A REASONABLE PROBABILITY IS THE ONLY CERTAINTY

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PROBABILITY

1 Probability

If an experiment results in a total of (m + n) outcomes which are equally likely and if 'm' outcomes are favourable to an event 'A' while 'n' are unfavorable, then the probability of occurrence of the event 'A' denoted by P(A), is defined by

$$P(A) = \frac{m}{m+n} = \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$$

2 Random Experiment

An Experiment is called random experiment if it satisfies the following two conditions:

1. It has more than one possible outcome.
2. It is not possible to predict the outcome in advance.

Outcome: A possible result of a random experiment is called its outcome.
Sample Space: Set of all possible outcomes of a random experiment is called sample space. It is denoted by symbol ' S '.

3 Algebra of Events

- Event A or B or $A \cup B = \{w: w \in A \text{ or } w \in B\}$
- Event A and B or $A \cap B = \{w: w \in A \text{ and } w \in B\}$
- Event A but not B or $A-B = A \cap B'$

4 Types of Events

1. Impossible and Sure Event: The empty set φ is called an Impossible event, where as the whole sample space ' S ' is called 'Sure event'.

2. Simple Event: If an event has only one sample point of a sample space, it is called a 'simple event'.

3. Compound Event: If an event has more than one sample point, it is called a 'compound event'.

4. Complementary Event: Complement event to A = 'not A'

5. Exhaustive Events: Many events that together form sample space are called exhaustive events.

6. Mutually Exclusive: Events A & B are called mutually exclusive events if occurrence of any one of them excludes occurrence of other event, i.e. they cannot occur simultaneously.

7. Mutually exclusive and exhaustive: The events which are not mutually exclusive are known as compatible events or mutually non exclusive events. Mutually exclusive and exhaustive system of events: Let S be the sample space associated with a random experiment. Let $A_1, A_2, \ldots$, An be subsets of S such that
(i) $A_i \cap A_j = \varphi$ for $i \neq j$ and
(ii) $A_1 \cup A_2 \cup \ldots \cup A_n = S$
Then, the collection of events $A_1, A_2, \ldots$, An is said to form a mutually exclusive and exhaustive system of events.

8. Independent Events

(i) If E&F are independent, then

$(P(E \cap F) = P(F)P(E/F) = P(E), P(F) \neq 0 P(F/E) = P(F), P(E) \neq 0)$

(ii) Three events A,B&C are said to be mutually independent, if

$(P(A \cap B) = P(A)P(B), P(A \cap C) = P(A)P(C), P(B \cap C) = P(B)P(C)$ & $P(A \cap B \cap C) = P(A)P(B)P(C))$

If at least one of the above is not true for three given events, we say that the events are not independent.

5 Probability of AUB, A ∩ B and P (notA)

If A and B are any two events, then

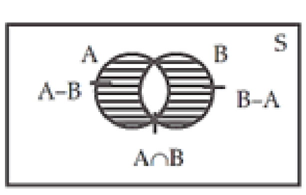
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(AUB)$$

If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

Probability of the event 'not A'

$$P(A') = P(\text{not } A) = 1 - P(A)$$



6 Conditional Probability

If E and F are two events associated with the sample space of a random experiment, the conditional probability of the event given that it has occured is given as:

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{n(E \cap F)}{n(F)}, P(F) \neq 0$$

Properties of Conditional Probability

1. Let E & F be events of sample space of an experiment, then we have

$$P(S/F) = P(F/F) = 1$$

2. If A and B are any two events of a sample space S & F is an event of such that

$P(F) \neq 0$, then $P((A \cup B)/F) = P(A/F) + P(B/F) - P((A \cap B)/F)$

In particular if A and B are disjoint events, then

$$P((A \cup B)/F) = P(A/F) + P(B/F)$$

3. $P(E/F') = 1 - P(E/F)$

7 Multiplication Theorem On Probability

For two events E & F associated with a sample space S, we have

$(P(E \cap F) = P(E)\ P(F/E) = P(F)P(E/F))$

provided $P(E) \neq 0$ & $P(F) \neq 0$

The above result is known as Multiplication Rule of Probability.

