

Assignment 3: Question 1

March 22, 2015

Consider a surface with Lambertian reflectance map, known geometry (example: sphere) and unknown but constant albedo. Given an image of such a surface taken with a point light source of unknown power, show how you will determine the lighting direction. Assume there are no shadows. Write down all necessary equations. [3 points]

Answer:

For a Lambertian surface we have,

$$I(x, y) = L\rho\mathbf{N}^t(x, y)\mathbf{d}$$

where \mathbf{d} is a lighting direction (unknown), L is the power of the light source (unknown), ρ is the albedo (unknown) and $\mathbf{N}(x, y)$ is the unit surface normal vector at pixel (x, y) (known).

Let $\tilde{\mathbf{d}} = L\rho\mathbf{d}$.

Therefore, $\mathbf{I} = \mathbf{N}^t\tilde{\mathbf{d}}$.

Using pseudo-inverse,

$$\tilde{\mathbf{d}} = (\mathbf{N}\mathbf{N}^t)^{-1}\mathbf{N}\mathbf{I}$$

$$L\rho\mathbf{d} = \frac{(\mathbf{N}\mathbf{N}^t)^{-1}\mathbf{N}\mathbf{I}}{\sqrt{\tilde{\mathbf{d}}_1^2 + \tilde{\mathbf{d}}_2^2 + \tilde{\mathbf{d}}_3^2}}$$

(Since, \mathbf{d} is a unit vector).

Therefore, $\mathbf{d} = ((\mathbf{N}\mathbf{N}^t)^{-1}\mathbf{N}\mathbf{I})/L\rho$.