

# Assignment

EE22BTECH11053 - Tanmay Vishwasrao

## Question 59

Let  $X(t)$  be a Gaussian noise with power spectral density  $\frac{1}{2} W/Hz$ . If  $X(t)$  is input to an LTI system with impulse response  $e^{-tu(t)}$ . The average power of the system is (rounded off to two decimal places). (GATE EC 2023)

**Solution:** The output power spectral density of a LTI system with impulse response  $h(t)$  and input  $X(t)$  and input power spectral density  $S_X(f)$  is given by:

$$S_Y(f) = |H(f)|^2 S_X(f) \quad (1)$$

where  $H(f)$  is frequency response of the system.

$H(f)$  can be found by taking fourier transform of  $h(t)$

$$H(f) \xrightarrow{\mathcal{F}} \frac{1}{j2\pi f + 1} \quad (2)$$

The average power of a signal with power spectral density  $S(f)$  is given by:

$$P_Y(f) = \int_{-\infty}^{\infty} S_Y(f) df \quad (3)$$

Substituting  $S_Y(f)$  in the equation we get:

$$P_Y(f) = \int_{-\infty}^{\infty} |H(f)|^2 \cdot S_X(f) df \quad (4)$$

$$= \int_{-\infty}^{\infty} \left| \frac{1}{j2\pi f + 1} \right|^2 \cdot \frac{1}{2} df \quad (5)$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{(2\pi f)^2 + 1} df \quad (6)$$

$$= \frac{1}{2} * 2 \int_0^{\infty} \frac{1}{(2\pi f)^2 + (1)^2} df \quad (7)$$

$$= \int_0^{\infty} \frac{1}{(2\pi f)^2 + (1)^2} df \quad (8)$$

$$= \frac{1}{2\pi} \tan^{-1}(x) \Big|_0^{\infty} \quad (9)$$

$$= \frac{1}{2\pi} (\tan^{-1} \infty - \tan^{-1} 0) \quad (10)$$

$$= \frac{1}{2\pi} \left( \frac{\pi}{2} - 0 \right) \quad (11)$$

$$= \frac{1}{2\pi} \left( \frac{\pi}{2} \right) \quad (12)$$

$$= \frac{1}{4} \quad (13)$$

Rounded off to two decimal places, the average power of the system output is  $0.25W$ .