Options, Futures & Swaps

A Quantitative Approach to Derivatives

Finance & Economics Series • Topic 2

Tailored for Quantitative Trading

Contents

1. Derivatives Fundamentals

A **derivative** is a financial contract whose value is derived from an underlying asset, index, or reference rate. For quant traders, derivatives are essential tools for expressing views on price direction, volatility, correlation, and other market factors with precise risk-reward profiles.

Quant Perspective

Derivatives allow you to isolate and trade specific risk factors. Want to bet on volatility without directional exposure? Use a straddle. Think correlations will spike? Trade dispersion. Believe the yield curve will flatten? Use swap spreads. This granular risk decomposition is what makes derivatives indispensable for systematic trading.

1.1 Key Terminology

Term	Definition	Notation
Underlying	Asset from which derivative derives value	S, S(t)
Strike Price	Pre-agreed price for exercise	K
Expiration	Date contract expires	T
Time to Maturity	Time remaining until expiration	$\tau = T - t$
Spot Price	Current market price of underlying	S■ or S(0)
Forward Price	Agreed future delivery price	F(t,T)
Risk-Free Rate	Return on riskless investment	r
Volatility	Standard deviation of returns	σ
Dividend Yield	Continuous dividend rate	q

2. Forward Contracts

A **forward contract** is an agreement to buy or sell an asset at a specified future date for a price agreed upon today. Forwards are traded over-the-counter (OTC), allowing customization but introducing counterparty risk.

2.1 Forward Pricing

The forward price is determined by no-arbitrage arguments. For an asset with no income:

$$F(t,T) = S(t) \times e^{(r\times\tau)}$$

For an asset paying continuous dividend yield q:

$$F(t,T) = S(t) \times e^{((r-q)\times\tau)}$$

No-Arbitrage Derivation

The forward price must equal the cost of carrying the underlying asset until maturity. If $F > S\times e^{(r\tau)}$, arbitrageurs would short the forward, buy the spot, and finance at r—locking in profit. If $F < S\times e^{(r\tau)}$, they'd do the reverse. Competition eliminates these opportunities.

2.2 Value of a Forward Contract

At inception, a forward has zero value. As spot prices move, the contract gains or loses value:

$$V(t) = (F(t,T) - K) \times e^{(-r\times(T-t))} = S(t)\times e^{(-q\times\tau)} - K\times e^{(-r\times\tau)}$$

where K is the originally agreed forward price.

2.3 Forward Rate Agreements (FRAs)

FRAs are forwards on interest rates. A "3x6 FRA" locks in the 3-month rate starting in 3 months. The payoff at settlement (time T■) is:

$$\text{Payoff} = N \times (R - K) \times \alpha / (1 + R \times \alpha)$$

where N is notional, R is the realized rate, K is the FRA rate, and α is the day count fraction.

Quant Application: Yield Curve Trading

FRAs and their exchange-traded equivalents (Eurodollar futures, SOFR futures) are fundamental for expressing views on interest rate movements and yield curve shape. Quant strategies often involve curve trades: steepeners, flatteners, and butterfly spreads across different maturities.

3. Futures Contracts

Futures are standardized forward contracts traded on exchanges. The key innovation is daily settlement (marking-to-market), which eliminates counterparty risk through the clearing house mechanism.

3.1 Futures vs Forwards

Feature	Forwards	Futures
Trading Venue	OTC (bilateral)	Exchange
Standardization	Customizable	Standardized contracts
Counterparty Risk	Direct exposure	Clearinghouse guarantee
Settlement	At maturity	Daily mark-to-market
Liquidity	Variable	Generally high
Margin	Negotiated	Required (initial + variation)
Price Transparency	Limited	Public quotes

3.2 The Margin System

Initial Margin: Deposit required to open a position (typically 5-15% of contract value). This is a performance bond, not a down payment.

Maintenance Margin: Minimum account balance before margin call is triggered (typically 75% of initial margin).

