Inverse Trigonometric Functions

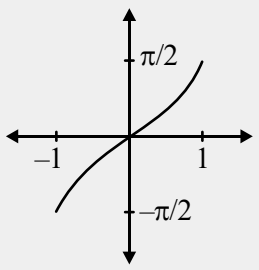
01

Inverse function	Domain	Principal Value Branch
$y = \sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$y = \text{cosec}^{-1}x$	$R-(-1,1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]-\{0\}$
$y = \sec^{-1}x$	$R-(-1,1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$y = \tan^{-1}x$	R	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1}x$	R	$(0,\pi)$

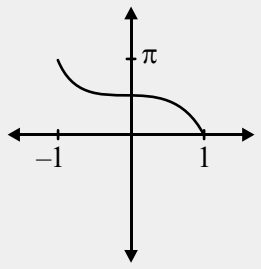
02

Graph

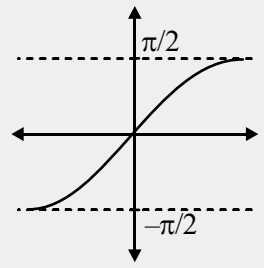
1. $y = \sin^{-1}x$



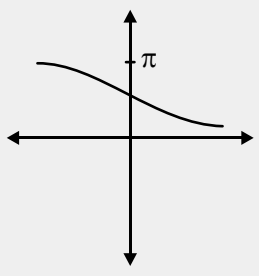
2. $y = \cos^{-1}x$



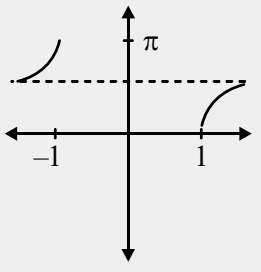
3. $y = \tan^{-1}x$



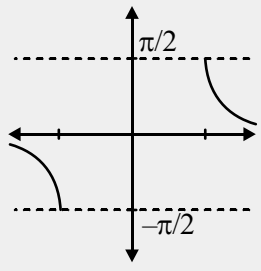
4. $y = \cot^{-1}x$



5. $y = \sec^{-1}x$



6. $y = \text{cosec}^{-1}x$



Properties Of Inverse Trigonometric Functions

03

Property -01

(i) $\sin^{-1}(\sin\theta) = \theta$ if $\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$

(ii) $\cos^{-1}(\cos\theta) = \theta$ if $0 \le \theta \le \pi$

(iii) $\tan^{-1}(\tan\theta) = \theta$ if $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$

(iv) $\cot^{-1}(\cot\theta) = \theta$ if $0 < \theta < \pi$

(v) $\sec^{-1}(\sec\theta) = \theta$ if $0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$

(vi) $\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$, if $-\frac{\pi}{2} \le \theta < 0$ or $0 < \theta \le \frac{\pi}{2}$

Property -02

(i) $\sin\left(\sin^{-1}x\right) = x$, if $-1 \le x \le 1$

(ii) $\cos\left(\cos^{-1}x\right) = x$, if $-1 \le x \le 1$

(iii) $\tan\left(\tan^{-1}x\right) = x$, if $-\infty < x < \infty$

(v) $\sec\left(\sec^{-1}x\right) = x$, if $-\infty < x \le -1$ or $1 \le x < \infty$

(iv) $\cot\left(\cot^{-1}x\right) = x$, if $-\infty < x < \infty$

(vi) $\text{cosec}\left(\text{cosec}^{-1}x\right) = x$, if $-\infty < x \le -1$ or $1 \le x < \infty$

Property -03

(i) $\sin^{-1}(-x) = -\sin^{-1}x$, if $-1 \le x \le 1$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$, if $-1 \le x \le 1$

(iii) $\tan^{-1}(-x) = -\tan^{-1}x$, if $-\infty < x < \infty$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x$, if $-\infty < x < \infty$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$, if $-\infty < x \le -1$ or $1 \le x < \infty$

(vi) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$, if $-\infty < x \le -1$ or $1 \le x < \infty$

Property -04

(i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, x \in [-1,1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, \quad x \in R$

(iii) $\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}, x \in (-\infty,-1] \cup [1,\infty)$

Property -05

(i) $\sin^{-1}x = \text{cosec}^{-1}\left(\frac{1}{x}\right), -1 \le x \le 1 - \{0\}$

(ii) $\text{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right), x \in R - (-1,1)$

(iii) $\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right), -1 \le x \le 1 - \{0\}$

