

$$3.22. \quad f_1 = f_c - \Delta f - W$$

$$f_2 = f_c + \Delta f$$

$$\begin{aligned} v_1(t)v_2(t) &= A_1 A_2 \cos(2\pi f_1 t + \phi_1) \cos(2\pi f_2 t + \phi_2) \\ &= \frac{A_1 A_2}{2} [\cos(2\pi(f_1 - f_2)t + \phi_1 - \phi_2) + \cos(2\pi(f_1 + f_2)t + \phi_1 + \phi_2)] \end{aligned}$$

The low-pass filter will only pass the first term.

$$\therefore LFP(v_1(t)v_2(t)) = \frac{1}{2} A_1 A_2 [\cos(-2\pi(W + \Delta f)t + \phi_1 - \phi_2)]$$

Let  $v_0(t)$  be the final output, before band-pass filtering.

$$\begin{aligned} v_n(t) &= \frac{1}{2} A_1 A_2 \left[ \cos\left(-2\pi \left(\frac{W + 2\Delta f}{W/\Delta f + 2}\right)t + \frac{\phi_1 - \phi_2}{W/\Delta f + 2}\right) \cdot A_2 \cos(2\pi f_2 t + \phi_2) \right] \\ &= \frac{1}{2} A_1 A_2^2 \left[ \cos\left(-2\pi \Delta f t + \frac{\phi_1 - \phi_2}{n+2} - \phi_2\right) \cdot \cos\left(2\pi f_2 t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2\right) \right] \\ &= \frac{1}{4} A_1 A_2^2 \left[ \cos(-2\pi(f_c + 2\Delta f)t + \frac{\phi_1 - \phi_2}{n+2} - \phi_2) + \cos(-2\pi f_c t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2) \right] \end{aligned}$$

After band-pass filtering, retain only the second term.

$$\therefore v_n(t) = \frac{1}{4} A_1 A_2^2 \left[ \cos(-2\pi f_c t + \frac{\phi_1 - \phi_2}{n+2} + \phi_2) \right]$$

$$\frac{\phi_1}{n+2} - \frac{\phi_2}{n+2} + \phi_2 = 0$$

rearranging and solving for  $\phi_2$ :

$$\phi_2 = -\frac{\phi_1}{n+1}$$

(b) At the second multiplier, replace  $v_2(t)$  with  $v_1(t)$ . This results in the following expression for the phase:

$$\frac{\phi_1}{n+2} - \frac{\phi_2}{n+2} + \phi_1 = 0$$

$$\phi_2 = \frac{\phi_1}{n+3}$$