

Q2 - Determine the single bit error detection and correction for BCD No - 10110 with odd Parity.

Message = 10110 given to bits for S.C. w/ odd parity

No. of message bit = $n=5$

$2^k \geq n+1$ $\Rightarrow k \geq 3$ bits required

$2^k \geq n+5+1$ $\Rightarrow k \geq 5$ bits required

$2^k \geq n+6$ $\Rightarrow k \geq 6$ bits required

$k=4$ satisfies all conditions

To satisfy the conditions given

D₁ D₂ D₃ D₄ P₁ P₂ P₃ P₄ P₅ P₆ P₇ P₈ P₉

P₁ P₂ P₃ P₄ P₅ P₆ P₇ P₈ P₉ P₁₀ P₁₁ P₁₂ P₁₃ P₁₄ P₁₅

P₁ P₂ 1 P₃ 0 1 1 P₄ 0

$$(P_1, D_3, D_5, D_7, D_9) = (P_1, 1, 0, 1, 0) = (1, 1, 0, 1, 0)$$

$$(P_2, D_3, D_6, D_7) = (P_2, 1, 1, 1) = (0, 1, 1, 1)$$

$$(P_3, D_4, D_5, D_6, D_7) = (P_3, 0, 1, 1) = (1, 0, 1, 1)$$

$$(P_4, D_9) = (P_4, 0) = (1, 0)$$

Final Code

P₁, P₂, P₃, P₄, D₁, D₂, D₃, D₄, D₅, D₆, D₇, D₈, D₉

(1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0, 0)

(1, 0, 0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0)

BCD - 1101

Unit - II Minimization Techniques

There are two types of expression :-

i) SOP \rightarrow sum of Product \rightarrow Min Term (m)

ii) POS \rightarrow Product of Sum \rightarrow Max Term (M)

$$y = ABC + CA\bar{B} \quad (\text{Min Term})$$

$$y = (\bar{A} + \bar{B} + C) (A + \bar{B} + C) \quad (\text{Max Term})$$

Digit	Variable	Min Terms SOP (m)	Max Terms POS (M)
0	0 0 0	$\bar{A}\bar{B}\bar{C}$	=
1	0 0 1	$\bar{A}\bar{B}C$	=
2	0 1 0	$\bar{A}B\bar{C}$	=
3	0 1 1	$\bar{A}B\bar{C}$	=
4	1 0 0	$A\bar{B}\bar{C}$	=
5	1 0 1	$A\bar{B}C$	=
6	1 1 0	$AB\bar{C}$	=
7	1 1 1	ABC	=

Min term = $\bar{A}\bar{B}C + \bar{A}BC + ABC$

Max term = $A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}C + ABC$

$$(M) \text{ min term} = \text{m}_1 + \text{m}_2 + \text{m}_3 + \text{m}_4 = 902 \quad (i)$$

$$(M) \text{ max term} = \text{m}_0 + \text{m}_5 + \text{m}_6 + \text{m}_7 = 209 \quad (ii)$$

$$\text{min term} = ABC + AC\bar{B} + A\bar{B}C$$

$$(M) \text{ min term} = (\bar{A} + \bar{B} + C)(\bar{A} + B + \bar{C})(A + B + \bar{C}) = 902$$

$$\begin{aligned} & \text{Ansatz: } m_0, m_1, m_2, m_3, m_4, m_5, m_6, m_7 \\ & \text{Ansatz: } \bar{A}\bar{B}\bar{C} = 000 \quad 9 \\ & \text{Ansatz: } \bar{A}\bar{B}C = 001 \quad 1 \\ & \text{Ansatz: } \bar{A}B\bar{C} = 010 \quad 5 \\ & \text{Ansatz: } \bar{A}BC = 011 \quad 4 \\ & \text{Ansatz: } AB\bar{C} = 100 \quad 2 \\ & \text{Ansatz: } ABC = 101 \quad 8 \\ & \text{Ansatz: } A\bar{B}\bar{C} = 110 \quad 6 \\ & \text{Ansatz: } A\bar{B}C = 111 \quad 3 \\ & \text{Ansatz: } AB\bar{C} = 101 \quad 7 \\ & \text{Ansatz: } ABC = 111 \quad 1 \\ & \text{Ansatz: } A\bar{B}C = 011 \quad 10 \\ & \text{Ansatz: } A\bar{B}\bar{C} = 001 \quad 11 \\ & \text{Ansatz: } A\bar{B}\bar{C} = 000 \quad 12 \\ & \text{Ansatz: } A\bar{B}C = 010 \quad 13 \\ & \text{Ansatz: } AB\bar{C} = 110 \quad 14 \\ & \text{Ansatz: } ABC = 111 \quad 15 \end{aligned}$$

