Assignment 1

EE22BTECH11053 - Tanmay Vishwasrao

Question 10.13.3.25
A coin is tossed 3 times. List the possible outcomes. Find the probability of getting (i) all heads (ii) at least 2 heads

Solution: As the coin is tossed 3 times we will get 8 different outcomes.Let us define a random variable X, where getting heads is success(1). The list of possble outcomes is:

$$\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{1}$$

$$X \sim \text{Ber}\,(p) \tag{2}$$

$$\tag{3}$$

Suppose $X_i (1 \le i \le n)$ represent each of the n tosses. Then we can define Z as:

$$Z = \sum_{i=1}^{n} X_i \tag{4}$$

The pmf of Z is given by :

$$Z \sim \text{Bin}\,(n, p) \tag{5}$$

$$\tag{6}$$

The cdf of Z is

$$F_Z(k) = \Pr(Z < k) \tag{7}$$

$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 1 \le k \le n \\ 1 & k \ge n \end{cases} \tag{8}$$

Parameter	Value	Description
n	3	no. of tosses
p	$\frac{1}{2}$	probability of getting heads

1) To get all heads: To get all heads Z should be equal to 3. So we need

$$\Pr(Z = 3) = \binom{n}{3} p^3 (1-p)^{n-3} \tag{9}$$

$$= \frac{1}{8} \tag{10}$$

2) To get atleast 2 heads: To get atleast two heads the value of $Z \ge 2$.

$$\Pr(Z \ge 2) = 1 - \Pr(Z < 2) \tag{11}$$

$$= F_Z(3) - F_Z(1) \tag{12}$$

$$\Pr(Z \ge 2) = \sum_{k=2}^{3} \binom{n}{k} p^k (1-p)^{n-k} \tag{13}$$

$$= \frac{1}{2} \tag{14}$$