EE313 Digital Signal Processing **Jul-Dec 2021**

Assignment2

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1 Solution

Part A

(Problem 1)

- IRR Filters
- Kaiser window based FIR filter

(Solution)

Specifications for the low pass filter:

- Sampling Frequency (F_s) = 700 Hz
- Passband ripple gain (δ_p) = -1 dB = 0.89
- Passband edge frequency ($\Omega_p^{'}$) = ($\Omega_c^{'}$) = 10Hz
- Stopband ripple gain (δ_s) = -40 dB = 0.01
- Stopband edge frequency ($\Omega_s^{'}$) = 20Hz = $40\pi\frac{rad}{sec}$
- Sampled Passband edge frequency ($\omega_p^{'}$) = $(\frac{\Omega_p^{'}}{F_s}) = \frac{\pi}{36}\frac{rad}{sample}$
- Sampled Stopband edge frequency ($\omega_s^{'}$) = $(\frac{\Omega_s^{'}}{F_s}) = \frac{\pi}{18}\frac{rad}{sample}$
- $\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = 2$

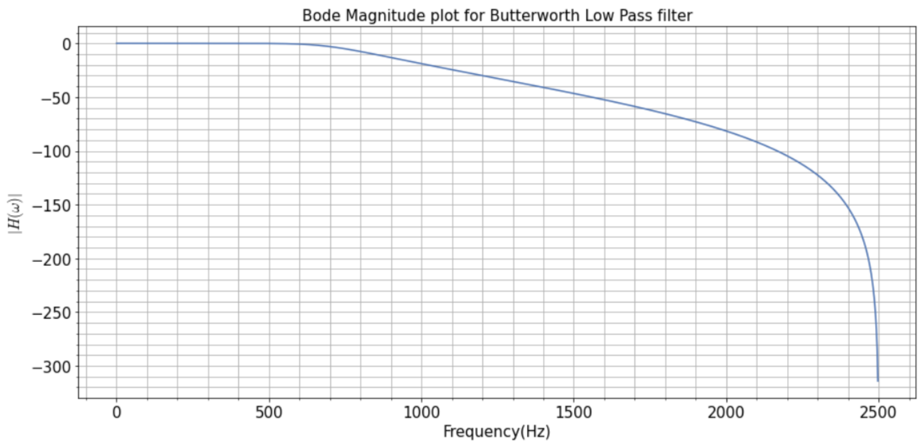
IIR Filter - Butterworth

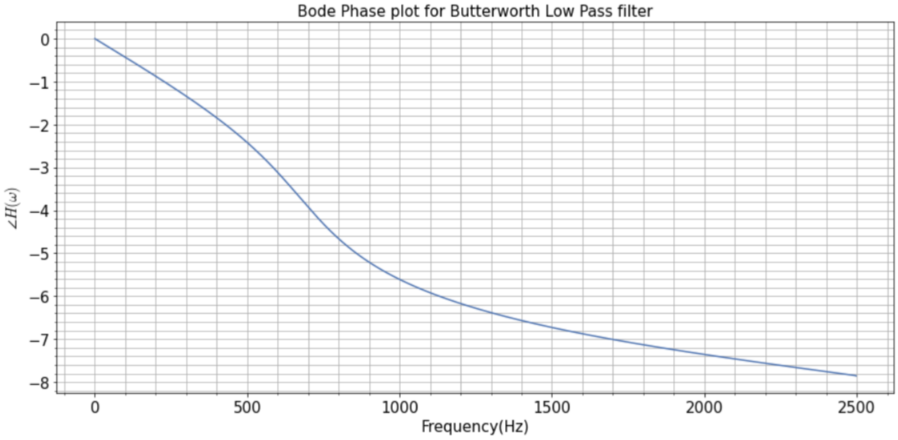
$$\delta_p = 0.89$$
$$\delta_s = 0.01$$
$$\epsilon = \sqrt{\frac{1-\delta_p^2}{\delta_p^2}}$$
$$\implies \epsilon = 0.5088$$
$$N = \frac{log(\frac{1}{\epsilon}\sqrt{\frac{1-\delta_s^2}{\delta_s^2}})}{log(\frac{\Omega_s}{\Omega_p})}$$
$$\implies N = 8$$

