

Assignment 1

EE22BTECH11053 - Tanmay Vishwasrao

Question 1.5.9

Find the other points of contact \mathbf{E}_3 and \mathbf{F}_3 .

Solution

From the previous references we have the value of Incentre \mathbf{I} is

$$\mathbf{I} = \begin{pmatrix} -1.4775 \\ -0.7949 \end{pmatrix} \quad (1)$$

And the value of inradius r is 1.8969.

The parametric equation of line is:

$$= \mathbf{A} + k\mathbf{m} \quad (2)$$

The equation of Incircle is given by:

$$\|\mathbf{x} - \mathbf{I}\|^2 = r^2 \quad (3)$$

Since its a parametric equation we can substitute (3) as \mathbf{x} in (4).

$$\|\mathbf{A} + k\mathbf{m} - \mathbf{I}\|^2 = r^2 \quad (4)$$

$$(\mathbf{A} + k\mathbf{m} - \mathbf{I})^T(\mathbf{A} + k\mathbf{m} - \mathbf{I}) = r^2 \quad (5)$$

On simplifying the above equation:

$$k^2 \|\mathbf{m}\|^2 + 2k(\mathbf{m})^T(\mathbf{A} - \mathbf{I}) + \|\mathbf{I}\|^2 + \|\mathbf{A}\|^2 - 2(\mathbf{A}^T\mathbf{I}) - r^2 = 0 \quad (6)$$

1) Finding the point \mathbf{E}_3 .

The equation of \mathbf{E}_3 :

$$\mathbf{E}_3 = \mathbf{A} + k(\mathbf{m}) \quad (7)$$

where $\mathbf{m} = \mathbf{A} - \mathbf{B}$

Now putting the values of $\mathbf{A}, \mathbf{m}, \mathbf{I}$ in eq. (7)

$$74k^2 + 27.6463k + 2.5821 = 0 \quad (8)$$

Discriminant of the above equation is:

$$D = (27.6463)^2 - 4(74)(2.5821) \quad (9)$$

$$D = 764.3179 - 764.3179 \quad (10)$$

$$D = 0 \quad (11)$$

Since the discriminant is 0. The value of k will be $-b/2a$.

$$\therefore k = -\frac{27.6463}{148} \quad (12)$$

$$k = -0.1867 \quad (13)$$

Now we can find \mathbf{E}_3 using above results:

$$\mathbf{E}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.1867 \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (14)$$

$$\mathbf{E}_3 = \begin{pmatrix} 0.066 \\ 0.307 \end{pmatrix} \quad (15)$$

2) Finding the point \mathbf{F}_3 .

For the point \mathbf{F}_3 the value of $\mathbf{m} = \mathbf{A} - \mathbf{C}$.

$$\mathbf{F}_3 = \mathbf{A} + k(\mathbf{m}) \quad (16)$$

Now putting the values of $\mathbf{A}, \mathbf{m}, \mathbf{I}$ in eq. (7)

$$32k^2 + 18.1801k + 2.5821 = 0 \quad (17)$$

Discriminant of the above equation is:

$$D = (18.1801)^2 - 4(32)(2.5821) \quad (18)$$

$$D = 330.51 - 330.51 \quad (19)$$

$$D = 0 \quad (20)$$

Since the discriminant is 0. The value of k will be $-b/2a$. Using equation (8) we can find the value of k .

$$\therefore k = -\frac{18.1801}{64} \quad (21)$$

$$k = -0.2840 \quad (22)$$

Now we can find \mathbf{F}_3 using above results:

$$\mathbf{F}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.2840 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (23)$$

$$\mathbf{F}_3 = \begin{pmatrix} -0.136 \\ -2.136 \end{pmatrix} \quad (24)$$

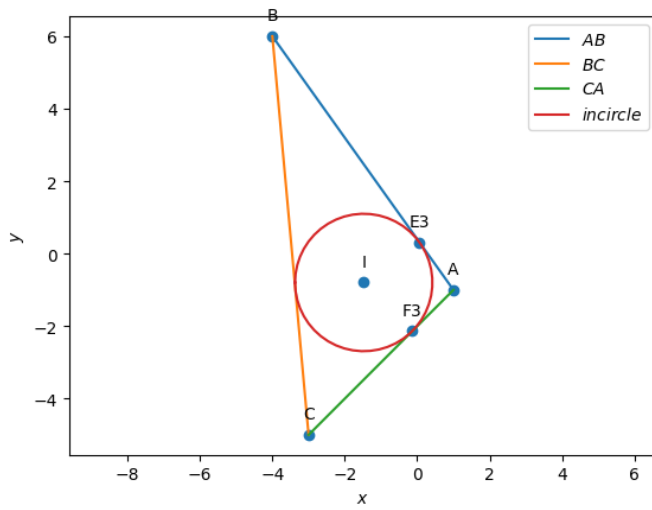


Fig. 2. Points E_3 and F_3 plotted using python