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Unit -1

Probability here defines the surety.

Uncertainty : The idea of information is related to uncertainty or surprise. If the probability of x_K is $P_K = 0$, then such symbol is impossible. Similarly, when probability $P_K = 1$ then such symbol is sure. In both cases no information is produced. When prob. of P_K is low then there is more surprise & uncertainty it means it contains more informa^n. If P_K ↑ses the amount of informa^n ↓ses.

Amount of Information :

$$I_K = \log_2 \left(\frac{1}{P_K} \right)$$

Q) Calculate the AOI for 1 PCM sys. w contains 4 (+ve) levels with 4 different probability.

$P_1 = 1/8$

$P_2 = 1/8$

$P_3 = 3/8$

$P_4 = 3/8$

→ $I_1 = \log_2 \left(\frac{1}{P_K} \right) \Rightarrow \log_2 \left(\frac{1}{\frac{1}{8}} \right)$

$\log_2 (2^3) \Rightarrow 3 \log_2 (2)$

$\Rightarrow 3$ bits

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Similarly, $I_2 = 3$ bits

$$I_3 = \log_2\left(\frac{1}{P_k}\right) \Rightarrow \log_2\left(\frac{1}{3/8}\right)$$

$$\Rightarrow \log_2\left(\frac{8}{3}\right)$$

$$\Rightarrow \log_2(8) - \log_2(3)$$

$$\Rightarrow 3 - \frac{\log_{10}(3)}{\log_{10}(2)}$$

$$\Rightarrow 3 - 1.58$$

$$\Rightarrow 1.42 \text{ bits}$$

Similarly, $I_4 = 1.42$ bits

Total information $= 3 + 3 + 1.42 + 1.42$

$\Rightarrow 6 + 2.84$

$\Rightarrow 8.84$ bits

Q) Calculate the AOI for:

$P_1 = 1/4$
$P_2 = 1/4$
$P_3 = 1/4$
$P_4 = 1/4$

→ $I_1 = \log_2\left(\frac{1}{\frac{1}{4}}\right) \Rightarrow \log_2(2^2)$

$\Rightarrow 2$ bits

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Similarly, I_2 & I_3 & $I_4 = 2$ bits

Total information $= 2 + 2 + 2 + 2 = 8$ bits

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Definition of Information:

Let us consider a communication system which transmit msg m_1, m_2, m_3 ------ with the probability of occurrence p_1, p_2, p_3 ------ p_k. The amount of information transmitted then the msg m_k with prob. p_k is given by:

Amount of information:

$$I_k = \log_2\left(\frac{1}{P_k}\right)$$

unit of information = bits

Properties of Information:

① If there is more uncertainity about the msg information carried is also more.

② If receiver knows the msg being transmitted the amount of information carried is zero.

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③ If I_1 is the information carried by msg m_1 & I_2 is the information carried by msg m_2, then amount of information carried completely due to m_1 & m_2 is $I_1 + I_2$.

④ If there are $M = 2^N$ equally likely msg then amount of information carried by each msg will be N bits.

eg: ① $M = 8 = 2^3$

$I_1 = 3$ bits

$I_2 = 3$ bits

$\vdots$

$I_8 = 3$ bits

② $M = 4 = 2^2$

$I_1 = 2$ bits

$\vdots$

$I_4 = 2$ bits

Q) Calculate the AOI if P_K is $\frac{1}{4}$.

→ $P_K = 1/4$

$$I_K = \log_2\left(\frac{1}{P_K}\right)$$

$$I_K = \log\left(\frac{1}{\frac{1}{4}}\right)$$

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$$I_K = \log_2(4) \Rightarrow \log_2(2^2)$$

$$\Rightarrow 2\log_2(2)$$

$$I_K = 2 \text{ bits}$$

Q) Calculate the AOI if binary digits occur with equally likely in binary PCM.

→ We know that in binary PCM there are only 2 binary levels 1 or 0. There prob. of occurance will be equally likely so that:

$$P_1(\text{'0' level}) = 1/2$$

$$P_2(\text{'1' level}) = 1/2$$

$$I_1 = \log_2\left(\frac{1}{P_1}\right) \Rightarrow \log_2\left(\frac{1}{\frac{1}{2}}\right)$$

$$I_1 = 1 \text{ bit}$$

Similarly, $I_2 = 1$ bit

II Alternate Method: $M = 2^N$ (equally likely property)

Then $I = N$ bits

$M = 2 = 2^1$

$N = 1$

$I_1 = I_2 = 1$ bit

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Q) If there are M equally likely independent msg, then prove the AOI carried by each msg will be I = N bits if $M = 2^N$.

→ Given:
M equally likely independent msg

probability of occurance of M equally likely msg is given by:

$$P_K = 1/M$$

Show that, $I_K = N$

If $M = 2^N$

$$I_K = \log_2\left(\frac{1}{P_K}\right) = \log_2\left(\frac{1}{\frac{1}{M}}\right)$$

$$I_K = \log_2(M)$$

$$\Rightarrow \log_2(2^N)$$

$$\Rightarrow N \log_2(2)$$

$$I_K = N \text{ bits}$$

Q) In the binary PCM, if 0 occurs with prob. of 1/4 & binary 1 occurs with prob. of 3/4. Then calculate the amount of inforⁿ in each level.

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→ P_1 ('0' level) = 1/4

P_2 ('1' level) = 3/4

$$I_1 = \log_2\left(\frac{1}{\frac{1}{4}}\right) \Rightarrow \log_2(4)$$

$$\Rightarrow 2 \text{ bits}$$

$$I_2 = \log_2\left(\frac{1}{3/4}\right) \Rightarrow \log_2(4/3)$$

$$\Rightarrow \log_2(4) - \log_2(3)$$

$$\Rightarrow 2 - \frac{\log_{10}(3)}{\log_{10}(2)}$$

$$\Rightarrow 2 - 1.5$$

$$\Rightarrow 0.5 \text{ bits}$$

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Q) If I_1 is the information carried by msg m_1 & I_2 is the information carried by msg m_2, then prove that AOI completely due to m_1 & m_2 is $I_{1,2} = I_1 + I_2$.

→ The definition of AOI is:

$$I_K = \log_2\left(\frac{1}{P_K}\right)$$

The individual AOI is carried by msg m_1 & m_2 is given by:

$$I_1 = \log_2\left(\frac{1}{P_1}\right)$$

$$I_2 = \log_2\left(\frac{1}{P_2}\right)$$

$$I_{1,2} = \log_2\left(\frac{1}{P_1 P_2}\right)$$

$$I_{1,2} = \log_2\left(\frac{1}{P_1}\right) + \log_2\left(\frac{1}{P_2}\right)$$

$$I_{1,2} = I_1 + I_2$$

Q) Prove the following statement:

→ If receiver knows the msg being transmitted. The AOI carried by msg is zero.

→ Here, it is stated that receiver knows the msg this means only one msg is transmitted. The prob. of occurance of dis msg will be $P_K = 1$. This is bcz only one msg, the AOI carried by dis type ~~only~~ ~~one~~ of msg is:

$$I_K = \log_2\left(\frac{1}{P_K}\right)$$

$$P_K = 1$$

$$I_K = \log_2\left(\frac{1}{1}\right)$$

$$I_K = \log_2(1) = 0$$

Entropy (Average Information):

Consider that we have M different msg. Let the msg are $m_1, m_2, m_3 - - - - m_M$, & they have prob. of occurance as $P_1, P_2, P_3 - - - - P_M$. Suppose that a sequence of L msg is transmitted then if L is very very large than we say that;

$P_1 L$ msg of m_1 is transmitted

⋮

$P_M L$ msg of m_M is transmitted

Hence, information due to m_1 msg

$$I_1 = \log_2\left(\frac{1}{P_1}\right)$$

Due to L no. of msg m_1 informaⁿ is given by:

$$I_1(\text{total}) = P_1 L \log_2\left(\frac{1}{P_1}\right)$$

Similarly, $I_M(\text{total}) = P_M L \log_2\left(\frac{1}{P_M}\right)$

$$I(\text{total}) = I_1(\text{total}) + - - - - - - + I_M(\text{total})$$

$$I_{total} = P_1 L \log_2\left(\frac{1}{P_1}\right) + - - - - - - + P_M L \log_2\left(\frac{1}{P_M}\right)$$

$$I_{total} = L\left[P_1 \log_2\left(\frac{1}{P_1}\right) + - - - + P_M \log_2\left(\frac{1}{P_M}\right)\right]$$

$$I_{total} = L \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

Average Information :

$$I_{avg} = \frac{\text{total information}}{\text{No. of msg}}$$

$$I_{avg} = \frac{\cancel{L} \sum_{K=1}^{M} \log_2\left(\frac{1}{P_K}\right)}{\cancel{L}}$$

$$I_{avg} = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right) = H$$

$H \rightarrow$ Entropy

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Properties of Entropy :

→ Entropy is zero, if the event is sure or it is impossible.

$H = 0$, if $P = 1$ or $P = 0$

→ If prob. of all msg are equal then it is called equi probable or equally likely msg. If there are M msgs then prob. of each msg is $1/M$, at that time entropy is given by $H = \log_2 M$.

→ The range of the entropy is :

$$0 \leq H \leq \log_2 M$$

Q) Calculate the entropy when $P_K = 0$ & when $P_K = 1$

→ Given $P_K = 0$

$$H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

→ only one msg

$$H = 0 \log_2\left(\frac{1}{0}\right)$$

$$\boxed{H = 0}$$

$P_K = 1$

$$H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$H = 1 \times \log_2\left(\frac{1}{1}\right)$$

$$H = 1 \times \log_2(1)$$

$$\boxed{H = 0}$$

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Q) Show that if there are M number of equally likely msg then entropy of source is $\log_2 M$.

→ Given :

M no. of equally likely msg probability of each msg $= 1/M$.

$$P_1 = P_2 = P_3 = P_4 ----- P_M = 1/M$$

$$H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + P_3 \log_3\left(\frac{1}{P_3}\right) + ---$$

$$+ P_M \log_2\left(\frac{1}{P_M}\right)$$

$$\Rightarrow H = \frac{1}{M} \log_2\left(\frac{1}{\frac{1}{M}}\right) + \frac{1}{M} \log_2\left(\frac{1}{\frac{1}{M}}\right) + \frac{1}{M} \log_2\left(\frac{1}{\frac{1}{M}}\right) + ----- + \frac{1}{M} \log_2\left(\frac{1}{\frac{1}{M}}\right)$$

$$H = \frac{1}{M}\left[\log_2(M) + \log_2(M) + ------ + \log_2(M)\right]$$

$$H = 1/\cancel{M} \times \cancel{M} \times \log_2 M$$

$$\boxed{H = \log_2 M}$$

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Q) A source transmit to independent msg with prob. of P & 1-p resp. Prove that the entropy is maximum when both msgs are equally likely.

