

Assignment 1

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Question 1.5.9

Find the other points of contact \mathbf{E}_3 and \mathbf{F}_3 .

$$(\mathbf{I} - \mathbf{F}_3) \cdot (\mathbf{A} - \mathbf{C}) = 0 \quad (12)$$

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{C} - \mathbf{F}_3 \cdot \mathbf{A} + \mathbf{F}_3 \cdot \mathbf{A} = 0 \quad (13)$$

Solution

From the previous references we have the value of Incentre \mathbf{I} is

$$\mathbf{I} = \begin{pmatrix} -1.4775 \\ -0.7949 \end{pmatrix} \quad (1)$$

The parametric equation of line AB is:

$$\mathbf{A} + k(\mathbf{A} - \mathbf{B}) \quad (2)$$

Now for the point E_3 let the value of k be k_1 .

$$\mathbf{E}_3 = \mathbf{A} + k_1(\mathbf{A} - \mathbf{B}) \quad (3)$$

Since the line AB and IE_3 are perpendicular to each other the dot product of the two lines will be 0.

$$(\mathbf{I} - \mathbf{E}_3) \cdot (\mathbf{A} - \mathbf{B}) = 0 \quad (4)$$

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{B} - \mathbf{E}_3 \cdot \mathbf{A} + \mathbf{E}_3 \cdot \mathbf{A} = 0 \quad (5)$$

Substituting the value of E_3 in the above equation and solving it:

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{B} - (\mathbf{A} + k_1(\mathbf{A} - \mathbf{B})) \cdot \mathbf{A} + (\mathbf{A} + k_1(\mathbf{A} - \mathbf{B})) \cdot \mathbf{A} = 0 \quad (6)$$

$$k_1(\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B}) = (\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{B}) \quad (7)$$

$$k_1 = \frac{(\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{B})}{(\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B})} \quad (8)$$

The value of k_1 comes out to be -0.1867 . Now we can find E_3 using the above results:

$$\mathbf{E}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.1867 \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (9)$$

Therefore, the point E_3 is

$$\mathbf{E}_3 = \begin{pmatrix} 0.066 \\ 0.307 \end{pmatrix} \quad (10)$$

Similarly, we will find the point \mathbf{F}_3 . For the point \mathbf{F}_3 let the value of k be k_2 .

$$\mathbf{F}_3 = \mathbf{A} + k_2(\mathbf{A} - \mathbf{C}) \quad (11)$$

Substituting the value of \mathbf{F}_3 in the above equation and solving it:

$$\mathbf{I} \cdot \mathbf{A} - \mathbf{I} \cdot \mathbf{C} - (\mathbf{A} + k_2(\mathbf{A} - \mathbf{C})) \cdot \mathbf{A} + (\mathbf{A} + k_2(\mathbf{A} - \mathbf{C})) \cdot \mathbf{A} = 0 \quad (14)$$

$$k_2 = (\mathbf{A} - \mathbf{C}) \cdot (\mathbf{A} - \mathbf{C}) = (\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{C}) \quad (15)$$

$$k_2 = \frac{(\mathbf{I} - \mathbf{A}) \cdot (\mathbf{A} - \mathbf{C})}{(\mathbf{A} - \mathbf{C}) \cdot (\mathbf{A} - \mathbf{C})} \quad (16)$$

The value of k_2 comes out to be -0.2840 . Now we can find \mathbf{F}_3 using the above results:

$$\mathbf{F}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.2840 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (17)$$

Therefore, the point \mathbf{F}_3 is

$$\mathbf{F}_3 = \begin{pmatrix} -0.136 \\ -2.136 \end{pmatrix} \quad (18)$$