Assignment

EE22BTECH11053 - Tanmay Vishwasrao

Question 59
Let $X(t)$ be a Gaussian noise with power spectral density $\frac{1}{2} W/Hz$. If $X(t)$ is input to an LTI system with impulse response $e^{-tu(t)}$. The average power of the system is (rounded off to two decimal places). (GATE EC 2023)

Solution: The output power spectral density of a LTI system with impulse response $h(t)$ and input $X(t)$ and input power spectral density $S_X(f)$ is given by:

$$S_Y(f) = |H(f)|^2 S_X(f) \tag{1}$$

where $H(f)$ is frequency response of the system.
$H(f)$ can be found by taking fourier transform of $h(t)$

$$H(f) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{j2\pi f + 1} \tag{2}$$

The average power of a signal with power spectral density $S(f)$ is given by:

$$P_Y(f) = \int_{-\infty}^{\infty} S_Y(f)\, df \tag{3}$$

Substituting $S_Y(f)$ in the equation we get:

$$P_Y(f) = \int_{-\infty}^{\infty} |H(f)|^2 \cdot S_X(f)\, df \tag{4}$$
$$= \int_{-\infty}^{\infty} \left| \frac{1}{j2\pi f + 1} \right|^2 \cdot \frac{1}{2}\, df \tag{5}$$
$$= \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{(2\pi f)^2 + 1}\, df \tag{6}$$
$$= \frac{1}{2} \times 2 \int_{0}^{\infty} \frac{1}{(2\pi f)^2 + (1)^2}\, df \tag{7}$$
$$= \int_{0}^{\infty} \frac{1}{(2\pi f)^2 + (1)^2}\, df \tag{8}$$
$$= \frac{1}{2\pi} \tan^{-1}(x) |_0^{\infty} \tag{9}$$
$$= \frac{1}{2\pi} \left(\tan^{-1}\infty - \tan^{-1} 0 \right) \tag{10}$$
$$= \frac{1}{2\pi} \left(\frac{\pi}{2} - 0 \right) \tag{11}$$
$$= \frac{1}{2\pi} \left(\frac{\pi}{2} \right) \tag{12}$$
$$= \frac{1}{4} \tag{13}$$

Rounded off to two decimal places, the average power of the system output is $0.25W$.