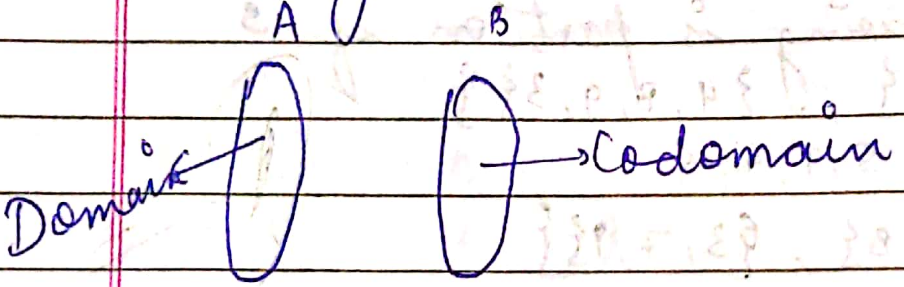
⇨ Function :-

A mapping f from $A \rightarrow B$ is said to be function if it satisfy following properties:-

1. Every element of A has image in B
2. An element ~~of~~ ~~a~~ of A have unique image in B.



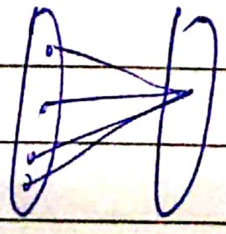
* Range is a subset of codomain

- If f is a function from $A \rightarrow B$ than set A is domain, set B is codomain & Range of f is given by $f = \{f(a) \mid \forall a \in A\}$

* Constant Function:-

$f: N \rightarrow N$

$f(x) \Longrightarrow 3$



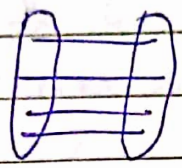
A funn f from $A \rightarrow B$ such that $\boxed{f(x) = c}$

where $c \in B$ is called constant function.

Identity Function:-

A funⁿ $f: A \longrightarrow A$ is said to be identity funⁿ if every element of A is image of itself. It is only define if domain = codomain.

$$f(x) = x$$



Equal Function:-

Two funⁿ f & g are said to be equal funⁿ if:-

i) codomain of f = codomain of g

ii) Domain of f = Domain of g

iii) $f(x) = g(x)$; $\forall$ x belonging to their common domain.

$A = \{1, 2\}$; $B = \{3, 6\}$

Let

$f: A \rightarrow B$ s.t $\Rightarrow$ $f(x) = x^2 + 2$ $\qquad f(1) = 3$, $f(2) = 6$

$g: A \rightarrow B$ s.t $\Rightarrow$ $g(x) = 3x$

$g(1) = 3$

$g(2) = 6$

Here

$f(1) = g(1)$

$f(2) = g(2)$

$\Rightarrow f = g$

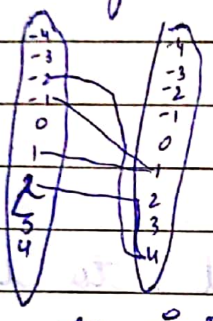
Properties of function:-

1. INTO function:-

A funⁿ $f: A \to B$ is s.T atleast one element of B is not f image of any element of A then it is called into mapping

$\boxed{\text{Range} \neq \text{codomain}}$

Eg:- let f be a funⁿ ~~defn~~ define from $\mathbb{Z} \to \mathbb{Z}$ s.t $f(x) = x^2$



← here -ve integers are not f image of any element
it is onto funⁿ

2. Onto / Surjective function:-

A funⁿ $f: A \to B$ is said to be onto if every element of B is f image of atleast one element of A then f is called onto function.

Eg:- $A = \{1, 2, 3, 4\}$ $B = \{1, 4, 9, 16\}$

$f: A \to B$

then $f(1) = 1$

$f(2) = 4$

$f(3) = 9$

$f(4) = 16$

$\therefore$ f is onto.

