Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1}$$

1 Vectors

Parameters	Values	Description
$\mathbf{m}_1$	$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$	**AB**
$\mathbf{m}_2$	$\begin{pmatrix} -6 \\ -1 \end{pmatrix}$	**BC**
$\mathbf{m}_3$	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$	**CA**
$\|\mathbf{B}-\mathbf{A}\|$	$\sqrt{32}$	length of AB
$\|\mathbf{C}-\mathbf{B}\|$	$\sqrt{37}$	length of BC
$\|\mathbf{A}-\mathbf{C}\|$	$\sqrt{13}$	length of CA
rank	3	non-collinear
$\mathbf{n}_1$	$\begin{pmatrix} 4 \\ -4 \end{pmatrix}$	**AB**
$\mathbf{c}_1$	12	
$\mathbf{n}_2$	$\begin{pmatrix} -1 \\ 6 \end{pmatrix}$	**BC**
$\mathbf{c}_2$	2	
$\mathbf{n}_3$	$\begin{pmatrix} -3 \\ -2 \end{pmatrix}$	**CA**
$\mathbf{c}_3$	6	
area	10	area of ΔABC
$\angle A$	78.69°	angles of triangle
$\angle B$	35.54°	
$\angle C$	65.77°	

TABLE 1: Vectors.

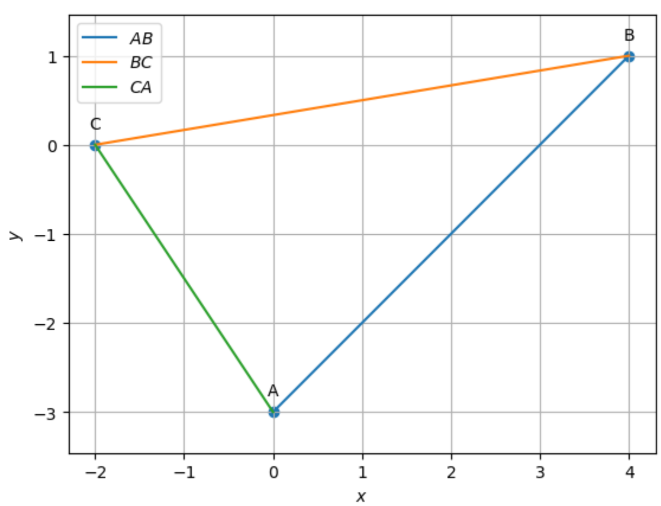

Fig. 1: Triangle with vertices ABC

2 Median

Parameters	Values	Description
$\mathbf{D}$	$\begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$	$\frac{\mathbf{B}+\mathbf{C}}{2}$
$\mathbf{E}$	$\begin{pmatrix} -1 \\ -1.5 \end{pmatrix}$	$\frac{\mathbf{C}+\mathbf{A}}{2}$
$\mathbf{F}$	$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$	$\frac{\mathbf{A}+\mathbf{B}}{2}$
$\mathbf{m}_4$	$\begin{pmatrix} 1 \\ 3.5 \end{pmatrix}$	Line AD
$\mathbf{n}_4$	$\begin{pmatrix} 3.5 \\ -1 \end{pmatrix}$	
c_4	3	
$\mathbf{m}_5$	$\begin{pmatrix} -5 \\ -2.5 \end{pmatrix}$	Line BE
$\mathbf{n}_5$	$\begin{pmatrix} -2.5 \\ 5 \end{pmatrix}$	
c_5	-5	
$\mathbf{m}_6$	$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$	Line CF
$\mathbf{n}_6$	$\begin{pmatrix} -1 \\ -4 \end{pmatrix}$	
c_6	2	
$\mathbf{G}$	$\frac{2}{3}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	centroid of ΔABC
AG : DG	2	Division by centroid
BG : EG	2	
CG : FG	2	
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	points are collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$	2	
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$	2	
$\mathbf{F}-\mathbf{A}$	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	$\therefore AFDE$ is a quadrilateral
$\mathbf{D}-\mathbf{E}$	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	

TABLE 2: Medians.