Variation Margin: Daily cash flows based on mark-to-market. Gains are credited; losses are debited. This is what makes futures "daily settled" contracts.

Implementation Note

Daily settlement creates subtle differences between futures and forward prices when interest rates are correlated with the underlying. For most equity and FX contracts, this "convexity adjustment" is negligible. For interest rate futures (Eurodollar, SOFR), it matters and creates trading opportunities.

3.3 Basis and Convergence

The **basis** is the difference between spot and futures prices:

$$\text{Basis} = S(t) - F(t,T)$$

As expiration approaches, the basis converges to zero (or to the cost of delivery for physical commodities). Basis risk—uncertainty in how basis evolves—is a key concern in hedging.

3.4 Key Futures Markets

Market	Major Contracts	Quant Applications
Equity Index	ES (S&P 500), NQ (Nasdaq)	Beta hedging, stat arb
Interest Rate	ZN (10Y Note), SR3 (SOFR)	Duration, curve trades
FX	EUR/USD (6E), JPY/USD (6J)	Carry trades, macro
Commodities	CL (Crude), GC (Gold)	Roll yield, term structure
Volatility	VX (VIX futures)	Vol trading, tail hedging

Quant Strategy: Futures Roll

Futures expire, requiring positions to be "rolled" to the next contract. In contango (F > S), rolling costs money; in backwardation (F < S), rolling earns money. Systematic roll strategies in commodities and VIX have historically generated significant returns.

4. Options: Mechanics & Payoffs

An **option** gives the holder the right, but not the obligation, to buy (call) or sell (put) an underlying asset at a specified strike price on or before expiration. This asymmetric payoff structure is what makes options unique and powerful.

4.1 Option Types

Call Option: Right to BUY at strike K. Valuable when $S > K$.

Put Option: Right to SELL at strike K. Valuable when $S < K$.

European: Can only be exercised at expiration.
American: Can be exercised any time before expiration.

4.2 Payoff Diagrams

At expiration, option values are determined by intrinsic value only:

Position	Payoff at Expiration	Max Gain	Max Loss
Long Call	max(S - K, 0)	Unlimited	Premium paid
Short Call	-max(S - K, 0)	Premium received	Unlimited
Long Put	max(K - S, 0)	K - Premium	Premium paid
Short Put	-max(K - S, 0)	Premium received	K - Premium

4.3 Moneyness

State	Call Option	Put Option	Intrinsic Value
In-the-Money (ITM)	$S > K$	$S < K$	Positive
At-the-Money (ATM)	$S \approx K$	$S \approx K$	Zero
Out-of-the-Money (OTM)	$S < K$	$S > K$	Zero

4.4 Put-Call Parity

The Most Important Relationship in Options

For European options on the same underlying with the same strike and expiration:

$$C - P = S \times e^{-q\tau} - K \times e^{-r\tau}$$

This relationship must hold by no-arbitrage. Violations create "box spread" arbitrage opportunities. Put-call parity also means you can synthetically create any position:

• Synthetic Long Stock = Long Call + Short Put + PV(K)
• Synthetic Long Call = Long Stock + Long Put - PV(Dividends)
• Synthetic Long Put = Long Call - Stock + PV(K)

Quant Application: Arbitrage Detection

Put-call parity violations indicate either arbitrage opportunities or data errors. In practice, bid-ask spreads, borrowing costs, and execution risk usually explain apparent violations. However, monitoring parity is essential for options market making and relative value trading.

5. Option Pricing: Black-Scholes-Merton

The Black-Scholes-Merton (BSM) model, published in 1973, revolutionized finance by providing the first closed-form solution for European option prices. Fischer Black, Myron Scholes, and Robert Merton received the Nobel Prize for this work.