- $H_{LP}(s) = \frac{1}{(s^2+0.390181s+1)(s^2+1.111140s+1)(s^2+1.663s+1)(s^2+1.961571s+1)}$
- Now apply low pass to low pass filter transformation: s' = $\Omega_p \frac{s}{\Omega_p'}$
- $\Omega_p = \epsilon^{\frac{1}{N}} = 0.95935$; $s' = \frac{0.95935s}{20\pi}$
- Bilateral transformation to $H_{LP}(s')$ to get $H(z)$ where $s' \rightarrow \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

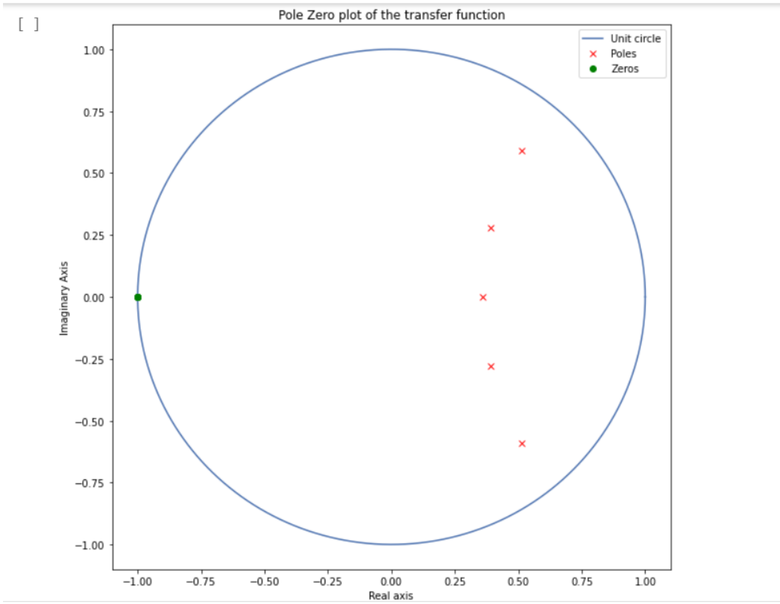
$$H(z) = \frac{4.798e+14}{\begin{array}{c} z^8 + 350.7z^7 + 6.148*10^4z^6 + 6.995*10^6z^5 + 5.627*10^8z^4 + \\ 3.274*10^10z^3 + 1.347*10^2z^2 + 3.595*10^13z + 4.798*10^{14} \end{array}}$$

Bode plot of the designed Butterworth Filter:



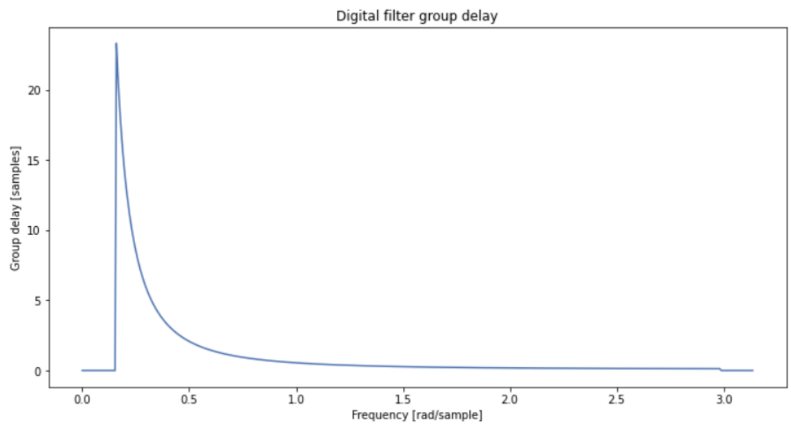


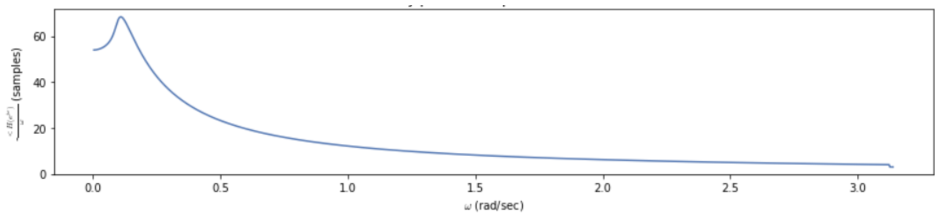
Pole zero plot of the designed Butterworth Filter:



All the poles lies inside of the circle and hence, the system is stable.
Number of filter coefficients =Filter order = 8.