→ Two msgs are there :

$$P_1 = P \quad \& \quad P_2 = 1-p$$

$$H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right)$$

$$H = P \log_2\left(\frac{1}{P}\right) + (1-P) \log_2\left(\frac{1}{1-P}\right)$$

If both msg are equal probable then :

$$P = 1-P \quad ; \quad P = 1/2$$

from maxima Condition :

$$H = P \log_2\left(\frac{1}{P}\right) + (1-P) \log_2\left(\frac{1}{1-P}\right)$$

$$H = \frac{P \log_{10}\left(\frac{1}{P}\right)}{\log_{10} 2} + \frac{(1-P) \log_{10}\left(\frac{1}{1-P}\right)}{\log_{10} 2}$$

$$H = \frac{1}{\log_{10} 2}\left[P \log_{10}\left(\frac{1}{P}\right) + (1-P) \log_{10}\left(\frac{1}{1-P}\right)\right] \quad \{\because (u \times v)\}$$

$$\frac{dH}{dP} = \frac{1}{\log_{10} 2}\left[P \times P + \log_{10}\left(\frac{1}{P}\right) - (1-P)(1-P) - \log_{10}\left(\frac{1}{1-P}\right)\right]$$

Put $\frac{dH}{dP} = 0$

$$0 = \frac{1}{\log_{10} 2}\left[P^2 + \log_{10}\left(\frac{1}{P}\right) - (1-P)^2 \log_{10}\left(\frac{1}{1-P}\right)\right]$$

$$0 = P^2 + [\log_{10}(1) - \log_{10}(P) - (1-2P+P^2) - (\log_{10}(1) - \log_{10}(1-P)] \quad \{\log_{10}(1) = 0\}$$

$$0 = P^2 + (0 - \log_{10}(P)] - 1 + 2P - P^2 - (0 - \log_{10}(1-P)]$$

$$0 = P^2 - P^2 - 1 + 2P - \log_{10} P + \log_{10}(1-P)$$

$$\Rightarrow 0 = 0 - 1 + 2P - \log_{10} + \log_{10} P$$

$$\boxed{0 = -1 + 2P}$$

$$1 = 2P$$

$$\Rightarrow \frac{1}{2} = P$$

$$\frac{d^2H}{dP^2} = \frac{1}{\log_{10} 2}\left[2P + P + 2(1-P) + (1-P)\right]$$

$$\frac{d^2H}{dP^2} = \log_{10} 2\left[3P - 2 - 2P + 1 - P\right]$$

$$\frac{d^2H}{dP^2} = \log_{10} 2\left[-2 + 1\right]$$

$$\frac{d^2H}{dP^2} = -\log_{10} 2$$

$$\frac{d^2H}{dP^2} < 0$$

at $P = 1/2$, H is Maximum

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Q) For a discrete memoryless source there are three symbols with probabilities $P_1 = \alpha$, $P_2 = P_3$. Determine the entropy of the source.

→ $P_1 = \alpha$, $P_2 = P_3 = x$

$$\Rightarrow P_1 + P_2 + P_3 = 1$$

$$\Rightarrow \alpha + x + x = 1$$

$$2x = 1 - \alpha$$

$$x = 1/2\,(1-\alpha)$$

$$P_2 = P_3 = \frac{1}{2}(1-\alpha)$$

$$H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$M = 3$$

$$H = \sum_{K=1}^{3} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$\Rightarrow H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + P_3 \log_2\left(\frac{1}{P_3}\right)$$

$$\Rightarrow H = \alpha \log_2\left(\frac{1}{\alpha}\right) + \frac{1}{2}(1-\alpha)\log_2\left(\frac{2}{(1-\alpha)}\right) + \frac{1}{2}(1-\alpha) \log_2\left(\frac{2}{1-\alpha}\right)$$

$$\Rightarrow H = \alpha \log_2\left(\frac{1}{\alpha}\right) + (1-\alpha)\log_2\left(\frac{2}{1-\alpha}\right)\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$\Rightarrow H = \alpha \log_2\left(\frac{1}{\alpha}\right) + (1-\alpha)\log_2\left(\frac{2}{1-\alpha}\right)$$

Information Rate :

$$R = rH$$

$R \rightarrow$ Information rate

$H \rightarrow$ Entropy

$r \rightarrow$ rate at which msg is generated

Informa^n rate R is represented in avg. no. of bits of information per sec. is calculated as follows:

$$R = \left(r \text{ in } \frac{\text{msg}}{\text{second}}\right) \times \left(H \text{ in } \frac{\text{informa}^n \text{ bits}}{\text{msg}}\right)$$

R = Informa^n bits per sec

Q) An analog signal is banned limited to B Hz and sampled at nyquist rate. The samples are quantized into 4 levels each level represents one msg. There are four msgs, the prob. of occurrences of each level are given by $P_1 = P_4 = \frac{1}{8}$, $P_2 = P_3 = 3/8$. Find out informa^n rate of the source.

$\rightarrow$ $M = 4$

$$R = rH$$

① $$H = \sum_{k=1}^{M} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$H = \sum_{k=1}^{4} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + P_3 \log_2\left(\frac{1}{P_3}\right) + P_4 \log_2\left(\frac{1}{P_4}\right)$$

$$H = \frac{1}{8}\log_2(8) + 3/8 \log_2(8/3) + 3/8 \log_2\left(\frac{8}{3}\right) + \frac{1}{8}\log_2(8)$$

$$H = 1.8 \text{ bit/msg}$$

② r = Nyquist rate $= 2 f_m$

$$f_m = B \text{ Hz}$$

$$r = 2B$$

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③ $R = rH$

$= 1.8 \times 2B = 3.6B$ bits / sec

① $\log_2(8) = \log_2(2^3)$
$\Rightarrow 3\log_2(2) = 3$

② $\log_2(3/2) = \frac{\log_{10}(3/2)}{\log_{10}(2)} = 1.415$

RTU

Q) Consider a telegraph source having a 2 symbols dot & dash. The dot duration is 0.2 sec & the dash duraⁿ is three 3 times of the dash duraⁿ. The prob. of the dot occuring is twice of the dash & the time b/w symbol is 0.2 ~~0.5~~ sec. Calculate the informaⁿ rate of the telegraph source.

→ Information rate :
$R = rH$

① dash $= P_1 = x$
dot $= P_2 = 2x$

$x + 2x = 1$
$3x = 1$
$x = \frac{1}{3}$

$P(\text{dash})\ P_1 = 1/3, \quad P_2 = 2 \times \frac{1}{3} = 2/3 \ P(\text{Dot})$

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② entropy of source :
$M = 2$

$$H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$H = \sum_{K=1}^{2} P_K \log_2\left(\frac{1}{P_K}\right)$$

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$$H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) \cancel{+ P_3 \log_2\left(\frac{1}{P_3}\right)}$$

$$H = \frac{1}{3}\log_2(3) + \frac{2}{3}\log_2(3/2) \cancel{+ 2/3 \log_2(3/2)}$$

$H = 0.9183$ bits / symbol

③ Calculate symbol rate :

dot duration $= T_{dot} = 0.2$ sec

Dash duration $= T_{dash} = 3 \times 0.2 = 0.6$ sec

Total duration of 1 symbol = dot duration + dash duration + time b/w symbol.

$r = 0.2 + 0.6 + 0.2$
$= 1$ sec.

④ Information rate :
$R = rH$
$\Rightarrow 1 \times 0.9183 = 0.9183$ bit/sec

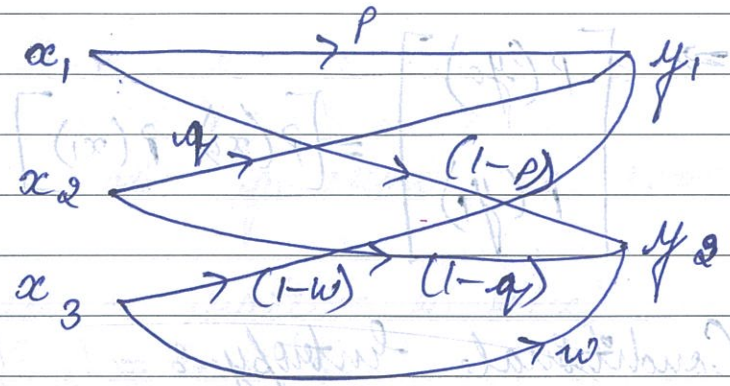
Discrete Memoryless Channels:

This channel has a 'X' input & 'Y' o/p. 'X' & 'Y' both are random variables. The channel is discrete when both 'x' & 'y' are discrete. This channel is ~~be~~/a memoryless (zero memory) when current o/p depends only on current i/p. The channel is described in terms of i/p alphabet & o/p alphabet & the set of transition probability. The transi^n prob.:

$$P\left(\frac{Y_i}{X_i}\right)$$

is the conditional prob. $P\left(\frac{Y_j}{X_i}\right)$ is received dat X_i was transmitted. If $i = j$ then $P\left(\frac{Y_j}{X_i}\right)$ is the conditional prob. of the recep^n & if $i \neq j$ then $P\left(\frac{Y_j}{X_i}\right)$ represents the conditional prob. of the error. The transi^n prob. of the channel can be represented by the matrix.

eg: Discrete memoryless ~~chanl~~ channels having a three transmitter & two receiver.



Transition matrix:

$$P = \begin{bmatrix} P\left(\frac{Y_1}{X_1}\right) & P\left(\frac{Y_2}{x_1}\right) \\ P\left(\frac{Y_1}{x_2}\right) & P\left(\frac{Y_2}{X_2}\right) \\ P\left(\frac{Y_1}{x_3}\right) & P\left(\frac{Y_2}{X_3}\right) \end{bmatrix} = \begin{bmatrix} p & (1-p) \\ q & (1-q) \\ (1-w) & w \end{bmatrix}$$

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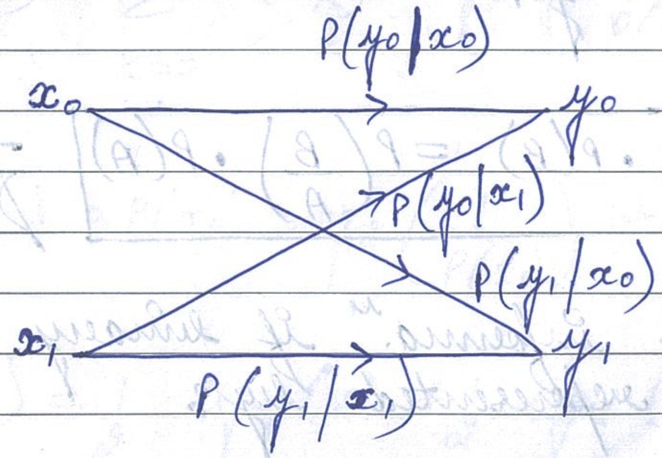
a) Binary Communica^n Discrete Memoryless Channel:



Binary memoryless channel represented by:

$$\Rightarrow \begin{bmatrix} P(y_0|x_0) & P(y_1|x_0) \\ P(y_0|x_1) & P(y_1|x_1) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} P(x_0) & P(x_1) \end{bmatrix} \begin{bmatrix} P(y_0/x_0) & P(y_1/x_0) \\ P(y_0/x_1) & P(y_1/x_1) \end{bmatrix}$$

