Assignment 1: Question 3

Due: 5th February before 11:59 pm

Question: Prove that the vanishing points of three coplanar lines are collinear. [2 points]

Answer:

Case 1: No two lines are parallel to each other.

Let two of the lines be $L_1 = (l_{1x}, l_{1y}, l_{1z})$ and $L_2 = (l_{2x}, l_{2y}, l_{2z})$.
The third line can be represented as a linear combination of L_1 and L_2.
Therefore,
$L3 = t_1 L_1 + t_2 L_2 = (t_1 l_{1x} + t_2 l_{2x}, t_1 l_{1y} + t_2 l_{2y}, t_1 l_{1z} + t_2 l_{2z})$

Now, the vanishing point of L_1 is $V_{L1}(f\frac{l_{1x}}{l_{1z}}, f\frac{l_{1y}}{l_{1z}})$ and the vanishing point of L_2 is $V_{L2}(f\frac{l_{2x}}{l_{2z}}, f\frac{l_{2y}}{l_{2z}})$.
And the vanishing point of L_3 is,

$$\begin{aligned} V_{L3} &= (f\frac{t_1 l_{1x} + t_2 l_{2x}}{t_1 l_{1z} + t_2 l_{2z}}, f\frac{t_1 l_{1y} + t_2 l_{2y}}{t_1 l_{1z} + t_2 l_{2z}}) \\ &= \frac{t_1 l_{1z}}{t_1 l_{1z} + t_2 l_{2z}} V_{L1} + \frac{t_2 l_{2z}}{t_1 l_{1z} + t_2 l_{2z}} V_{L2} \end{aligned}$$

Therefore, V_{L3} lies on the line passing through V_{L1} and V_{L2}.

Case 2: Two lines are parallel and one line intersecting them.

Two parallel lines will have same vanishing point and the third line will have a different vanishing point.
Only two distinct vanishing points and two points are always collinear.

Case 3: All three lines are parallel.

Only one distinct vanishing point.