Min term = $\bar{A}\bar{B}C + \bar{A}BC + ABC$

0	0	1	0	0	1	0	00
1	0	1	0	1	0	1	10
2	1	0	1	0	1	0	01
3	1	0	1	1	0	1	11

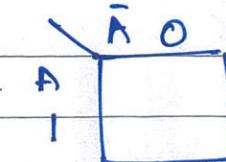
Max term = $A\bar{B}\bar{C} + A\bar{B}C + A\bar{B}C + ABC$

0	0	1	0	0	1	0	00
1	0	1	0	1	0	1	10
2	1	0	1	0	1	0	01

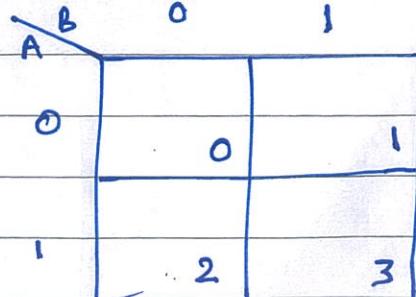
Digit	Variable	Min Terms (SOP) (m)	Max Terms (POS) (M)
A	B	C	
0	0	0	$\bar{A}\bar{B}\bar{C} = M_0$
1	0	0	$\bar{A}\bar{B}C = M_1$
2	1	0	$\bar{A}B\bar{C} = M_2$
3	1	0	$\bar{A}BC = M_3$
4	1	1	$A\bar{B}\bar{C} = M_4$
5	1	1	$A\bar{B}C = M_5$
6	1	0	$AB\bar{C} = M_6$
7	1	1	$ABC = M_7$

KMaps

i) One variable K-map



ii) Two variable K-map



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iii) Three variable K-map

A, B \ C	0	1
00	0	1
01	2	3
10	6	7
11	4	5

A \ B, C	00011110
0	0 1 3 2
1	4 5 7 6

iv) Four Variable K-map

A, B \ C, D	00	01	11	10
00	0	1	2	3
01	4	5	7	6
11	8	9	11	10
10	12	13	15	14

Minimization of equation

$$Q1. Y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

Sol^A-

$$Y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

↳ SOP form

so, we use min term

$$\bar{A}\bar{B}C = 101 \rightarrow 5$$

$$\bar{A}\bar{B}C = 001 \rightarrow 1$$

$$\bar{A}BC = 011 \rightarrow 3$$

$$A\bar{B}\bar{C} = 100 \rightarrow 4$$

$$\bar{A}\bar{B}\bar{C} = 000 \rightarrow 0$$

$$\Rightarrow Y = \sum m(0, 1, 3, 4, 5) = Y = \bar{B}\bar{A}C + \bar{A}\bar{B}\bar{C}$$

K-map of three variables

AB \ C	0	1
00	1	0
01	2	1
11	6	7
10	4	5

$$Y = \bar{B} + \bar{A}C$$

$$Y = \bar{B} + \bar{A}C$$

Q2. $y = \sum m(0, 3, 5, 6, 7, 8, 11, 13, 15)$

$\bar{A} \bar{B}$	$C \bar{D}$	$\bar{C} D$	$C D$	$\bar{C} \bar{D}$	$\bar{A} \bar{B} + \bar{C} \bar{D} + C \bar{D} + \bar{C} \bar{D}$
00	10	1	13	2	$\bar{C} \bar{D}$
01	4	15	17	16	$\bar{A} \bar{B} + \bar{C} \bar{D}$
10	12	13	15	14	$\bar{B} \bar{D}$
11	18	9	11	10	$\bar{A} \bar{B} + \bar{C} \bar{D} + C \bar{D}$

$$y = \bar{B} \bar{C} \bar{D} + \bar{A} \bar{B} \bar{C} + C \bar{D} + B \bar{D}$$