• Whenever we wish to prove f is a

onto mapping then we show that
$\forall\, b \in B$, $\exists\, a \in A$, s.t $f(a) = b$

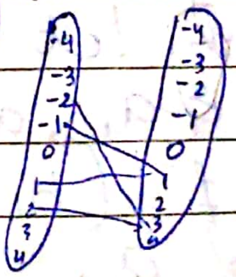
$$\boxed{\text{Range} = \text{Codomain}}$$

3. One - One / Injective Mapping :-
A funⁿ $f : A \rightarrow B$ is said to be one-One if different element of A have diff. f images in B. whenever we wish to prove $f : A \rightarrow B$ is one-one mapping then we should that
let $f(a) = f(b)$
$\Rightarrow a = b$
OR
$f(\alpha)$. $a \neq b$
$\Rightarrow f(a) \neq f(b)$

Eg:- Identity funⁿ.

4. Many - One mapping :-
A mapping $f : A \longrightarrow B$ is said to be many one if 2 or more elements of A have same f image i.e $a \neq b$
$\Rightarrow f(a) = f(b)$

Eg:- $f : I \longrightarrow J$
$f(x) = x^2$



5. Bijective Function :-
A funⁿ f from $A \to B$ is said to be bijective if it is both one-one and onto.

6. Cardinally Equivalent Set :-
If their exist a mapping f from $A \to B$ which is one-one & onto both then the 2 sets are said to be cardinally equivalent and it is denoted by $\boxed{A \sim B}$

Inverse of Function :-

If f is a funⁿ from $A \to B$ such that it is one-one onto than inverse of funⁿ f is defined as

$$f^{-1} : B \to A$$

s.t $f^{-1}(b) = a$

i.e $f(a) = b$

where $a \in A$, $b \in B$

Theorem 1 :- The inverse map $f^{-1} : B \to A$ is also one-one onto.

for one-one :
$f : A \to B$
$f(a_1) = f(a_2)$
$\Rightarrow a_1 = a_2$

$f^{-1} : B \to A$

Proof :- Let $b_1, b_2 \in B$ & $a_1, a_2 \in A$

s.t $f^{-1}(b_1) = a_1$ & $f^{-1}(b_2) = a_2$

f^{-1} is one one as

Let $f^{-1}(b_1) = f^{-1}(b_2)$
$\Rightarrow a_1 = a_2$
$\Rightarrow f(a_1) = f(a_2)$. [as f is a fn]
$\Rightarrow b_1 = b_2$

Also f^{-1} is onto
let $a \in A$
Then $\exists\, b \in B$ s.t $f(a) = b$ [as f is a fn]
$\Rightarrow f^{-1}(b) = a$
$\therefore f^{-1}$ is onto

eorem 2 :- If $f : A \longrightarrow B$ & it is one - one onto then show that inverse of f is unique

$f : A \longrightarrow B$

Proof :- Let inverse of f is not unique let g & h be inverse of f then

$g : B \longrightarrow A$
$h : B \longrightarrow A$

Now, we show g & h are equal funn.
First two properties are satisfied.
i.e Domain of g = Domain of h
Codomain of g = Codomain of h

Now let. ~~b₁~~ $b_2 \in B$ $a_1, a_2 \in A$
S.t
$g(b) = a_1 \Rightarrow f(a_1) = b$
$h(b) = a_2 \Rightarrow f(a_2) = b$
~~$\therefore b_1 = b_2$~~
~~$\Rightarrow g(b_1) = h(b_2)$~~
~~$\Rightarrow g = h$~~

$\therefore f(a_1) = f(a_2)$
$\left[\begin{array}{c}\text{as } f \text{ is}\\ \text{one-one}\end{array}\right] \Rightarrow a_1 = a_2$
$g(b) = h(b)$
$g = h$

Q. Let $f: R \to R$ s.t $f(x) = ax + b$ where $a, b \in R$ than show that f is inveatible.

$x_1, x_2 \in R$ (Domain)

s.t $f(x_1) = f(x_2)$

$ax_1 + b = ax_2 + b$

$\boxed{x_1 = x_2}$

$y \in R$ (codomain)