Group and phase plot of Butterworth filter:




(samples)
ω (rad/sec)

IIR Filter - Chebyshev Filter

$$\delta_p = 0.89$$
$$\delta_s = 0.01$$
$$N = \frac{cosh^{-1}(\frac{1}{\epsilon}\sqrt{\frac{1-\delta_s^2}{\delta_s^2}})}{cosh^{-1}(\frac{\Omega_s}{\Omega_p})} = 5$$
$$\implies N = 5$$

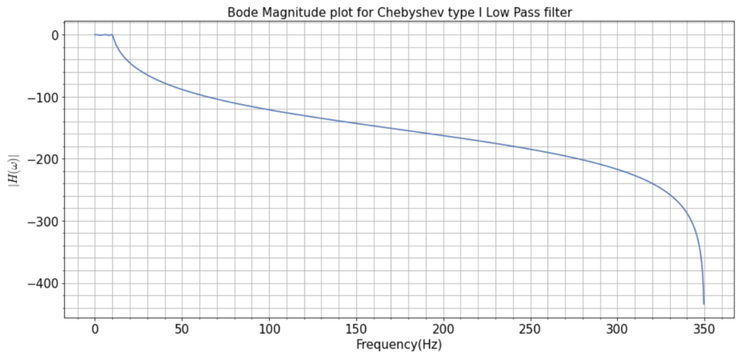
Number of filter coefficients =Filter order= 5.

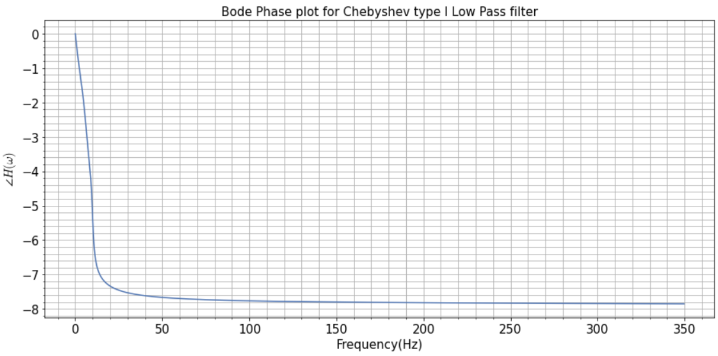
(Solution)

Chebyshev filter Transfer function :

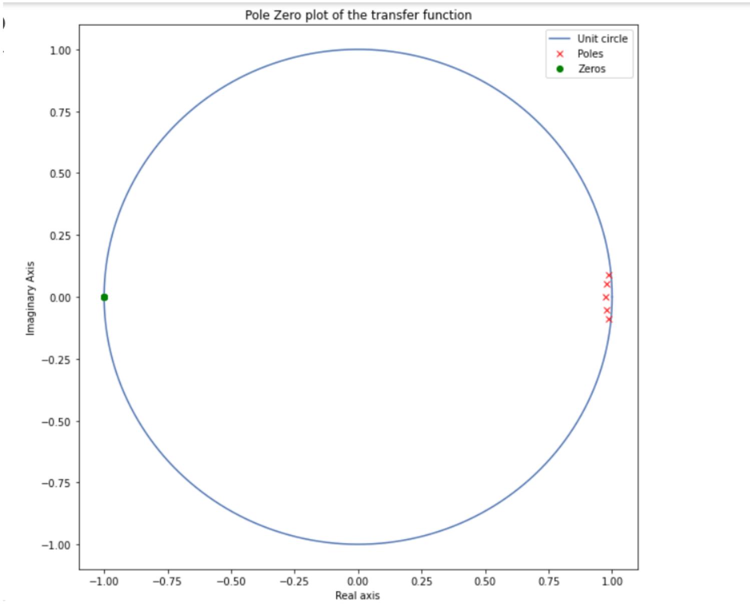
$$H(z) = \frac{1.207 * 10^8}{z^5 + 58.9z^4 + 6676z^3 + 2.422 * 10^5 z^2 + 9.071 * 10^6 z + 1.207 * 10^8}$$

Bode plot for the Chebyshev filter.

