"IN MATHEMATICS
THE ART OF PROPOSING A QUESTION
MUST BE HELD OF HIGHER VALUE THAN SOLVING IT."
– Georg Cantor

FUNCTION

1 Classification of function

01. Constant function

$f(x) = k$, k is a constant.

02. Identity function

The function $y = f(x) = x$, $\forall x \in R$
Here domain & Range both R

03. Polynomial function

$y = f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n$, n is non negative integer, a_i are real constants. Given $a_0 \neq 0$, n is the degree of polynomial function

There are two polynomial functions, $f(x) = 1 + x^n$ & $f(x) = 1 - x^n$
satisfying the relation: $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$
where 'n' is a positive integer.

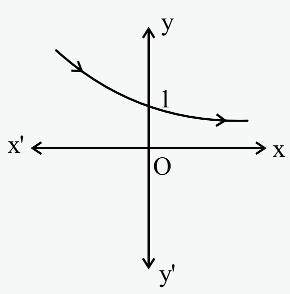
4. Rational functions

It is defined as the ratio of two polynomials.
$f(x) = \frac{P(x)}{Q(x)}$ provided $Q(x) \neq 0$
Dom $\{f(x)\}$ is all real numbers except when denominator is zero $[i, e, Q(x) \neq 0]$

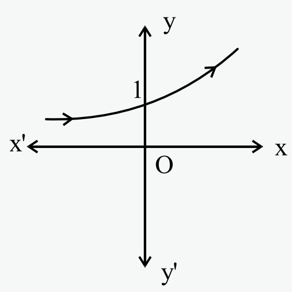
2 Exponential function

$f(x) = a^x, a > 0, a \neq 1.$

0<a<1



a > 1

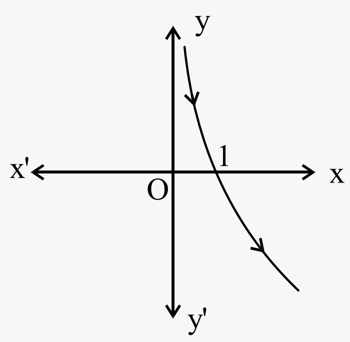


Domain =R, Range = (0, ∞)

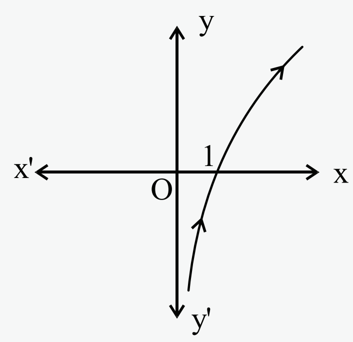
3 Logarithmic function

$f(x) = \log_a x \; [a > 0, a \neq 1]$

0 < a < 1



a > 1



Domain =(0, ∞), Range =R

Proprieties of Log. Functions

1. $\log_a(xy) = \log_a|x| + \log_a|y|$, where $a > 0, a \neq 1$ and $xy > 0$

2. $\log_a x = \frac{1}{\log_x a}$ for $a > 0, a \neq 1$ and $x > 0, x \neq 1$

3. $\log_a\left(\frac{x}{y}\right) = \log_a|x| - \log_a|y|$, where $a > 0, a \neq 1$ and $\frac{x}{y} > 0$

4. $\log_a\left(x^n\right) = n\log_a|x|$, where $a > 0, a \neq 1$ and $x^n > 0$

5. $\log_{a^n} x^m = \frac{m}{n}\log_{|a|}|x|$, where $a > 0, a \neq 1$ and $x > 0$

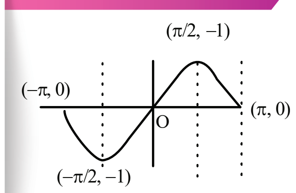
6. $x^{\log_a y} = y^{\log_a x}$ where $x > 0, y > 0, a > 0, a \neq 1$

