4 Solution of system of differential equations

4.1 Objective

- To solve the system of n-th order differential equations of the form $X^n = AX+B$ using diagonalization method.
- To develop code for solving physical problems which can be modeled in the above form.

4.2 Mathematical Model

System of differential equations arise in modeling of various engineering application. A system of first order differential equation can be represented as

$$\begin{aligned} x_1' &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_1 \\ x_2' &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_2 \\ &\vdots \qquad\quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ x_n' &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_n \end{aligned} \tag{4}$$

or in matrix form

$$X' = AX + B$$

where A and B are coefficient matrix.
If A is diagonalisable, i.e. if there exist a nonsingular $n \times n$ matrix P such that

$$D = P^{-1}AP$$

then in order to solve the given system of equation, assume

$$X = PY \tag{5}$$

then, the system of differential equation (1) becomes

$$Y' = DY + C \tag{6}$$

or

$$\begin{aligned} y_1' &= \lambda_1 y_1 + c_1 \\ y_2' &= \lambda_2 y_2 + c_2 \\ &\vdots \qquad\quad \vdots \\ y_n' &= \lambda_n y_n + c_n \end{aligned} \tag{7}$$

where D is the diagonal matrix similar to A and $C = P^{-1}B$ is a $n \times 1$ column vector. Equation (3) is a decoupled differential equation which can be solved using any of the method for solving differential equations. After solving (3) for Y, we can get x by (2).

4.3 MATLAB Functions

MATLAB provides many matrix functions for various matrix/vector manipulations; see Table 3.3 for some of these functions. Use the online help of MATLAB to and how to use these functions.

Command	Explanation
inv	$inv(A)$ returns inverse of matrix A.
round	Y = round(X,N) rounds X to N digits
\	$X = A\backslash B$ solves the system of linear equations $AX = B$. The matrices A and B must have the same number of rows.
det	det(A), returns the determinant of matrix A
diag	diag(A), returns an equivalent diagonal matrix of A
eig	[V,D] = eig(A) returns matrices V and D. The columns of V present eigenvectors of A. The diagonal matrix D contains eigenvalues.
null	Z = null(A) returns an orthonormal basis for the null space of A.

Exercise:

Write a MATLAB code for solving the following system of differential equations

1. Solve the system of homogeneous differential equations

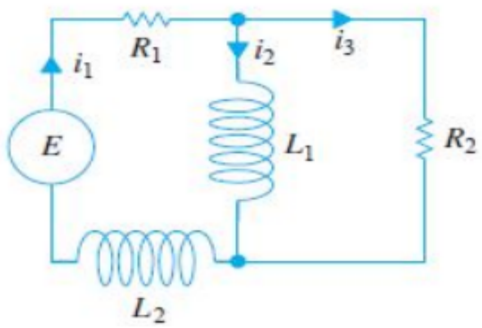
$$\begin{aligned} x_1'(t) &= 2x_1(t) - 2x_2(t) + x_3(t) \\ x_2'(t) &= -x_1(t) + 3x_2(t) - x_3(t) \\ x_3'(t) &= -2x_1(t) - 4x_2(t) + 3x_3(t) \end{aligned}$$

   with initial conditions
   $x_1(0) = 1, x_2(0) = 0, x_3(0) = 0.$

2. Solve the system of non-homogeneous differential equations

$$\begin{aligned} x_1'(t) &= 2x_1(t) - 2x_2(t) + x_3(t) \\ x_2'(t) &= -x_1(t) + 3x_2(t) - x_3(t) + 3 \\ x_3'(t) &= -2x_1(t) - 4x_2(t) + 3x_3(t) + 3t \end{aligned}$$

3. Write the mathematical Model of the given electrical network and solve using matrix diagonalization method, if, $R_1 = 8\Omega$, $R_2 = 3\Omega$, $L_1 = 1h$, $L_2 = 1h$, $E(t) = 100\sin(t)$ V, $i_1(0) = 0$, and $i_2(0) = 0$.



4. Write the mathematical Model of the given mass spring system and solve using matrix diagonalization method.