$B = 4$ kHz.

$\therefore$ Nyquist rate $= 2B = 2 \times 4000 = 8000$ Hz.

Hence the sampling rate is, $r = 1.25 \times$ Nyquist rate

$= 1.25 \times 8000 = 10000$ messages/second.

Since the samples are quantized into 256 equally likely levels, there will be $M = 256$ samples. Each sample will have a probability of occurrence as $\frac{1}{256}$. The entropy for such messages is given by equation 2.6.9. i.e.,

$$H = \log_2(M) = \log_2(256) = 8 \text{ bits/sample}$$

i) To obtain information rate (R) :

The information rate (R) is given by equation 2.7.1 as,

$$R = r\,H = 10000 \times 8 = 80{,}000 \text{ bits/second}$$

ii) To check for error-free transmission of B = 10 kHz and $\frac{S}{N} = 20$ dB :

The signal to noise power ratio is 20 dB.

Hence,
$$\left(\frac{S}{N}\right)_{dB} = 10 \log \frac{S}{N}$$

$$\therefore \quad 20 = 10 \log \frac{S}{N}$$

$$\therefore \quad \frac{S}{N} = 100$$

The capacity of AWGN channel is given by equation 2.14.2 as,

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

$$= 10000 \log_2(1+100) = 10000 \log_2(101)$$

$$= 10000 \frac{\log_{10} 101}{\log_{10} 2} = 66.582 \text{ k bits/sec.}$$

If $R \le C$, then error-free transmission is possible. Here R = 80000 and C = 66.582 k bits/sec. Hence R > C. Therefore error-free transmission is not possible,

iii) $\frac{S}{N}$ ratio for error-free transmission :

In part (ii) we have R = 80000 and B = 10000. For error-free transmission $R \le C$.

Hence,
$$R \le C = B \log_2\left(1 + \frac{S}{N}\right)$$

i.e.
$$R \le B \log_2\left(1 + \frac{S}{N}\right)$$

$$80000 \le 10000 \log_2\left(1 + \frac{S}{N}\right)$$

$$\therefore \quad 8 \le \log_2\left(1 + \frac{S}{N}\right)$$

$$8 \le \frac{\log_{10}\left(1 + \frac{S}{N}\right)}{\log_{10} 2}$$

$$2.40824 \le \log_{10}\left(1 + \frac{S}{N}\right)$$

$$\frac{S}{N} \ge 255$$

or
$$\left(\frac{S}{N}\right)_{dB} = 10 \log 255 = 24 \text{ dB}$$

iv) To determine bandwidth :

The given $\frac{S}{N}$ ratio is 20 dB. Hence,

$$\left(\frac{S}{N}\right)_{dB} = 10 \log \frac{S}{N}$$

$$20 = 10 \log \frac{S}{N}$$

$$\therefore \quad \frac{S}{N} = 100 \text{ dB}$$

We know that for error-free transmission $R \le C$. Hence,

$$R \le C = B \log_2\left(1 + \frac{S}{N}\right)$$

i.e.
$$R \le B \log_2\left(1 + \frac{S}{N}\right)$$

Here R = 80000 bits/sec and $\frac{S}{N} = 100$. The bandwidth can be obtained from above equation i.e.

$$80000 \le B \log_2(1+100)$$

$$\le B \log_2(101)$$

$$\therefore \quad B \ge \frac{80000}{\log_2(101)}$$

$$\therefore \quad B \ge 11.974 \text{ kHz.}$$

This is the bandwidth required for errorfree transmission.

Ex.2.14.5 *For an AWGN channel with 4 kHz bandwidth and noise power spectral density $\frac{N_0}{2} = 10^{-12}$ W/Hz, the signal power required at the receiver is 0.1 mW. Calculate capacity of this channel.* **[Dec-2001, 4 Marks]**

Sol. : From equation 2.13.2, the channel capacity is given as,

$$C = B \log_2\left(1 + \frac{S}{N}\right) \text{ bits/sec}$$

Here
$$B = 4000 \text{ Hz}$$

$$S = 0.1 \times 10^{-3} \text{ W}$$

And noise power can be obtained as,

$$N = N_0B = 10^{-12} \times 2 \times 4000 = 8 \times 10^{-9} \text{ W}$$

$$\therefore \quad C = 4000 \log_2\left(1 + \frac{0.1 \times 10^{-3}}{8 \times 10^{-9}}\right) = 4000 \log_2(12501)$$

$$= 4000 \frac{\log_{10}(12501)}{\log_{10} 2} = 54.44 \text{ kbits/sec}$$

This the capacity of the channel.