5.1 Model Assumptions

The BSM model assumes:

- Stock price follows geometric Brownian motion: $dS = \mu S\, dt + \sigma S\, dW$
- Volatility σ is constant
- No dividends (or continuous dividend yield q)
- Risk-free rate r is constant
- No transaction costs or taxes
- Continuous trading is possible
- No arbitrage opportunities exist

5.2 The Black-Scholes Formulas

The Black-Scholes-Merton Pricing Formulas

$$\text{Call: } C = S \times e^{(-q\tau)} \times N(d■) - K \times e^{(-r\tau)} \times N(d■)$$

$$\text{Put: } P = K \times e^{(-r\tau)} \times N(-d■) - S \times e^{(-q\tau)} \times N(-d■)$$

where:

$$d■ = [\ln(S/K) + (r - q + \sigma^2/2)\tau] / (\sigma\sqrt{\tau})$$

$$d■ = d■ - \sigma\sqrt{\tau}$$

N(·) is the standard normal cumulative distribution function.

5.3 Intuition Behind the Formula

The call price can be interpreted as:

$$C = e^{(-r\tau)} \times E^*[\max(S_T - K, 0)]$$

where E* denotes expectation under the risk-neutral measure. Alternatively:

- $S \times e^{(-q\tau)} \times N(d■)$: Present value of receiving stock if option finishes ITM
- $K \times e^{(-r\tau)} \times N(d■)$: Present value of paying strike if option finishes ITM
- N(d■): Risk-neutral probability of finishing ITM
- N(d■): Delta—shares needed to hedge one option

5.4 The Risk-Neutral Framework

Key Insight for Quants

The actual expected return μ of the stock doesn't appear in the BSM formula! This is because derivatives can be perfectly hedged, making their price independent of risk preferences. Under the risk-neutral measure Q, all assets earn the risk-free rate:

$dS = rS\, dt + \sigma S\, dW^Q$

This allows pricing by simulation: simulate risk-neutral paths, calculate payoffs, and discount at the risk-free rate.

5.5 BSM Implementation

Python Implementation

```
import numpy as np
from scipy.stats import norm

def black_scholes(S, K, T, r, sigma, q=0, option='call'):
    d1 = (np.log(S/K) + (r - q + sigma**2/2)*T) / (sigma*np.sqrt(T))
    d2 = d1 - sigma*np.sqrt(T)

    if option == 'call':
        price = S*np.exp(-q*T)*norm.cdf(d1) - K*np.exp(-r*T)*norm.cdf(d2)
    else:
        price = K*np.exp(-r*T)*norm.cdf(-d2) - S*np.exp(-q*T)*norm.cdf(-d1)
    return price
```

5.6 Model Limitations

Critical Limitations

• **Volatility Smile:** Implied volatility varies by strike, contradicting constant σ
• **Fat Tails:** Returns have excess kurtosis; extreme moves are more common
• **Volatility Clustering:** High volatility periods cluster together
• **Jumps:** Prices can gap without continuous trading
• **Transaction Costs:** Continuous hedging is impossible in practice

These limitations drive the development of more sophisticated models (Heston, SABR, Local Vol) that quants use in practice.

6. The Greeks: Risk Sensitivities

The Greeks measure how option prices change with respect to various parameters. Understanding Greeks is essential for risk management, hedging, and constructing trading strategies. For quant traders, Greeks are the language of options risk.

6.1 First-Order Greeks

Delta (Δ): Sensitivity to Underlying Price

$$\Delta = \partial V/\partial S$$

For calls: $\Delta = e^{(-q\tau)} \times N(d■) \in [0, 1]$
For puts: $\Delta = -e^{(-q\tau)} \times N(-d■) \in [-1, 0]$

Delta represents: (1) hedge ratio—shares needed to delta-hedge one option, (2) approximate probability of finishing ITM, (3) sensitivity to $1 move in underlying.

Theta (Θ): Time Decay

$$\Theta = \partial V/\partial t = -\partial V/\partial \tau$$

Theta measures daily time decay. For ATM options:

$$\Theta\_call \approx -S \times \sigma \times e^{(-q\tau)} / (2\sqrt{(2\pi\tau)}) - rKe^{(-r\tau)}N(d■) + qSe^{(-q\tau)}N(d■)$$

Theta is typically negative for long options (time decay hurts) and positive for short options.

