

Assignment 1

EE22BTECH11053 - Tanmay Vishwasrao

Question 10.13.3.25

A coin is tossed 3 times. List the possible outcomes. Find the probability of getting (i) all heads (ii) at least 2 heads

Solution: As the coin is tossed 3 times we will get 8 different outcomes. Let us define a random variable X , where getting heads is success.

$$X \sim \text{Ber}(p) \quad (1)$$

$$(2)$$

The list of possible outcomes is:

$$\Pr(X) = \begin{cases} \frac{1}{8} & X = 0 \\ \frac{3}{8} & X = 1 \\ \frac{3}{8} & X = 2 \\ \frac{1}{8} & X = 3 \end{cases} \quad (3)$$

Suppose $X_i (1 \leq i \leq n)$ represent each of the n tosses. Then we can define Z as:

$$Z = \sum_{i=1}^n X_i \quad (4)$$

The pmf of Z is given by :

$$Z \sim \text{Bin}(n, p) \quad (5)$$

$$(6)$$

Parameter	Value	Description
n	3	no. of tosses
p	$\frac{1}{2}$	probability of getting heads

1) To get all heads: To get all heads Z should be equal to 3. So we need

$$\Pr(Z = 3) = \binom{n}{3} p^3 (1-p)^{n-3} \quad (7)$$

$$= \frac{1}{8} \quad (8)$$

2) To get atleast 2 heads: To get atleast two heads the value of $Z \geq 2$.

$$\Pr(Z \geq 2) = 1 - \Pr(Z < 2) \quad (9)$$

$$= F_Z(3) - F_Z(1) \quad (10)$$

$$\Pr(Z \geq 2) = \sum_{k=2}^3 \binom{n}{k} p^k (1-p)^{n-k} \quad (11)$$

$$= \frac{1}{2} \quad (12)$$