Assignment

EE22BTECH11053 - Tanmay Vishwasrao

Question 12.13.3.47
By examining the chest X ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB?
Solution: To find the probability that the person actually has TB given that they were diagnosed with TB, we can use Bayes' theorem.

To find the probability that a person actually has TB given that they were diagnosed with TB, we can use Bayes' theorem. Bayes' theorem relates conditional probabilities and can be expressed as:

$$\Pr(A|B) = \frac{\Pr(B|A) \cdot \Pr(A)}{\Pr(B)} \tag{1}$$

In this case, we want to find the probability that the person has TB (A) given that they were diagnosed with TB (B). Let's define the following probabilities:

Parameter	Description	Probability
$\Pr(A)$	Person actually having TB	0.001
$\Pr(A|B)$	Diagnosed TB given has TB	0.99
$\Pr(B|\neg A)$	Diagnosed TB given is healthy	0.001
$\Pr(\neg A)$	Person not having TB	0.999

Now, we can calculate $P(B)$, the probability of being diagnosed with TB:

$$\Pr(B) = \Pr(B|A) \cdot \Pr(A) + \Pr(B|\neg A) \cdot \Pr(\neg A) \tag{2}$$

$$= (0.99 \cdot 0.001) + (0.001 \cdot 0.999) = 0.001989 \tag{3}$$

Now, we can use Bayes' theorem to calculate $\Pr(A|B)$, the probability that the person actually has TB given the diagnosis:

$$\Pr(A|B) = \frac{\Pr(B|A) \cdot \Pr(A)}{\Pr(B)} \tag{4}$$

$$= \frac{0.99 \cdot 0.001}{0.001989} \tag{5}$$

$$= \frac{0.00099}{0.001989} \tag{6}$$

$$= 0.4987 \tag{7}$$

So, the probability that the person actually has TB given the diagnosis is approximately 0.4987.