Assignment 1: Question 1

Question: In class, we have seen image formation on a flat screen (i.e. image plane) with a pinhole camera. Now suppose the screen was wrapped on the surface of a sphere and hence, the 3D points were projected onto a spherical surface. Derive a relationship between the coordinates of a 3D point $P = (X, Y, Z)$ and its image on such a screen (both in camera coordinate system). If you had to calibrate this sort of a system, what are the additional intrinsic parameters of the camera as compared to the case of an image plane ? [4 points]

Answer:

Let $C = (X_0, Y_0, Z_0)$ and R be the center and radius of the spherical surface S respectively.

Using parametric form of line, the image of point P on S is given by $P_{img} = (tX, tY, tZ)$.

P_{img} lies on S.

$$\begin{aligned} R^2 &= (tX - X_0)^2 + (tY - Y_0)^2 + (tZ - Z_0)^2 \\ 0 &= t^2(X^2 + Y^2 + Z^2) - 2t(XX_0 + YY_0 + ZZ_0) + X_0^2 + Y_0^2 + Z_0^2 - R^2 \\ t &= \frac{2(XX_0 + YY_0 + ZZ_0) \pm \sqrt{4(XX_0 + YY_0 + ZZ_0)^2 - 4(X^2 + Y^2 + Z^2)(X_0^2 + Y_0^2 + Z_0^2 - R^2)}}{2(X^2 + Y^2 + Z^2)(X_0^2 + Y_0^2 + Z_0^2 - R^2)} \end{aligned}$$

Since P_{img} and P are on opposite side of pinhole (origin), t should be negative.

Therefore,

$$t = \frac{(XX_0 + YY_0 + ZZ_0) - \sqrt{(XX_0 + YY_0 + ZZ_0)^2 - (X^2 + Y^2 + Z^2)(X_0^2 + Y_0^2 + Z_0^2 - R^2)}}{(X^2 + Y^2 + Z^2)(X_0^2 + Y_0^2 + Z_0^2 - R^2)}$$

Instrinsic Parameters:

Center of the spherical surface ($C = (X_0, Y_0, Z_0)$)

Radius of the sphere (R)

Pixel Aspect Ratios (s_θ, s_ϕ in spherical co-ordinates)