Week 3-4 Report

Check Point 2

What kind of pattern did you obtain in the observed points? Can you relate this with the above data? What can be the uses of generative models according to you?

- With increasing X, Y observed is also increasing.
- Same pattern observed as given in the document, linear regression will fit the same line to this data also.
- To evaluate models with the realistic data by generating them with generative models.

Check Point 3

MLE

The core idea of MLE is to choose the parameters (denoted as θ) that maximize the likelihood of the observed data.
Given: n independent and identically distributed (IID) samples $X_1, X_2, \ldots, X_n$,
Assumption: data follow either the same PMF if they are discrete, or the same PDF if they are continuous. This shared distribution is denoted by $f(X|\theta)$.

$$\text{Likelihood Function: } L(\theta) = \prod_{i=1}^{n} f(X_i|\theta)$$

$$\text{Maximization: } \hat{\theta} = \arg\max_{\theta} L(\theta)$$

Since logarithm is a monotonic function we can maximize the log likelihood function and then compute the first derivative of the log-likelihood function and get our θ (MLE).

Check Point 4

Problem Description

Brain MRI images often suffer from intensity inhomogeneities due to the spatial distribution imperfections of the b1 magnetic field, resulting in a smoothly varying bias field across the image. Additionally, vignetting, a reduction of brightness or saturation at the image periphery, further complicates accurate tissue segmentation.

Model Assumptions

- **MRI Image and Voxel Intensities:**
  - The MRI image consists of N voxels.
  - At voxel i, the true (uncorrupted) intensity is x_i (unknown).
  - The bias field at voxel i is b_i (unknown) and varies smoothly over the spatial domain.
- **Noise Model:**
  - The observed intensity at voxel i is y_i.
  - The noise η_i is modeled as i.i.d. additive zero-mean Gaussian noise: $\eta_i \sim \mathcal{N}(0, \sigma^2)$.
- **Observation Model:**

$$y_i = x_i b_i + \eta_i$$

- **Class Assumptions:**
  - The image is composed of K distinct tissue classes (K is known).
  - Each class k has a constant uncorrupted intensity c_k.
- **Spatial Smoothness:**
  - For a voxel i, if its neighbor j belongs to the same class k:

$$x_j = c_k \quad \text{and} \quad b_j \approx b_i \implies x_j b_j \approx c_k b_i$$