Vega (ν): Volatility Sensitivity

$$\nu = \partial V/\partial \sigma$$

$$\text{Vega} = S \times e^{(-q\tau)} \times \sqrt{\tau} \times n(d■)$$

where n(·) is the standard normal PDF. Vega is positive for all long options—they benefit from volatility increases. Vega is highest for ATM options and increases with time to expiration.

Rho (ρ): Interest Rate Sensitivity

$$\rho = \partial V/\partial r$$

$\rho\_call = K \times \tau \times e^{(-r\tau)} \times N(d■) > 0$
$\rho\_put = -K \times \tau \times e^{(-r\tau)} \times N(-d■) < 0$

6.2 Second-Order Greeks

Gamma (Γ): Delta Sensitivity

$$\Gamma = \partial^2 V/\partial S^2 = \partial \Delta/\partial S$$

$$\Gamma = e^{(-q\tau)} \times n(d■) / (S \times \sigma \times \sqrt{\tau})$$

Why Gamma Matters for Quants

Gamma measures the convexity of the option position. High gamma means delta changes rapidly, requiring frequent rehedging. Gamma is highest for ATM options near expiration—these positions are the most volatile and expensive to hedge. The P&L; from gamma is:

Gamma P&L; $\approx \frac{1}{2} \times \Gamma \times (\Delta S)^2$

This is always positive for long gamma positions, explaining why you pay theta to own gamma.

Vanna: Delta-Volatility Cross-Sensitivity

$$\text{Vanna} = \partial^2 V/\partial S \partial\sigma = \partial\Delta/\partial\sigma = \partial\nu/\partial S$$

$$\text{Vanna} = -e^{(-q\tau)} \times n(d■) \times d■/\sigma$$

Vanna measures how delta changes with volatility. Important for managing portfolios with significant skew exposure.

Volga (Vomma): Vega Convexity

$$\text{Volga} = \partial^2 V/\partial\sigma^2 = \partial\nu/\partial\sigma$$

$$\text{Volga} = \nu \times d■ \times d■ / \sigma$$

Volga measures the convexity of vega. Positive for OTM options (wing options), explaining why the volatility smile exists—OTM options need extra premium for their volga exposure.

6.3 Greeks Summary Table

Greek	Formula	Measures	Sign (Long Call)
Delta Δ	$\partial V/\partial S$	Price sensitivity	+ (0 to 1)
Gamma Γ	$\partial^2 V/\partial S^2$	Delta sensitivity	+ (always)
Theta Θ	$\partial V/\partial t$	Time decay	- (usually)
Vega ν	$\partial V/\partial\sigma$	Vol sensitivity	+ (always)
Rho ρ	$\partial V/\partial r$	Rate sensitivity	+ (calls)
Vanna	$\partial^2 V/\partial S\partial\sigma$	Delta-vol cross	± (by strike)
Volga	$\partial^2 V/\partial\sigma^2$	Vega convexity	+ (wings)

6.4 The Theta-Gamma Relationship

Fundamental Trading Relationship

For a delta-hedged option position, the BSM PDE implies:

$\Theta + \frac{1}{2}\sigma^2 S^2\Gamma + rS\Delta - rV = 0$

For a delta-neutral position ($\Delta = 0$), this simplifies to the crucial insight:

$\Theta \approx -\frac{1}{2}\sigma^2 S^2\Gamma$

Long gamma = negative theta (you pay time decay to own convexity)
Short gamma = positive theta (you collect premium but face convexity risk)

This is the fundamental tradeoff in options trading.

7. Volatility: Implied vs Realized

For quant traders, volatility is perhaps the most important variable in options markets. Understanding the distinction between implied and realized volatility—and how to trade it—is fundamental to systematic options strategies.

