Assignment:4 Question 2

1 Reason to do normalization

When we are solving for the essential matrix we are trying to find a matrix f such that $Af = 0$ where A is the matrix as described in the slides. As per the homogeneous representation of the points

$$p = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{1}$$

Both x and y will lie in range of the size of the image where as the last coordinate is constant. Further if the features points are found in a very small region of the image the first principal singular will be large as all the p's are more of less in the same direction affecting the remaining singular values which will be small.

2 Normalization expressed as Matrices

$$p_l = \begin{pmatrix} x_l \\ y_l \\ 1 \end{pmatrix} \tag{2}$$

$$p_r = \begin{pmatrix} x_r \\ y_r \\ 1 \end{pmatrix} \tag{3}$$

Let T_l and T_r be the normalization matrices for the left and right set of points.

$$\bar{p_l} = T_l \begin{pmatrix} x_l \\ y_l \\ 1 \end{pmatrix} \tag{4}$$

$$\bar{p_r} = T_r \begin{pmatrix} x_r \\ y_r \\ 1 \end{pmatrix} \tag{5}$$

Let the essential matrix for initial points be F and for normalized points be $\bar{F}$. Since

$$p_l' F p_r = 0 \tag{6}$$

$$\bar{p_l}'((T_l)')^{-1} F (T_r)^{-1} \bar{p_r} = 0 \tag{7}$$

$$\bar{p_l}' \bar{F} \bar{p_r} = 0 \tag{8}$$

$$\bar{F} = (T_l')^{-1} F (T_r')^{-1} \tag{9}$$

$$(T_l') \bar{F} (T_r') = F \tag{10}$$

Structure for Translation & Scaling matrices

$$T_l = \begin{bmatrix} \frac{1}{\sigma_l} & 0 & \bar{x_l} \\ 0 & \frac{1}{\sigma_l} & \bar{y_l} \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$T_r = \begin{bmatrix} \frac{1}{\sigma_r} & 0 & \bar{x_r} \\ 0 & \frac{1}{\sigma_r} & \bar{y_r} \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Similarly we can write the formula for T_r and $\sigma_{l,x}$ represents the variance for the x co-ordinates for the points in the left image. $\bar{x_l}$ represents the mean for the x coordinates of the for the x co-ordinates for the points in the left image. Similar notation for other symbols too.

$$\bar{x_l} = \frac{\sum_{i=1}^{N} x_{l,i}}{N} \tag{13}$$

$$\bar{y_l} = \frac{\sum_{i=1}^{N} y_{l,i}}{N} \tag{14}$$

$$\bar{x_r} = \frac{\sum_{i=1}^{N} x_{r,i}}{N} \tag{15}$$

$$\bar{y_r} = \frac{\sum_{i=1}^{N} y_{r,i}}{N} \tag{16}$$

$$\sigma_l = \frac{\sqrt{\sum_{i=1}^{N} (x_{l,i} - x_l)^2 + (\bar{y}_{l,i} - \bar{y}_l)^2}}{N} \tag{17}$$

$$\sigma_r = \frac{\sqrt{\sum_{i=1}^{N} (x_{r,i} - x_r)^2 + (\bar{y}_{r,i} - \bar{y}_r)^2}}{N} \tag{18}$$