EE386 Digital Signal Processing Lab **Jul-Dec 2021**

8: Experiment

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1 Introduction

This week is about modelling real-world data, the growth, stabilisation and saturation of epidemics and studying effects such as virulence and social distancing. We primarily focus on the SARS-CoV-2 virus.

2 Solution

Question 1-First-Order Model

The propagation mechanism of an epidemic, such as the one caused by the SARS-CoV-2 virus, can be modelled, at least in its initial phase, as a process in which each infected individual will eventually transmit the disease to an average of R0 healthy people; these newly infected patients will, in turn, infect R0 healthy individuals each, and so on, creating a pernicious positive feedback in the system. The constant R0 is called the basic reproduction number for a virus.

In signal processing terms, the infection mechanism is equivalent to a first-order recursive filter. Assume that each infected person spreads the virus over a single day and then recovers and assume that an initial patient zero appears at day n = 0. The number of newly infected people per day is described by the difference equation

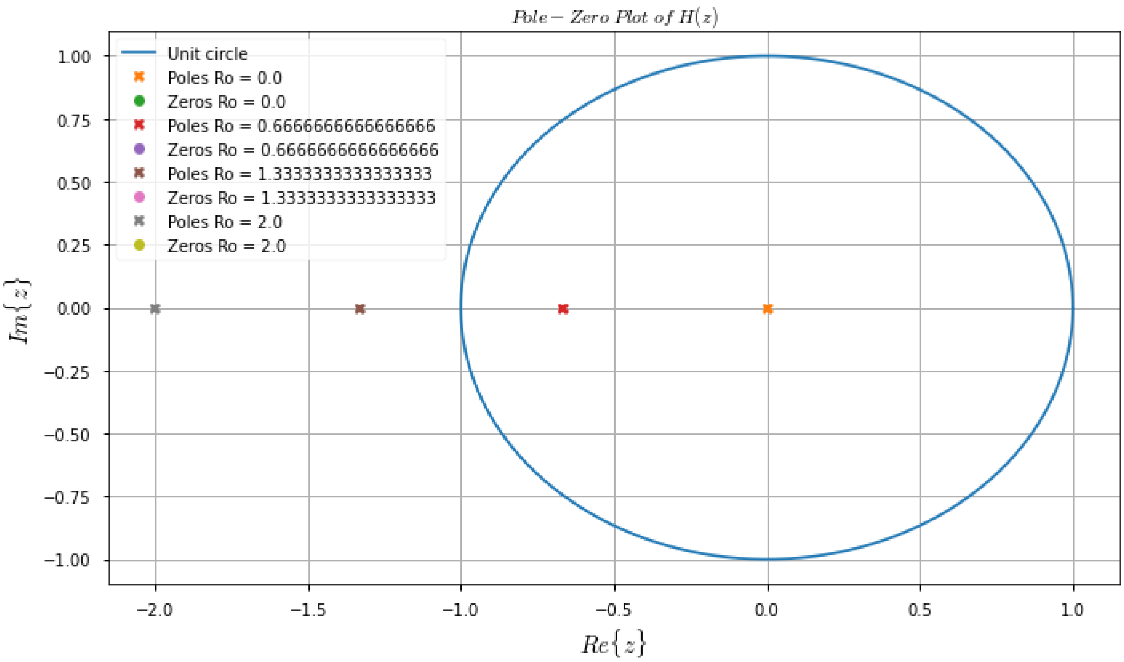
$$y[n] = \delta[n] + R_0 * y[n-1]$$

(Problem 1)
We need to find the transfer function $H_1(z)$ for the given system.
(Solution)

By applying z - transform on both the sides and rearranging terms we can get the transfer function

$$Y[z] = 1 + R_0 Y[z] * z^{-1}$$
$$\implies H(z) = \frac{z}{z - R_0}$$

Transfer function depends on the value of R_o, we'll create the pole zero plot varying R_o so that we can infer the stability of the system based on the value of R_o.
Pole-zero plot of the transfer function with varying R_o:



From the plot that as the value of R_o increases, the poles of the system moves left. Hemce as long as the value of R_o remains below 1(the poles of the system remains within the unit circle, the system is stable.)