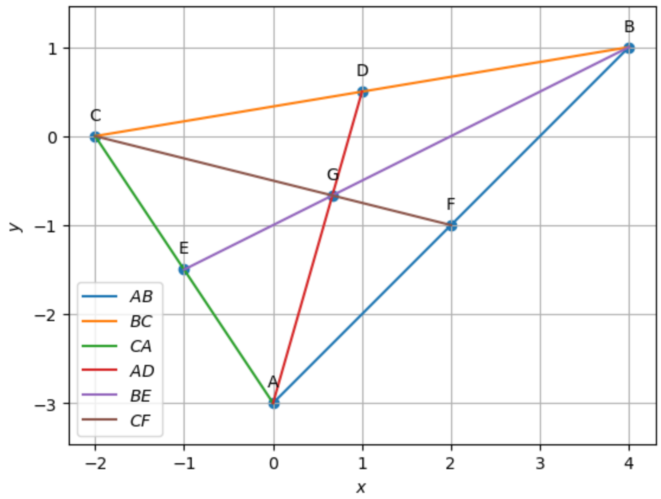

Fig. 2: Medians of triangle ABC

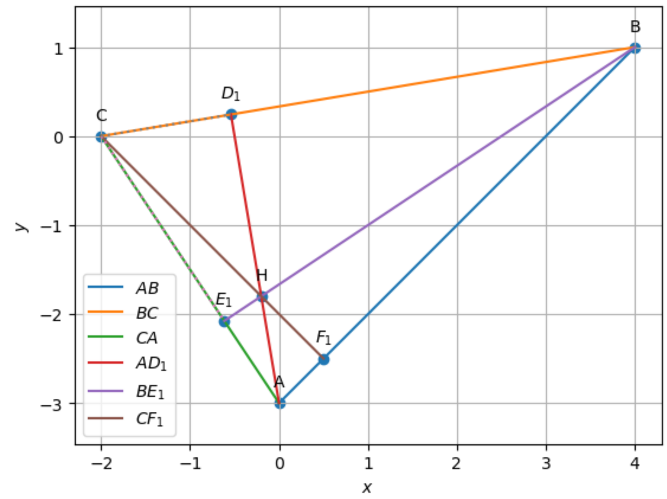

Fig. 3: Altitude of triangle ABC

3 Altitude

Parameters	Values	Description
$\mathbf{D}_1$	$\begin{pmatrix}-0.54\\0.24\end{pmatrix}$	Foots of Altitude
$\mathbf{E}_1$	$\begin{pmatrix}-0.62\\-2.08\end{pmatrix}$	
$\mathbf{F}_1$	$\begin{pmatrix}0.5\\-2.5\end{pmatrix}$	
$\mathbf{m}_7$	$\begin{pmatrix}-0.54\\3.24\end{pmatrix}$	Line AD_1
$\mathbf{n}_7$	$\begin{pmatrix}3.24\\0.54\end{pmatrix}$	
c_7	-1.62	
$\mathbf{m}_8$	$\begin{pmatrix}-4.62\\-3.08\end{pmatrix}$	Line BE_1
$\mathbf{n}_8$	$\begin{pmatrix}-3.08\\4.62\end{pmatrix}$	
c_8	-7.69	
$\mathbf{m}_9$	$\begin{pmatrix}2.5\\-2.5\end{pmatrix}$	Line CF_1
$\mathbf{n}_9$	$\begin{pmatrix}-2.5\\-2.5\end{pmatrix}$	
c_9	5	
$\mathbf{H}$	$\begin{pmatrix}-0.2\\-1.8\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

Parameters	Values	Description
$\mathbf{m}_{10}$	$\begin{pmatrix}-1\\6\end{pmatrix}$	Line OD
$\mathbf{n}_{10}$	$\begin{pmatrix}-6\\-1\end{pmatrix}$	
c_{10}	-6.5	
$\mathbf{m}_{11}$	$\begin{pmatrix}3\\2\end{pmatrix}$	line OE
$\mathbf{n}_{11}$	$\begin{pmatrix}-2\\3\end{pmatrix}$	
c_{11}	-2.5	
$\mathbf{m}_{12}$	$\begin{pmatrix}-4\\4\end{pmatrix}$	line OF
$\mathbf{n}_{12}$	$\begin{pmatrix}-4\\-4\end{pmatrix}$	
c_{12}	-4	
$\mathbf{O}$	$\begin{pmatrix}1.1\\-0.1\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	3.1	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$	3.1	
$\|\mathbf{O}-\mathbf{C}\|$	3.1	
$\angle BOC$	157.38°	$\angle BOC = 2\angle BAC$
$\angle BAC$	78.69°	
$\angle AOC$	71.07°	$\angle AOC = 2\angle ABC$
$\angle ABC$	35.5°	
$\angle AOB$	228.45°	$\angle AOB = 2\angle BCA$
$\angle BCA$	65.77°	

TABLE 4: Perpendicular Bisector.

