Assignment 3: Question 2

March 22, 2015

A Lambertian object illuminated by a point source has a reflectance map of the form given by

$$R(p,q) = \frac{1 + pp_s + qq_s}{\sqrt{1+p_s^2+q_s^2}\sqrt{1+p^2+q^2}} \tag{1}$$

where the surface normal is $(-p, -q, 1)$ and the light source direction is $(-p_s, -q_s, 1)$. What value(s) of (p, q) will maximize $R(p, q)$? For what values, will you get $R(p, q) = 0$? [2 points]

Answer:

$$R(p,q) = \frac{1 + pp_s + qq_s}{\sqrt{1+p_s^2+q_s^2}\sqrt{1+p^2+q^2}} \tag{2}$$

where the surface normal is $(-p, -q, 1)$ and the light source direction is $(-p_s, -q_s, 1)$.

Then,
$R(p, q) = cos(\theta)$ where θ is the angle between surface normal and the light source direction.
Maximum value of $R(p, q)$ occurs when $cos(\theta) = 1$, that is, $(-p, -q, 1)$ and $(-p_s, -q_s, 1)$ are parallel.
Therefore, $p = p_s$ and $q = q_s$.

$R(p, q) = 0$ implies $\theta = 90°$.
$1 + pp_s + qq_s = 0$
$p = -(1 + qq_s)/p_s$
Therefore, $R(p, q) = 0$ for any pair $(-(1 + qq_s)/p_s, q)$ for any q.