

Assignment 1: Question 4

Question: Consider a triangle (in 3D) whose side lengths are known to you. You capture an image of this triangle and mark out the positions of its three vertices in the image (assume all vertices were visible when you took the picture). Suppose, that you knew the 3D coordinates of exactly one vertex of the triangle, in the camera coordinate system. Explain how you will determine the 3D coordinates of the other two vertices, and write down the key equations (I do not expect you to solve the equations). Assume the pixel resolution of your camera to be 1 in both directions and the optical center to be $(0, 0)$. Do not assume that you knew the focal length. [4 points]

Answer:

Let the known point be $P_0 = (X_0, Y_0, Z_0)$ and its coordinates in the image co-ordinate system be $P_{img,0} = (X_{img,0}, Y_{img,0})$.

Therefore,

$$f = \frac{X_{img,0}Z_0}{X_0}$$

Now, let the other co-ordinates be $P_1 = (X_1, Y_1, Z_1)$ and $P_2 = (X_2, Y_2, Z_2)$, their images $P_{img,1} = (X_{img,1}, Y_{img,1})$ and $P_{img,2} = (X_{img,2}, Y_{img,2})$.

We have,

$$\begin{aligned} X_1 &= \frac{X_{img,1}Z_1}{f} \\ Y_1 &= \frac{Y_{img,1}Z_1}{f} \\ X_2 &= \frac{X_{img,2}Z_2}{f} \\ Y_2 &= \frac{Y_{img,2}Z_2}{f} \end{aligned}$$

$$\begin{aligned} (X_1 - X_0)^2 + (Y_1 - Y_0)^2 + (Z_1 - Z_0)^2 &= \ell(P_1P_0)^2 \\ (X_2 - X_0)^2 + (Y_2 - Y_0)^2 + (Z_2 - Z_0)^2 &= \ell(P_2P_0)^2 \end{aligned}$$

Six equations and six unknowns. Unique solution exists.