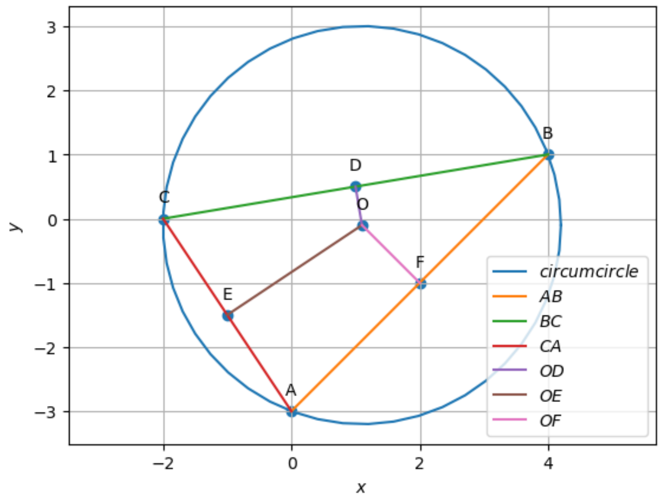

Fig. 4: Circumcircle of triangle ABC

5 Angle Bisector

Parameters	Values	Description
$\mathbf{m}_{13}$	$\begin{pmatrix}-0.15\\-1.54\end{pmatrix}$	AI
$\mathbf{n}_{13}$	$\begin{pmatrix}-1.54\\0.15\end{pmatrix}$	
c_{13}	-0.46	
$\mathbf{m}_{14}$	$\begin{pmatrix}-1.69\\-0.87\end{pmatrix}$	BI
$\mathbf{n}_{14}$	$\begin{pmatrix}0.87\\-1.69\end{pmatrix}$	
c_{14}	1.79	
$\mathbf{m}_{15}$	$\begin{pmatrix}-1.54\\0.67\end{pmatrix}$	CI
$\mathbf{n}_{15}$	$\begin{pmatrix}-0.67\\-1.54\end{pmatrix}$	
c_{15}	1.33	
$\mathbf{I}$	$\begin{pmatrix}0.20\\-0.95\end{pmatrix}$	Incentre
$\mathbf{D}_3$	$\begin{pmatrix}-0.01\\0.33\end{pmatrix}$	POC with BC
$\mathbf{E}_3$	$\begin{pmatrix}-0.88\\-1.68\end{pmatrix}$	POC with AC
$\mathbf{F}_3$	$\begin{pmatrix}1.12\\-1.88\end{pmatrix}$	POC with AB
$\|\mathbf{I}-\mathbf{D}_3\|$	1.3	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E}_3\|$	1.3	
$\|\mathbf{I}-\mathbf{F}_3\|$	1.3	
r	1.3	
$\angle BAI$	39.34°	$\angle BAI = \angle CAI$
$\angle CAI$	39.34°	
$\angle ABI$	17.77°	$\angle ABI = \angle CBI$
$\angle CBI$	17.77°	
$\angle ACI$	32.89°	$\angle ACI = \angle BCI$
$\angle BCI$	32.89°	

TABLE 5: Angle Bisector.

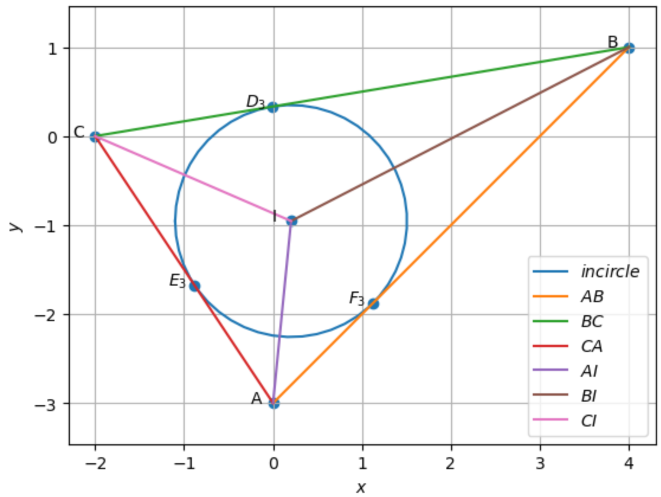

Fig. 5: Incircle of triangle ABC