Conditional Entropy :

$$H\left(\frac{X}{Y}\right) = \sum_{i=1}^{M} \sum_{j=1}^{N} P(x_i, y_j) \log_2 \left[\frac{1}{P(x_i/y_j)}\right]$$

$$H\left(\frac{Y}{X}\right) = \sum_{i=1}^{M} \sum_{j=1}^{N} P(x_i, y_j) \log_2 \left[\frac{1}{P(y_j/x_i)}\right]$$

Joint Entropy :

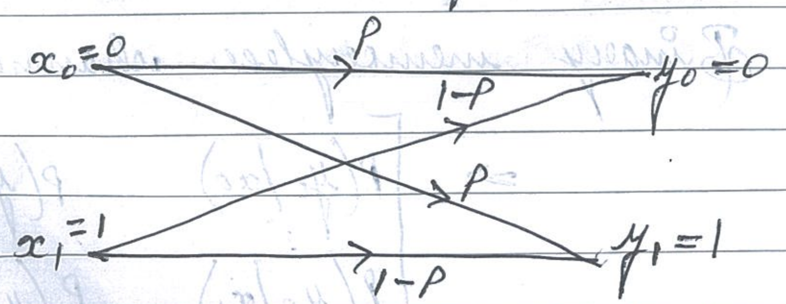
$$H(X,Y) = \sum_{i=1}^{M} \sum_{j=1}^{N} P(x_i, y_j) \log_2 \left(\frac{1}{P(y_j/x_i)}\right)$$

$$\boxed{P(AB) = P\left(\frac{A}{B}\right) \cdot P(B) = P\left(\frac{B}{A}\right) \cdot P(A)}$$ from Baye's Theorem

Q) Find (i) rate of informa^n^ of binary symmetric channel is represented by :

$P(x_0) = \frac{1}{2}$

$P(x_1) = \frac{1}{2}$



Find : (ii) $H\left(\frac{X}{Y}\right)$

Symbols are generate at 1000 symbols/sec.

→ (i) Information rate $= r\, H(x)$

$P(x_0) = 0.5 / 1/2$

$P(x_1) = 0.5 / 1/2$

$P\left(\frac{y_0}{x_0}\right) = 0.6 / P$ $\qquad P\left(\frac{y_1}{x_0}\right) = 0.4 / 1-P$

$P\left(\frac{y_0}{x_1}\right) = 0.8 / 1-P$ $\qquad P\left(\frac{y_1}{x_1}\right) = 0.2 / P$

$$H(X) = \sum_{K=1}^{M} P(x_K) \log_2 \left(\frac{1}{P(x_K)}\right)$$

$$H(X) = \sum_{K=0}^{M=2} P(x_K) \log_2 \left(\frac{1}{P(x_K)}\right)$$

$$H(X) = P(x_0) \log_2 \left(\frac{1}{P(x_0)}\right) + P(x_1) \log_2 \left(\frac{1}{P(x_1)}\right)$$

$$H(X) = \frac{1}{2} \log_2 (2) + \frac{1}{2} \log_2 (2)$$

$$H(X) = \frac{1}{2} + \frac{1}{2} = 1$$

$$R = r\, H(x)$$

$r \to$ Symbol rate $= 1000$

$R = 1000 \times 1 = 1000$ bits/sec

(3) $P(y_0)$, $P(y_1)$

$\to$ finding of $P(y_0)$ and $P(y_1)$

$$\Rightarrow \begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} P(x_0) & P(x_1) \end{bmatrix} \begin{bmatrix} P\left(\frac{y_0}{x_0}\right) & P\left(\frac{y_1}{x_0}\right) \\ P\left(\frac{y_0}{x_1}\right) & P\left(\frac{y_1}{x_1}\right) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} P & 1-P \\ 1-P & P \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \times P + \frac{1}{2}(1-P) \\ \frac{1}{2}(1-P) + \frac{1}{2}P \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P(y_0) \\ P(y_1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$P(y_0) = \frac{1}{2}$

$P(y_1) = 1/2$

(2) $$H\left(\frac{X}{Y}\right) = \sum_{i=1}^{M} \sum_{j=1}^{N} P(x_i, y_j) \log_2 \left(\frac{1}{P(x_i/y_j)}\right)$$

$M = 2$, $N = 2$

$$H\left(\frac{X}{Y}\right) = P(x_0, y_0) \log_2 \left(\frac{1}{P(x_0/y_0)}\right) + P(x_0, y_1) \log_2 \left(\frac{1}{P(x_0/y_1)}\right) +$$

$$P(x_1, y_0) \log_2 \left(\frac{1}{P\left(\frac{x_1}{y_0}\right)}\right) + P(x_1, y_1) \log_2 \left(\frac{1}{P(x_1/y_1)}\right)$$

$\Rightarrow P\left(\frac{x_0}{y_0}\right) = P$, $P\left(\frac{x_1}{y_0}\right) = 1-P$

$P\left(\frac{x_0}{y_1}\right) = 1-P$, $P\left(\frac{x_1}{y_1}\right) = P$

$P(AB) = P\left(\frac{A}{B}\right) \cdot P(B)$

$P(x_0, y_0) = P\left(\frac{x_0}{y_0}\right) \cdot P(y_0)$

$\Rightarrow P \times \frac{1}{2} \Rightarrow \frac{1}{2}P$

$P(x_0, y_1) = P\left(\frac{x_0}{y_1}\right) \cdot P(y_1)$

$\Rightarrow 1-P \times \frac{1}{2} \Rightarrow \frac{1-P}{2}$

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$$P(x_1, y_1) = P\left(\frac{x_1}{y_1}\right) \times P(y_1)$$

$$\Rightarrow P \times \frac{1}{2} \quad \Rightarrow P/2$$

$$P(x_1, y_0) = P\left(\frac{x_1}{y_0}\right) \times P(y_0)$$

$$\Rightarrow 1-P \times \frac{1}{2}$$

$$\Rightarrow \frac{1-P}{2}$$

$$H\left(\frac{X}{Y}\right) = \frac{1}{2} P \log_2\left(\frac{1}{P}\right) + \frac{1}{2}(1-P) \log_2\left(\frac{1}{1-P}\right) + \frac{1}{2}(1-P)\log_2\left(\frac{1}{1-P}\right) + \frac{1}{2}P\log_2\left(\frac{1}{P}\right)$$

$$H\left(\frac{X}{Y}\right) = P\log_2\left(\frac{1}{P}\right) + (1-P)\log_2\left(\frac{1}{1-P}\right)$$

Mutual Information :-

The mutual Information is defined as amount of Information when x_i is transmitted and y_j is received.
It is represented by $I(x_i, y_j)$. It is given by

$$I(x_i, y_j) = \log_2\left[\frac{P(x_i/y_j)}{P(x_i)}\right] \text{ bits} \quad \Big\} \text{Imp}$$

where $I(x_i, y_j) \rightarrow$ mutual Information

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The average mutual Information is Represented by $I(X;Y)$. It is calculated in bits/symbol. The average mutual information is defined as the amount of source infor. gained Per receive symbol.
Here, note that average Mutual Information is different from Entropy.

$$I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i, y_j)\log_2\left[\frac{P(x_i/y_j)}{P(x_i)}\right] \quad \Big\} \text{Imp}$$

→ Properties of Mutual Information :-

(i) The Mutual Information of the channel is Symmetric.

$$I(X;Y) = I(Y;X)$$

(ii) The Mutual Information is always positive.

$$I(X;Y) \geq 0$$

(iii) The average Mutual Information is Represented in terms of Entropy of the channel Input or output and Conditional Entropy is given by

$$I(X;Y) = H(X) - H(X/Y)$$

$$= H(Y) - H(Y/X)$$

$H(X/Y)$ or $H(Y/X)$ are conditional Entropy.

(iv) The average mutual Information is related to the

Joint Entropy. It is given by

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

$H(X,Y) \rightarrow$ Joint Entropy

Ques Prove that the mutual information of the channel is symmetric. $[I(X;Y) = I(Y;X)]$

Ans

$$\Rightarrow I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{P(x_i/y_j)}{P(x_i)}\right)$$

$$\Rightarrow I(Y;X) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left[\frac{P(y_j/x_i)}{P(y_j)}\right]$$

$$\Rightarrow P(x_i \cap y_j) = P\left(\frac{x_i}{y_j}\right) \times P(y_j) = P\left(\frac{y_j}{x_i}\right) \cdot P(x_i)$$

$$\Rightarrow \text{If } P\left(\frac{x_i}{y_j}\right) \cdot P(y_j) = P\left(\frac{y_j}{x_i}\right) \cdot P(x_i)$$

Then

$$I(X;Y) = I(Y;X) \qquad \text{H.P}$$

Ques Prove that Following Statments :-

(a) $I(X;Y) = H(X) - H(X/Y)$

(b) $I(Y;X) = H(Y) - H(Y/X)$

(c) $I(X;Y) = H(X) + H(Y) - H(X,Y)$

Sol^n

(a) $I(X;Y) = H(X) - H(X/Y)$

$$\Rightarrow H(X/Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{1}{P\left(\frac{x_i}{y_j}\right)}\right)$$

formula $\left\{ \log_2\left(\frac{a}{b}\right) = \log_2\left(\frac{1}{b}\right) + \log_2(a) = \log_2\left(\frac{1}{b}\right) - \log_2\left(\frac{1}{a}\right) \right.$

$$\Rightarrow I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left[\frac{P(x_i/y_j)}{P(x_i)}\right]$$

$$\Rightarrow I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\left[\log_2[P(x_i/y_j)] - \log_2[P(x_i)]\right]$$

$$\Rightarrow I(X;Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j) \cdot \log_2\left(P\left[\frac{x_i}{y_j}\right]\right) - \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2 P(x_i)$$

$$\Rightarrow I(X;Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{1}{P\left(\frac{x_i}{y_j}\right)}\right) + \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{1}{P(x_i)}\right)$$

$$\Rightarrow \sum_{j=1}^{m} P(x_i,y_j) = P(x_i)$$

$$\Rightarrow I(X;Y) = -\sum_{i=1}^{n} P(x_i)\log_2\left(\frac{1}{P\left(\frac{x_i}{y_j}\right)}\right) + \sum_{i=1}^{n} P(x_i)\log_2\left(\frac{1}{P(x_i)}\right)$$

$$\Rightarrow I(X;Y) = -H\left(\frac{X}{Y}\right) + H(X)$$

$$= H(X) - H(X/Y)$$

(b) $I(X;Y) = H(Y) - H(Y/X)$

Sol^n

$$I(Y;X) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left[\frac{P(y_j/x_i)}{P(y_j)}\right]$$

$$\Rightarrow I(Y;X) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{1}{P(y_j)}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{1}{P(y_j/x_i)}\right)$$

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow P(AB) = P(A\cap B)$$

$$\Rightarrow \sum_{i=1}^{n} P(x_i, y_j) = P(y_j)$$

$$\Rightarrow I(Y;X) = \sum_{j=1}^{m} P(y_j) \log_2\left(\frac{1}{P(y_j)}\right) - \sum_{j=1}^{m} P(y_j) \log_2\left[\frac{1}{P(y_j/x_i)}\right]$$

$$\Rightarrow I(Y;X) = H(Y) - H(Y/X)$$ H.P

(c) $I(X;Y) = H(X) + H(Y) - H(X/\cancel{Y})$

Solⁿ

$$H(X;Y) = H\left(\frac{X}{Y}\right) + H(Y)$$

$$\Rightarrow H(X;Y) - H(Y) = H\left(\frac{X}{Y}\right) \quad \text{---(i)}$$

$$\Rightarrow I(X;Y) = H(X) - H(X|Y) \quad \text{---(ii)}$$

put the value of $H\left(\frac{X}{Y}\right)$ from eq (i) to eq (ii)

$$\Rightarrow I(X;Y) = H(X) - [H(X;Y) - H(Y)]$$

$$\Rightarrow I(X;Y) = H(X) - H(X;Y) + H(Y)$$

$$\Rightarrow I(X;Y) = H(X) + H(Y) - H(X;Y)$$ H.P.

