Assignment 1

EE22BTECH11053 - Tanmay Vishwasrao

Question 1.5.9
Find the other points of contact $\mathbf{E}_3$ and $\mathbf{F}_3$.
Solution:
From the previous references we have the value of
Incentre $\mathbf{I}$ is

$$\mathbf{I} = \begin{pmatrix} -1.4775 \\ -0.7949 \end{pmatrix} \tag{1}$$

And the value of inradius r is 1.8969.
The parametric equation of line is:

$$= \mathbf{A} + k\mathbf{m} \tag{2}$$

The equation of Incircle is given by:

$$\|\mathbf{x} - \mathbf{I}\|^2 = r^2 \tag{3}$$

Since its a parametric equation we can substitute (3)
as $\mathbf{x}$ in (4).

$$\|\mathbf{A} + k\mathbf{m} - \mathbf{I}\|^2 = r^2 \tag{4}$$

$$(\mathbf{A} + k\mathbf{m} - \mathbf{I})^\top (\mathbf{A} + k\mathbf{m} - \mathbf{I}) = r^2 \tag{5}$$

On simplifying the above equation:

$$k^2 \|\mathbf{m}\|^2 + 2k(\mathbf{m})^\top (\mathbf{A} - \mathbf{I}) + \|\mathbf{I}\|^2$$
$$+ \|\mathbf{A}\|^2 - 2(\mathbf{A}^\top \mathbf{I}) - r^2 = 0 \tag{6}$$

1) Finding the point $\mathbf{E}_3$.
   The equation of $\mathbf{E}_3$:

$$\mathbf{E}_3 = \mathbf{A} + k(\mathbf{m}) \tag{7}$$

   where $\mathbf{m} = \mathbf{A} - \mathbf{B}$
   Now putting the values of $\mathbf{A}, \mathbf{m}, \mathbf{I}$ in eq. (7)

$$74k^2 + 27.6463k + 2.5821 = 0 \tag{8}$$

   Discriminant of the above equation is:

$$D = (27.6463)^2 - 4(74)(2.5821) \tag{9}$$
$$D = 764.3179 - 764.3179 \tag{10}$$
$$D = 0 \tag{11}$$

   Since the discriminant is 0. The value of k will
   be:

$$k = -\frac{2(\mathbf{m})^\top (\mathbf{A} - \mathbf{I})}{2 \|\mathbf{m}\|^2} \tag{12}$$

.

$$\therefore k = -\frac{27.6463}{148} \tag{13}$$

$$k = -0.1867 \tag{14}$$

Now we can find $\mathbf{E}_3$ using above results:

$$\mathbf{E}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.1867 \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{15}$$

$$\mathbf{E}_3 = \begin{pmatrix} 0.066 \\ 0.307 \end{pmatrix} \tag{16}$$

2) Finding the point $\mathbf{F}_3$.
   For the point $\mathbf{F}_3$ the value of $\mathbf{m} = \mathbf{A} - \mathbf{C}$.

$$\mathbf{F}_3 = \mathbf{A} + k(\mathbf{m}) \tag{17}$$

   Now putting the values of $\mathbf{A}, \mathbf{m}, \mathbf{I}$ in eq. (7)

$$32k^2 + 18.1801k + 2.5821 = 0 \tag{18}$$

   Discriminant of the above equation is:

$$D = (18.1801)^2 - 4(32)(2.5821) \tag{19}$$
$$D = 330.51 - 330.51 \tag{20}$$
$$D = 0 \tag{21}$$

   Since the discriminant is 0. The value of k will
   be:

$$k = -\frac{2(\mathbf{m})^\top (\mathbf{A} - \mathbf{I})}{2 \|\mathbf{m}\|^2} \tag{22}$$

$$\therefore k = -\frac{18.1801}{64} \tag{23}$$

$$k = -0.2840 \tag{24}$$

Now we can find $\mathbf{F}_3$ using above results:

$$\mathbf{F}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.2840 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{25}$$

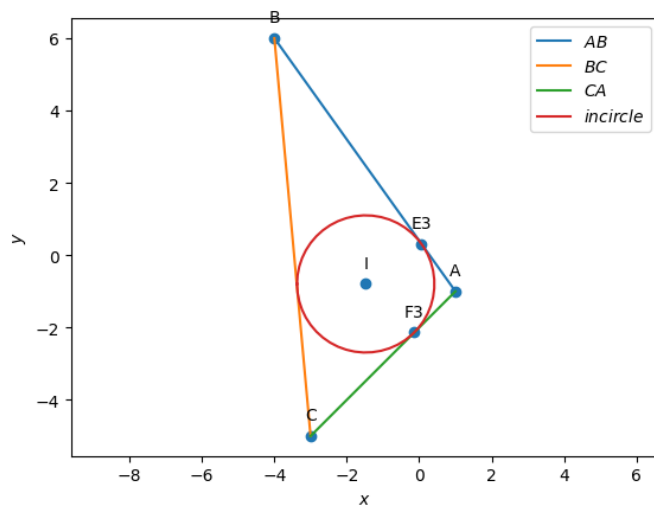$$\mathbf{F}_3 = \begin{pmatrix} -0.136 \\ -2.136 \end{pmatrix} \tag{26}$$

Fig. 2. Points $\mathbf{E}_3$ and $\mathbf{F}_3$ plotted using python