7.1 Realized (Historical) Volatility

Realized volatility measures actual price fluctuations over a historical period. The standard estimator uses log returns:

$$\sigma\_\text{realized} = \sqrt{(252/n \times \Sigma(r■ - r■)^2)}$$

where $r■ = \ln(S■/S■■■)$ are daily log returns and 252 annualizes the result.

Alternative Volatility Estimators

- **Parkinson:** Uses high-low range: $\sigma^2 = \ln(H/L)^2 / (4\times\ln(2))$
- **Garman-Klass:** Uses OHLC data for efficiency
- **Yang-Zhang:** Handles overnight jumps
- **EWMA:** Exponentially weighted for responsiveness

Range-based estimators are more efficient (lower variance) than close-to-close but sensitive to microstructure noise.

7.2 Implied Volatility

Implied volatility is the volatility that, when plugged into the BSM formula, produces the observed market price. It's backed out numerically (BSM has no analytical inverse for σ):

$$\text{IV: } C\_\text{market} = \text{BSM}(S, K, T, r, \sigma\_\text{IV}, q)$$

Implied volatility represents the market's expectation of future realized volatility, plus a risk premium. It's forward-looking and incorporates all available information.

7.3 The Volatility Surface

In practice, implied volatility varies by both strike (smile/skew) and expiration (term structure), forming a surface:

The Volatility Smile and Skew

- **Smile:** Symmetric—both OTM puts and calls have higher IV than ATM
- **Skew:** Asymmetric—OTM puts have higher IV than OTM calls (equity markets)
- **Smirk:** Combination of smile and skew

The skew in equity markets reflects:
1. Crash protection demand (put buying)
2. Leverage effect (volatility rises when prices fall)

3. Fat left tail in return distributions

7.4 The Variance Risk Premium

The Core Options Trading Opportunity

Historically, implied volatility exceeds realized volatility by 2-4 vol points on average. This "variance risk premium" exists because:

1. Investors pay for crash protection (insurance premium)
2. Volatility spikes are painful—they correlate with market losses
3. Selling volatility has negative skew—small gains, occasional large losses

$VRP = E[IV] - E[RV] > 0$

Systematically selling volatility (short straddles, iron condors, variance swaps) harvests this premium but requires careful risk management.

7.5 Trading the Vol Surface

Strategy	View	Greeks Exposure
Long Straddle	Realized > Implied	Long Gamma, Long Vega
Short Straddle	Realized < Implied	Short Gamma, Short Vega
Risk Reversal	Skew too steep/flat	Vanna exposure
Butterfly	Wings mispriced	Volga exposure
Calendar Spread	Term structure move	Long/Short Vega by tenor
Dispersion	Correlation view	Short index vol, long single stock

8. Swaps: Interest Rate & Beyond

A **swap** is an agreement to exchange cash flows between two parties. Swaps are the largest derivatives market by notional value, with interest rate swaps alone exceeding \$400 trillion notional outstanding.

8.1 Interest Rate Swaps (IRS)

The vanilla interest rate swap exchanges fixed rate payments for floating rate payments:

Fixed Leg: $N \times c \times \alpha$ (periodic fixed payment)
Float Leg: $N \times L \times \alpha$ (periodic floating payment based on SOFR, etc.)

where N is notional, c is the fixed rate, L is the floating rate, and α is the day count fraction.

8.2 Swap Valuation

A swap can be valued as the difference between a fixed-rate bond and a floating-rate bond:

$$V_{swap} = B_{fixed} - B_{float}$$

For the fixed leg:

$$B_{fixed} = c \times \Sigma\, \alpha_i \times DF(t_i) + DF(T)$$

where $DF(t)$ is the discount factor to time t.

At inception, the swap rate c is set so that $V_{swap} = 0$:

$$c = (1 - DF(T)) / \Sigma\, \alpha_i \times DF(t_i)$$

8.3 Swap Risk Measures

DV01 (Dollar Value of 01): Change in swap value for 1 basis point parallel shift:

$$DV01 = \Sigma\, \alpha_i \times DF(t_i) \times 0.0001 \times N$$

Duration: Weighted average time of cash flows, measuring interest rate sensitivity.