Ques Prove that :-

(a) $H(X,Y) = H\left(\frac{X}{Y}\right) + H(Y)$

(b) $H(X,Y) = H\left(\frac{Y}{X}\right) + H(X)$

Solⁿ. (a) $H(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2\left[\frac{1}{P(x_i, y_j)}\right]$

$$\Rightarrow H(X;Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 [P(x_i, y_j)]$$

$$\Rightarrow P(AB) = P\left(\frac{A}{B}\right) \cdot P(B)$$

$$\Rightarrow P(x_i y_j) = P\left(\frac{x_i}{y_j}\right) \cdot P(y_j)$$

$$\Rightarrow H(X;Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \left[P(y_j) \cdot P\left(\frac{x_i}{y_j}\right)\right]$$

$$\Rightarrow H(X,Y) = -\sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 [P(y_j)] + \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \left[\frac{1}{P\left(\frac{x_i}{y_j}\right)}\right]$$

$$\Rightarrow \sum_{i=1}^{n} P(x_i, y_j) = P(y_j)$$

$$\Rightarrow H(X,Y) = \sum_{j=1}^{m} P(y_j) \log_2 \left[\frac{1}{P(y_j)}\right] + H\left(\frac{X}{Y}\right)$$

$$\Rightarrow H(X,Y) = H(Y) + H\left(\frac{X}{Y}\right)$$ H.P

(b) $H(X,Y) = H\left(\frac{Y}{X}\right) + H(X)$

$$\Rightarrow H(X,Y) = \sum_{i=1}^{m} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \left[\frac{1}{P(x_i, y_j)}\right]$$

$$\Rightarrow \boxed{P(x_i, y_j) = P\left(\frac{y_j}{x_i}\right) \cdot P(x_i)}$$

$$\Rightarrow H(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \left[\frac{1}{P\left(\frac{y_j}{x_i}\right) \cdot P(x_i)}\right]$$

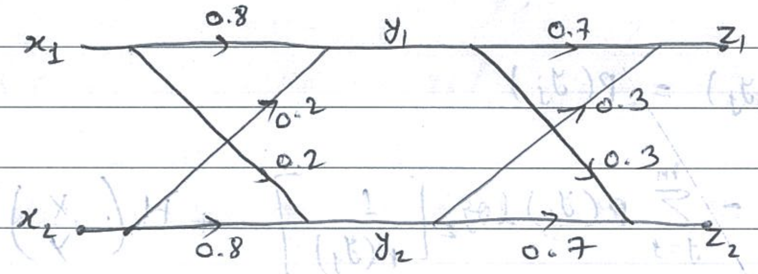
$$\Rightarrow H(X,Y) = -\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(P\left(\frac{y_j}{x_i}\right)\right) - \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2(P(x_i))$$

$$\Rightarrow \sum_{j=1}^{m} P(x_i,y_j) = P(x_i)$$

$$\Rightarrow H(X,Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{1}{P(y_j/x_i)}\right) + \sum_{i=1}^{n} P(x_i)\log_2\left[\frac{1}{P(x_i)}\right]$$

$$\Rightarrow H(X,Y) = H\left(\frac{Y}{X}\right) + H(X) \qquad \underline{\text{H.P}}$$

Ques Two BSC's are connected in cascade as shown in fig :-



(i) Find the channel Matrix of Resultant channel.
(ii) Find $P(z_1)$ and $P(z_2)$ if $P(x_1) = 0.6$ and $P(x_2) = 0.4$

Sol^n (i)

$$P(Y/X) = \begin{bmatrix} P\left(\frac{y_1}{x_1}\right) & P\left(\frac{y_2}{x_1}\right) \\ P\left(\frac{y_1}{x_2}\right) & P\left(\frac{y_2}{x_2}\right) \end{bmatrix}$$

$$\Rightarrow P(Y/X) = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

$$\Rightarrow P(Z/y) = \begin{bmatrix} P(z_1/y_1) & P(z_2/y_1) \\ P(z_1/y_2) & P(z_2/y_2) \end{bmatrix}$$

$$\Rightarrow P(Z/y) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

$$\Rightarrow P\left(\frac{Z}{X}\right) = P\left(\frac{Y}{X}\right) \cdot P(Z/y)$$

$$\Rightarrow P\left(\frac{Z}{X}\right) = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}\begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

$$\Rightarrow P\left(\frac{Z}{X}\right) = \begin{bmatrix} 0.62 & 0.38 \\ 0.38 & 0.62 \end{bmatrix}$$

$$P\left(\frac{z_1}{x_1}\right) = 0.62, \quad P\left(\frac{z_2}{x_1}\right) = 0.38$$

$$P\left(\frac{z_1}{x_2}\right) = 0.38, \quad P\left(\frac{z_2}{x_2}\right) = 0.62$$

(ii) To obtain $P(z_1)$ and $P(z_2)$

$$\Rightarrow P(Z) = P(X) \cdot P(Z/X)$$

$$\Rightarrow P(Z) = \begin{bmatrix} P(x_1) & P(x_2) \end{bmatrix} \cdot \begin{bmatrix} 0.62 & 0.38 \\ 0.38 & 0.62 \end{bmatrix}$$

$$\Rightarrow P(Z) = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}\begin{bmatrix} 0.62 & 0.38 \\ 0.38 & 0.62 \end{bmatrix} = \begin{bmatrix} 0.524 & 0.476 \end{bmatrix}$$

$$\Rightarrow P(z_1) = 0.524, \quad P(z_2) = 0.476$$

Ques A binary matrix channel is given by :-

$$\begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 2/3 & 1/3 \\ x_2 & 1/10 & 9/10 \end{array}$$

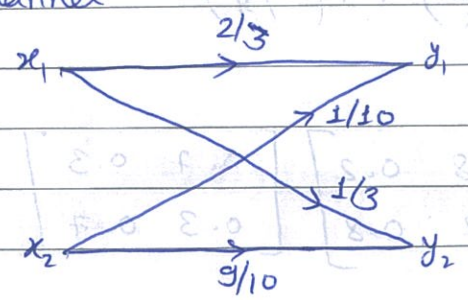
If $P(x_1) = 1/3$, $P(x_2) = 2/3$

(a) Draw binary channel

(b) Find $H(X)$, $H(X/Y)$, $H(Y/X)$ and $I(X,Y)$

Sol^n

(a) Binary channel



$P\left(\frac{y_1}{x_1}\right) = 2/3$, $P\left(\frac{y_2}{x_1}\right) = \frac{1}{3}$

$P\left(\frac{y_1}{x_2}\right) = \frac{1}{10}$, $P\left(\frac{y_2}{x_2}\right) = \frac{9}{10}$

(b) (i) To determine $H(X)$:-

$$H(X) = \sum_{i=1}^{2} P(x_i)\log_2\left[\frac{1}{P(x_i)}\right]$$

$$H(X) = P(x_1)\log_2\left(\frac{1}{P(x_1)}\right) + P(x_2)\log_2\left(\frac{1}{P(x_2)}\right)$$

$$\Rightarrow H(X) = \frac{1}{3}\log_2\left(\frac{3}{1}\right) + \frac{2}{3}\log_2\left(\frac{3}{2}\right)$$

$\Rightarrow H(X) = 0.9182$ bits/symbol

(ii) To determine $H(Y)$

$$\Rightarrow H(Y) = \sum_{j=1}^{2} P(y_j)\log_2\left(\frac{1}{P(y_j)}\right)$$

$$\Rightarrow P(Y) = P(X)\cdot P\left(\frac{Y}{X}\right)$$

$$P(Y) = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix} = \begin{bmatrix} 0.2889 \\ 0.7111 \end{bmatrix}$$

$P(y_1) = 0.2889$, $P(y_2) = 0.7111$

$$H(Y) = P(y_1)\log_2\left(\frac{1}{P(y_1)}\right) + P(y_2)\log_2\left(\frac{1}{P(y_2)}\right)$$

$H(Y) = 0.8672$ bits/symbol

(iii) To determine $H(X,Y)$

$$\Rightarrow H(X,Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)\log_2\left(\frac{1}{P(x_i,y_j)}\right)$$

$$P(x_i,y_j) = P\left[\frac{y_i}{x_i}\right]\cdot P(x_i)$$

$$P(x_1,y_1) = P\left(\frac{y_1}{x_1}\right)\cdot P(x_1) = \frac{2}{3}\times\frac{1}{3} = \frac{2}{9}$$

$$P(x_1,y_2) = P\left(\frac{y_2}{x_1}\right)\cdot P(x_1) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$$

$$P(x_2,y_1) = P\left(\frac{y_1}{x_2}\right)\cdot P(x_2) = \frac{1}{10}\times\frac{2}{3} = \frac{2}{30}$$

$$P(x_2,y_2) = P\left(\frac{y_2}{x_2}\right)\cdot P(x_2) = \frac{9}{19}\times\frac{2}{3} = \frac{18}{30}$$

$$\Rightarrow H(X,Y) = P(x_1,y_1)\log_2\left(\frac{1}{P(x_1,y_1)}\right) + P(x_1,y_2)\log_2\left(\frac{1}{P(x_1,y_2)}\right) +$$

$$P(x_2,y_1)\log_2\left(\frac{1}{P(x_2,y_1)}\right) + P(x_2,y_2)\log_2\left(\frac{1}{P(x_2,y_2)}\right)$$

$$H(x,y) = \frac{2}{9}\log_2\left(\frac{9}{2}\right) + \frac{1}{9}\log_2 9 + \frac{2}{30}\log_2\left(\frac{30}{2}\right) + \frac{18}{30}\log_2\left(\frac{30}{18}\right)$$

$$= 1.5365 \text{ bits/symbol}$$

(iv) $H(Y/X) = H(X,Y) - H(X)$

$$= 1.5365 - 0.9182 = 0.6183 \text{ bits/symbol}$$

(v) $H(X/Y) = H(X,Y) - H(Y)$

$$= 1.5365 - 0.8672 = 0.6692 \text{ bits/symbol}$$

(vi) $I(X,Y) = H(X) - H(X/Y)$

$$= 0.918 - 0.6692 = 0.249 \text{ bits/symbol}$$

$I(X,Y) = H(Y) - H\left(\frac{Y}{X}\right)$

$$= 0.8672 - 0.6183 = 0.249 \text{ bits/symbol}$$

Source Coding Theorem :- (Shannon's First Theorem)

⇒ For a given discrete memoryless source of entropy H, the average code word length $\bar{N}$ for any distortionless source encoding is bounded as $\bar{N} \geq H$

$H \rightarrow$ Entropy $\rightarrow$ Average Information
$\bar{N} \rightarrow$ Average Number of bits per symbol

⇒ If the average no. of bits/symbol $\bar{N}$ is less than entropy H, then error is created in transmitted message by discrete memoryless source.