Q3. $y = \bar{A} \bar{B} \bar{C} + A \bar{B} \bar{D} + A \bar{B} C \bar{D}$

Sol^a $y = \bar{A} \bar{B} \bar{C} \cdot 1 + A \bar{B} \bar{D} \cdot 1 + A \bar{B} C \bar{D} = \bar{C} \bar{D} \bar{A}$

$$y = \bar{A} \bar{B} \bar{C} (0 + \bar{D}) + A \bar{B} \bar{D} (C + \bar{C}) + A \bar{B} C \bar{D}$$

$$y = \bar{A} \bar{B} C \bar{D} + \bar{A} \bar{B} C \bar{D} + A \bar{B} C \bar{D} + A \bar{B} \bar{C} \bar{D} + A \bar{B} C \bar{D}$$

$$\bar{A} \bar{B} C \bar{D} = 0111 \rightarrow 7$$

$$\bar{A} \bar{B} C \bar{D} = 0110 \rightarrow 6$$

$$A \bar{B} C \bar{D} = 1110 \rightarrow 14$$

$$A \bar{B} \bar{C} \bar{D} = 1100 \rightarrow 12$$

$$A \bar{B} C \bar{D} = 1111 \rightarrow 15$$

$$y = \sum m(6, 8, 12, 14, 15)$$

$\bar{A} \bar{B}$	$C \bar{D}$	00	01	11	10
00	0	1	3	2	
01	4	5	17	16	$\bar{B} \bar{C}$
11	12	13	15	14	$A \bar{B} \bar{D}$
10	8	9	11	10	

$$y = A \bar{B} \bar{D} + \bar{B} \bar{C}$$

Q4. $y = \sum m(3, 4, 5, 7, 9) + d(1, 6, 11, 13)$

don't care

$$(0+1+3+7). (0+5+9+11) (6+1+8+13) =$$

$$(\bar{A} \bar{B} \bar{C} \bar{D}) (0011) (1010)$$

$$00 \quad 0 \quad | \quad X \quad 1 \quad 3 \quad 2 \quad 209$$

$$01 \quad 14 \quad | \quad 5 \quad 17 \quad X \quad 6 \quad \bar{A} \bar{B}$$

$$11 \quad 12 \quad | \quad X \quad 13 \quad 15 \quad 14 \quad \bar{C} D$$

$$10 \quad 8 \quad | \quad 1 \quad X \quad 11 \quad 10 \quad A + B + \bar{C} + \bar{D}$$

$$y = \bar{A} \bar{B} + \bar{B} \bar{D} + \bar{C} D$$

Q1. $y = (\bar{A} + \bar{B} + C + D)(\bar{A} + \bar{B} + \bar{C} + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

$$= (\bar{A} + \bar{B} + C + \bar{D})(A + B + \bar{C} + D)(A + \bar{B} + \bar{C} + D)(A + B + C + D)$$

pos

Sol -

$$(\bar{A} + \bar{B} + C + D) = 1100 \rightarrow 12$$

$$(\bar{A} + \bar{B} + \bar{C} + D) = 1110 \rightarrow 14$$

$$(\bar{A} + \bar{B} + \bar{C} + \bar{D}) = 1111 \rightarrow 15$$

$$(\bar{A} + \bar{B} + C + \bar{D}) = 1101 \rightarrow 13$$

$$(A + B + \bar{C} + D) = 0010 \rightarrow 2$$

$$(A + \bar{B} + \bar{C} + D) = 0110 \rightarrow 6$$

$$(A + B + C + D) = 0000 \rightarrow 0$$

$$y = \pi M(0, 2, 6, 12, 13, 14, 15)$$

$$AB \quad C \bar{D} \quad 00 \quad 01 \quad 11 \quad 10$$

$$AB \quad 00 \quad 0 \quad 1 \quad 3 \quad \bar{0} \quad 2$$

$$AB \quad 01 \quad 4 \quad 5 \quad 7 \quad \bar{0} \quad 6$$

$$\bar{A} \bar{B} \quad 11 \quad \bar{0} \quad 12 \quad \bar{0} \quad 13 \quad \bar{0} \quad 14$$

$$\bar{A} \bar{B} \quad 10 \quad \bar{0} \quad 8 \quad 9 \quad 11 \quad 10$$

$$B \bar{D} \quad A + B + D$$

$$A + \bar{C} \bar{B} + D$$

$$AB \quad \bar{A} + \bar{B}$$

$$y = (\bar{A} + B)(A + \bar{C} + D)(A + B + D)$$

Q2.