$\frac{y-b}{a} \in R$ (domain)

s.t $f\left(\frac{y-b}{a}\right) = y$

$y = ax + b$

$x = \frac{y-b}{a}$

$f\left(\frac{y-b}{a}\right) = a\left(\frac{y-b}{a}\right)$

$f\left(\frac{y-b}{a}\right) = y$

Q. Determine whether the foll^n fun^n are bijection or not where $f: R \to R$

i) $f(x) = -3x + 4$

ii) $f(x) = -3x^2 + 7$

iii) $f(x) = \frac{x+1}{x+2}$

iv) $f(x) = x^5 + 1$

i) $f(x) = -3x + 4$

$f(x_1) = f(x_2)$

$-3x_1 + 4 = -3x_2 + 4$

$\boxed{x_1 = x_2}$

$y \in R$ (codomain)

$\frac{y-4}{-3} \in R$ (domain)

s.t $f\left(\frac{y-4}{-3}\right) = y$.

$y = -3x + 4$

$\frac{y-4}{-3} = x$

$f\left(\frac{y-4}{-3}\right) = -3\left(\frac{y-4}{-3}\right) +$

$f\left(\frac{y-4}{-3}\right) = y$

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$(-1)^2 = (1)^2$

$(-1)^{15} = (1)^{15}$

ii) $f(x) = -3x^2 + 7$

$f(x_1) = f(x_2)$

$-3x_1^2+7 = -3x_2^2 + 7$

$x_1 = \pm x_2$

∴ this funⁿ is not one - one

$\left.\begin{matrix} f(1) = 4 \\ f(-1) = 4 \end{matrix}\right\}$ not possible

iii) $f(x) = \frac{x+1}{x+2}$

this funⁿ will be one - one when range is defined from $f: R \to R - \{2\}$

As if $x = -2$

$f(-2) = \frac{-2+1}{-2+2} = \frac{-1}{0}$

function will be infinite and ∞ does not belong in the range of real no. ∴ funⁿ is not one - one

iv) $f(x) = x^5 + 1$

$f(x_1) = f(x_2)$

$x_1^5 + 1 = x_2^5 + 1$

$x_1^5 = x_2^5 \Rightarrow x_1 = (x_2^5)^{1/5}$

$\boxed{x_1 = x_2}$

$y \in R$ (codomain)

$y = x^5 + 1$

$(y-1)^{1/5} =$

$f[(y-1)^{1/5}] = (y-1)$

y

$$\text{s.t } (y-1)^{1/5} \in R \text{ (domain)}$$
$$f[(y-1)^{1/5}] = y$$

Composite of Function :-

If $f: A \to B$, $g: B \to C$ than composite of the fun^n f & g is defined as $\boxed{g \circ f: A \to C}$
s.t $g \circ f(a) = g[f(a)]$

1. $\boxed{g \circ f \neq f \circ g}$ [It is not commutative]
2. $f \circ (g \circ h) = (f \circ g) \circ h$ [It is Associative]

Properties of composite function :-

1. composite fun^n is not commutative
$f: A \to B$
$g: B \to C$
then $f \circ g \neq g \circ f$
as $f \circ g$ is not defined because domain of one is not equal to codomain of another & $f \circ g$ will be defined only if $A = C$

2. The composite fun^n is associative i.e
$f \circ (g \circ h) = (f \circ g) \circ h$
let f, g, h be the 3 fun^n defined as
$h: A \to B$

$g : B \longrightarrow C$
$f : C \longrightarrow D$

Now, here $f o (g o h) : A \longrightarrow D$
$(f o g) o h : A \longrightarrow D$

we have to show $[f o (g o h)]$ & $[(f o g) o h]$ are equal fun"

domain of $f o (g o h)$ = domain of $(f o g) o h$
also codomain of $f o (g o h)$ = codomain of $(f o g) o h$

Now let $a \in A$, $b \in B$, $c \in C$, $d \in D$
s.t $h(a) = b$
$g(b) = c$
$f(c) = d$