(i) Code efficiency :-

$$\eta = \frac{H}{\bar{N}} \qquad \because \eta = \text{code efficiency}$$

(ii) Code Redundency :-

$$V = 1 - \eta$$

(iii) Code Variance :-

$$\sigma^2 = \sum_{K=0}^{L-1} P_K(n_K - \bar{N})$$

$\sigma^2 \rightarrow$ Variance of code
$L \rightarrow$ Number of symbol
$P_K \rightarrow$ Probability of K^{th} symbol
$n_K \rightarrow$ Number of bits assigned to K^{th} symbol
$\bar{N} \rightarrow$ Average codeword length

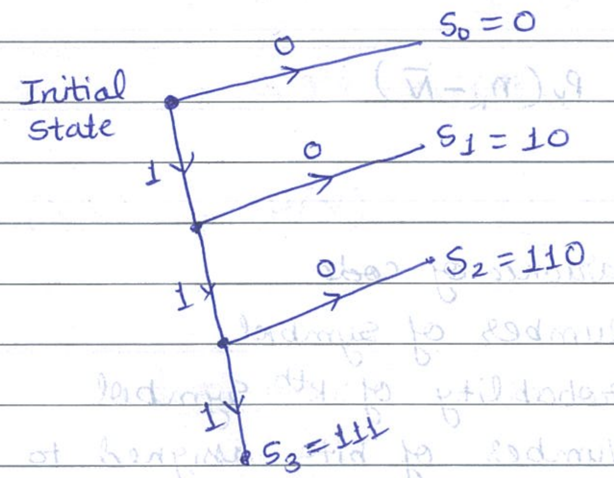
Unit-2

Variable Length Coding

(a) Prefix Coding (Instaneous Coding) :-

***

Source symbol	Probability of occurence	Prefix code
S_0	0.5	0 ← code word
S_1	0.25	10 ← code word (1 = Prefix)
S_2	0.125	110 ← code word (11 = Prefix)
S_3	0.125	111 ← code word (11 = Prefix)

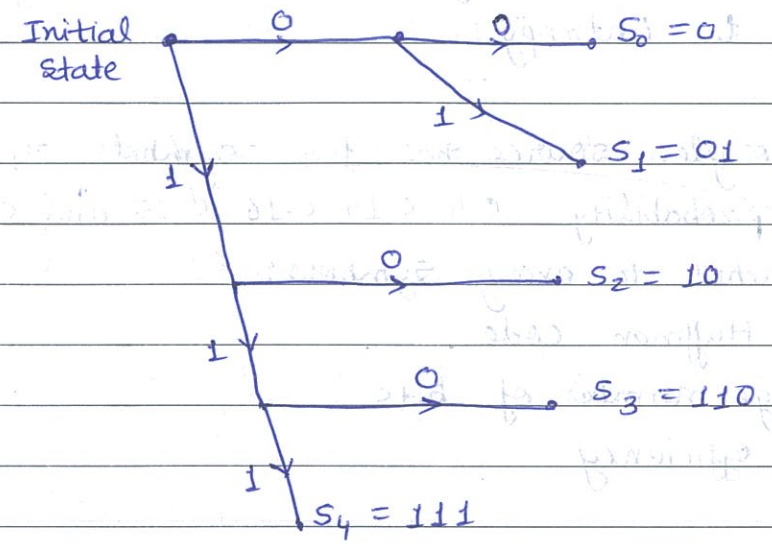


∴ Direction tree for decoding Prefix code

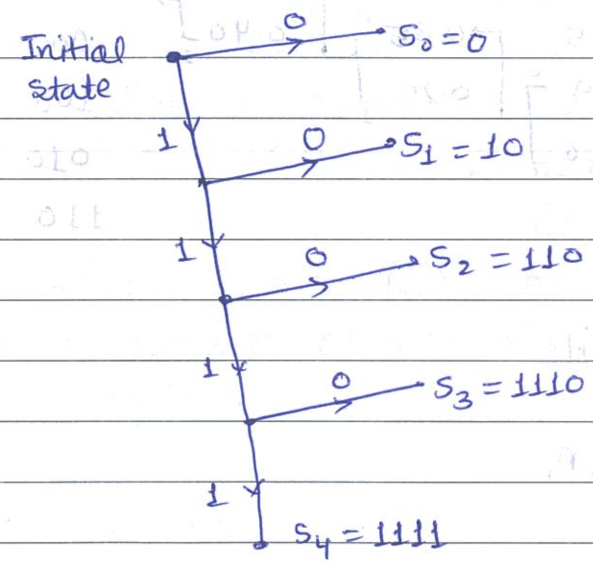
Que State condition for unique decodability of codes four codes given below :-

Symbol	Code A	Code D
S_0	0	00
S_1	10	01
S_2	110	10
S_3	1110	110
S_4	1111	111

Sol^n ⇒ Decision Tree for D :-



⇒ Decision Tree for A :-



*** This is variable length coding algorithm. It assigns binary digits to the msgs as per their probabilities of occurance. Prefix of the code word means any sequence in which is initial part of sequence.

(b) Huffman Coding :- It is also variable length coding. This type of coding makes average number of binary digits nearly equal to Entropy.

Ques A discrete memoryless source has five symbols x_1, x_2, x_3, x_4 and x_5 with probability 0.4, 0.19, 0.16, 0.15 and 0.10 respectively attached to every symbols :-
(a) To obtain Huffman code.
(b) Find average number of bits.
(c) Find code efficiency.

Sol^n

(a)

Message	S-I	S-II	S-III	S-IV	Binary bits obtained	Code Word	n_k
x_1	0.40	0.40	0.40	0.60	1	1	1
x_2	0.19	0.25	0.35	0.40	000	000	3
x_3	0.16	0.19	0.25		100	001	3
x_4	0.15	0.16			010	010	3
x_5	0.10				110	011	3

(b) Average number of bits :

$$\bar{N} = \sum_{K=1}^{L} P_K n_K$$

$$\bar{N} = P_1 n_1 + P_2 n_2 + P_3 n_3 + P_4 n_4 + P_5 n_5$$

$$\bar{N} = (0.4 \times 1) + (0.19 \times 3) + (0.16 \times 3) + (0.15 \times 3) + (0.10 \times 3)$$

$$\bar{N} = 2.2$$

(c) Code efficiency $= \eta = \frac{H}{\bar{N}}$

$$\Rightarrow H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right) = \sum_{K=1}^{5} \log_2\left(\frac{1}{P_K}\right)$$

$$\Rightarrow H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + \cdots + P_5 \log_2\left(\frac{1}{P_5}\right)$$

$$\Rightarrow H = 0.4 \log_2\left(\frac{1}{0.4}\right) + 0.19 \log_2\left(\frac{1}{0.19}\right) + 0.16 \log_2\left(\frac{1}{0.16}\right) + 0.15 \log_2\left(\frac{1}{0.15}\right) + 0.10 \log_2\left(\frac{1}{0.10}\right)$$

$$\Rightarrow H = 2.2281$$

$$\Rightarrow \eta = \frac{H}{\bar{N}} = \frac{2.2281}{2.2} = 1.012$$

(c) Shannon - Fanno Algorithm :- It is used to encode the message depending upon their Probability. This Algorithm allots less number of bits for highly Probable message.

Ques Obtain codeword for message if the Probability of message are given by $m_1 = 16/32$, $m_2 = 4/32$, $m_3 = \frac{4}{32}$, $m_4 = \frac{2}{32}$, $m_5 = \frac{2}{32}$, $m_6 = 2/32$, $m_7 = 1/32$, $m_8 = 1/32$
(a) average number of bits $\bar{N}$?
(b) Code efficiency.

Sol^n

Message	Probability	S-I	S-II	S-III	S-IV	S-V	Code word	n_K
m_1	16/32	0					0	1
m_2	4/32	1	0	0			100	3
m_3	4/32	1	0	1			101	3
m_4	2/32	1	1	0	0		1100	4
m_5	2/32	1	1	0	1		1101	4
m_6	2/32	1	1	1	0		1110	4
m_7	1/32	1	1	1	1	0	11110	5
m_8	1/32	1	1	1	1	1	11111	5

a) Average Number of bits :

$$\bar{N} = \sum_{i=1}^{L} P_K n_K = P_1 n_1 + P_2 n_2 + P_3 n_3 + P_4 n_4 + P_5 n_5 + P_6 n_6 + P_7 n_7 + P_8 n_8$$

$$\bar{N} \Rightarrow 1 \times \frac{16}{32} + 3 \times \frac{4}{32} + 3 \times \frac{4}{32} + 4 \times \frac{2}{32} + 4 \times \frac{2}{32} + 4 \times \frac{2}{32} + 5 \times \frac{1}{32} + 5 \times \frac{1}{32}$$

$$\bar{N} \Rightarrow \frac{37}{16}$$

b) Code efficiency :

$$\bar{\eta} = \frac{H}{\bar{N}}$$

$$H = \sum_{K=1}^{8} P_K \log_2 \left(\frac{1}{P_K}\right)$$

$$H = P_1 \log_2 \left(\frac{1}{P_1}\right) + \cdots + P_8 \log_2 \left(\frac{1}{P_8}\right)$$

Channel Capacity :

The channel capacity of any channel is the max. informa^n rate with error of probability is within the tolerable limits.

Channel capacity is given by :

$$C = B \log_2 \left(1 + S/N\right) \text{ bits / sec.}$$

$B \longrightarrow$ Bandwidth of the channel
$S \longrightarrow$ Signal Power
$N \longrightarrow$ Noise Power

$$N = \int_{B}^{B} \frac{N_0}{2} \cdot df = N_0 B$$

$\frac{N_0}{2} \longrightarrow$ Power spectral density of white noise

Shannon's Theorem on channel Capacity :

Statement : For a given source of M equally likely msg m is generating informa^n at a rate R.

Given channel capacity C, Then :

if $R \leq C$

Then output of informa^n may be transmitted over the channel with a prob. of error in the received msg may be arbitarily small.

Explanation :
The theorem says that if $R \le C$, it is possible to transmit informa^n without any error if noise is present. Coding techniques are used to detect and correct the error.

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Q) The data is transmitted at the rate of 1000 bits / sec over a channel having bandwidth $B = 3000$ Hz. Determine the signal to the noise ratio required. If the BW is increased to 10000 Hz, then determine a signal to noise ratio.

