Assignment 1

EE22BTECH11053 - Tanmay Vishwasrao

Question 1.5.9
Find the other points of contact $\mathbf{E}_3$ and $\mathbf{F}_3$.

Solution
From the previous references we have the value of Incentre $\mathbf{I}$ is

$$\mathbf{I} = \begin{pmatrix} -1.4775 \\ -0.7949 \end{pmatrix} \tag{1}$$

And the value of inradius r is 1.8969.
The parametric equation of line $\mathbf{AB}$ is:

$$= \mathbf{A} + k(\mathbf{A} - \mathbf{B}) \tag{2}$$
$$= \mathbf{A} + k\mathbf{m} \tag{3}$$

The equation of Incircle is given by:

$$\|\mathbf{x} - \mathbf{I}\|^2 = r^2 \tag{4}$$

Since its a parametric equation we can substitute (3) as $\mathbf{x}$ in (4).

$$\|\mathbf{A} + k\mathbf{m} - \mathbf{I}\|^2 = r^2 \tag{5}$$

$$(\mathbf{A} + k\mathbf{m} - \mathbf{I})^\tau \cdot (\mathbf{A} + k\mathbf{m} - \mathbf{I}) = r^2 \tag{6}$$

On simplifying the above equation:

$$k^2 \|\mathbf{m}\|^2 + 2k(\mathbf{m})^\tau \cdot (\mathbf{A} - \mathbf{I}) + \|\mathbf{I}\|^2 + \|\mathbf{A}\|^2 - 2(\mathbf{A}^\tau \cdot \mathbf{I}) - r^2 = 0 \tag{7}$$

The discriminant of the above quadratic equation is 0. So solution of the equation is $-b/2a$.

$$\therefore k = -\frac{\mathbf{m}^\tau \cdot (\mathbf{A} - \mathbf{I})}{2\|\mathbf{m}\|^2} \tag{8}$$

Now the equation of $\mathbf{E}_3$ wil be

$$\mathbf{E}_3 = \mathbf{A} + k(\mathbf{m}) \tag{9}$$

where $\mathbf{m} = \mathbf{A} - \mathbf{B}$
Now putting the values of $\mathbf{A}, \mathbf{m}, \mathbf{I}$

$$\therefore k = -\frac{27.6463}{148} \tag{10}$$

$$k = -0.1867 \tag{11}$$

Now we can find $\mathbf{E}_3$ using above results:

$$\mathbf{E}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.1867 \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{12}$$

$$\mathbf{E}_3 = \begin{pmatrix} 0.066 \\ 0.307 \end{pmatrix} \tag{13}$$

Similarily, we will find the point $\mathbf{F}_3$. For the point $\mathbf{F}_3$ the value of $\mathbf{m} = \mathbf{A} - \mathbf{C}$.

$$\mathbf{F}_3 = \mathbf{A} + k(\mathbf{m}) \tag{14}$$

Using equation (8) we can find the value of k.

$$\therefore k = -\frac{18.1801}{64} \tag{15}$$

$$k = -0.2840 \tag{16}$$

Now we can find $\mathbf{F}_3$ using above results:

$$\mathbf{F}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.2840 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{17}$$

$$\mathbf{F}_3 = \begin{pmatrix} -0.136 \\ -2.136 \end{pmatrix} \tag{18}$$