Now $[f o (g o h)] a$
$= f[(g o h) a]$
$= f[g(h(a))]$
$= f[g(b)]$
$= f(c)$
$= d$

$[(f o g) o h] a$
$= f o g [h(a)]$
$= f o g [b]$
$= f[g(b)]$
$= f[c]$
$= d$

$\therefore f o (g o h) = (f o g) o h$

3. If f & g are 2 bijection than show that gof is also a bijection :-

Proof :- Let $f : A \to B$
$g : B \to C$

then $gof : A \xrightarrow{} C$

Now we have to show gof is one - one & onto

i) gof is one - one :-

Let $a_1, a_2 \in A$ & $b_1, b_2 \in B$, & $c_1, c_2 \in C$

S.T $f(a_1) = b_1$, $f(a_2) = b_2$
& $g(b_1) = c_1$, $g(b_2) = c_2$

let $(gof)a_1 = (gof)a_2$

$\Rightarrow g[f(a_1)] = g[f(a_2)]$

$\Rightarrow f(a_1) = f(a_2)$ [as g is one - one]

$\Rightarrow a_1 = a_2$ [as f is one - one]

ii) gof is onto

let $c \in C$ [codomain]

$\exists\ b \in B$ S.T $g(b) = c$ [as g is onto]

let $B \Rightarrow \exists\ a \in A$ S.T $f(a) = b$ [as f is onto]

$\Rightarrow \forall\ c \in C$

$\exists\ a \in A$

S.T $(gof)a$
$= g[f(a)]$
$= g[b]$
c

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Q. let f, g & h be the mappings defined from $f: N \to N$ where N is set of natural no. s.T, $g: N \to N$, $f: N \to N$ & $h: N \to N$

$f(n) = n+1$, $g(n) = 2n$

$h(n) = \begin{cases} 0 ; & n \text{ is even} \\ 1 ; & n \text{ is odd} \end{cases}$

Show that i) f, g & h are funⁿ

ii) Also find fog, goh, $(fog)oh$, fof, hog

Ans. • $f(n_1) \neq f(n_2)$
$n_1 + 1 \neq n_2 + 1$
$n_1 \neq n_2$

• $g(n_1) \neq g(n_2)$
$2n_2 \neq 2n_2$
$n_1 \neq n_2$

• h is not a function as zero is not a natural no.
∴ range of $h: N \to N + \{0\}$

i) $fog = f(g(n))$
$= f(2n)$
$= 2[n+1] = 2n+2$

ii) $goh = g[h(n)]$

$= \begin{cases} g(0) ; & n \text{ is even} \\ g(1) ; & n \text{ is odd} \end{cases}$

$= \begin{cases} 0 ; & n \text{ is even} \\ 2 ; & n \text{ is odd.} \end{cases}$

iv) $f \circ f = f(f(n))$

$= f(n+1) = n+1+1 = n+2$

v) $h \circ g = h(g(n))$

$= h(2n)$

$= \begin{cases} 0 & n \text{ is even} \\ 1 & n \text{ is odd} \end{cases}$

$= \begin{cases} 0 & n \text{ is even} \\ 2 & n \text{ is odd} \end{cases}$

iii) $f \circ g) \circ h = [f \circ g) [\cancel{f} h(n)]$

$= [f \circ g] \begin{cases} 0 & ; \text{ when } n \text{ is even} \\ 1 & ; \text{ when } n \text{ is odd} \end{cases}$

$\begin{cases} f \circ g(0) = \cancel{f g(0)}\ 1 \\ f \circ g(1) = 3 \end{cases}$

→ Countable & Uncountable Set

Countable Set

finite

$A = \{1, 2, 3, 4\}$

$B = \{c, d, e\}$

Infinite

I

$f : I \to N$

one-one

onto

- Set of +ve even no. are countable infinite set

Defination:-
A set is said to be countable if it is either finite or their exist a bijection b/w a given set and set of natural no. If the given set is not countable then it is called Uncoutable set. Cardianlity of Natural no. is denoted by α. & hence cardianlity of every ∞ countable set is denoted by α.

Q. Show that set of even +ve integers is countable infinite set.

$$E = \{2, 4, 6, 8 - - - \}$$

let us define a mapping

$$f : N \longrightarrow E$$
$$\text{s.t} \quad f(x) = 2x$$

Now if f is one-one onto then E will be infinite countable set

1. f is one one
   let $x_1, x_2 \in N$
   s.t $f(x_1) = f(x_2)$
   $\Rightarrow 2x_1 = 2x_2$
   $\Rightarrow x_1 = x_2$

2. f is onto
   let $y \in E$ (codomain)
   then $y/2 \in N$ (Domain)
   s.t $f(y/2) = 2 \times y/2$
   $= y$

$x = y/2$

$f(y/2)$ $f\, 2 \times \frac{y}{2}$

$\therefore$ from (1) & (2)
f is bijection
& hence E is infinite countable set.

