Assignment 1

EE22BTECH11053-Tanmay Vishwasrao

Question 1.5.9
Find the other points of contact $\mathbf{E_3}$ and $\mathbf{F_3}$.

Solution
From the previous references we have the value of Incentre $\mathbf{I}$ is

$$\mathbf{I} = \begin{pmatrix} -1.4775 \\ -0.7949 \end{pmatrix} \tag{1}$$

The parametric equation of line AB is:

$$\mathbf{A} + k(\mathbf{A} - \mathbf{B}) \tag{2}$$

Now for the point E_3 let the value of k be k_1.

$$\mathbf{E_3} = \mathbf{A} + k_1(\mathbf{A} - \mathbf{B}) \tag{3}$$

Since the line AB and IE_3 are perpendicular to each other the dot product of the two lines will be 0.

$$(\mathbf{I} - \mathbf{E_3}) \cdot (\mathbf{A} - \mathbf{B}) = 0 \tag{4}$$

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{B} - \mathbf{E_3} \cdot \mathbf{A} + \mathbf{E_3} \cdot \mathbf{A} = 0 \tag{5}$$

Substituting the value of E_3 in the above equation and solving it:

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{B} - (\mathbf{A} + k_1(\mathbf{A} - \mathbf{B})) \cdot \mathbf{A} + (\mathbf{A} + k_1(\mathbf{A} - \mathbf{B})) \cdot \mathbf{A} = 0 \tag{6}$$

$$k_1(\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = (\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{B}) \tag{7}$$

$$k_1 = \frac{(\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{B})}{(\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B})} \tag{8}$$

The value of k_1 comes out to be -0.1867. Now we can find E_3 using the above results:

$$\mathbf{E_3} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.1867 \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{9}$$

Therefore, the point E_3 is

$$\mathbf{E_3} = \begin{pmatrix} 0.066 \\ 0.307 \end{pmatrix} \tag{10}$$

Similarily, we will find the point $\mathbf{F_3}$. For the point $\mathbf{F_3}$ let the value of k be k_2.

$$\mathbf{F_3} = \mathbf{A} + k_2(\mathbf{A} - \mathbf{C}) \tag{11}$$

$$(\mathbf{I} - \mathbf{F_3}) \cdot (\mathbf{A} - \mathbf{C}) = 0 \tag{12}$$

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{C} - \mathbf{F_3} \cdot \mathbf{A} + \mathbf{F_3} \cdot \mathbf{A} = 0 \tag{13}$$

Substituting the value of $\mathbf{F_3}$ in the above equation and solving it:

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{C} - (\mathbf{A} + k_2(\mathbf{A} - \mathbf{C})) \cdot \mathbf{A} + (\mathbf{A} + k_2(\mathbf{A} - \mathbf{C})) \cdot \mathbf{A} = 0 \tag{14}$$

$$k_2 = (\mathbf{A} - \mathbf{C}) \cdot (\mathbf{A} - \mathbf{C}) = (\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{C}) \tag{15}$$

$$k_2 = \frac{(\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{C})}{(\mathbf{A} - \mathbf{C}) \cdot (\mathbf{A} - \mathbf{C})} \tag{16}$$

The value of k_2 comes out to be $- -0.2840$. Now we can find $\mathbf{F_3}$ using the above results:

$$\mathbf{F_3} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.2840 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{17}$$

Therefore, the point $\mathbf{F_3}$ is