Assignment

EE22BTECH11053 - Tanmay Vishwasrao

Question 12.13.3.47

By examining the chest X ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB?

Solution: Let us define two random variables A and B, where A is probability of a person being healthy and B is the probability of a person getting diagnosed with TB.

Random Variable	Value	Description
A	0	Person not having TB
	1	Person having TB
B	0	not diagnosed with TB
	1	diagnosed with TB

The following information is given to us:

$$\Pr(A = 1) = 0.001 \tag{1}$$

$$\Pr(A = 0) = 0.999 \tag{2}$$

$$\Pr(B = 1|A = 1) = 0.99 \tag{3}$$

$$\Pr(B = 1|A = 0) = 0.001 \tag{4}$$

To find the probability that the person actually has TB given that they were diagnosed with TB, we can use Bayes' theorem.

$$\Pr(A = 1|B = 1) = \frac{\Pr(B = 1|A = 1) \cdot \Pr(A = 1)}{\Pr(B = 1)} \tag{5}$$

We can calculate $\Pr(B = 1)$, the probability of being diagnosed with TB:

$$\Pr(B = 1) = \Pr(B = 1|A = 1) \cdot \Pr(A = 1) + \Pr(B = 1|A = 0) \cdot \Pr(A = 0) \tag{6}$$

$$= (0.99 \cdot 0.001) + (0.001 \cdot 0.999) \tag{7}$$

$$= 0.001989 \tag{8}$$

Now, we can use Bayes' theorem to calculate $\Pr(A = 1|B = 1)$, the probability that the person actually has TB given the diagnosis:

$$\Pr(A = 1|B = 1) = \frac{\Pr(B = 1|A = 1) \cdot \Pr(A = 1)}{\Pr(B = 1)} \tag{9}$$

$$= \frac{0.99 \cdot 0.001}{0.001989} \tag{10}$$

$$= \frac{0.00099}{0.001989} \tag{11}$$

$$= 0.4987 \tag{12}$$

So, the probability that the person actually has TB given the diagnosis is approximately 0.4987.