3.22. $f_1 = f_c - \Delta f - W$

$$f_2 = f_c + \Delta f$$

$$v_1(t)v_2(t) = A_1 A_2 \cos(2\pi f_1 t + \phi_1)\cos(2\pi f_2 t + \phi_2)$$

$$= \frac{A_1 A_2}{2}[\cos(2\pi(f_1 - f_2)t + \phi_1 - \phi_2) + \cos(2\pi(f_1 + f_2)t + \phi_1 + \phi_2)]$$

The low-pass filter will only pass the first term.

$$\therefore LFP(v_1(t)v_2(t)) = \frac{1}{2}A_1 A_2[\cos(-2\pi(W + 2\Delta f)t + \phi_1 - \phi_2)]$$

Let $v_0(t)$ be the final output, before band-pass filtering.

$$v_o(t) = \frac{1}{2}A_1 A_2[\cos(-2\pi\left(\frac{W + 2\Delta f}{W/\Delta f + 2}\right)t + \frac{\phi_1 - \phi_2}{W/\Delta f + 2}) \cdot A_2 \cos(2\pi f_2 t + \phi_2)]$$

$$= \frac{1}{2}A_1 A_2^2[\cos(-2\pi\Delta f t + \frac{\phi_1 - \phi_2}{n+2} - \phi_2) \cdot \cos(2\pi f_2 t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2)]$$

$$= \frac{1}{4}A_1 A_2^2[\cos(-2\pi(f_c + 2\Delta f) + \frac{\phi_1 - \phi_2}{n+2} - \phi_2) + \cos(-2\pi f_c t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2)]$$

After band-pass filtering, retain only the second term.

$$\therefore v_o(t) = \frac{1}{4}A_1 A_2^2[\cos(-2\pi f_c t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2)$$

$$\frac{\phi_1}{n+2} - \frac{\phi_2}{n+2} + \phi_2 = 0$$

rearranging and solving for $\phi_2$:

$$\phi_2 = -\frac{\phi_1}{n+1}$$

(b) At the second multiplier, replace $v_2(t)$ with $v_1(t)$. This results in the following expression for the phase:

$$\frac{\phi_1}{n+2} - \frac{\phi_2}{n+2} + \phi_1 = 0$$

$$\phi_1 = \frac{\phi_2}{n+3}$$

3.22. $f_1 = f_c - \Delta f - W$

$f_2 = f_c + \Delta f$

$$v_1(t)v_2(t) = A_1A_2\cos(2\pi f_1 t + \phi_1)\cos(2\pi f_2 t + \phi_2)$$
$$= \frac{A_1A_2}{2}[\cos(2\pi(f_1 - f_2)t + \phi_1 - \phi_2) + \cos(2\pi(f_1 + f_2)t + \phi_1 + \phi_2)]$$

The low-pass filter will only pass the first term.

$$\therefore LFP(v_1(t)v_2(t)) = \frac{1}{2}A_1A_2[\cos(-2\pi(W + 2\Delta f)t + \phi_1 - \phi_2)]$$

Let $v_0(t)$ be the final output, before band-pass filtering.

$$v_o(t) = \frac{1}{2}A_1A_2[\cos(-2\pi\left(\frac{W + 2\Delta f}{W/\Delta f + 2}\right)t + \frac{\phi_1 - \phi_2}{W/\Delta f + 2}) \cdot A_2\cos(2\pi f_2 t + \phi_2)]$$
$$= \frac{1}{2}A_1A_2^2[\cos(-2\pi\Delta f t + \frac{\phi_1 - \phi_2}{n+2} - \phi_2) \cdot \cos(2\pi f_2 t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2)]$$
$$= \frac{1}{4}A_1A_2^2[\cos(-2\pi(f_c + 2\Delta f) + \frac{\phi_1 - \phi_2}{n+2} - \phi_2) + \cos(-2\pi f_c t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2)]$$

After band-pass filtering, retain only the second term.

$$\therefore v_o(t) = \frac{1}{4}A_1A_2^2[\cos(-2\pi f_c t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2)$$

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