(Problem 2)
Solve the difference equation and give the time-domain equation for the number of newly infected people.

(Solution)

$$
\begin{aligned}
y[n] &= \delta[n] + R_0 * y[n-1] \\
y[0] &= \delta[0] + R_o * y[-1] \\
\implies y[0] &= 1(y[-1]) \\
\&\ \ y[1] &= R_o^1 \\
\&\ \ y[2] &= R_o^2 \\
\therefore y[n] &= R_o^n
\end{aligned}
$$

(Problem 3)

Suppose R0 = 2.5, how many days will it take to reach 1 million new daily infections?.

(Solution) Given $y[n] = 1,000,000$, the value of n is:

$$
\begin{aligned}
R_o^n &= 1,000,000 \\
\&\ \ R_o &= 2.5 \\
\implies n &= ceil(\frac{log(1000000)}{log(2.5)}) \\
\implies n &= 16
\end{aligned}
$$

(Problem 4)

Estimate the value of R_0 in the initial phase of the first wave of infections in India using the data available from Covid-19 in India

(Solution)

Collect the data points from the *covid19india.org* website and tried to fit the curve for an exponential function.

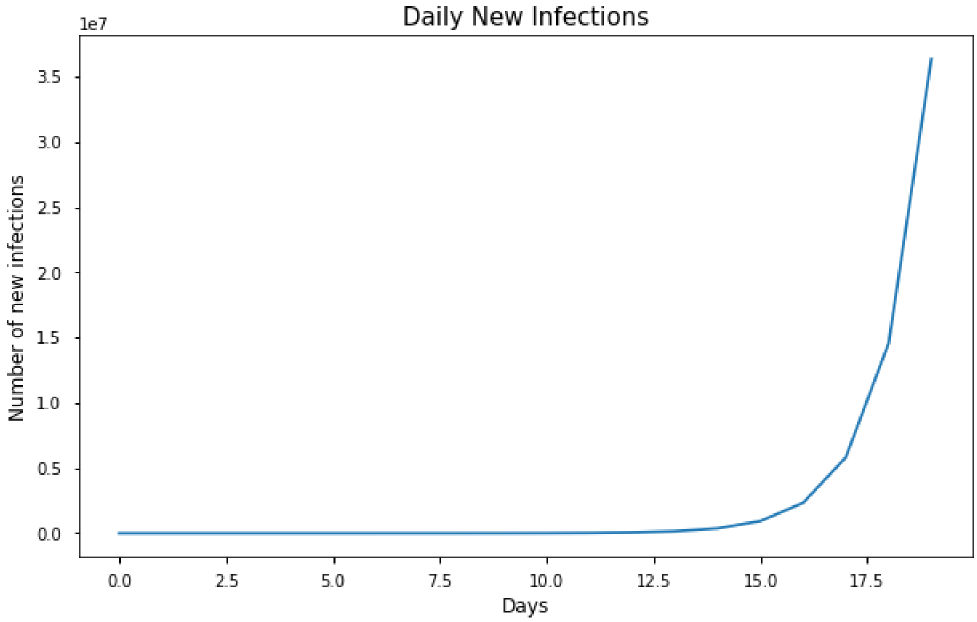
On fitting the values, the value of R_o was found to be 1.15.

(Problem 5)

With $R_o = 2.5$, we have to plot the new daily infections for the first n = 20 days. Also, we will design an integrator to obtain the total number of infections for the first n = 20 days.