8 Total Probability Theorem

If an even A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots$, Bn and the probabilities $P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known, then

$$P(A) = \sum_{i=1}^{n} P(B_i)P(\mathbf{A}|B_i)$$

9 Baye's Theorem

Partition of a Sample Space

A set of events $E_1, E_2, \ldots, E_n$ is said to represent a partition of the sample space S if

(a) $E_i \cap E_j = \phi, i \neq j, i, j = 1, 2, 3, \ldots n$

(b) $E_1 \cup E_2 \cup \ldots \cup E_n = S$

(c) $P(E_i) > 0$ for all $i = 1, 2, \ldots n$

Theorem of Total Probability. Let $\{E_1, E_2, \ldots .E_n\}$ be a partition of the sample space S and suppose that each of the events $E_1, E_2, \ldots ., E_n$ has nonzero probability of occurence. Let A be any event associated with S then

$$P(A) = \sum_{j=1}^{n} P(E_j)P(A/E_j)$$

Baye's Theorem: If $E_1, E_2, \ldots, E_n$ are non-empty events which constitute a partition of sample space S & A is any event of non-zero probability.

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)} \quad \text{for any } i = 1, 2, 3, \ldots, n$$

10 Random Variable & Its Probability Distributions

A random variable is a real valued function whose domain is the sample space of a random experiment. The probability distribution of a random variable

$$X: x_1 \quad x_2 \quad \cdots \quad x_n$$
$$P(X): \quad p_1 \quad p_2 \quad \cdots \quad p_n$$

where, $p_i > 0, \sum_{i=1}^{n} p_i = 1, i = 1, 2, \ldots, n$

The real numbers $x_1, x_2, \ldots, x_n$ are the possible values of the random variable X and $p_i (i = 1, 2, \ldots, n)$ is the probability of the random variable i.e.,

$$P(X = x_i) = p_i$$

11 Mean Of A Random Variable

The mean (μ) of a random variable X is also called the expectation of X denoted by $E(X)$

$$E(X) = \mu = \sum_{i=1}^{n} x_i p_i$$

Here $x_1, x_2, \ldots, x_n$ are possible values of random variable X, occuring with probabilities $p_1, p_2, \ldots, p_n$ respectively.

12 Variance Of Random Variable

Let X be a random variable whose possible values $x_1, x_2, \ldots, x_n$ occur with probabilities $p(x_1), p(x_2) \ldots, p(x_n)$ respectively. Also let $\mu = E(X)$ be the mean of X then the variance of X is given as:

$$\text{Var}(X) \text{ or } \sigma_X^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i) = E(X-\mu)^2 = E(X^2) - [E(X)]^2$$

The non-negative number $\sigma_X = \sqrt{\text{Var}(X)}$ is called the Standard Deviation of random variable X

13 Bernoulli Trials & Binomial Distribution

Bernoulli Trials :
Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:
(i) There should be a finite number of trials.
(ii) The trials should be independent.
(iii) Each trial has exactly two outcomes: success or failure.
(iv) The probability of success remains same in each trial.

Binomial Distribution :
The probability distribution of number of successes in an experiment consisting of n Bernoulli trials may be obtained by the binomial expansion $(q + p)^n$ where p is probability of success in each trial and p + q = 1 . Hence, this distribution (also called Binomial distribution B(n, p) of number of successes X can be written as:

X	0	1	2	---	x	n
P(x)	${}^nC_0q^n$	${}^nC_1q^{n-1}p^1$	${}^nC_2q^{n-2}p^2$		${}^nC_xq^{n-x}p^x$	${}^nC_np^n$

The probability of X successes $P(X = x)$ is also denoted by $P(x)$ is given as:

$$P(x) = {}^nC_x q^{n-x} p^x, x = 0, 1, \quad \cdots, n \quad (q = 1 - p)$$

This $P(x)$ is called the probability function of the binomial distribution.