Assignment

EE22BTECH11053 - Tanmay Vishwasrao

Question 59
Let $X(t)$ be a Gaussian noise with power spectral density $\frac{1}{2}W/Hz$. If $X(t)$ is input to an LTI system with impulse response $e^{-tu(t)}$. The average power of the system is (rounded off to two decimal places). (GATE EC 2023)

Solution: The output power spectral density of a LTI system with impulse response $h(t)$ and input $X(t)$ and input power spectral density $S_X(f)$ is given by:

$$S_Y(f) = |H(f)|^2 S_X(f) \tag{1}$$

where $H(f)$ is frequency response of the system.
$H(f)$ can be found by taking fourier transform of $h(t)$

$$H(f) \overset{\mathcal{F}}{\rightleftharpoons} F\{h(t)\} = \frac{1}{j2\pi f + 1} \tag{2}$$

The average power of a signal with power spectral density $S(f)$ is given by:

$$P_Y(f) = \int_{-\infty}^{\infty} S_Y(f)\, df \tag{3}$$

Substituting $S_Y(f)$ in the equation we get:

$$P_Y(f) = \int_{-\infty}^{\infty} |H(f)|^2 \cdot S_X(f)\, df \tag{4}$$

$$= \int_{-\infty}^{\infty} \left|\frac{1}{j2\pi f + 1}\right|^2 \cdot \frac{1}{2}\, df \tag{5}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{(2\pi f)^2 + 1}\, df \tag{6}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{(2\pi f)^2 + (1)^2}\, df \tag{7}$$

$$= \frac{1}{2}\frac{1}{2\pi} tan^{-1}(x)|_{-\infty}^{\infty} \tag{8}$$

$$= \frac{1}{4\pi}(tan^{-1}(\infty) - tan^{-1}(-\infty)) \tag{9}$$

$$= \frac{1}{4\pi}(\frac{\pi}{2} - \frac{-\pi}{2}) \tag{10}$$

$$= \frac{1}{4\pi}(\pi) \tag{11}$$

$$= \frac{1}{4} \tag{12}$$

Rounded off to two decimal places, the average power of the system output is $0.25W$.