

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (1)$$

1 VECTORS

Parameters	Values	Description
\mathbf{m}_1	$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$	AB
\mathbf{m}_2	$\begin{pmatrix} -6 \\ -1 \end{pmatrix}$	BC
\mathbf{m}_3	$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$	CA
$\ \mathbf{B} - \mathbf{A}\ $	$\sqrt{32}$	length of AB
$\ \mathbf{C} - \mathbf{B}\ $	$\sqrt{37}$	length of BC
$\ \mathbf{A} - \mathbf{C}\ $	$\sqrt{13}$	length of CA
rank	3	non-collinear
\mathbf{n}_1	$\begin{pmatrix} 4 \\ -4 \end{pmatrix}$	AB
\mathbf{c}_1	12	
\mathbf{n}_2	$\begin{pmatrix} -1 \\ 6 \end{pmatrix}$	BC
\mathbf{c}_2	2	
\mathbf{n}_3	$\begin{pmatrix} -3 \\ -2 \end{pmatrix}$	CA
\mathbf{c}_3	6	
area	10	area of $\triangle ABC$
$\angle A$	78.69°	angles of triangle
$\angle B$	35.54°	
$\angle C$	65.77°	

TABLE 1: Vectors.

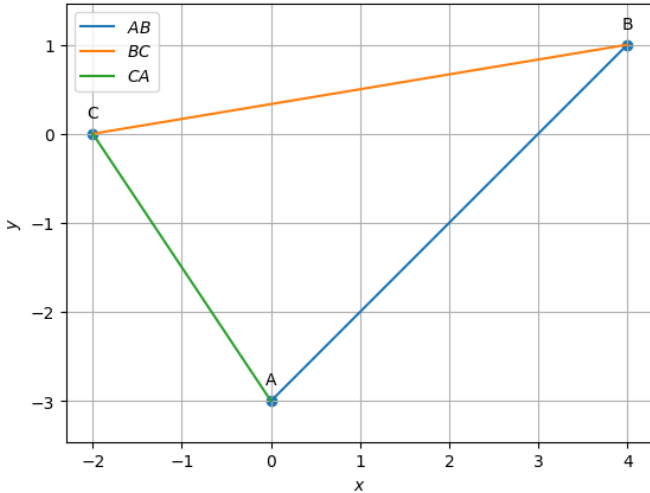


Fig. 1: triangle plotted using python

2 MEDIAN

Parameters	Values	Description
D	$\begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$	$\frac{\mathbf{B}+\mathbf{C}}{2}$
E	$\begin{pmatrix} -1 \\ -1.5 \end{pmatrix}$	$\frac{\mathbf{C}+\mathbf{A}}{2}$
F	$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$	$\frac{\mathbf{A}+\mathbf{B}}{2}$
\mathbf{m}_4	$\begin{pmatrix} 1 \\ 3.5 \end{pmatrix}$	Line AD
\mathbf{n}_4	$\begin{pmatrix} 3.5 \\ -1 \end{pmatrix}$	
c_4	3	
\mathbf{m}_5	$\begin{pmatrix} -5 \\ -2.5 \end{pmatrix}$	Line BE
\mathbf{n}_5	$\begin{pmatrix} -2.5 \\ 5 \end{pmatrix}$	
c_5	-5	
\mathbf{m}_6	$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$	Line CF
\mathbf{n}_6	$\begin{pmatrix} -1 \\ -4 \end{pmatrix}$	
c_6	2	
G	$\frac{2}{3} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	centroid of $\triangle ABC$
AG : DG	2	Division by centroid
BG : EG	2	
CG : FG	2	
$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	points are collinear
$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$	2	
$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$	2	
$\mathbf{F} - \mathbf{A}$	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	$\therefore AFDE$ is a quadrilateral
$\mathbf{D} - \mathbf{E}$	$\begin{pmatrix} 2 \\ 2 \end{pmatrix}$	

TABLE 2: Medians.

