nabla u|}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2}$$

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial \xi}\left[g(|\nabla u|)\frac{\partial u}{\partial \xi}\right] + \frac{\partial}{\partial \eta}\left[g(|\nabla u|)\frac{\partial u}{\partial \eta}\right]$$

$$= g(|\nabla u|)\frac{\partial^2 u}{\partial \xi^2} + g'(|\nabla u|)\frac{\partial |\nabla u|}{\partial \xi}\frac{\partial u}{\partial \xi} + g(|\nabla u|)\frac{\partial^2 u}{\partial \eta^2} + g'(|\nabla u|)\frac{\partial |\nabla u|}{\partial \eta}\frac{\partial u}{\partial \eta}$$

$$= g(|\nabla u|)u_{\xi\xi} + [g(|\nabla u|) + g'(|\nabla u|)|\nabla u|]u_{\eta\eta}$$

$$= g(|\nabla u|)u_{\xi\xi} + \phi'(|\nabla u|)u_{\eta\eta}$$

$$\boxed{\frac{\partial u}{\partial \xi} = 0, \frac{\partial u}{\partial \eta} \geq 0}$$

$$\boxed{g(r) = \frac{1}{1 + \left(\frac{r}{K}\right)^p}, \qquad p = 1,2}$$

$$\boxed{|\nabla u| = \frac{\partial u}{\partial \eta}}$$

$$\boxed{\phi'(r) = \frac{1 - (p-1)\left(\frac{r}{K}\right)^p}{\left[1 + \left(\frac{r}{K}\right)^p\right]^2}}$$

$$\boxed{\phi(r) = rg(r)}$$

$$\boxed{\phi'(r) = g(r) + rg'(r)}$$

$$\boxed{\frac{\partial |\nabla u|}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2}}$$