7. If a>1, then the values of f(x)= $\log_a x$ increase with the increase in x.
i.e. $x < y \Leftrightarrow \log_a x < \log_a y$
Also, $\log_a x \begin{cases} < 0 & \text{for } 0 < x < 1 \\ = 0 & \text{for } x = 1 \\ > 0 & \text{for } x > 1. \end{cases}$

8. If , 0<a<1, then the values of f (x) = $\log_a x$ decrease with the increase in x .
i.e. $x < y \Leftrightarrow \log_a x > \log_a y$
Also, $\log_a x \begin{cases} > 0 & \text{for } 0 < x < 1 \\ = 0 & \text{for } x = 1 \\ < 0 & \text{for } x > 1 \end{cases}$

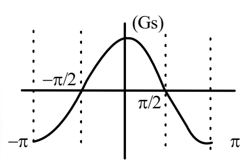
4 Trigonometric Functions

Sine function



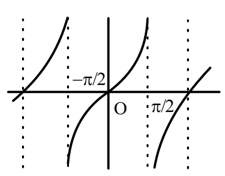
f(x) = sin x.
Dom (f) = R
Ran (f) = [–1, 1]

Cosine function



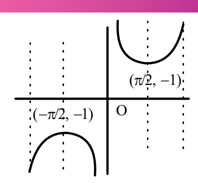
f(x) = cos x
Dom (f) = R
Ran (f) = [-1, 1].

Tangent function



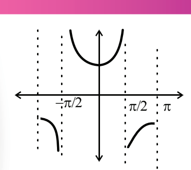
$f(x) = \tan x$
$\text{Dom}(f) = R - \left\{\frac{(2n+1)\pi}{2}, n \in z\right\}$
$\text{Ran}(f) = R$

Cosecant Function



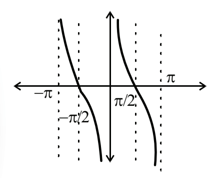
f(x) = cosec x
Dom (f) = R–{mR, n e z }
Ran (f) = R–(–1, 1)

Secant Function



f(x)=sec x
Dom (s) = R - $\{(2n+1)\frac{\pi}{2} | n \in Z\}$
Ran (f) = R - (-1, 1).

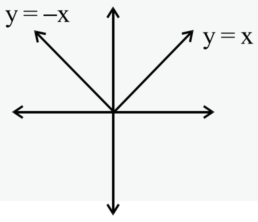
Cotangent function



f(x) = cot (x)
Dom (f) = R –$\{n\pi \mid n \in Z\}$
Ran (f) = R.

5 Absolute Value Function

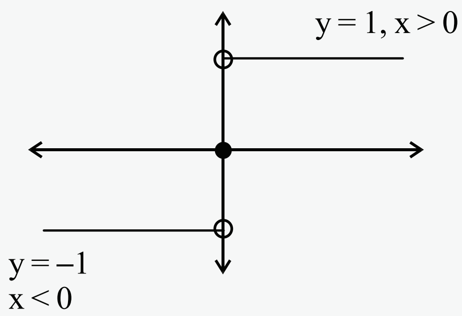
$$y = f(x) = |x| = \begin{cases} x, & x \geqslant 0 \\ -x, & x < 0 \end{cases}$$



1. $|x|^2 = x^2$
2. $\sqrt{x^2} = |x|$
3. $|x| = \max\{-x, x\}$
4. $-|x| = \min\{-x, x\}$
5. $\max(a,b) = \dfrac{a+b}{2} + \left|\dfrac{a-b}{2}\right|$
6. $\min(a,b) = \dfrac{a+b}{2} - \left|\dfrac{a-b}{2}\right|$
7. $|x+y| \leq |x| + |y|$
8. $|x+y| = |x| + |y|$ if $xy > 0$
9. $|x-y| = |x| + |y|$ if $xy \leqslant 0$
10. $|x| \geqslant a$ (is – ve) $x \in R$
11. $a \leqslant |x| \leqslant b \Rightarrow -b \leq x \leq -a$ or $a \leqslant x \leqslant b$. $x \in [-b, -a] \cup [a, b]$.

