Assignment 1

EE22BTECH11053-Tanmay Vishwasrao

Question 1.5.9

Find the other points of contact $\mathbf{E_3}$ and $\mathbf{F_3}$.

Solution

From the previous references we have the value of Incentre $\mathbf{I}$ is

$$\mathbf{I} = \begin{pmatrix} -1.4775 \\ -0.7949 \end{pmatrix} \tag{1}$$

And the value of inradius r is 1.8969.

The parametric equation of line AB is:

$$\mathbf{A} + k(\mathbf{A} - \mathbf{B}) \tag{2}$$

Now for the point E_3 let the value of k be k_1.

$$\mathbf{E_3} = \mathbf{A} + k_1(\mathbf{A} - \mathbf{B}) \tag{3}$$

$$\|\mathbf{x} - \mathbf{I}\|^2 = r^2 \tag{4}$$

Since its a parametric equation we can substitute (3) in (4) as $\mathbf{E_3}$ will also lie on the incirlce.

$$\|\mathbf{A} + k_1(\mathbf{A} - \mathbf{B}) - \mathbf{I}\|^2 = r^2 \tag{5}$$

$$(\mathbf{A} + k_1(\mathbf{A} - \mathbf{B}) - \mathbf{I})^\tau \cdot (\mathbf{A} + k_1(\mathbf{A} - \mathbf{B}) - \mathbf{I}) = r^2 \tag{6}$$

On simplifying the above equation:

$$k_1^2\|\mathbf{A} - \mathbf{B}\|^2 + 2k_1(\mathbf{A} - \mathbf{B})^\tau(\mathbf{A} - \mathbf{I}) + \|\mathbf{I}\|^2 + \|\mathbf{A}\|^2 - 2(\mathbf{A}^\tau\mathbf{I}) - r^2 = 0 \tag{7}$$

On putting the values $\mathbf{A}, \mathbf{B}, \mathbf{I}$ in the above equation:

$$74k_1^2 + 2k_1(13.8231) + 2.81513 + 2 - 2(-0.6826) - 3.5982 = 0 \tag{8}$$

$$74k_1^2 + 27.6463k_1 + 2.5821 = 0 \tag{9}$$

Solving the quadratic equation for k_1.The value of k_1 comes out to be -0.1867. Now we can find E_3 using the above results:

$$\mathbf{E_3} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.1867 \begin{pmatrix} 5 \\ -7 \end{pmatrix} \tag{10}$$

Therefore, the point E_3 is

$$\mathbf{E_3} = \begin{pmatrix} 0.066 \\ 0.307 \end{pmatrix} \tag{11}$$

Similarily, we will find the point $\mathbf{F_3}$. For the point $\mathbf{F_3}$ let the value of k be k_2.

$$\mathbf{F_3} = \mathbf{A} + k_2(\mathbf{A} - \mathbf{C}) \tag{12}$$

Substituting (12) as $\mathbf{x}$ in (4).

$$\|\mathbf{A} + k_2(\mathbf{A} - \mathbf{C}) - \mathbf{I}\|^2 = r^2 \tag{13}$$

$$k_2^2\|\mathbf{A} - \mathbf{C}\|^2 + 2k_2(\mathbf{A} - \mathbf{C})^\tau(\mathbf{A} - \mathbf{I}) + \|\mathbf{I}\|^2 + \|\mathbf{A}\|^2 - 2(\mathbf{A}^\tau\mathbf{I}) - r^2 = 0 \tag{14}$$

On putting the values $\mathbf{A}, \mathbf{C}, \mathbf{I}$ in the above equation:

$$74k_2^2 + 2k_2(13.8231) + 2.81513 + 2 - 2(-0.6826) - 3.5982 = 0 \tag{15}$$

$$32k_2^2 + 18.1801k_2 + 2.5821 = 0 \tag{16}$$

The value of k_2 comes out to be -0.2840. Now we can find $\mathbf{F_3}$ using the above results:

$$\mathbf{F_3} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0.2840 \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{17}$$

Therefore, the point $\mathbf{F_3}$ is

$$\mathbf{F_3} = \begin{pmatrix} -0.136 \\ -2.136 \end{pmatrix} \tag{18}$$