(Solution)

Given below is the plot of daily infections for 20 days:



Differece equation for the integrator filter is $y[n] = y[n-1] + x[n]$.
Take the z-transform:

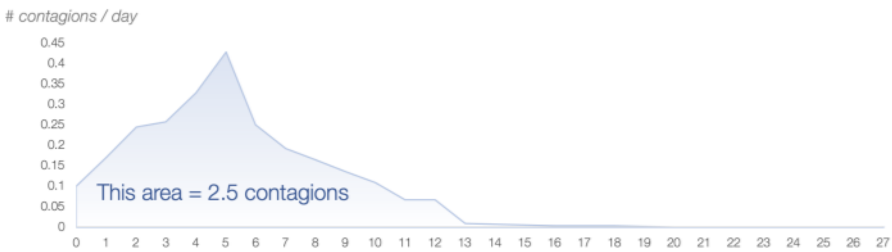
$$H(z) = \frac{z}{z-1}$$

Total number of cases (method elaborated in *.ipnb* file) is 60632979.

Question 2-Increasing the complexity

The actual infectiousness of SARS-CoV-2 doesn't follow the model we found out in the problem 1. A more closer infectiousness of it will have the following transfer function:

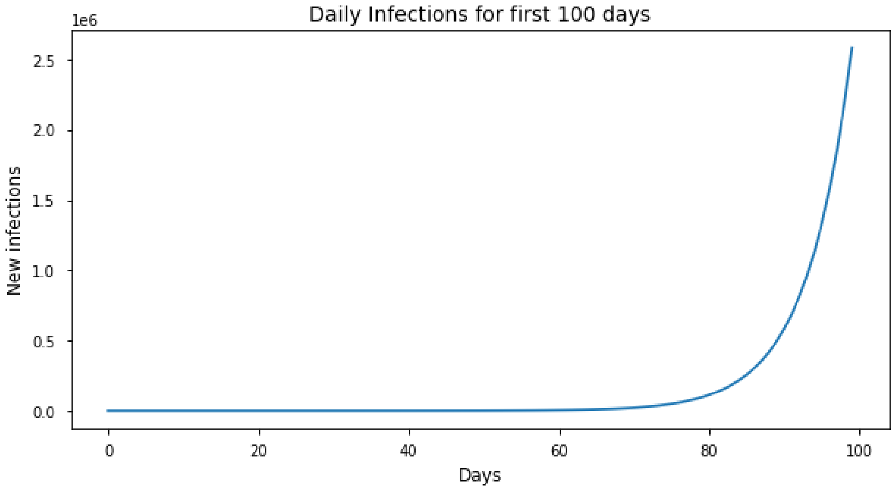
$$H_M(z) = \frac{1}{1 - \Sigma_{k=1}^{M} a_k z^{-k}}$$



Take z-transform, we get the impulse response which is also the time domain equation of number of infected people.

$$y(n) = \delta(n) + \Sigma_{k=1}^{M} a_k h(n-k)$$

(Problem 1) Plot the new daily infections for the first n = 100 days is given below:



After using the integrator filter, the total number of infections found on 100th day was 16995618.

(Problem 2) Comment on the differences between the trends that are obtained with the first-order model.

(Solution) The first-order model is purely exponential while model in problem 2 wasn't which is why the second model is more realistic.

Question 3-Effects of Social Distancing

IIR filter with transfer function :

$$H_M(z;) = \frac{1}{1 - \Sigma_{k=1}^{M}(1-)a_k z^{-k}}$$

(Problem 1)

