

# Assignment 1

EE22BTECH11053 - Tanmay Vishwasrao

## Question 1.5.9

Find the other points of contact  $\mathbf{E}_3$  and  $\mathbf{F}_3$ .

**Solution:** From the previous references we have the value of Incentre  $\mathbf{I}$  is

$$\mathbf{I} = \begin{pmatrix} -1.4775 \\ -0.7949 \end{pmatrix} \quad (1)$$

And the value of inradius  $r$  is 1.8969. The parametric equation of line is:

$$= \mathbf{A} + k\mathbf{m} \quad (2)$$

The equation of Incircle is given by:

$$\|\mathbf{x} - \mathbf{I}\|^2 = r^2 \quad (3)$$

Since its a parametric equation we can substitute (3) as  $\mathbf{x}$  in (4).

$$\|\mathbf{A} + k\mathbf{m} - \mathbf{I}\|^2 = r^2 \quad (4)$$

$$(\mathbf{A} + k\mathbf{m} - \mathbf{I})^\top (\mathbf{A} + k\mathbf{m} - \mathbf{I}) = r^2 \quad (5)$$

On simplifying the above equation:

$$k^2 \|\mathbf{m}\|^2 + 2k (\mathbf{m})^\top (\mathbf{A} - \mathbf{I}) + \|\mathbf{I}\|^2 + \|\mathbf{A}\|^2 - 2 (\mathbf{A}^\top \mathbf{I}) - r^2 = 0 \quad (6)$$

1) Finding the point  $\mathbf{E}_3$ .

The equation of  $\mathbf{E}_3$ :

$$\mathbf{E}_3 = \mathbf{A} + k\mathbf{m} \quad (7)$$

Where

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \quad (8)$$

Now putting the values of  $\mathbf{A}, \mathbf{m}, \mathbf{I}$  in (6).

$$74k^2 + 27.6463k + 2.5821 = 0 \quad (9)$$

Discriminant of the above equation is:

$$D = (27.6463)^2 - 4 (74) (2.5821) \quad (10)$$

$$= 764.3179 - 764.3179 \quad (11)$$

$$= 0 \quad (12)$$

Since the discriminant is 0. The value of  $k$  will be:

$$k = -\frac{2 (\mathbf{m})^\top (\mathbf{A} - \mathbf{I})}{2 \|\mathbf{m}\|^2} \quad (13)$$

$$= -\frac{27.6463}{148} \quad (14)$$

$$= -0.1867 \quad (15)$$

Now we can find  $\mathbf{E}_3$  using above results:

$$\mathbf{E}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.1867 \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (16)$$

$$= \begin{pmatrix} 0.066 \\ 0.307 \end{pmatrix} \quad (17)$$

2) Finding the point  $\mathbf{F}_3$ .

For the point  $\mathbf{F}_3$  the value of  $\mathbf{m} = \mathbf{A} - \mathbf{C}$ .

$$\mathbf{F}_3 = \mathbf{A} + k\mathbf{m} \quad (18)$$

Now putting the values of  $\mathbf{A}, \mathbf{m}, \mathbf{I}$  in (6).

$$32k^2 + 18.1801k + 2.5821 = 0 \quad (19)$$

Discriminant of the above equation is:

$$D = (18.1801)^2 - 4 (32) (2.5821) \quad (20)$$

$$= 330.51 - 330.51 \quad (21)$$

$$= 0 \quad (22)$$

Since the discriminant is 0. The value of  $k$  will be:

$$k = -\frac{2 (\mathbf{m})^\top (\mathbf{A} - \mathbf{I})}{2 \|\mathbf{m}\|^2} \quad (23)$$

$$= -\frac{18.1801}{64} \quad (24)$$

$$= -0.2840 \quad (25)$$

Now we can find  $\mathbf{F}_3$  using above results:

$$\mathbf{F}_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.2840 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (26)$$

$$= \begin{pmatrix} -0.136 \\ -2.136 \end{pmatrix} \quad (27)$$

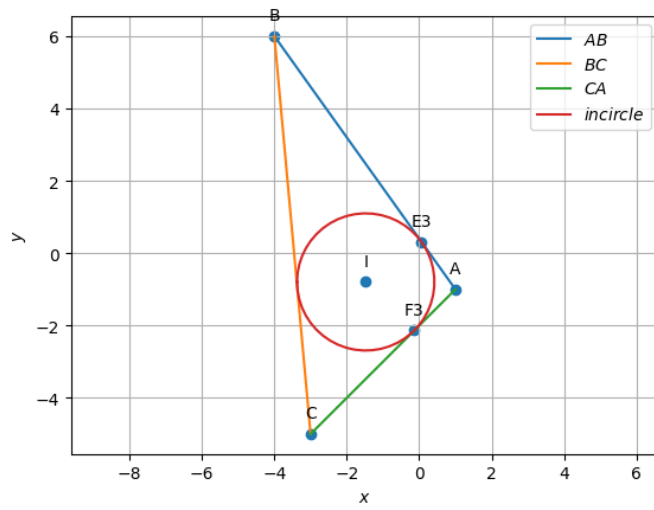


Fig. 2. Points  $E_3$  and  $F_3$  plotted using python