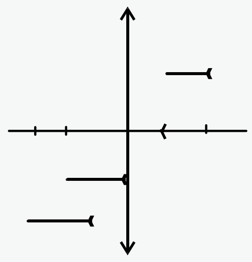
6 Signum Function

$$y = \operatorname{sgn}(x) = \begin{cases} \dfrac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$



7 Greatest Integer Function

f(x) = [x] the integral part of x, which is nearest & smaller integer



1. $[x] \leqslant x < [x] + 1$
2. $x - 1 < [x] < x$
3. $I \leq x < I + 1 \Rightarrow [x] = I$
4. $[x] - [-x] = \begin{cases} 2x & , x \in I \\ 2x + 1 & , 2 \notin I. \end{cases}$
5. $[x] + [-x] = \begin{cases} 0, & x \in I \\ -1, & x \in I, \end{cases} \Bigg| \begin{matrix} 2x, & x \in I \\ 2x+1, & , x \notin I \end{matrix}$
6. $[x] \leq n \Leftrightarrow x < n + 1, n \in I$
7. $[x] < n \Leftrightarrow x < n$
8. $[x] = \left[\dfrac{x}{2}\right] + \left[\dfrac{x+1}{2}\right]$
9. $\left[\dfrac{n+1}{2}\right] + \left[\dfrac{n+2}{4}\right] + \left[\dfrac{n+4}{8}\right] + \cdots = n$
10. $[x] + [y] \leqslant [x+y] \leqslant [x] + [y] + 1$
11. $[x] + \left[x + \dfrac{1}{n}\right] + \left[x + \dfrac{2}{n}\right] + \cdots + \left[x + \dfrac{n-1}{n}\right] = [nx]$

8 Fractional Part Function

: y={x} fractional past of x.

$y = \{x\} = x - [x]$

1. $\{x\} = x, 0 \leq x < 1.$
2. $\{x\} = 0, x \in I$
3. $\{-x\} = 1 - \{x\}, x \notin I$
4. $\{x \pm \text{integer}\} = \{x\}$

9 Odd and Even Function

1. if f(-x) =−f(x) ∀ x ∈R then f is an odd function, odd functions are symmetrical in opposite quadrants or about origin.

2. If f(-x) = f(x). then even. It is symmetric about y axis.

Properties

1. Product of two odd or two even function is an even function.
2. Product of odd & even function is an odd function.
3. Every function can be expressed as the sum of an even and odd function, i.e,

$$f(x)=\frac{f(x)+f(-x)}{2}+\frac{f(x)-f(-x)}{2}$$

4. Derivative of an odd function is an even function and of an even is odd.

10 Periodic function

f(x) is periodic if f(x + T) = f(x) $\forall$ x $\in$ R , T = period

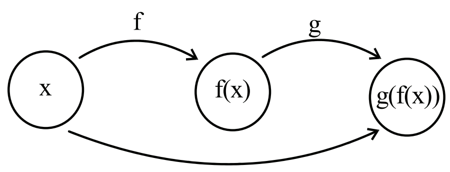
Functions	Period
$\sin^n x, \cos^n x, \sec^n x, \text{cosec}^n x$	π(n is even), 2π(n odd/ fraction)
$\tan^n x, \cot^n x$	π
|trig function|	π
$x-[x]$	1
$f(x)=$ constant	Periodic with no fundamental period.

Properties of Periodic functions

If f(x) is periodic with period T, then

1. c. f(x) is periodic with period T
2. f(x + c) is periodic with period T.
3. f(x) ± c is periodic with period T.
4. $kf(cx+d)$ has period $\frac{T}{|c|}$ period is only affected by coefficient of x.

11 Composition of Function



- h(x) = g f(x) = (gof) (x).
- gof ≠ fog.
- Composition of two bijection is a bijection.