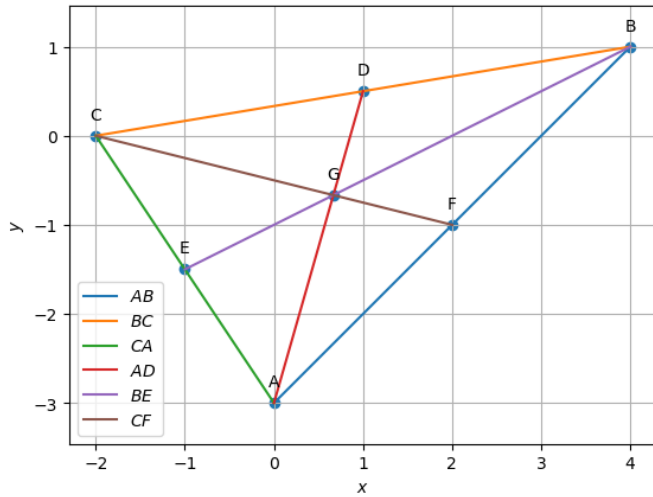


Fig. 2: triangle plotted using python

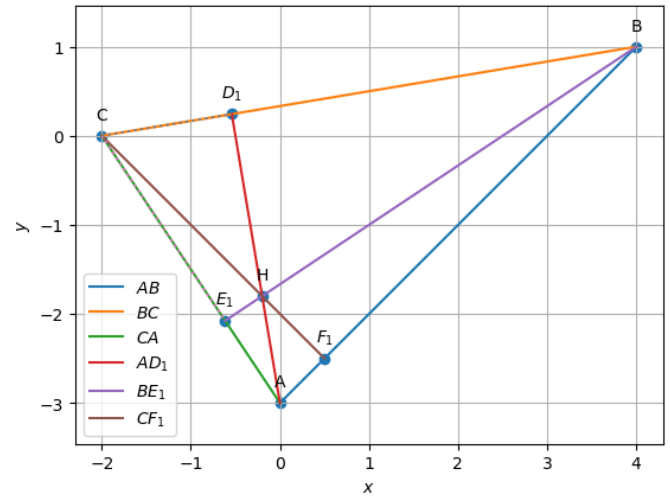


Fig. 3: triangle plotted using python

3 ALTITUDE

Parameters	Values	Description
D_1	$\begin{pmatrix} -0.54 \\ 0.24 \end{pmatrix}$	Foots of Altitude
E_1	$\begin{pmatrix} -0.62 \\ -2.08 \end{pmatrix}$	
F_1	$\begin{pmatrix} 0.5 \\ -2.5 \end{pmatrix}$	
m_7	$\begin{pmatrix} -0.54 \\ 3.24 \end{pmatrix}$	Line AD_1
n_7	$\begin{pmatrix} 3.24 \\ 0.54 \end{pmatrix}$	
c_7	-1.62	
m_8	$\begin{pmatrix} -4.62 \\ -3.08 \end{pmatrix}$	Line BE_1
n_8	$\begin{pmatrix} -3.08 \\ 4.62 \end{pmatrix}$	
c_8	-7.69	
m_9	$\begin{pmatrix} 2.5 \\ -2.5 \end{pmatrix}$	Line CF_1
n_9	$\begin{pmatrix} -2.5 \\ -2.5 \end{pmatrix}$	
c_9	5	
H	$\begin{pmatrix} -0.2 \\ -1.8 \end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 PERPENDICULAR BISECTOR

Parameters	Values	Description
m_{10}	$\begin{pmatrix} -1 \\ 6 \end{pmatrix}$	Line OD
n_{10}	$\begin{pmatrix} -6 \\ -1 \end{pmatrix}$	
c_{10}	-6.5	
m_{11}	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	line OE
n_{11}	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$	
c_{11}	-2.5	
m_{12}	$\begin{pmatrix} -4 \\ 4 \end{pmatrix}$	line OF
n_{12}	$\begin{pmatrix} -4 \\ -4 \end{pmatrix}$	
c_{12}	-4	
O	$\begin{pmatrix} 1.1 \\ -0.1 \end{pmatrix}$	Circumcentre
$\ O - A\ $	3.1	$OA = OB = OC = R$
$\ O - B\ $	3.1	
$\ O - C\ $	3.1	
$\angle BOC$	157.38°	$\angle BOC = 2\angle BAC$
$\angle BAC$	78.69°	
$\angle AOC$	71.07°	$\angle AOC = 2\angle ABC$
$\angle ABC$	35.5°	
$\angle AOB$	228.45°	$\angle AOB = 2\angle BCA$
$\angle BCA$	65.77°	

TABLE 4: Perpendicular Bisector.

