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PHL 741 QFT&QED Major exam answer sheet
Qn no 6 To quantize or not to quantize

Let us understand that quantizing gravity is a fundamentally possible task. In the low-energy limit, there can exist a tree-level analysis of gravitons under general relativity. But the problem arises when you try to normalize it using field-theory methods. When we take a Feynman diagram with graviton exchanges in loops, we find that your answer goes to infinity, and cannot be resolved in a finite number of steps. What this means is that, despite offering a more unified counterpart to the standard model, when accounting for quantum effects the theory has no predictive power. Hence, it might as well be useless for computation at practical energy levels.
In other words, quantizing gravity beyond the Planck scale has definitive, hugely monumental implications for a unified theory, but atleast with simply general relativity it looks to be impossible.
Perhaps one particular theory of quantized gravity will atleast mature enough to make preliminary predictions.
Different challenges are involved in this: renormalization, as mentioned earlier, taming any singularities in GR, a solution to the measurement problem, a resolved description of time as a vector with more than one direction, and the unification of fundamental forces.

5) (Once upon a time) the Young BRST was mapping out their plans for the future. They knew that they were very confused - they had a strong sense of self identity but ~~so~~ they also knew this: no matter what aspect of their personality was chosen, each variable gave infinitely many paths to follow.

"I know not what I want to be," said BRST "but I know what I'm not. I'm not Standard. This allows me to see the world in greater detail, but I am not able to pick what is right and wrong. I have a set of assumptions, but are they real or ~~so~~ have I made them up just to fit into the idea of the world that I have?"

BRST's hugely supportive teacher was Ms. Yang-Mills. They had a very close bond. ~~Together~~ With their help, BRST was able to tackle ~~the~~ personal struggles, ~~s~~ with quantum chromodynamics and electroweak theory: both of which needed BRST to build fundamental assumptions in isolated states, then see if those assumptions held up in the bigger picture: in other words, they took a normal situation, reduced it to base states and then see if they still worked upon renormalizing

But BRST's current problems was inward: their past was Lagrangian, but their future had to be Hamiltonian. However, to do so, they must look inwards, and pray they find an answer. They need to fix individual parameters ~~in~~ ~~eqn~~ for ALL their possible paths, and then find a way to associate with their relevance to the real (physical) world.

This inward prayer of BRST granted their wish for help. A ghost appeared. "Hello BRST," said the ghost, "my name is Faddeev-Popov".
BRST wasn't perturbed (but only because BRST was very perturbation-friendly). They said "If you are a ghost, how can you be helpful?"

"Oh I am a good ghost," chortled Faddeev-Popov. "I will help you regularise your options. Of course, I am not real in a physical sense, but that does not mean I can't be helpful. Dumbledore once said just because it is all happening in your head doesnt mean it isn't real."
BRST said, "are there bad ghosts?" and Faddeev-Poppov nodded. "Yes. The Pauli-Villars ghost. But that doesn't mean they are evil. I am just additively meaningful in a situation, while they must be exorcised to provide physical analogies.
Now, BRST, let us look at your problem. I see that you have mapped out all your possible paths. But then your assumptions are all over the place. You are therefore overwhelmed. I want you to stop looking at the big picture altogether. This puts you in your very own "bubble chamber". Instead, please focus on each individual idea, and how they contribute to your final goal. I call this method of mapping it the "Feynman diagram". Now you try it".
So BRST did that. They took each individual assumption and tried their usual vetting process for that "interaction" – they slightly perturbed their base states/assumptions, and kept only the ones which held up. This "Descartesian" approach yielded

a brand new set of ideas that could be fully stable in the field they were working in

"So you have taught yourself how to fix your ~~thought~~ mapping and thus you have taught yourself how to find symmetry and cohomology within yourself", Faddeev-Popov beamed proudly.

"But you have helped me here. Is it truly consistent if I can do it only with the help of a ghost?"

~~"Yes, BRST. Your thoughts have crests & troughs, physical & aspects that give symmetry & stability but also ones that take it away bat's sc~~

"Well, BRST, when you are working under a gauge/particular worldview, you may decide SOME assumptions are like bosons & the others are like fermions. Now if you fix the fermionic variables with their ghosts, which will now help you as they become bosonic. To return to the symmetry of the old assumptions, just use a c-ghost!"