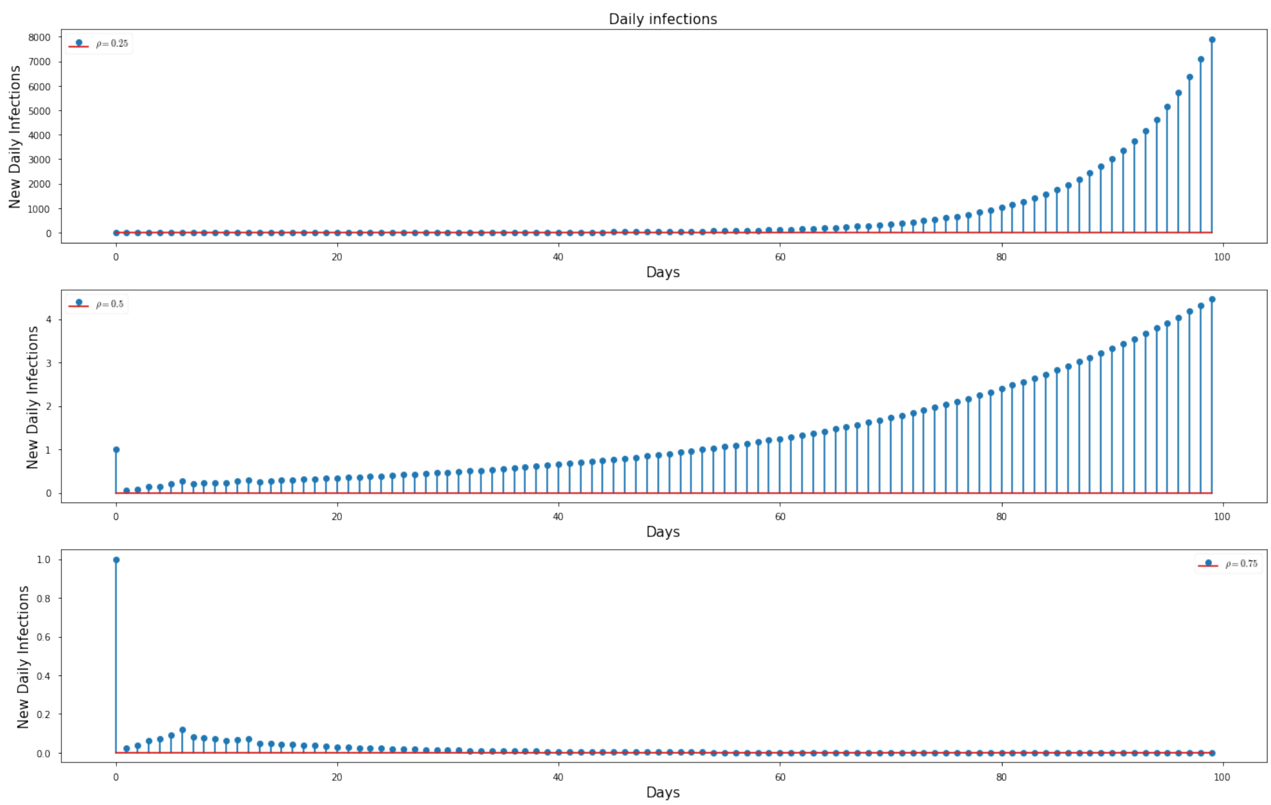
(Solution) ρ is the measure of degree by which the social interaction of every individual is reducced. ρ is multiplied with the coefficients which means that, more the value of less will be the value of $(1-\rho) * a_k$, which is justified because lesser social interaction should imply lesser infection.

When $\rho = 1$, this means that the social interaction is reduced completely.

(Problem 2)

Plot new daily infections for n = 100 days with the Kronecker delta as input for $\rho = 0.25, 0.5, 0.75$.

(Solution)