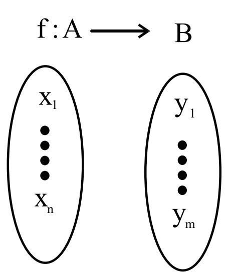
Properties of Composite Function

f	g	fog
even	even	even
odd	odd	odd
even	odd	ever
odd	even	even

12 Kinds of Mapping

1. **One-one/Injective/Homomorphic**: f(x) = f(y) ⇒ x=y, then one-one. Graphically, if no line parallel to x-axis meets the graph of function at more than one point.
2. **Onto/Surjective**: If range = co-domain. Method to show subjectivity: Finding the range of y = f(x) & Showing range of f = c o-domain of f
3. **Many-one mapping**: If two or more element in domain have same image in co-domain.
4. **Into Function**: There's an element in B not having a pre image in A under f. [f : A → B].

13 Number of functions



Total no of functions $= m^n$

Number of One to one functions $=\begin{cases} {}^mP_n, & m \ge n \\ 0, & m < n \end{cases}$

No. of many one functions $=\begin{cases} m^n - {}^mP_n, & m \geqslant n \\ m^n & , m < n \end{cases}$

No. of constant function $= m$

No. of onto function $=\begin{cases} \sum_{r=0}^{n} (-1)^r \, {}^mC_r \, (m-r)^n, & n \ge m \\ m^n & , n < m \end{cases}$

No. of one-to-one onto functions = n!, if m= n

14 Inverse of a function

g:B→A , f(x) = y ⇔ g(y) = x ∀ x ∈ A and y ∈ B.
Then g is inverse of f

1. Inverse of a bijection is unique.
2. If f : A → B is a bijection g : B →A is the inverse of f, then fog = I_B gof = I_A, where I_A & I_B are identity function
3. The inverse of a bijection is also a bijection $(gof)^{-1} = f^{-1} \circ g^{-1}$.

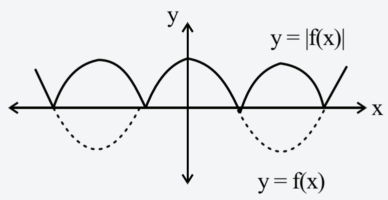
15 Hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

16 Elementary transformation of graphs

01

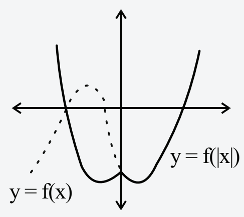
Drawing the graph of y = |f(x)| from the



$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0. \end{cases}$$

02

Drawing graph of y=f(|x|) from the known graph of y=f(x).

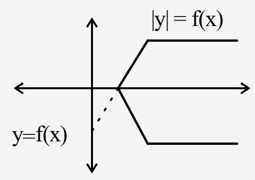


$$f(|x|) = \begin{cases} f(x), & x \geq 0 \\ f(-x), & x < 0 \end{cases}$$

Neglect the curve for x<0 & take the images of curve for x⩾0 about y axis.

03

Drawing graph of |y|=f(x) from the know m graph of y=f(x).



Remove portion that lies below x axis. Plot the remaining portion of the graph & also its mirror image in x-axis.

17 Things to remember

Range of $a\cos x + b\sin x$ is $\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

Range of $f(x) = \sqrt{a-x} + \sqrt{x-b}$ if $a > b > 0$ is $\sqrt{a-b} + \sqrt{2(a-b)}$

Range of $\left(x+\frac{1}{x}\right) + \frac{1}{\left(x+\frac{1}{x}\right)}$ is $(-\infty, -2.5] \cup [2.5, \infty)$

$-\sin 1 < \sin(\cos x) < \sin 1$

$\cos 1 < \cos(\sin x) < 1$

18 Functional Equation

1) $f(x+y) = f(x)f(y)$, then $f(x) = a^x$

2) $f(xy) = f(x) + f(y)$, then $f(x) = \log_a x$

3) $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$, then $f(x) = mx + c$

4) $f(x)f\left(\frac{1}{x}\right) = 1$, then $f(x) = \pm x^n$