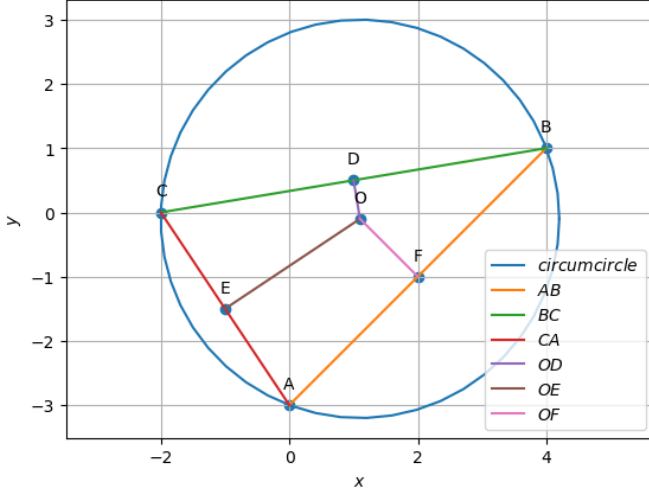


Fig. 4: triangle plotted using python

5 ANGLE BISECTOR

Parameters	Values	Description
\mathbf{m}_{13}	$\begin{pmatrix} -0.15 \\ -1.54 \end{pmatrix}$	AI
\mathbf{n}_{13}	$\begin{pmatrix} -1.54 \\ 0.15 \end{pmatrix}$	
c_{13}	-0.46	
\mathbf{m}_{14}	$\begin{pmatrix} -1.69 \\ -0.87 \end{pmatrix}$	BI
\mathbf{n}_{14}	$\begin{pmatrix} 0.87 \\ -1.69 \end{pmatrix}$	
c_{14}	1.79	
\mathbf{m}_{15}	$\begin{pmatrix} -1.54 \\ 0.67 \end{pmatrix}$	CI
\mathbf{n}_{15}	$\begin{pmatrix} -0.67 \\ -1.54 \end{pmatrix}$	
c_{15}	1.33	
\mathbf{I}	$\begin{pmatrix} 0.20 \\ -0.95 \end{pmatrix}$	Incentre
\mathbf{D}_3	$\begin{pmatrix} -0.01 \\ 0.33 \end{pmatrix}$	POC with BC
\mathbf{E}_3	$\begin{pmatrix} -0.88 \\ -1.68 \end{pmatrix}$	POC with AC
\mathbf{F}_3	$\begin{pmatrix} 1.12 \\ -1.88 \end{pmatrix}$	POC with AB
$\ \mathbf{I} - \mathbf{D}_3\ $	1.3	$ID_3 = IE_3 = IF_3 = r$
$\ \mathbf{I} - \mathbf{E}_3\ $	1.3	
$\ \mathbf{I} - \mathbf{F}_3\ $	1.3	
r	1.3	
$\angle BAI$	39.34°	$\angle BAI = \angle CAI$
$\angle CAI$	39.34°	
$\angle ABI$	17.77°	$\angle ABI = \angle CBI$
$\angle CBI$	17.77°	
$\angle ACI$	32.89°	$\angle ACI = \angle BCI$
$\angle BCI$	32.89°	

TABLE 5: Angle Bisector.

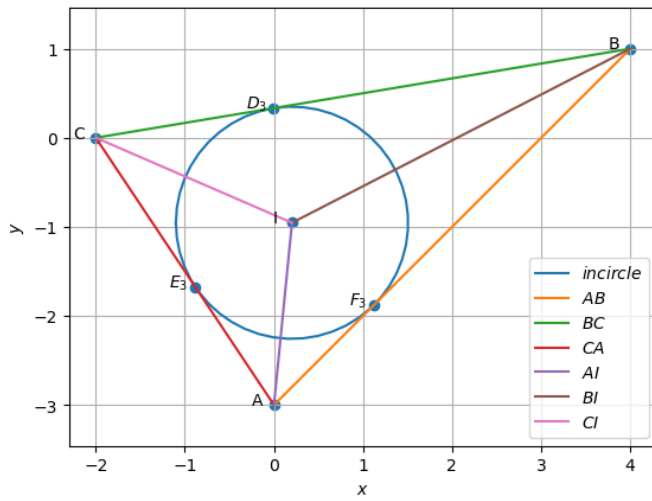


Fig. 5: triangle plotted using python