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Student Satellite Project
Indian Institute of Technology, Bombay
Powai, Mumbai - 400076, INDIA

Website: www.aero.iitb.ac.in/satlab

README - rbd.pdf

Guidance, Navigation and Controls Subsystem

General Practice: *"Main" function should be written first*

Function_1 qx_dot (t,qx,qy,qz,qs)

Code author: Tanmay Ganguli
Created on: 01/05/2022
Last modified: 01/05/2022
Reviwed by: Not reviewed yet
Description:
It is the function for the rate of change of the x component of the quaternion representing attitude of the satellite. It is defined in the main function rbd_state_t. it depends on the various components of angular velocity at an instant and the various components of the quaternion.
Formula & References:
$\frac{d(qx)}{dt} = 0.5 * (qs * wx - qz * wy + qy * wz)$
Where qx, qy, qz represent the three vector components of the quaternion and qs represents the scalar part.
wx, wy, wz represent the x,y and z components of angular velocity at a given time, they may themselves be a function of time. We use them as constants at a given time.They are not parameters to the function, they are assumed to be defined in the block of the function.
Reference:F. Landis Markley, John L. Crassidis - Fundamentals of Spacecraft Attitude Determination and Control
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of the quaternion. *Seconds*
2. **qx** : (Float) - x-component of the vector part of quaternion at time t. *Dimensionless*
3. **qy** : (Float) - y-component of the vector part of quaternion at time t. *Dimensionless*
4. **qz** : (Float) - z-component of the vector part of quaternion at time t. *Dimensionless*
5. **qs** : (Float) - scalar part of the quaternion at time t. *Dimensionless*

Output:
Returns the rate of change of the x component of the vector part of the quaternion at time t, passed as parameter.

Function_2 qy_dot (t,qx,qy,qz,qs)

Code author: Tanmay Ganguli
Created on: 01/05/2022
Last modified: 01/05/2022
Reviwed by: Not reviewed yet
Description:
It is the function for the rate of change of the y component of the quaternion representing attitude of the satellite. It is defined in the main function rbd_state_t. it depends on the various components of angular velocity at an instant and the various components of the quaternion.
Formula & References:
$\frac{d(qy)}{dt} = 0.5 * (qz * wx + qs * wy - qx * wz)$
Where qx, qy, qz represent the three vector components of the quaternion and qs represents the scalar part.
wx, wy, wz represent the x,y and z components of angular velocity at a given time, they may themselves be a function of time. We use them as constants at a given time.They are not parameters to the function, they are assumed to be defined in the block of the function.
Reference:F. Landis Markley, John L. Crassidis - Fundamentals of Spacecraft Attitude Determination and Control
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of the quaternion. *Seconds*
2. **qx** : (Float) - x-component of the vector part of quaternion at time t. *Dimensionless*
3. **qy** : (Float) - y-component of the vector part of quaternion at time t. *Dimensionless*
4. **qz** : (Float) - z-component of the vector part of quaternion at time t. *Dimensionless*
5. **qs** : (Float) - scalar part of the quaternion at time t. *Dimensionless*

Output:
Returns the rate of change of the y component of the vector part of the quaternion at time t, passed as parameter.

Function_3 qz_dot (t,qx,qy,qz,qs)

Code author: Tanmay Ganguli
Created on: 01/05/2022
Last modified: 01/05/2022
Reviwed by: Not reviewed yet
Description:
It is the function for the rate of change of the z component of the quaternion representing attitude of the satellite. It is defined in the main function rbd_state_t. it depends on the various components of angular velocity at an instant and the various components of the quaternion.
Formula & References:
$\frac{d(qz)}{dt} = 0.5 * (-qy * wx + qx * wy + qs * wz)$
Where qx, qy, qz represent the three vector components of the quaternion and qs represents the scalar part.

wx, wy, wz represent the x,y and z components of angular velocity at a given time, they may themselves be a function of time. We use them as constants at a given time.They are not parameters to the function, they are assumed to be defined in the block of the function.
Reference:F. Landis Markley, John L. Crassidis - Fundamentals of Spacecraft Attitude Determination and Control
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of the quaternion. *Seconds*
2. **qx** : (Float) - x-component of the vector part of quaternion at time t. *Dimensionless*
3. **qy** : (Float) - y-component of the vector part of quaternion at time t. *Dimensionless*
4. **qz** : (Float) - z-component of the vector part of quaternion at time t. *Dimensionless*
5. **qs** : (Float) - scalar part of the quaternion at time t. *Dimensionless*

Output:
Returns the rate of change of the z component of the vector part of the quaternion at time t, passed as parameter.