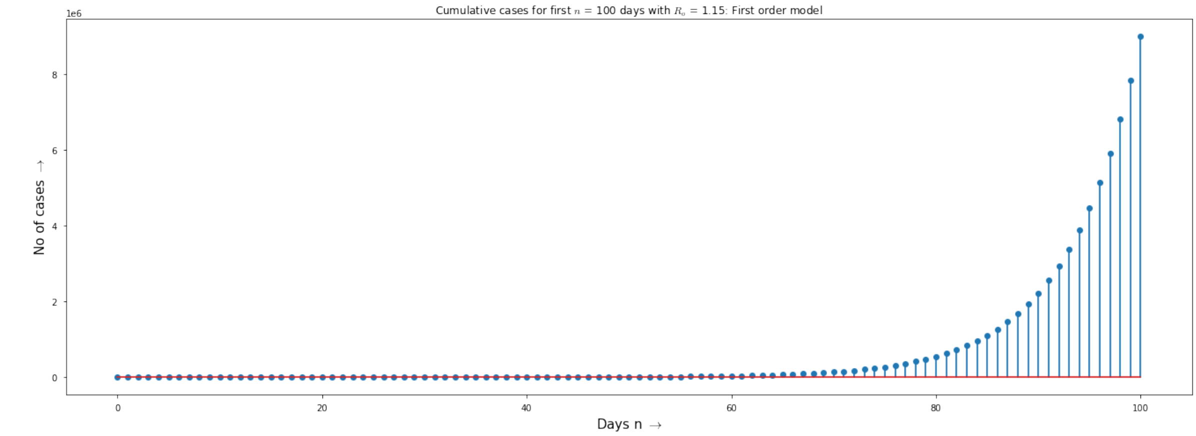


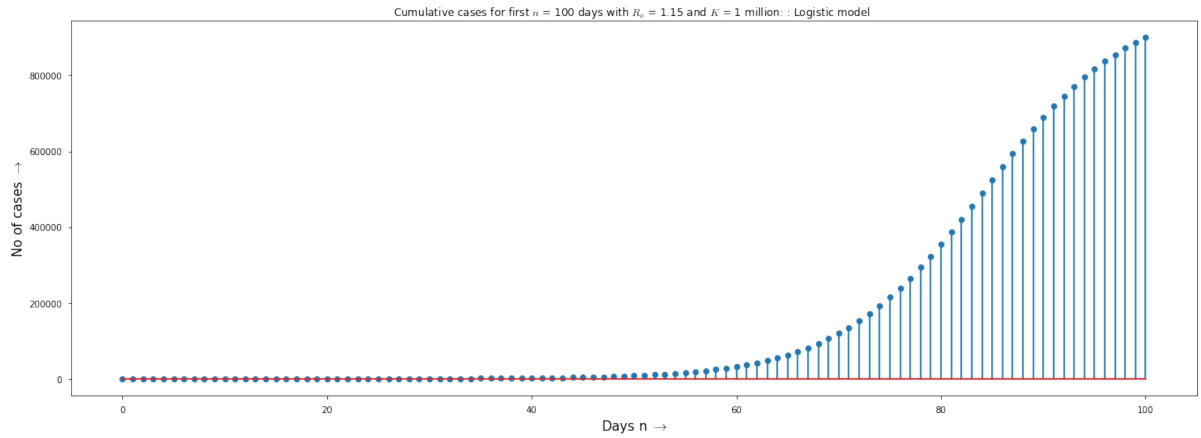
From the above plot social distancing has a great impact if we are able to implement it to certain extent.When the social interactions factor is 0.25, it doesn't affects much in decreasing the number of infections but as the factor increases, the number of infections decreases drastically.

Question 4-Saturation And Towards Normality

(Problem 1)
Plot the total number of cases using the logistic function as well as the first-order.

(Solution)





In the first order model, cumulative cases keep increasing whereas in case of logistic model, the slope of the curve slows down.

(Problem 2) Find the point of inflection using the global maximum of the first derivative and the zero-crossing of the second derivative.

(Solution)
According to the code:

```
The point of inflection obatained by first derivative:  85 day
The point of inflection obtained by second derivative:  85 day
```

The point of inflection by first derivative and second derivative is 85 days.

A Code Repositories

Refrain from including any or all code in this document. Upload codes to your repository and include the links to executed nbviewer files here as – The codes to reproduce the results can be found in the GitHub repository `https://github.com/TanmayRanaware/Digital-Signal-Processing-La`