"Okay. Thank you ghost! But this has really thrown me for a loop about what's real & what isn't. What if my worldview isn't all there is? What if I pick the wrong gauge-slice?"

"Oh you almost certainly know it isn't all there is. ~~By~~ There are other global symmetries. BUT what matters is that you did not pick WRONG. It's symmetrical to YOU, ~~and~~ under BRST. And I think that's that matters!"

1) Let the final state be a normalized plane wave $\frac{1}{L^{3/2}} e^{i\vec{k}\cdot\vec{r}}$

$$\sigma_{n\to i}(\omega) = \frac{4\pi^2\alpha}{m^2\omega} |\langle i | e^{i\vec{k}_{ph}\cdot\vec{r}_n} \hat{e}\cdot\hat{p} | n\rangle|^2 \delta(\omega - \omega_{in})$$

where $k_{ph} = \frac{\omega}{c}\hat{n}$, $\omega_{in} = \frac{E_i - E_n}{\hbar}$

Assuming the energy of photon $\hbar\omega \gg (E_{1s})$ Ionisation potential

$$\langle \vec{r} | i\rangle = \frac{1}{L^{3/2}} e^{i\vec{k}\cdot\vec{r}}$$ $\Rightarrow e^-$ is a free particle

we know $E_i = \frac{\hbar^2 k^2}{2m} = \hbar\omega + E_{1s}$

Summation over k values

$$d\sigma_n(\omega) = \int_{\Delta\omega} \sigma_{n\to i}(\omega)\, d\omega = \frac{4\pi^2\alpha}{m^2\omega} |\langle i | e^{i\vec{k}\cdot\vec{r}_n} \hat{e}\cdot p | n\rangle|^2 \rho(\omega)$$

$k = k_{photon}$ $\rho(\omega) = \rho(\omega_{k i s})$

$\Delta\omega$ is centered at $E_i - E_{1s}$. $\rho(E)$ is the density of a free e^-

$$\rho(E_k) = \frac{d\Omega}{4\pi} \frac{m^{3/2}\sqrt{E_k}\, L^3}{\sqrt{2}\,\pi^2 \hbar^2} \; ; \; E_k = \frac{\hbar^2 k^2}{2m}$$

where $d\Omega$ is the differential solid angle for emission in the differential crossection $d\sigma_n(\omega)$

~~where~~ we know for charge Z

$$\langle \bar{r} | 1s \rangle = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a}\right)^{3/2} e^{-Zr/a}$$

a: Bohr radius

$$\Rightarrow \frac{d\sigma_{1s}(\omega)}{d\Omega} = \frac{e^2 k Z^3}{2mc\hbar^2 \pi^2 \omega a^3} \left| \int e^{-i\bar{k}\cdot\bar{r} + i\bar{k}_{ph}\cdot\bar{r}}\, \hat{e}\cdot \frac{\hbar}{i}\nabla e^{-Zr/a} d^3r \right|^2$$

$$\int e^{-i\bar{k}\cdot\bar{r} + i\bar{k}_{ph}\bar{r}}\, \hat{e}\cdot\frac{\hbar}{i}\nabla e^{-Zr/a} d^3r =$$

$$= \hat{e}\cdot\bar{k}\left(\frac{8\pi Z\hbar}{a}\right)\left(\frac{Z^2}{a^2} + q^2\right)^{-2}$$

$$\frac{d\sigma_{1s}(\omega)}{d\Omega} = \frac{32 e^2 k (\hat{e}\cdot\bar{k})^2}{mc\omega} \frac{Z^5}{a^5} \left(\frac{Z^2}{a^2} + q^2\right)^{-4}$$

$$= \frac{32 e^2 k (\hat{e}\cdot\bar{k})^2}{mc\omega} \frac{Z^5}{a^5} \left(\frac{Z^2}{a^2} + \left(\bar{k} - \frac{\omega}{c}\hat{n}\right)^2\right)^{-4}$$