→ a) given :

$C = 10000$

$B = 3000$ Hz

$$C = B \log_2 (1 + S/N)$$

$$10000 = 3000 \log_2 (1 + S/N)$$

$$\Rightarrow \frac{10000}{3000} = \log_2 (1 + S/N)$$

$$\Rightarrow 10/3 = \log_2 (1 + S/N)$$

$$\Rightarrow 2^{10/3} = 1 + S/N$$

$$\Rightarrow S/N = 9$$

b) If BW ↑sed to 10,000 Hz then :

$B = 10000$

$$C = B \log_2 (1 + S/N)$$

$$\Rightarrow 10000 = 10000 \log_2 (1 + S/N)$$

$$1 = \log_2 (1 + S/N)$$

$$\Rightarrow 2^1 = 1 + S/N$$

$$\Rightarrow 2 - 1 = S/N$$

$$\Rightarrow S/N = 1$$

Comment :
If BW ↑sed to 10,000 Hz then signal to noise ratio is decreased to 1.

Imp

Shanon limit for Channel Capacity (Derivation) :

channel capacity is given by –

$$C = B \log_2 (1 + S/N)$$

In above equation when the signal power is fixed & white gaussian noise is present, the channel capacity approaches an upper limit wid ↑se in BW 'B'.

$$N = N_0 B$$

$N \rightarrow$ Noise Power

$\frac{N_0}{2} \rightarrow$ white gaussian noise

$$C = B \log_2 \left(1 + \frac{S}{N_0 B}\right)$$

$$C = \frac{S}{N_0} \cdot \frac{N_0 B}{S} \log_2 \left(1 + \frac{S}{N_0 B}\right)$$

$$C = \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 B}\right)^{\frac{N_0 B}{S}}$$

$$C = S/N_0 \log_2 \left(1 + \frac{S}{N_0 B}\right)^{1/S/N_0 B}$$

for shanon limit of channel capacity: $B \rightarrow \infty$

$$C_\infty = \lim_{B \to \infty} C = \lim_{B \to \infty} S/N_0 \log_2 \left(1 + \frac{S}{N_0 B}\right)^{1/S/N_0 B}$$

$\Rightarrow$ Let $x = \frac{S}{N_0 B}$

$$C_\infty = \lim_{B \to \infty} \frac{S}{N_0} \log_2 (1+x)^{1/x}$$

If $B \rightarrow \infty$, $x \rightarrow 0$

$$C_\infty = S/N_0 \lim_{x \to 0} \log_2 (1+x)^{1/x}$$

$$\lim_{x \to 0} \log_2 (1+x)^{1/x} = e$$

$$C_\infty = S/N_0 \log_2 e$$

$$C_\infty = S/N_0 \times \left(\frac{\log_{10} e}{\log_{10} 2}\right)$$

$$C_\infty = \frac{S}{N_0} \times 1.44 = 1.44 \frac{S}{N_0}$$

C_∞ gives the upper limit on channel capacity. It is the shanon limit of chanel capacity.

8) An analog signal having 4 KHz BW is sampled at 1.25 times the nyquist rate & each sample is quantized into one of 256 equally likely levels. Assuming the samples to be statically independent:

① What is informa^n rate of this source?

② Can the o/p of this source transmitted widout error over an AWGN channel wid a BW of 10 KHz & S/N ratio of 20 dB?

③ Find S/N ratio required for error free transmission of part ②.

④ Find the BW required for an AWGN channel for error free transmission of o/p of the source if S/N ratio is 20 dB.

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→ given $B = 4\,KHz$
$M = 256$

$M = 256$ equally likely msg

entropy of equally likely msg is given by,

$$H = \log_2 M$$

$$H = \log_2 (256)$$

$$= \log_2 (2^8) = 8 \text{ bit / sample}$$

$$\text{Nyquist rate} = 2B$$
$$= 2 \times 4$$
$$= 8\,KHz$$

$$\text{Sampling rate} = r = 1.25 \text{ time of Nyquist rate}$$

$$r = 1.25 \times 8$$
$$\Rightarrow 10\,KHz$$
$$\Rightarrow 10000$$

① Information rate :
$$R = rH = 10000 \times 8$$
$$= 80{,}000 \text{ bits / sec}$$

② To check for error free transmission,
If $R < C$, the transmission is error free acc. to shanon ~~first~~ second theorem.

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$$C = \log_2 (1 + S/N)$$

$$\Rightarrow \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} (S/N)$$

$$\Rightarrow 20 = 10 \log_{10} (S/N)$$

$$2 = \log_{10} (S/N)$$

$$\Rightarrow 10^2 = S/N$$

$$\Rightarrow 100 = S/N \quad , B = 10\,KHz$$

$$C = B \log_2 (1 + S/N)$$

$$C = 10000 \log_2 (1 + 100)$$

$$\Rightarrow 10000 \log_2 (101)$$

$$C = 10000 \times \left(\frac{\log_{10}(101)}{\log_{10} 2}\right)$$

$$= 66587 \text{ bits / sec}$$

$$\Rightarrow R > C$$

Transmission suffers from error

③ S/N ratio for error free transmission :

$$\Rightarrow R < C$$

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$$\Rightarrow 80{,}000 < B \log_2\left(1+S/N\right)$$

$$\Rightarrow B = 10000$$

$$\Rightarrow 80{,}000 < 10000 \log_2\left(1+S/N\right)$$

$$\Rightarrow 8 < \log_2\left(1+S/N\right)$$

$$\Rightarrow 8 < \frac{\log_{10}\left(1+S/N\right)}{\log_{10} 2}$$

$$\Rightarrow 8 \times \log_{10} 2 < \log_{10}\left(1+S/N\right)$$

$$\Rightarrow 8 \times 0.3010 < \log_{10}\left(1+S/N\right)$$

$$\Rightarrow 2.40824 = \log_{10}\left(1+S/N\right)$$

$$\Rightarrow 10^{2.40824} = 1+S/N$$

$$256 = 1+S/N$$

$$\Rightarrow 256-1 = S/N$$

$$\Rightarrow 255 = S/N$$

$$\Rightarrow (S/N)_{dB} = 10\log_{10}(255)$$

$$\Rightarrow 24\ dB$$

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(4) $(S/N) = 20\ dB$

$$(S/N)_{dB} = 10\log_{10}\left(1+S/N\right)$$

$$\Rightarrow 20 = 10\log_{10}\left(1+S/N\right)$$

$$\Rightarrow \frac{20}{10} = \log_{10}\left(1+S/N\right)$$

$$\Rightarrow 2 = \log_{10}\left(1+S/N\right)$$

$$\Rightarrow 10^2 = 1+S/N$$

$$100 = 1+S/N$$

$$99 = S/N$$

$\Rightarrow$ for error free transmission

$$\Rightarrow R < C$$

$$\Rightarrow 8000 < B\log_2\left(1+S/N\right)$$

$$\Rightarrow 8000 < B\log_2(1+99)$$

$$\Rightarrow 8000 < B\log_2(100)$$

$$\Rightarrow 8000 < B\frac{\log_{10}(10)}{\log_{10}(2)}$$

$$\Rightarrow 11974\ HZ < B$$

Lempel - ZIV - Algorithm :

The logic behind lempel ZIV universal coding is as follows. The compression of an arbitrary sequence of bits is possible by coding 0's & 1's as same previous such string plus one new bit. Then, the new string formed by adding the new bit to the previously used prefix string becomes a potential prefix strings for future strings. This variable length blocks are k/a phrases. The phrases are listed in a dictionary w stores the existing phrases & their locations.

eg: let the sequence : (Phrases)

10101101101101010101
1,01011011011010101 0101
1,0, 1011011011010101 01
1,0,10, 11 011 0110101010101
1,0,10,11, 0 110110101 0101
1,0,10,11,01, 1011010101 0101
1,0,10,11,01,101, 1010101 01
1,0,10,11,01,101,1010,10101
1,0,10,11,01,101,1010,10101

Dictionary for the Lempel - ZIV algorithm :

Dictionary Location	Dictionary Content	Fixed length Coding
001	1	00001
010	0	00000
011	10	00010
100	11	00011
101	01	11101
110	101	00101
111	1010	01010
—	10101	10101

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"Unit - 3"
Linear Block Codes

Linear Block Codes:

A code is linear if the sum of any two code vector produce another code vector. This shows that any code vector can be represented as linear combination of other code vectors.

Consider that particular code vector consist of $m_1, m_2, m_3, ----m_k$ (msg bits) & $c_1, c_2 ----c_q$ check bits.

Then, the code vector can be written as:

$$X = (m_1, m_2 -----m_k, c_1, c_2 ------c_q)$$

$\Rightarrow n = q + k$

$n \rightarrow$ Total no. of ~~msg~~ linear code vector bits

- $X = (M / C)$ —— ①

$M = k$ bits msg vector
$C = q$ bits check vector
$X \rightarrow$ Linear code vector of n bits

- $X = MG$ —— ②

$G \rightarrow$ generator matrix of $k \times n$ size

$$[X]_{1\times n} = [M]_{1\times k} [G]_{k\times n}$$

- $G = [I_k \,|\, P_{k\times q}]_{k\times n}$

I_k = Identity matrix of $k \times k$

$P \rightarrow k \times q$ sub matrix

- $C = MP$

$$[C]_{1\times q} = [M]_{1\times k} [P]_{k\times q}$$

- $H = [P^T : I_q]_{q\times n}$

$H \rightarrow$ Parity check Matrix

- $S = EH^T$; E = error pattern for single bit error

$S \rightarrow$ Syndrom vector

- $S = YH^T$

$Y \rightarrow$ Receiving msg

- If $YH^T = 0$
Then there is no error in receiving msg

- $XH^T = 0$ (always)

error detection Capabilities:
$d_{min} \geq S+1$

$s \rightarrow$ error detecting bits

$d_{min} \geq 2t + 1$

$t \rightarrow$ error Correcting bits

d_{min} = Mini distance of linear block code

q) The parity check matrix of particular (7,4) linear block code is given by:

$$[H] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

① Find the generator matrix G.

② List all the code vector

③ What is the mini distance b/w code vectors?

④ How many error can be detected? How many error can be corrected?