(iv) $\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right), x \in R - (-1,1)$

(v) $\tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right), x \in R - \{0\}$

(vi) $\tan^{-1}(1/x) = \begin{cases} \cot^{-1}x & \forall \quad x > 0 \\ -\pi + \cot^{-1}x & \forall \quad x < 0 \end{cases}$

Property -06

(i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$

$= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), x > 0, y > 0, xy > 1$

$= -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right), x < 0, y < 0, xy > 1$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right), xy > -1$

$= \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), x > 0, y > 0, xy < -1$

$= -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right), \text{if } x < 0, y < 0 \text{ and } xy > 1$

(iii) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$, if $x > 0, y > 0, z > 0$ and $(xy + yz + zx) < 1$, $\text{if } x < 0, y < 0 \text{ and } xy > 1$

Property -07

(i) $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\right\}$ *,if* $x^2+y^2\le 1$ *or if* $xy<0$ *and* $x^2+y^2>1$; *where* $x,y\in[-1,1]$

$=\pi-\sin^{-1}\left\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\right\}$ if $0<x\le 1, 0<y\le 1$ and $x^2+y^2>1$

$=-\pi-\sin^{-1}\left\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\right\}$, *if* $-1\le x<0, 0<y\le 1$ *and* $x^2+y^2>1$

(ii) $\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}$, $xy>0$, $x^2+y^2>1$ or $x^2+y^2\le 1$

$=\pi-\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}$, $0<x\le 1$, $-1\le y\le 0$, $x^2+y^2>1$

$=-\pi-\sin^{-1}\left\{x\sqrt{1-y^2}-y\sqrt{1-x^2}\right\}$, $-1\le x<0$, $0<y\le 1$, $x^2+y^2>1$

Property -08

(i) $\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\right\}$, $-1\le x, y\le 1$, $x+y\ge 0$

$=2\pi-\cos^{-1}\left\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\right\}$, $-1\le x, y\le 1$, $x+y<0$

(ii) $\cos^{-1}x-\cos^{-1}y=\cos^{-1}\left\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\right\}$, $-1\le x, y\le 1$, $x\le y$

$=-\cos^{-1}\left\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\right\}$, $-1\le y\le 0$, $0<x\le 1$, $x>y$

Property -09

(i) $2\sin^{-1}x=\sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{-1}{\sqrt{2}}\le x\frac{1}{\sqrt{2}}$

$=\pi-\sin^{-1}\left(2x\sqrt{1-2^2}\right), \quad \frac{1}{\sqrt{2}}\le x\le 1.$

$=-\pi-\sin^{-1}\left(2x\sqrt{1-x^2}\right), -1\le x\le\frac{-1}{\sqrt{2}}$

(ii) $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right), -1/2\le x\le 1/2$

$=\pi-\sin^{-1}\left(3x-4x^3\right), \ 1/2<x\le 1$

$=-\pi-\text{san}^{-1}\left(3x-4x^3\right), -1\le x<-1/2$

Property -10

(i) $2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right), \quad 0\le x\le 1. =2\pi-\cos^{-1}\left(2x^2-1\right), -1\le x\le 0$

(ii) $3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right), \quad 1/2\le x\le 1$

$=2\pi-\cos^{-1}\left(4x^3-3x\right), -1/2\le x\le 1/2$

$=2\pi+\cos^{-1}\left(4x^3-3x\right), -1\le x\le -1/2$

Property -11

(i) $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right), \ -1<x<1$

$=\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right), \quad x>1$

$=-\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right), \quad x<-1$

(ii) $2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right), \ -1\le x\le 1$

$=\pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right), \ x>1$

$=-\pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right), \ x<-1$

(iv) $3\tan^{-1}x=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \ \frac{-1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

$=\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \ x>\frac{1}{\sqrt{3}}$

$=-\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \ x<\frac{-1}{\sqrt{3}}$

(iii) $2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \quad 0\le x<\infty$

$=-\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), -\infty<x\le 0$

Property -12

(i) $\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{x}{\sqrt{1-x^2}}=\cot^{-1}\frac{\sqrt{1-x^2}}{x}=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)=\text{cosec}^{-1}\left(\frac{1}{x}\right), x>0$

$\cos^{-1}x=\sin^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{\sqrt{1-x^2}}{x}=\cot^{-1}\frac{x}{\sqrt{1-x^2}}=\sec^{-1}\left(\frac{1}{x}\right)=\text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right), x>0$