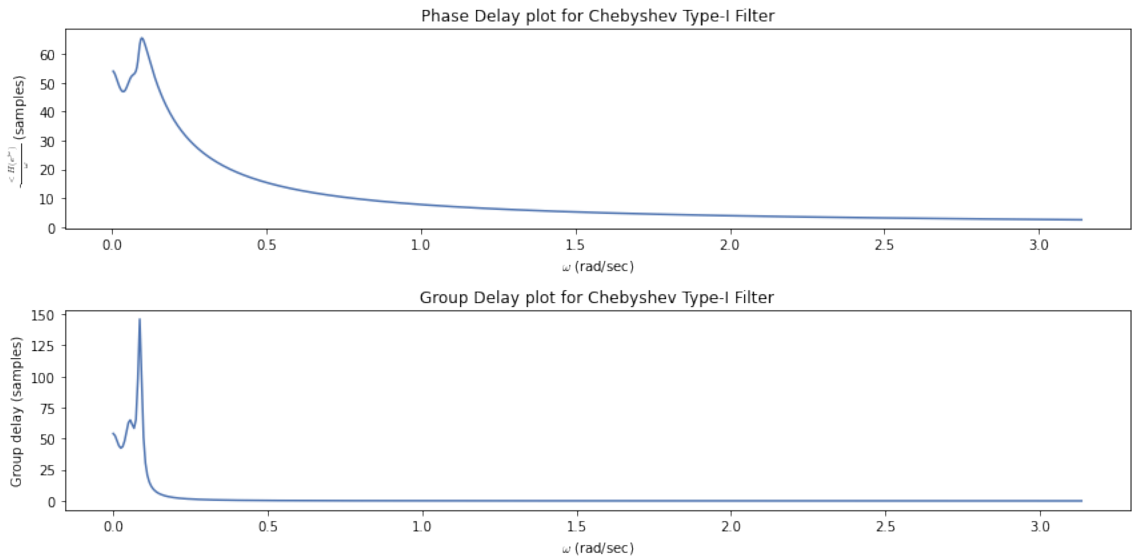




Given below is the pole zero plot for the Chebyshev filter.



Phase and group plot for Chebyshev Type-I filter.

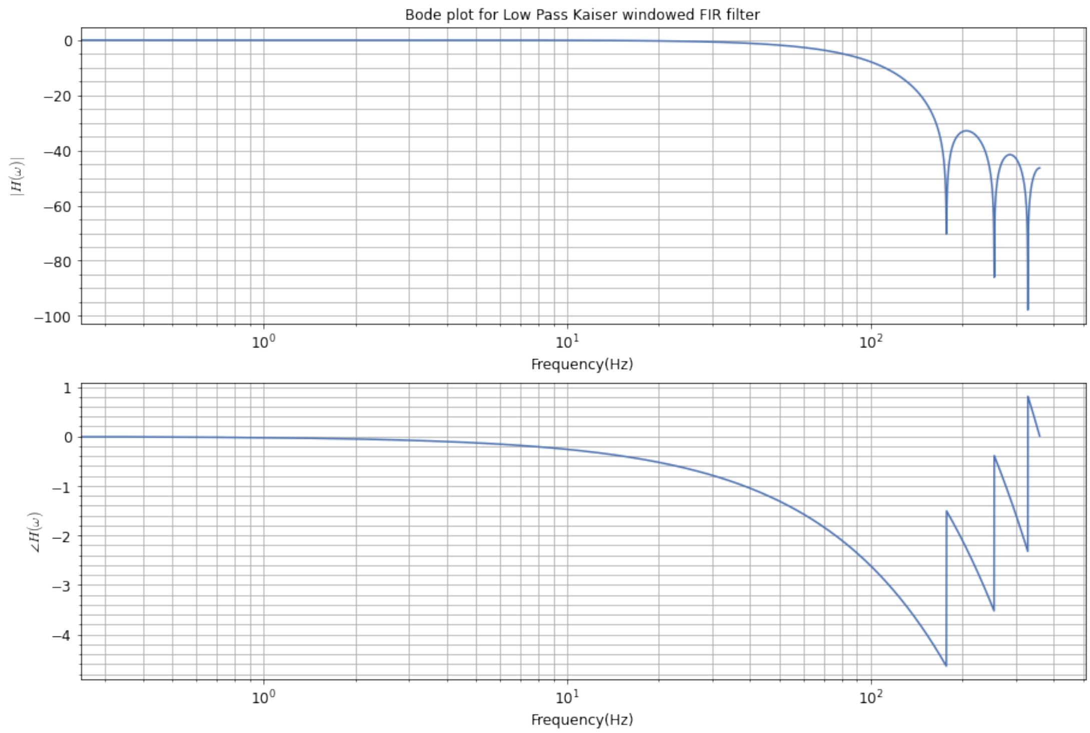


Butterworth has higher order but on the other hand, Chebyshev has more settling time this is observed