$$Y = (A + \bar{B} + D)(\bar{A} + B + C)$$

$$= (A + \bar{B} + D + CC) (\bar{A} + B + C + D\bar{D})$$

$$= (A + \bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)(\bar{A} + B + C + D)$$

$$(\bar{A} + B + C + \bar{D})$$

$$= POS$$

$$A + \bar{B} + C + D = 0100 = 4$$

$$A + \bar{B} + \bar{C} + D = 0110 = 6$$

$$\bar{A} + B + C + D = 1000 = 8$$

$$\bar{A} + B + C + \bar{D} = 1001 = 9$$

$$Y = \pi M(4, 6, 8, 9)$$

		C, D		A, B		
		00	01	10	11	
00	0	1	3	2		
01	0	4	5	7	6	$A + \bar{B} + D$
11	12	13	14			$(A + D + \bar{B} + \bar{A})$
10	0	8	9	11	10	$\bar{A} + B + C$

$$Y = (A + \bar{B} + D)(\bar{A} + B + C)$$

conversion of SOP to POS form

+ @) $Y = ABC + \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C}$ → POS form

$$\begin{aligned} &\Rightarrow Y = ABC + \bar{A}BC \\ &\Rightarrow \bar{Y} = (ABC + \bar{A}BC) \\ &\Rightarrow \bar{Y} = \overline{ABC} \cdot \overline{\bar{A}BC} \\ &\Rightarrow \bar{Y} = (\bar{A} + \bar{B} + \bar{C}) \cdot (\bar{\bar{A}} + \bar{B} + \bar{C}) \\ &\Rightarrow \bar{Y} = (\bar{A} + \bar{B} + \bar{C}) (A + \bar{B} + \bar{C}) \rightarrow POS form \end{aligned}$$

(B) $Y = (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + C)$ → POS form

$$Y = (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + C)$$

$$\Rightarrow \bar{Y} = (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + C)$$

$$\Rightarrow \bar{Y} = \overline{A + \bar{B} + \bar{C}} + \overline{\bar{A} + B + C}$$

$$= (\bar{A} \cdot \bar{B} \cdot \bar{C}) + (\bar{\bar{A}} \cdot \bar{B} \cdot \bar{C})$$

$$= (\bar{A} \cdot B \cdot C) + (A \cdot \bar{B} \cdot \bar{C})$$

$$\bar{Y} = (\bar{A} \cdot \bar{B} \cdot \bar{C}) + (A \cdot \bar{B} \cdot \bar{C})$$

$$= (\bar{A} \cdot B \cdot C) + (A \cdot \bar{B} \cdot \bar{C})$$

$$\bar{Y} = (\bar{A} \cdot \bar{B} \cdot \bar{C}) + (A \cdot \bar{B} \cdot \bar{C})$$

$$= (\bar{A} \cdot B \cdot C) + (A \cdot \bar{B} \cdot \bar{C})$$

$$= (\bar{A} \cdot \bar{B} \cdot \bar{C}) + (A \cdot \bar{B} \cdot \bar{C})$$

$$= (\bar{A} \cdot B \cdot C) + (A \cdot \bar{B} \cdot \bar{C})$$

$$= (\bar{A} \cdot \bar{B} \cdot \bar{C}) + (A \cdot \bar{B} \cdot \bar{C})$$

$$= (\bar{A} \cdot B \cdot C) + (A \cdot \bar{B} \cdot \bar{C})$$

Kmap or Graphical Method

Minimize using Quine-McCluskey method or Kmap

Tabular method

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D$$

$$\bar{A}\bar{B}\bar{C}\bar{D} = 0000 = 0$$

$$\bar{A}\bar{B}C\bar{D} = 0010 = 2$$

$$\bar{A}\bar{B}CD = 0011 = 3$$

$$\bar{A}B\bar{C}\bar{D} = 0110 = 6$$

$$\bar{A}BC\bar{D} = 0111 = 7$$

$$A\bar{B}\bar{C}\bar{D} = 1000 = 8$$

$$A\bar{B}C\bar{D} = 1010 = 10$$

$$AB\bar{C}\bar{D} = 1100 = 12$$

$$AB\bar{C}D = 1101 = 13$$

$$Y = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$$

Table 1

Binary Term	Min Term
0	0000
2	0010
3	0011
6	0110
7	0111
8	1000
10	1010
12	1100
13	1101

Table 2

Binary Term	Min Term
(0, 2)	00-0
(0, 8)	-000
(2, 3)	001-
(2, 6)	0-10
(2, 10)	-010
(3, 7)	0-11
(6, 7)	011-
(8, 10)	10-0
(8, 12)	1-00
(12, 13)	110-

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Table 3

Binary term	Min term
(0, 2, 8, 10)	-0-0
(2, 3, 6, 7)	0-1-

Table 4

Prime Implicate	Binary Representation
$\bar{B}\bar{D}$	(0, 2, 8, 10)
$\bar{A}C$	(2, 3, 6, 7)
$A\bar{C}\bar{D}$	(8, 12)
$AB\bar{C}$	(12, 13)