(iii) $\tan^{-1}x=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=\cot^{-1}\left(\frac{1}{x}\right)=\sec^{-1}\left(\sqrt{1+x^2}\right)=\text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

Property -13

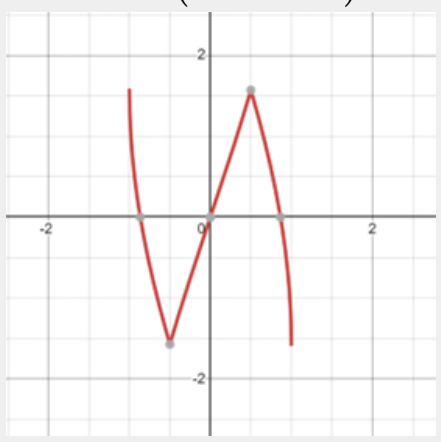
If $x_1, x_2, \ldots, x_n\in R$ then $\tan^{-1}x_1+\tan^{-1}x_2+\ldots+\tan^{-1}x_n=$

$$\tan^{-1}\left(\frac{s_1-s_3+s_5-s_7+\cdots}{1-s_2+s_4-s_6+\cdots}\right)$$

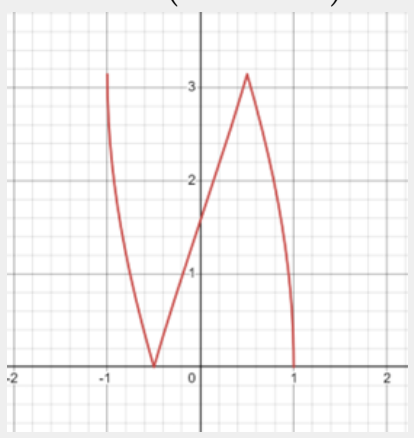
where, s_k = sum of products of $x_1, x_2, \ldots, k_n$ taken k at a time.

Some Important graphs

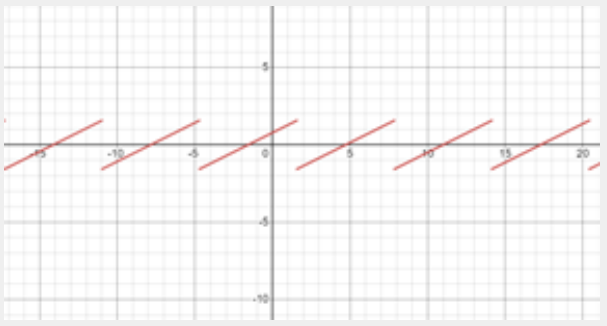
$\sin^{-1}\left(3x-4x^3\right)$



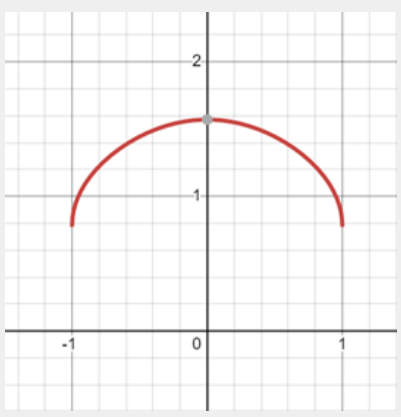
$\cos^{-1}\left(4x^3-3x\right)$



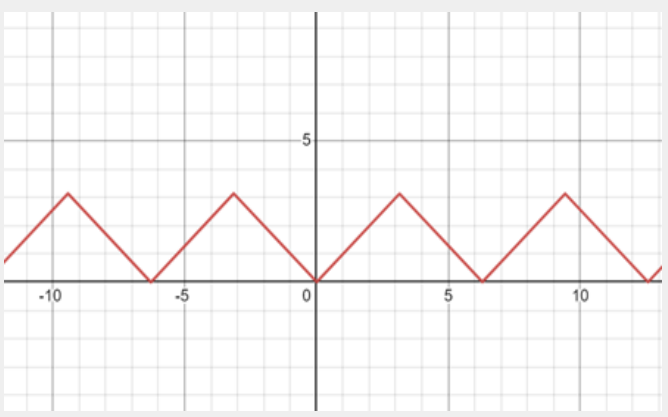
$\tan^{-1}\left(\sec x+\tan x\right)$



$\tan^{-1}\left(\frac{\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$



$\cos^{-1}\left(\cos x\right)$



$\sin^{-1}\left(\sin x\right)$