Function_4 qs_dot (t,qx,qy,qz,qs)

Code author: Tanmay Ganguli
Created on: 01/05/2022
Last modified: 01/05/2022
Reviwed by: Not reviewed yet
Description:
It is the function for the rate of change of the scalar part of the quaternion representing attitude of the satellite. It is defined in the main function rbd_state_t. it depends on the various components of angular velocity at an instant and the various components of the quaternion.
Formula & References:
$\frac{d(qs)}{dt} = 0.5 * (-qx * wx - qy * wy - qz * wz)$
Where qx, qy, qz represent the three vector components of the quaternion and qs represents the scalar part.
wx, wy, wz represent the x,y and z components of angular velocity at a given time, they may themselves be a function of time. We use them as constants at a given time.They are not parameters to the function, they are assumed to be defined in the block of the function.
Reference:F. Landis Markley, John L. Crassidis - Fundamentals of Spacecraft Attitude Determination and Control
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of the quaternion. *Seconds*
2. **qx** : (Float) - x-component of the vector part of quaternion at time t. *Dimensionless*
3. **qy** : (Float) - y-component of the vector part of quaternion at time t. *Dimensionless*
4. **qz** : (Float) - z-component of the vector part of quaternion at time t. *Dimensionless*
5. **qs** : (Float) - scalar part of the quaternion at time t. *Dimensionless*

Output:
Returns the rate of change of the scalar part of the quaternion at time t, passed as parameter.

Function_5 omega_dot (t,wx,wy,zz)

Code author: Tanmay Ganguli
Created on: 03/05/2022
Last modified: 10/05/2022
Reviwed by: Not reviewed yet
Description:
It calculates the rate of change of angular velocity vector(of satellite) with time and returns the same. The angular velocity is the absolute angular velocity as seen from an inertial frame but expressed in the body frame coordinates.
The Euler's equations are solved to get the omega vector.The function works for any generalised inertia tensor(of satellite), not necessarily diagonalised and also if its motion is not moment free, that is torque acting on the satellite is proportional to angular velocity.
Note: the inertia tensor and torque expression are not inputs to the function, but are assumed to be defined in the block of the function.
The function is moreover defined inside the function rbd_state_t. In the function block, inertia tensor and torque expression are available.
Formula & References:
$\vec{M} = \frac{d(\vec{L})}{dt}\hat{l} + \vec{w} \times \vec{L}$
$\vec{M}$ is the torque acting on the satellite as seen in inertial frame.
1st term on RHS is the rate of change of angular momentum as seen from the body frame.
$\hat{l}$ is the unit vector along $\vec{L}$ in body frame.
$\vec{w}$ is angular velocity of satellite in body frame.
Further, $\vec{L} = [I]\vec{w}$
$[I]$ is the inertia tensor, assumed to be constant.
So $\dot{\vec{L}} = [I]\dot{\vec{w}}$
$\dot{\vec{w}}$ is what the function calculates.
Reference:F. Landis Markley, John L. Crassidis - Fundamentals of Spacecraft Attitude Determination and Control
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of angular velocity. *Seconds*
2. **wx** : (Float) - x-component of angular velocity at time t. *Radians/second*
3. **wy** : (Float) - y-component of angular velocity at time t. *Radians/second*
4. **wz** : (Float) - z-component of angular velocity at time t. *Radians/second*

Output:
Returns $\dot{\vec{w}}$ as a 3x1 numpy array.

Function_6 omega_x_dot (t,wx,wy,wz)

Code author: Tanmay Ganguli
Created on: 03/05/2022
Last modified: 03/05/2022
Reviwed by: Not reviewed yet
Description:

Returns the rate of change of x-component of angular velocity with time. The function omega_dot(t,wx,wy,wz) is assumed to be defined in the same block as this function definition.
Formula & References:
The function omega_dot() is called with the same parameters and the first (x) component of the vector returned by omega_dot() is simply picked out and returned.
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of x component of angular velocity. *Seconds*
2. **w_x** : (Float) - x-component of the angular velocity at time t. *Radians/second*
3. **w_y** : (Float) - y-component of the angular velocity at time t. *Radians/second*
4. **w_z** : (Float) - z-component of the angular velocity at time t. *Radians/second*

Output:
Returns x component of $\dot{\vec{w}}$ as a floating point number.

