Assignment 1

EE22BTECH11053 - Tanmay Vishwasrao

Question 10.13.3.25
A coin is tossed 3 times. List the possible outcomes. Find the probability of getting (i) all heads (ii) at least 2 heads
Solution: As the coin is tossed 3 times we will get 8 different outcomes.The list of possble outcomes is HHH,HHT,HTH,THH,HTT,THT,TTH,TTT.
Let us define a random variable X, where getting heads is success(1).

$$X \sim \{0,1\} \tag{1}$$
$$\text{where} \Pr(X = 1) = p \tag{2}$$

Here p is the probability of getting head i.e. $\frac{1}{2}$. Suppose $X_i (1 \leq i \leq n)$ represent each of the n tosses. Then we can define Z as:

$$Z = \sum_{i=1}^{n} X_i \tag{3}$$

The CDF of Z is

$$F_Z(k) = \Pr(Z < k) \tag{4}$$
$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 1 \leq k \leq n \\ 1 & k \geq n \end{cases} \tag{5}$$

For our question the value of n is 3 as the coin is tossed 3 times.

1) all heads
   To get all heads Z should be equal to 3. So we need

$$\Pr(Z = 3) = \binom{n}{3} p^3 (1-p)^{n-3} \tag{6}$$
$$= \frac{1}{8} \tag{7}$$

2) atleast 2 heads
   To get atleast two heads the value of $Z \geq 2$.

$$\Pr(Z \geq 2) = 1 - \Pr(Z < 2) \tag{8}$$
$$= F_Z(3) - F_Z(1) \tag{9}$$
$$\Pr(Z \geq 2) = \sum_{k=2}^{3} \binom{n}{k} p^k (1-p)^{n-k} \tag{10}$$
$$= \frac{1}{2} \tag{11}$$