⑤ Y = 1010110 is receiving msg. Check it is error free or not. If it not error free then correct the msg.

$$H = \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]_{3\times 7}$$

$q = 3 \rightarrow$ check bits

$n = 7 \rightarrow$ code vector bits

$n = q + k$

$7 = 3 + k$

$7 - 3 = k$

$4 = k$

$k = 4 \rightarrow$ msg bits

$$P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}_{q \times k = 3\times 4}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{4\times 3}$$

① $G = [I_k \mid P_{k\times q}]_{k\times n}$

$G = [I_4 \mid P_{4\times 3}]_{4\times 7}$

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & \phi & 1 \end{array}\right]_{4 \times 7}$$

(2) $C = MP$

$$[M]_{1 \times k} = [M]_{1 \times 4} = [m_1 \ m_2 \ m_3 \ m_4]$$

$$\Rightarrow [C]_{1 \times q} = [C]_{1 \times 3} = [m_1 \ m_2 \ m_3 \ m_4] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & \phi & 1 \end{bmatrix}$$

$$[c_1 \ c_2 \ c_3] = m_1 \oplus m_2 \oplus m_3$$

$$c_1 = m_1 \oplus m_2 \oplus m_3$$
$$c_2 = m_1 \oplus m_2$$
$$c_3 = m_1 \oplus m_3 \oplus m_4$$

Code vector Table:

								Code vector							
Sr. No.	m_1	m_2	m_3	m_4	c_1	c_2	c_3	m_1	m_2	m_3	m_4	c_1	c_2	c_3	d_{min}
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	1	1	0	0	0	1	0	1	1	3
3	0	0	1	0	1	0	1	0	0	1	0	1	0	1	3
4	0	0	1	1	1	1	0	0	0	1	1	1	1	0	4
5	0	1	0	0	1	1	0	0	1	0	0	1	1	0	3
6	0	1	0	1	1	0	1	0	1	0	1	1	0	1	4
7	0	1	1	0	0	1	1	0	1	1	0	0	1	1	4
8	0	1	1	1	0	0	0	0	1	1	1	0	0	0	3
9	1	0	0	0	1	1	1	1	0	0	0	1	1	1	4
10	1	0	0	1	1	0	0	1	0	0	1	1	0	0	3
11	1	0	1	0	0	1	0	1	0	1	0	0	1	0	3
12	1	0	1	1	0	0	ϕ	1	0	1	1	0	0	1	4
13	1	1	0	0	0	0	ϕ	1	1	0	0	0	0	1	3
14	1	1	0	1	0	ϕ	0	1	1	0	1	0	1	0	4
15	1	1	1	0	ϕ	0	0	1	1	1	0	ϕ	0	0	4
16	1	1	1	1	1	1	1	1	1	1	1	ϕ	1	1	7

(3) $d_{min} = 3$

$[d_{min} \neq 0]$

Mini distance code vector = 3

(4) $d_{min} \geq s+1$

$3 \geq s+1$

$3-1 \geq s$

$2 \geq s$

No. of error detected = 2

Now, no. of error corrected:

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$$\Rightarrow 3-1 \geq 2t$$
$$1 \geq t$$

No. of error corrected $= 1$

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(5) To find out Syndrome vector

$$S = E H^T$$

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E \rightarrow$ Error pattern matrix

E-Pattern matrix:

Sr. No.	Bit in error	B_1	B_2	B_3	B_4	B_5	B_6	B_7
		Non zero bit shows error						
1	1st	1	0	0	0	0	0	0
2	2nd	0	1	0	0	0	0	0
3	3rd	0	0	1	0	0	0	0
4	4th	0	0	0	1	0	0	0
5	5th	0	0	0	0	1	0	0
6	6th	0	0	0	0	0	1	0
7	7th	0	0	0	0	0	0	1

(1) Syndrome vector for 1st bit error:

$$S = [1000000] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = [101]$$

(2) Syndrome vector for error in 2nd bit:

$$S = [0100000] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = [111]$$

(3) Syndrome vector for error in 4th bit:

$$S = [0001000] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad S = 011$$

Error in bit	Syndrome vector
0	0 0 0
1	1 0 1
2	1 1 1
3	1 1 0
4	0 1 1
5	1 0 0
6	0 1 0
7	0 0 1

$S = YH^T$

$Y = 1010110$

$$S = [1010110]\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = 101$

Y is not error free msg. The error in 1st bit of Y.

Incorrect Message, $Y = 1010110$

Correct Message, $Y = 0010110$

Conversion of Non-Systematic form of matrices into Systematic form:

Non-Systematic generator matrix/polynomial —

$$G = \begin{bmatrix} 0 & 1 & 2 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \\ 1 & 2 & 3 & : & 1 & 0 & 1 \\ 1 & 0 & 2 & : & 0 & 1 & 1 \end{bmatrix}_{4\times 7}$$

$n = 4$

$k = 7$ It does not contain I_k

Systematic generator Polynomial —

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

It contain I_k.

I_k = Identity matrix

Method for Conversion of non-systematic matrices into systematic matrices:

1. Permutation of rows
2. multiplication of row by a non-scalar
3. Addition of scalar multiple of one row to another.

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④ Permutaⁿ of Columns

⑤ Multiplicaⁿ of any column by non-zero scalar

The 1st 3 operaⁿ are just row operation. The operaⁿ merely modify the basis. The last 2 operaⁿ are column operaⁿ convert the matrix one to produce equivalent code.

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Q) Consider the generator matrix of (4,3) code over GF(3).

$$G = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

Convert non-symmetric matrix into systematic matrix.

→ Multiplicaⁿ of any column by non-zero scalar method:

$$G = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$n = 4,\ k = 3$

$I_k = I_3$ (3 order identity matrix create)

$r_1 = r_1 - r_3$

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$$G = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$C_4 \rightarrow C_1$

$C_1 \rightarrow C_2$

$C_2 \rightarrow C_3$

$C_3 \rightarrow C_4$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{3 \times 4}$$

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$G = [I_k : P]_{k \times n}$$

$$G = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Q) For a systematic linear block code the three check digits $C_4 - C_5$ and C_6 are given by:

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$C_4 = m_1 \oplus m_2 \oplus m_3$
$C_5 = m_1 \oplus m_2$
$C_6 = m_1 \oplus m_3$

① Construct generator matrix G

② Construct generator matrix X

③ Construct parity check matrix H

④ Determine error correcting and detecting capability

⑤ Prepare the suitable decoding table or syndrome vector table.

⑥ Decode for received msg 101100.

Sol:
$K = 3 \, (m_1, m_2, m_3)$
$q = 3 \, (C_1, C_2, C_3)$
$n = K + q$
$= 3 + 3 = 6$

① To obtain generator matrix:

$C = MP$

$[C_4 \; C_5 \; C_6] = [m_1 \; m_2 \; m_3]_{1\times 3} \, [P]_{3\times 3}$

$$[C_4 \; C_5 \; C_6] = [m_1 \; m_2 \; m_3] \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

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$C_4 = m_1 P_{11} \oplus m_2 P_{21} \oplus m_3 P_{31} \rightarrow ①$

given:
$C_4 = m_1 \oplus m_2 + m_3 \longrightarrow ②$

$P_{11} = 1 \quad P_{21} = 1 \quad P_{31} = 1$

from matrix multiplication:

$C_5 = m_1 P_{12} \oplus m_2 P_{22} \oplus m_3 P_{32} \rightarrow ③$

given:
$C_5 = m_1 \oplus m_2 \longrightarrow ④$

Compare eqn ③ & ④
$P_{12} = 1 \quad P_{22} = 1 \quad P_{32} = 0$

from matrix multiplication:
$C_6 = m_1 P_{13} \oplus m_2 P_{23} \oplus m_3 P_{33} \rightarrow ⑤$

given:
$C_6 = m_1 \oplus m_3 \longrightarrow ⑥$

Compare eqn ⑤ & ⑥
$P_{13} = 1 \quad P_{23} = 0 \quad P_{33} = 1$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$G = [I_K : P_{K\times q}]$

$K = 3$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

② To obtain code vector :

Sr. No.	Msg vector (M)			check bits			Code vector (X)						Weight of code vector
	m_1	m_2	m_3	c_4	c_5	c_6	m_1	m_2	m_3	c_4	c_5	c_6	
1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	0	1	0	0	1	1	0	1	3
3	0	1	0	1	1	0	0	1	0	1	1	0	3
4	0	1	1	0	1	1	0	1	1	0	1	1	4
5	1	0	0	1	1	1	1	0	0	1	1	1	4
6	1	0	1	0	1	0	1	0	1	0	1	0	3
7	1	1	0	0	0	1	1	1	0	0	0	1	3
8	1	1	1	1	0	0	1	1	1	1	0	0	4

④ error detecting and correcting capability :

$$d_{min} = 3$$

a) error detecting :

$$d_{min} \geq s+1$$

$$3 \geq s+1$$

$$3-1 \geq s$$

$$2 \geq s$$

It is capable to maxi. two error bits

b) error correcting capability :

$$d_{min} \geq 2t+1$$

$$3 \geq 2t+1$$

$$3-1 \geq 2t$$

$$\frac{2}{2} \geq t$$

$$1 \geq t$$

error correcting capability is maximum

③ Parity check matrix H :

$$H = [P^T : I_q]_{q \times n}$$

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$q = 3$$

$$I_q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H = \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

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(5) Syndrom vector :

$S = EH^T$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E = error pattern matrix

Sr.	Error vector 'E' showing single bit error Patterns	Syndrome 's) vector	Comments
1	0 0 0 0 0 0	0 0 0	0 Error
2	1 0 0 0 0 0	1 1 1	1st bit
3	0 1 0 0 0 0	1 1 0	2nd bit
4	0 0 1 0 0 0	1 0 1	3rd bit
5	0 0 0 1 0 0	1 0 0	4th bit
6	0 0 0 0 1 0	0 1 0	5th bit
7	0 0 0 0 0 1	0 0 1	6th bit
8			

Syndrome vector for 4th bit

$$\Rightarrow S = [000100] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\ 0\ 0]$$

(6) Decode for received bit Message :-

$Y = 101100$

$\Rightarrow S = YH^T$

$$S = [101100] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\ 1\ 0]$$

Error in 2nd bit from syndrome vector table.

$\Rightarrow Y = 1\,0\,1\,1\,0\,0 \rightarrow$ Incorrect message

$\Rightarrow Y = 1\,1\,1\,1\,0\,0 \rightarrow$ Correct message

Linear block coder (6, 3) :

$n = 6$

$K = 3$, $d = n - k$

$= 6 - 3 = 3$

$n \rightarrow$ number of code vector bit

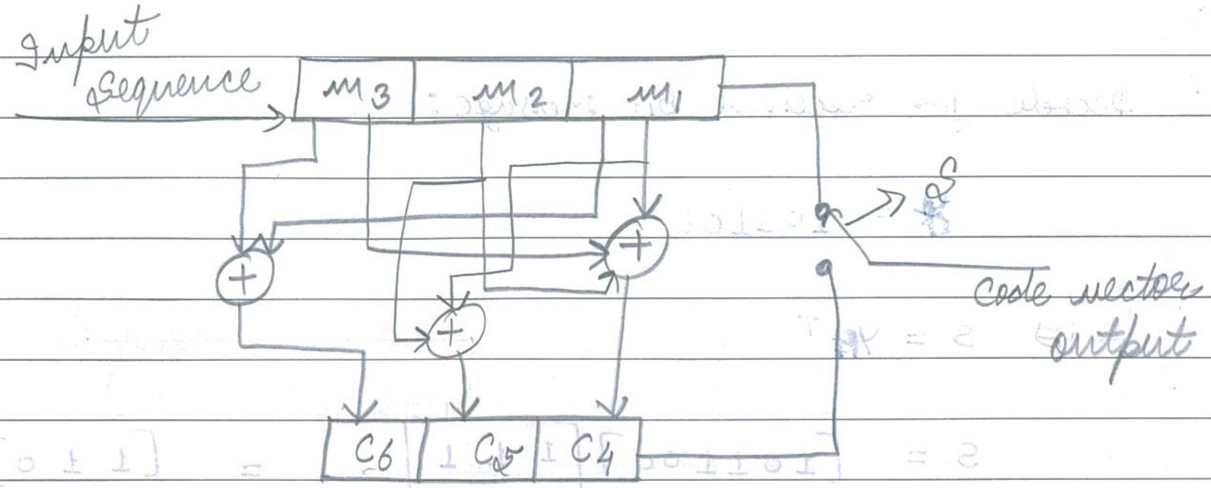
$K \rightarrow$ " " msg bit

$d \rightarrow$ " " check bit

$C_4 = m_1 \oplus m_2 \oplus m_3$

$C_5 = m_1 \oplus m_2$

$C_6 = m_1 \oplus m_3$



(6,4) Hamming coder or (6,4) Linear block Code

The linear block coder contains 2 reg. the upper shift reg. ~~contains~~ used for ~~store~~ the msg bits. The X-OR logic is used for generate the check bits, using msg bits. & finally generating check bit are stored in the lower shift reg. Both shift ~~&~~ reg. are u8 shift reg. The switch S is 1st connected to the msg bit shift reg. then connected to the check bit shift reg. In this way, it generates Linear Code vector.