Convexity: Second derivative—measures how duration changes with rates.

8.4 Other Swap Types

Swap Type	Exchange	Use Case
Currency Swap	Principal + interest in different currencies	FX hedging, funding
Basis Swap	Two floating rates (e.g., SOFR vs LIBOR)	Basis risk management
Total Return Swap	Total return vs funding rate	Synthetic equity exposure
Variance Swap	Realized variance vs fixed strike	Pure volatility trading

Credit Default Swap	Premium vs credit protection	Credit risk transfer

8.5 Variance Swaps

Pure Volatility Exposure for Quants

A variance swap pays the difference between realized variance and a fixed strike:

Payoff = $N_{var} \times (\sigma^2_{realized} - K^2_{var})$

where N_{var} is the variance notional. Variance swaps provide pure exposure to realized volatility without the path-dependency and Greeks management of vanilla options.

The fair strike K_{var} can be replicated by a portfolio of options across all strikes, weighted by $1/K^2$. This is the theoretical basis for the VIX index.

9. Trading Strategies for Quants

This section covers systematic strategies that quant traders employ using derivatives. These strategies exploit market inefficiencies, risk premia, or specific structural features.

9.1 Delta-Neutral Strategies

Straddle: Long call + Long put at same strike

• Position: Delta-neutral, Long gamma, Long vega
• Profits from: Large moves in either direction, volatility increase
• Risk: Time decay (theta), volatility decrease

Strangle: Long OTM call + Long OTM put

• Cheaper than straddle but needs larger move to profit
• Lower gamma/vega per dollar invested
• Wider breakeven points

Iron Condor: Short strangle + Long wings for protection

• Defined risk version of short strangle
• Profits from: Low realized volatility, time decay
• Max loss limited by wing strikes

9.2 Volatility Strategies

Systematic Vol Selling (Harvesting VRP)

Selling ATM straddles or strangles on indices systematically captures the variance risk premium. Key implementation considerations:

• Delta-hedge continuously or discretely
• Size positions by vega or expected shortfall, not notional
• Use stops or dynamic position sizing to manage tail risk
• Avoid earnings, FOMC, and other event dates

Volatility Timing: Enter short vol positions when IV is elevated relative to recent realized vol. VIX/VIX3M ratio, term structure slope, and put-call skew are useful signals.

9.3 Relative Value Strategies

Dispersion Trading:

• Short index volatility, long single-stock volatility
• Profits when implied correlation > realized correlation
• Index IV = weighted average single-stock IV × implied correlation
• Historically, implied correlation is too high (fear of systemic risk)

Term Structure Trading:

• Calendar spreads based on term structure shape
• In contango (normal): short front, long back
• In backwardation (fear): opposite direction
• VIX futures term structure is particularly tradeable

9.4 Event-Driven Strategies

Earnings Volatility Trading

Around earnings, options experience "volatility crush"—IV collapses post-announcement. Strategies include:

• Selling straddles before earnings (captures premium, risks gap)
• Buying straddles when IV seems too low relative to historical moves
• Using weekly options to isolate the event

The key metric: Historical earnings move vs. implied move (ATM straddle / stock price)

10. Risk Management & Implementation

Effective risk management separates successful quant traders from those who blow up. Derivatives, with their leverage and non-linear payoffs, require sophisticated risk frameworks.

10.1 Position Sizing

Size by Risk, Not Notional

Options positions should be sized by their risk contribution, not notional or delta equivalent:

• For directional: Size by delta dollars or VaR
• For volatility: Size by vega (how much you lose per 1 vol point decline)
• For gamma: Size by daily theta (your daily bleed)
• Use Expected Shortfall (CVaR) for tail risk positions

Rule of thumb: Vega should be small relative to capital, as vol can move 10+ points in crisis.