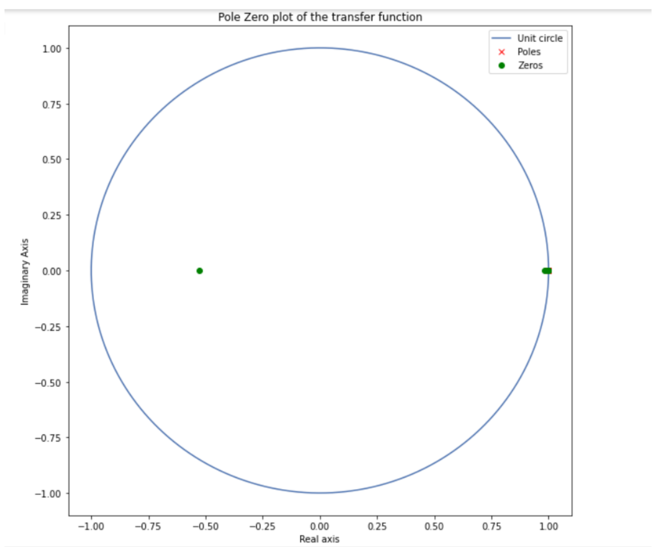
Kaiser windowed FIR Filter

- Filter length = 7
- Cuttoff frequency = $\frac{w_p + w_s}{2\pi} = 30$

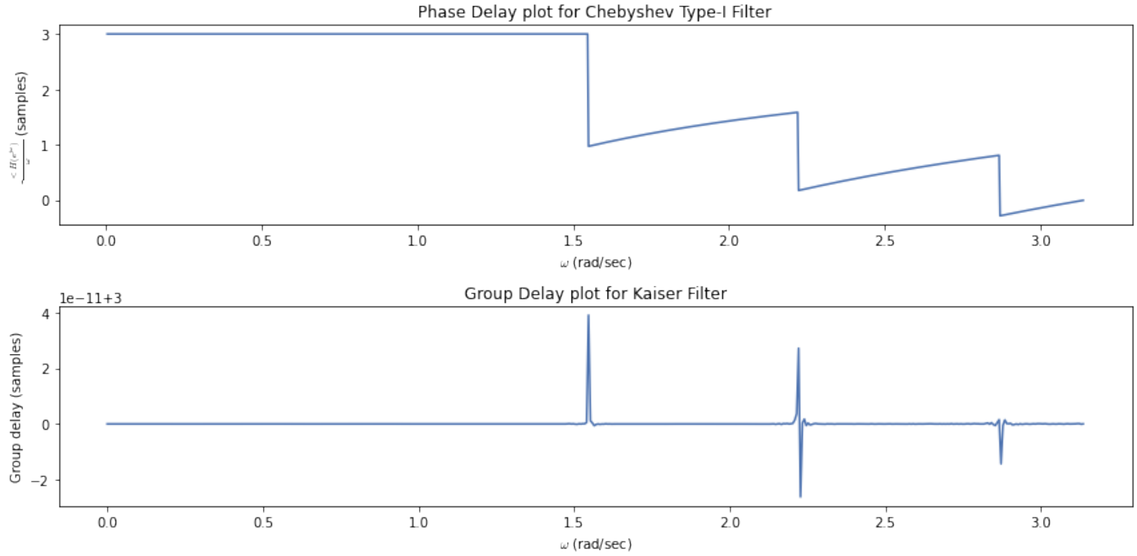
Using the *signal.scipy.firwin* and *signal.scipy.freqz* functions, we get the bode plot for Kaiser filter as follows:



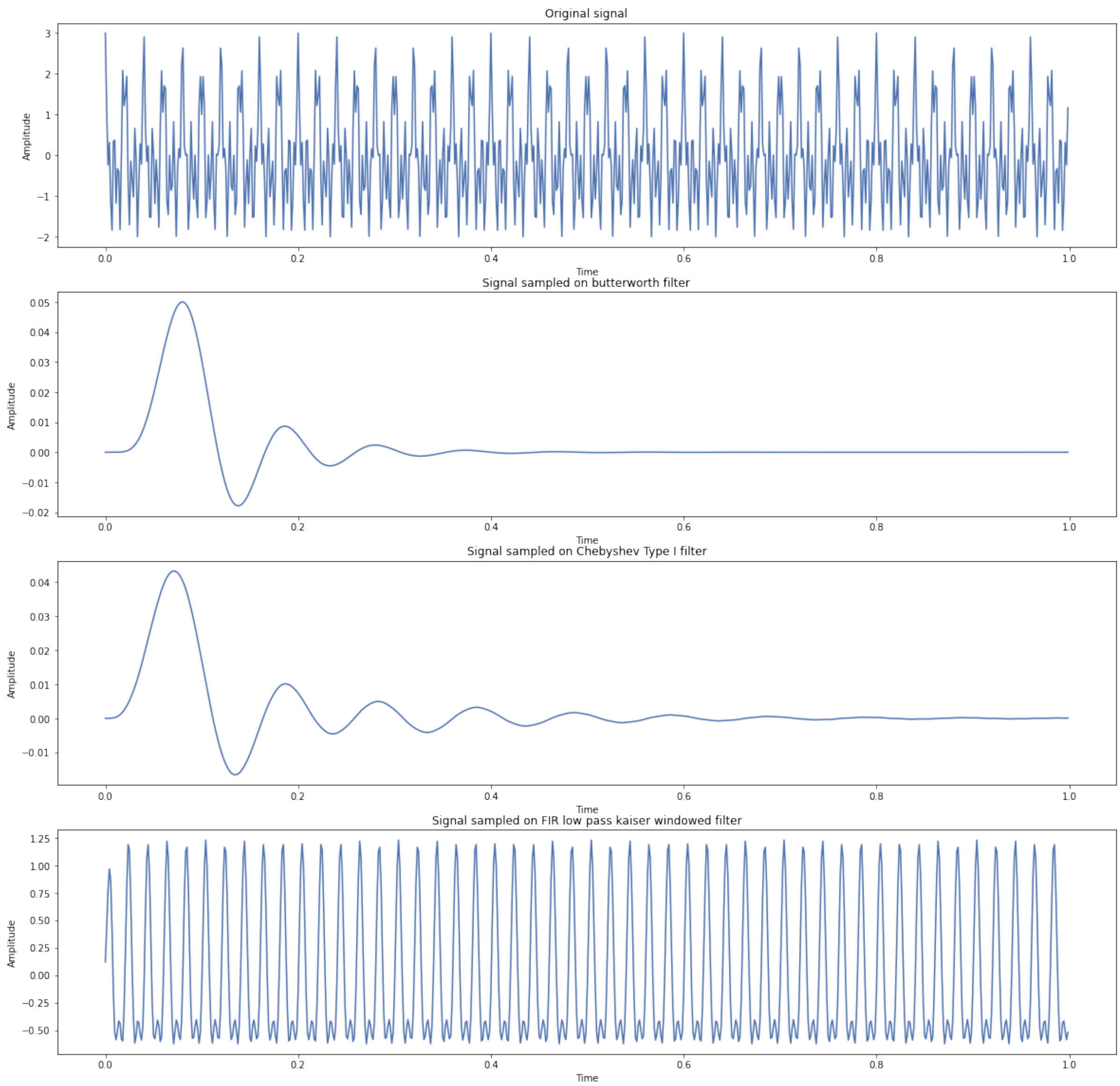
Pole zero plot for Kaiser window-based FIR filter:



Phase and group plot for the Kaiser window-based FIR filter:



Using random function $y(t) = \cos 100\pi t + \cos 200\pi t + \cos 300\pi t$, and passed it through Butterworth, Chebyshev Type-I and Kasier window-based FIR filter as shown below:

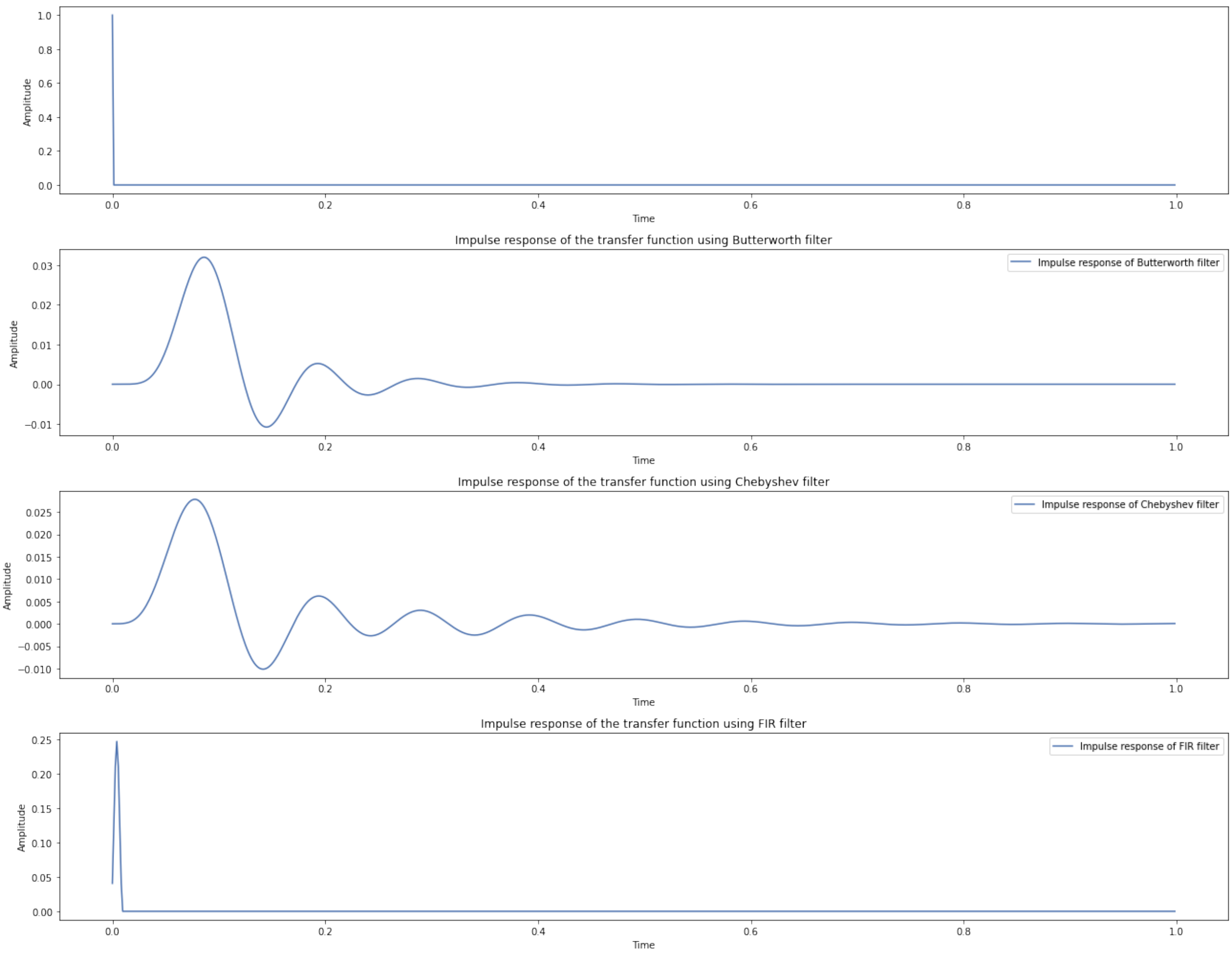


Comparing Butterwirth, Chebyshev Type-I and Kaiser Filter, it is observed that FIR filter has lowest peak overshoot and the least settling time. On the other hand, FIR requires more computation in comparision to Chebyshev and Butterworth filters.

Part B

Code for the implementation of FIR lattice filter is in the .ipnb file attached along with the report.

Given below is the impulse response of the Butterworth, Chebyshev type-I and lattice Filter.



A Code

The codes to reproduce the results can be found in the GitHub repository `https://github.com/TanmayRanaware/EE313Assignment2`.