Q. Show that set of +ve integers is a countable set.

$I = \{0, 1, 2, 3, \text{———}\}$

let us define a mapping

$f: N \longrightarrow I$

s.t $f(n) = \begin{cases} n/2 \ ; & n \text{ is even +ve integer} \\ \frac{1-n}{2} \ ; & n \text{ is odd +ve integer} \end{cases}$

we show f is one-one

let $n_1, n_2 \in N$ [Domain]

There are three cases

1. n_1, n_2 both are even

let $f(n_1) = f(n_2)$

$\Rightarrow n_1/2 = n_2/2$

$\Rightarrow n_1 = n_2$

$\therefore f$ is one one

2. n_1, n_2 both are odd

let $f(n_1) = f(n_2)$

$\Rightarrow \frac{1-n_1}{2} = \frac{1-n_2}{2}$

$n_1 = n_2$

$\therefore f$ is one one

3. n_1 is even & n_2 is odd

let $f(n_1) = f(n_2)$

$\frac{n_1}{2} = \frac{1-n_2}{2}$

or $n_1 = 1 - n_2$
+ve integer = -ve integer
not possible
$\Rightarrow n_1 = n_2$

$\therefore f$ is one one

2. f is onto :-
Again there are three cases.

1. If $n = 0$ ~~then~~ $\in I$ (codomain)
$\exists\, 1 \in$ Domain
s.t $f(1) = 0$

2. If $n > 0 \in I$ (codomain)
$\exists\, 2n \in N$ (Domain)
s.t $f(n) = n$

3. If $\cancel{n \geq 0}$ $n<0$ $\in I$ (codomain)
$\exists\, 1-2n$ (domain)
s.t $f(1-2n) = n$

Q. Show that odd +ve integer ~~no~~ is countable.

$f(n) = 2n-1$

Imp Pigeon Hole Principle:-
If their are $n+1$ pigeons & n pigeon holles then atleast one pigeon hole has more than one pigeon.

Eg:- 1. If their are 13 person than atleast one month is their in which their is birthday of more than one person.

2. In any grp of 27 english word their must be atleast 2 word which begin with same letter.

Generalize pigeon hole Principle:-
Let n pigeons are assigned r pigeon holes than one of the pigeon hole will contain at least
$$\left\lfloor \frac{n+1}{R} \right\rfloor + 1$$

Eg:- Where $\left\lfloor \frac{n+1}{R} \right\rfloor$ is the floor of $\frac{n+1}{R}$

Q. If their is a store which consists of 5 departments & total no. of employee are 36 ~~for~~ show that their is atleast one department which consists of 8 employee.

$n = 36 \quad , R = 5$

$$\left\lfloor \frac{36+1}{5} \right\rfloor + 1 = [7.4] + 1 = 7 + 1 = 8$$

Q. If their are $n+1$ integers selected from the set $\{1, 2, 3, \ldots 2n\}$ then show that one of them divides another integer that has been selected.

Ans. Let $x_1, x_2, \ldots x_{n+1}$ are integer selected from the given set now we express each x_i as product of power of 2 & odd integer i.e

$$x_i = 2^{p_i} b_i$$

where $b_i \leq 2n$

Now, there are exactly n odd no.s in the given set $\therefore$ there exist 2 integers x_i, x_j such that $x_i = 2^{p_i} b_i$

$$x_j = 2^{p_j} b_i$$

$$x_i = 2^{p_i} b_i \Rightarrow b_i = \frac{x_i}{2^{p_i}}$$

$$x_j = 2^{p_j} b_i \Rightarrow b_i = \frac{x_j}{2^{p_j}}$$

$$\therefore \frac{x_i}{2^{p_i}} = \frac{x_j}{2^{p_j}}$$

$$\boxed{\frac{x_i}{x_j} = 2^{p_i - p_j}}$$

$\therefore$ x_i is divisible in x_j