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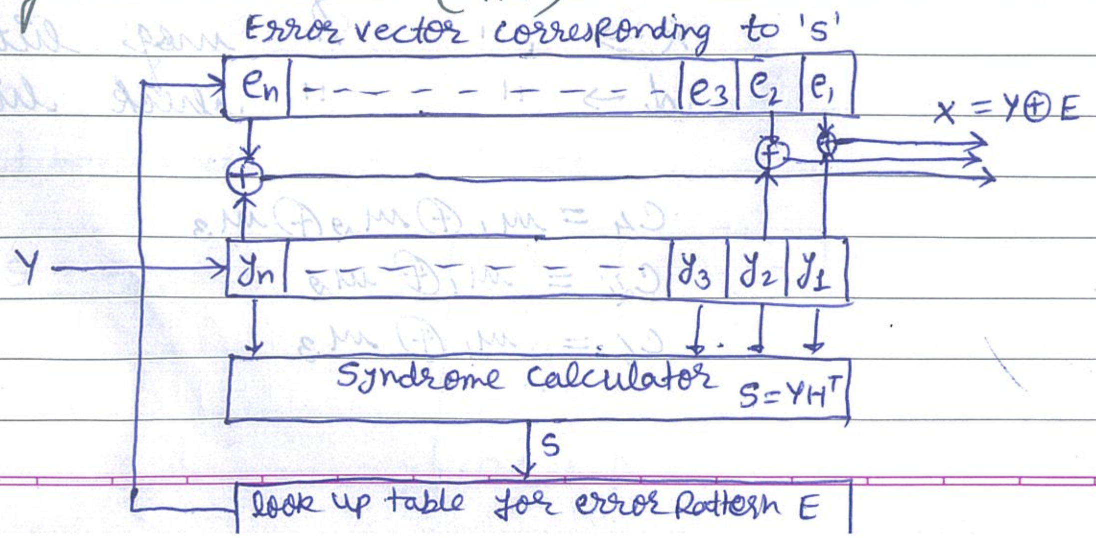
Syndrome decoder (n, k)



The Block diagrame for syndrome decoder for linear block code to correct error. The received n bit vector Y is stored in the n bit register. from these vector syndrome is calculated using

$$\Rightarrow S = YH^T$$

The H^T is stored in the syndrome vector. The q-bit syndrome vector than it is applied to the look up table for error pattern. depending upon the particular syndrome an error pattern is selected. this error pattern is added to the syndrome vector. Y.

The output is given by

$$\Rightarrow Y \oplus E = X$$

The block diagram shown above is correct only single bit error in above vector.

Ques# A generator Matrix of (6,3) linear block code is given by:-

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

(i) determine P, C, X, d_{min}, S, t, H and S

(ii) Check Y = 101101 is correct or incorrect. If incorrect then correct it.

Sol^n (i) Determine P :-

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 1 & ; & 0 & 1 & 1 \end{bmatrix} = [I_K : P]$$

$$\rightarrow \quad P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow n = 6$, $K = 3$, $q = n-K = 6-3 = 3$

$\rightarrow$ $\underline{\underline{C}}$:-

$$C = MP$$

$$[C_4 \; C_5 \; C_6] = [m_1 \; m_2 \; m_3] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow C_4 = m_1 \oplus m_2$

$\Rightarrow C_5 = m_1 \oplus m_2 \oplus m_3$

$\Rightarrow C_6 = m_1 \oplus m_3$

$\rightarrow$ S.No.	X = Code vector m_1	m_2	m_3	C_4	C_5	C_6	weight of code
1	0	0	0	0	0	0	0
2	0	0	1	0	1	1	3
3	0	1	0	1	1	0	3
4	0	1	1	1	0	1	4
5	1	0	0	1	1	1	4
6	1	0	1	1	0	0	3
7	1	1	0	0	0	1	3
8	1	1	1	0	1	0	4

$\rightarrow$ $\underline{\underline{d_{min}}} = 3$

$\rightarrow$ $d_{min} \geq s + 1$

$[\because 3] \geq s+1$

$2 \geq s$

$\rightarrow$ $d_{min} \geq 2t+1$

$3 \geq 2t+1$

$1 \geq t$

$\rightarrow$ $\underline{\underline{H}}$:-

$$H = [P^T : I_q] \qquad \because q = 3$$

$$P^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{H} = \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow$ $\underline{\underline{S}}$:-

$$S = E H^T$$

$\because E \rightarrow$ Error pattern matrix

S.No.	Error pattern						S
1	0	0	0	0	0	0	0 0 0
2	1	0	0	0	0	0	0 1 1
3	0	1	0	0	0	0	1 0 1
4	0	0	1	0	0	0	1 1 0
5	0	0	0	1	0	0	1 0 0
6	0	0	0	0	1	0	0 1 0
7	0	0	0	0	0	1	0 0 1

(ii) $Y = 101101$

$S = YH^T$

$$\Rightarrow [101101] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = 001$

error in 10110(1)

$\Rightarrow Y = 10110(1) \longrightarrow$ Incorrect - vector

$\Rightarrow Y = 101100 \longrightarrow$ Correct vector

"unit-2"

Q) Compute the Huffman coding and Shannon fano coding algo. for data compression. For a discrete memoryless source 'X' with six symbol $x_1, x_2, x_3, x_4, x_5, x_6$. Find a compact code for every symbol if the probability distribution is as follows :

$P(x_1) = 0.30$ $\quad P(x_4) = 0.12$

$P(x_2) = 0.25$ $\quad P(x_5) = 0.08$

$P(x_3) = 0.20$ $\quad P(x_6) = 0.05$

→ ① Entropy :

$$H = \sum_{K=1}^{M} P_K \log_2\left(\frac{1}{P_K}\right)$$

$M = 6$

$$H = \sum_{K=1}^{6} P_K \log_2\left(\frac{1}{P_K}\right)$$

$$H = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + \text{---}$$

$$\text{---} + P_6 \log_2\left(\frac{1}{P_6}\right)$$

$$H = 0.30 \log_2\left(\frac{1}{0.30}\right) + 0.25 \log_2\left(\frac{1}{0.25}\right) + 0.20 \log_2\left(\frac{1}{0.20}\right) + 0.12 \log_2\left(\frac{1}{0.12}\right) + 0.08 \log_2\left(\frac{1}{0.08}\right) + 0.06 \log_2\left(\frac{1}{0.06}\right)$$

$$H = 2.3568 \text{ bits / symbol}$$

② To obtain Shannon Fano code :

symbol	P_K	stage 1	stage 2	stage 3	stage 4	code word	No. of bits per msg n_K
x_1	0.30	0	0			00	2
x_2	0.25 (0.55)	0	1			01	2
x_3	0.20	1	0			10	2
x_4	0.12	1	1	0		110	3
x_5	0.08 (0.45)	1	1	1	0	1110	4
x_6	0.05	1	1	1	1	1111	4

- Code efficiency :

$$\eta = \frac{H}{\bar{N}}$$

$\bar{N} \rightarrow$ Avg. no. of bits

$$\bar{N} = \sum_{K=1}^{M} P_K n_K$$

$$M = 6$$

$$\bar{N} = \sum_{K=1}^{6} P_K n_K$$

$$\bar{N} = P_1 n_1 + \text{------} + P_6 n_6$$

$$\bar{N} = 0.30 \times 2 + 0.25 \times 2 + 0.20 \times 2 + 0.12 \times 3 + 0.08 \times 4 + 0.05 \times 4$$

$$\bar{N} = 2.38$$

$$\eta = \frac{H}{\bar{N}} = \frac{2.3658}{2.38} = 0.99$$

- Code redundancy :

$r \rightarrow$ code redundancy

$$r = 1 - \eta = 1 - 0.99 = 0.01$$

③ Huffman Coding :

symbol	stage 1	stage 2	stage 3	stage 4	stage 5	bits of code word	code word	n_K
x_1	0.30	0.30	0.30	0.45	0.55	00	00	2
x_2	0.25	0.25	0.25	0.30	0.45	10	01	2
x_3	0.20	0.20	0.25	0.25		11	11	2
x_4	0.12	0.13	0.20			101	101	3
x_5	0.08	0.12				0001	1000	4
x_6	0.05					1001	1001	4

- Code efficiency :

$$\eta = \frac{H}{\bar{N}}$$

$$\bar{N} = \sum_{K=1}^{M} P_K n_K = \sum_{K=1}^{6} P_K n_K$$

$$\bar{N} = P_1 n_1 + \text{------} + P_6 n_6$$

$$\bar{N} = 0.30 \times 2 + 0.25 \times 2 + 0.20 \times 2 + 0.12 \times 3 + 0.08 \times 4 + 0.05 \times 4$$

$$\bar{N} = 2.38$$

$$n = \frac{H}{\bar{N}}$$

$$n = \frac{2.2568}{2.38}$$

$$n = 0.99$$

- Code redundancy :

$$r = 1 - n$$

$$r = 1 - 0.99$$

$$r = 0.01$$

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"Unit - 4"

Cyclic Codes

Cyclic codes are the subset of linear block codes. The cyclic codes can be in systematic or non-systematic codes. In the systematic form check bits are calculated seperately & the code vector $X = (M : C)$.

Here, M represent the msg bit & C is check bit.

Defini^n of Cyclic Codes : A linear code is cyclic code if every cyclic shift of the code vector produces some other code vector. This defini^n includes 2 fundamental property of cyclic codes.

Properties of Cyclic Codes :

1. Linear property
2. Cyclic property

Linear property : This property states dat sum of any 2 code vector is also a valid code vector.

$$X_3 = X_1 \oplus X_2$$