Function_7 omega_y_dot (t,wx,wy,wz)

Code author: Tanmay Ganguli
Created on: 03/05/2022
Last modified: 03/05/2022
Reviwed by: Not reviewed yet
Description:
Returns the rate of change of y-component of angular velocity with time. The function omega_dot(t,wx,wy,wz) is assumed to be defined in the same block as this function definition.
Formula & References:
The function omega_dot() is called with the same parameters and the second (y) component of the vector returned by omega_dot() is simply picked out and returned.
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of y component of angular velocity. *Seconds*
2. **w_x** : (Float) - x-component of the angular velocity at time t. *Radians/second*
3. **w_y** : (Float) - y-component of the angular velocity at time t. *Radians/second*
4. **w_z** : (Float) - z-component of the angular velocity at time t. *Radians/second*

Output:
Returns y component of $\dot{\vec{w}}$ as a floating point number.

Function_8 omega_z_dot (t,wx,wy,wz)

Code author: Tanmay Ganguli
Created on: 03/05/2022
Last modified: 03/05/2022
Reviwed by: Not reviewed yet
Description:

Returns the rate of change of z-component of angular velocity with time. The function omega_dot(t,wx,wy,wz) is assumed to be defined in the same block as this function definition.
Formula & References:
The function omega_dot() is called with the same parameters and the third (z) component of the vector returned by omega_dot() is simply picked out and returned.
Input parameters:

1. **t** : (Float) - time when we are calculating the rate of change of y component of angular velocity. *Seconds*
2. **w_x** : (Float) - x-component of the angular velocity at time t. *Radians/second*
3. **w_y** : (Float) - y-component of the angular velocity at time t. *Radians/second*
4. **w_z** : (Float) - z-component of the angular velocity at time t. *Radians/second*

Output:
Returns z component of $\dot{\vec{w}}$ as a floating point number.

Function_9 state_t(t_i,t_f,omega_i,q_i ,inertia,const=0.1,wxplot=False,wyplot=False, wzplot=False)

Code author: Tanmay Ganguli
Created on: 03/05/2022
Last modified: 03/05/2022
Reviwed by: Not reviewed yet
Description:
Returns the angular velocity vector and the quaternion at time t_f, given the initial time t_i, and initial angular velocity and quaternion. Moment of inertia tensor is also an input, need not be diagonalised, assumed to be a constant throughout.
All the above mentioned functions are defined inside this function block.(This can be considered as the main function of this program).
The aim is to calculate the required quantities at time t_f using rk4 method of integration. So to calculate quaternion, 4 sets of simultaneous differential equations need to be solved, and 3 sets for angular velocity. The functions qx_dot, ..., qz_dot, omega_x_dot,... represent the functions in the simultaneous ODE's.
So first, quaternion is updated in the for loop. But rate of change of quaternion depends on angular velocity, which itself depends on time. So first the quaternion is updated considering the omega as constant for the small time step, then the omega itself is updated.In RK4 integration the independent variable also has to be incremented in each step. Here the independent variable is time. But time is incremented only after omega has been incremented. Otherwise we will be incrementing time twice in each step, while incrementing omega and quaternion separately, which will lead to errors.
Thus the for loop is run with a small time step, till t_f is reached.
There are also parameters wxplot, wyplot, wzplot, with default value False, to plot x,y,z component of omega with time. But plotting is time and memory consuming, so should be avoided, and if needed, only one or two plots should be preferably obtained.
There is one more parameter const with default value 0.1, which is the magnitude of constant of proportionality of magnetic field with angular velocity, as in the b-dot controller.
Formula & References:

The following references are referred:
F. Landis Markley, John L. Crassidis - Fundamentals of Spacecraft Attitude Determination and Control
The resources provided for attitude kinematics and rigid body dynamics mini projects.
Input parameters:

1. **t_i** : (Float) - initial time. *Seconds*
2. **t_f** : (Float) - final time. *Seconds*
3. **omega_i** : (List) - initial angular velocity, 3 element list *Radians/second*
4. **q_i** : (List) - initial quaternion representing frame transformation between body frame and inertial frame. *no units*
5. **inertia** : (List of lists,(2-d array)) - inertia tensor of satellite. *kg/*m^2
6. **const** : (float) - magnitude of ratio of magnetic field and angular velocity. Default value=0.1 *Tesla-seconds/radian*
7. **wxplot** : (Bool) - Plot x component of omega if True. Default:False. *no units*
8. **wyplot** : (Bool) - Plot y component of omega if True. Default:False. *no units*
9. **wzplot** : (Bool) - Plot z component of omega if True. Default:False. *no units*

Output:
Returns angular velocity as a 3 element list, followed by a 4 element quaternion at desired time. The tuple is returned where the function is called. Apart from this it may plot angular velocity as instructed, which shows up in the output section.