10.2 Greeks Management

Delta: Hedge at regular intervals or when delta exceeds threshold. Consider transaction costs vs. gamma P&L.;

Gamma: Monitor gamma/theta ratio. Large gamma near expiration requires careful management.

Vega: Set limits on total portfolio vega. Consider vega term structure.

Vanna/Volga: Important for exotic options and large positions. Monitor cross-Greeks.

10.3 Scenario Analysis

Stress Testing Framework

1. **Historical Scenarios:** 1987 crash, 2008 crisis, COVID-19, etc.
2. **Hypothetical Scenarios:** ±20% spot moves, vol spike to 80
3. **Greeks-Based:** What-if spot ±1%, vol ±5 points
4. **Correlation Stress:** Correlations go to ±1 in crisis
5. **Liquidity Stress:** Can you exit at theoretical prices?

10.4 Implementation Considerations

Execution: Use limit orders; options have wide bid-ask spreads. Market orders are expensive.

Slippage: Models assume frictionless trading. Budget for 1-2 bid-ask spreads of slippage.

Pin Risk: ATM options at expiration have uncertain exercise—hedge or close.

Early Exercise: American options on dividend-paying stocks may be exercised early.

Corporate Actions: Mergers, special dividends, and splits affect option terms.

11. Key Formulas Reference

Quick Reference for Quant Interviews & Implementation

Forwards & Futures

Forward Price (no income)	$F = S \times e^{(r\tau)}$
Forward Price (dividend yield)	$F = S \times e^{((r-q)\tau)}$
Forward Value	$V = (F - K) \times e^{(-r\tau)}$
Cost of Carry	$F = S \times e^{((r+u-y)\tau)}$ [u=storage, y=convenience]

Black-Scholes-Merton

Call Price	$C = Se^{(-q\tau)}N(d\blacksquare) - Ke^{(-r\tau)}N(d\blacksquare)$
Put Price	$P = Ke^{(-r\tau)}N(-d\blacksquare) - Se^{(-q\tau)}N(-d\blacksquare)$
$d\blacksquare$	$[\ln(S/K) + (r-q+\sigma^2/2)\tau] / (\sigma\sqrt{\tau})$
$d\blacksquare$	$d\blacksquare - \sigma\sqrt{\tau}$
Put-Call Parity	$C - P = Se^{(-q\tau)} - Ke^{(-r\tau)}$

Greeks (Call Options)

Delta	$\Delta = e^{(-q\tau)} N(d\blacksquare)$
Gamma	$\Gamma = e^{(-q\tau)} n(d\blacksquare) / (S\sigma\sqrt{\tau})$
Theta	$\Theta = -Se^{(-q\tau)}n(d\blacksquare)\sigma/(2\sqrt{\tau}) - rKe^{(-r\tau)}N(d\blacksquare) + qSe^{(-q\tau)}N(d\blacksquare)$
Vega	$\nu = Se^{(-q\tau)}\sqrt{\tau}\, n(d\blacksquare)$
Rho	$\rho = K\tau e^{(-r\tau)}N(d\blacksquare)$

Volatility

Realized Vol (annualized)	$\sigma = \sqrt{(252 \times \mathrm{Var(daily\ returns)})}$
Variance Swap Payoff	$N \times (\sigma^2\_\mathrm{realized} - K^2)$
Theta-Gamma Relation	$\Theta \approx -\frac{1}{2}\sigma^2 S^2\Gamma$ (delta-hedged)
VRP	$E[IV] - E[RV] \approx$ 2-4 vol points historically

Recommended Reading for Quants

- Hull, J. *Options, Futures, and Other Derivatives* (the "Bible")
- Taleb, N. *Dynamic Hedging* (practical trading wisdom)
- Gatheral, J. *The Volatility Surface* (advanced vol modeling)
- Sinclair, E. *Volatility Trading* (systematic strategies)
- Natenberg, S. *Option Volatility and Pricing* (market maker perspective)
- Wilmott, P. *Paul Wilmott on Quantitative Finance* (comprehensive theory)