Assignment

EE22BTECH11053 - Tanmay Vishwasrao

Question 59

Let $X(t)$ be a Gaussian noise with power spectral density $\frac{1}{2}W/Hz$. If $X(t)$ is input to an LTI system with impulse response $e^{-tu(t)}$. The average power of the system is (rounded off to two decimal places). (GATE EC 2023)

Solution: The output power spectral density of a LTI system with impulse response $h(t)$ and input $X(t)$ and input power spectral density $S_X(s)$ is given by:

$$S_Y(s) = \|H(s)\|^2 \cdot S_X(s) \tag{1}$$

where $H(s)$ is frequency response of the system.
$H(s)$ can be found by taking fourier transform of $h(t)$

$$H(s) = \mathcal{F}\{h(t)\} = \frac{1}{j2\pi s + 1} \tag{2}$$

The average power of a signal with power spectral density $S(s)$ is given by:

$$P_Y(s) = \int_{-\infty}^{\infty} S_Y(s)\, ds \tag{3}$$

Substituting $S_Y(s)$ in the equation we get:

$$P_Y(s) = \int_{-\infty}^{\infty} \|H(s)\|^2 \cdot S_X(s)\, ds \tag{4}$$

$$= \int_{-\infty}^{\infty} \left\| \frac{1}{j2\pi s + 1} \right\|^2 \cdot \frac{1}{2}\, ds \tag{5}$$

$$= \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{1}{(2\pi s)^2 + 1}\, ds \tag{6}$$

$$= \frac{1}{4\pi} \cdot \pi \tag{7}$$

$$= \frac{1}{4} \tag{8}$$

Rounded off to two decimal places, the average power of the system output is $0.25W$.