Table 5

Prime Implicate	m_0	m_2	m_3	m_6	m_7	m_8	m_{10}	m_{12}	m_{13}
$\bar{B}\bar{D}$ (0, 2, 8, 10)	0	0				0	0		1
$\bar{A}C$ (2, 3, 6, 7)		0	0	0	0				0
$A\bar{C}\bar{D}$ (8, 12)						0	0		0
$AB\bar{C}$ (12, 13)							0	0	0

$$Y = \bar{B}\bar{D} + \bar{A}C + AB\bar{C}$$

Q. $Y(A, B, C, D) = \sum m(1, 2, 3, 5, 9, 12, 14, 15) + \sum d(4, 8, 11)$

Minimize the expression using Quine-McCluskey method

Sol-

Table 1.

Min Term	Binary Representation
1	0001
2	0010
3	0011
4	0100
5	00101
8	1000
9	1001
11	100011
12	1100
14	1110
15	1111

Table 2

Min Term	Binary Representation
1, 3	00-1
1, 5	0-01
1, 9	-001
2, 3	001-
3, 11	-011
4, 5	010-
4, 12	-10010A
8, 9	100-0A
8, 12	1-00
9, 11	10-1
11, 15	1-11
12, 14	11-0
14, 15	111-

Tab. 3

Min Term	Binary Representation
(1, 3, 9, 11)	-0-1

Tab. 4

Primary Implicants	Min Term	Binary Representation
$\bar{B}D$	(1, 3, 9, 11)	-0-1
$\bar{A}\bar{C}D$	(1, 5)	0-01
$\bar{A}\bar{B}C$	(2, 3)	001-
$\bar{A}\bar{B}\bar{C}$	(4, 5)	010-
$B\bar{C}\bar{D}$	(4, 12)	-100
$A\bar{B}\bar{C}$	(8, 9)	100-
$\bar{A}\bar{C}\bar{D}$	(8, 12)	1-00
ACD	(11, 15)	
$AB\bar{D}$	(12, 14)	
ABC	(14, 15)	

Table 5

Prime Implicate	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{11}	m_{12}	m_{14}	m_{15}
$\bar{B}D$	(1, 3, 9, 11)	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}\bar{C}D$	(1, 5)	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}\bar{B}C$	(2, 3)	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}\bar{B}\bar{C}$	(4, 5)	0	0	0	0	0	0	0	0	0	0	0	0
$B\bar{C}\bar{D}$	(4, 12)	0	0	0	0	0	0	0	0	0	0	0	0
$A\bar{B}\bar{C}$	(8, 9)	0	0	0	0	0	0	0	0	0	0	0	0
$\bar{A}\bar{C}\bar{D}$	(8, 12)	0	0	0	0	0	0	0	0	0	0	0	0
ACD	(11, 15)	0	0	0	0	0	0	0	0	0	0	0	0
$AB\bar{D}$	(12, 14)	0	0	0	0	0	0	0	0	0	0	0	0
ABC	(14, 15)	0	0	0	0	0	0	0	0	0	0	0	0

$$Y = \bar{B}D + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{C}D + A\bar{B}\bar{C} + ACD + B\bar{C}\bar{D} + ABC$$

$$\frac{\bar{B}D + \bar{A}\bar{B}C + \bar{A}\bar{C}D + ABC + B\bar{C}\bar{D}}{\bar{A}\bar{B}\bar{C}}$$

$$\bar{A} + A\bar{A} = m_2$$

$$m_2 = \bar{A}(1 - A) + A(\bar{A}) = 0$$

$$m_4 = \bar{A}(\bar{A} - A) + A(\bar{A} - \bar{A}) = 0$$

$$m_6 = \bar{A}(\bar{A} - A) + A(\bar{A} - \bar{A}) = 0$$

$$m_8 = \bar{A}(\bar{A} - A) + A(\bar{A} - \bar{A}) = 0$$

$$m_{10} = \bar{A}(\bar{A} - A) + A(\bar{A} - \bar{A}) = 0$$

$$m_{12} = \bar{A}(\bar{A} - A) + A(\bar{A} - \bar{A}) = 0$$

$$m_{14} = \bar{A}(\bar{A} - A) + A(\bar{A} - \bar{A}) = 0$$

$$m_{15} = \bar{A}(\bar{A} - A) + A(\bar{A} - \bar{A}) = 0$$