Ex.2.14.6 *In a facsimile transmission of a picture, there are about 2.25×10^6 picture elements per frame. For good reception, twelve brightnen levels are necessary. Assuming all these levels to be equiprobable, calculate the channel bandwidth required to transmit one picture in every three minutes for a signal to noise power ratio of 30 dB. If SNR requirement increases by 40 dB, calculate the new bandwidth. Explain the trade-off between bandwidth and SNR by comparing two results.*

[May-2000, 8 Marks]

Sol. : i) Given data

Number of picture elements/frame = 2.25×10^6

Source levels or brightness levels, $M = 12$

Picture transmission rate = 1/3 minutes

$\left(\frac{S}{N}\right) dB = 30 dB$ and 40 dB

Calculate 'B'.

(ii) To calculate source entropy :

There are 12 distinct brightness levels. This means source has 12 different symbols of equal probability.

Hence M = 12 and $p_k = \frac{1}{M} = \frac{1}{12}$. For 'M' number of equally likely symbols, 'H' is given as,

$$H = \log_2 M \quad \text{from equation 2.6.9}$$

$$= \log_2 12$$

$$= 3.5849 \quad \text{bits/symbol (level)}$$

(iii) To calculate message rate 'r' :

One picture consists of 2.25×10^6 picture elements. And such one picture is transmitted in every 3 minutes. Hence,

$$r = \frac{2.25 \times 10^6}{3 \text{ minutes}} = \frac{2.25 \times 10^6}{180} \quad \text{picture elements/sec}$$

$$= 12500 \quad \text{picture elements/sec}$$

Here each picture element is a symbol. Hence,

$$r = 12500 \quad \text{symbols/sec}$$

(iv) To calculate information rate (R) :

Information rate is given as,

$$R = rH$$

$$= 12500 \quad \text{symbols/sec} \times 3.5849 \quad \text{bits/symbol}$$

$$= 44812 \quad \text{bits/sec}$$

(v) To calculate bandwidth (B) for $\frac{S}{N}$ = 30 dB :

We know that,

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \frac{S}{N}$$

$$30 \text{ dB} = 10 \log_{10} \frac{S}{N}$$

$$3 = \log_{10} \frac{S}{N}$$

$$\therefore \quad \frac{S}{N} = 10^3 = 1000$$

Channel capacity is given by equation 2.14.2 as,

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

From channel coding theorem,

$$R \leq C$$

$$\therefore \quad R \leq B \log_2\left(1 + \frac{S}{N}\right) \quad \text{... (2.14.22)}$$

Putting values in above equation,

$$44812 \leq B \log_2(1 + 1000)$$

$$\therefore \quad B \geq \frac{44812}{\log_2(1001)}$$

$$\therefore \quad B \geq 4495.93 \text{ Hz} \quad \text{or} \quad 4.5 \text{ kHz}$$

(vi) To calculate bandwidth (B) for $\frac{S}{N}$ = 40 dB :

We know that, $\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \frac{S}{N}$

$$\therefore \quad 40 \text{ dB} = 10 \log_{10} \frac{S}{N}$$

$$\therefore \quad \frac{S}{N} = 10,000$$

Putting values in equation 2.14.22,

$$44812 \leq B \log_2 (1+10,000)$$

$$B \geq 3372.4 \text{ Hz} \quad \text{or} \quad 3.37 \text{ kHz.}$$

Trade off between bandwidth and SNR :

The results are shown below

$\frac{S}{N}$ = 30 dB	B = 4.5 kHz
$\frac{S}{N}$ = 40 dB	B = 3.37 kHz

From the above result it is clear that the increase in signal to noise ratio by 10 times reduces the bandwidth requirement by more than 1 kHz.

Ex.2.14.7 *A black and white TV picture consists of about 2×10^6 picture elements with 16 different brightness levels, with equal probabilities. If pictures are repeated at the rate of 32 per second, calculate average rate of information conveyed by this TV picture source. If SNR is 30 dB, what is the maximum bandwidth required to support the transmission of the resultant video signal* **[May-2002, 7 Marks]**

Sol. : (i) Given data

Picture elements = 2×10^6

Source levels (symbols) = 16 i.e. M = 16

Picture repetition rate = 32/sec

$$\left(\frac{S}{N}\right)_{dB} = 30$$

(i) To obtain the source symbol entropy (H) :

Source emits any one of the 16 brightness levels. Here M = 16. These levels are equiprobable. Hence entropy of such source is given as,

$$H = \log_2 M$$

$$= \log_2 16 = 4 \text{ bits/symbol (level)}$$

(ii) To obtain symbol rate (r) :

Each picture consists of 2×10^6 picture elements. Such 32 pictures are transmitted per second. Hence number of picture elements per second will be,

$$r = 2 \times 10^6 \times 32 \text{ symbols/sec}$$

$$= 64 \times 10^6 \text{ symbols/sec}$$

(iii) To calculate average information rate (R) :

Information rate of the source is given as,

$$R = r H$$

$$= 64 \times 10^6 \times 4 \text{ bits/sec}$$

$$= 2.56 \times 10^8 \text{ bits/sec}$$

This is the average rate of information conveyed by TV picture.

(iv) To obtain the required bandwidth for $\frac{S}{N}$ = 30 dB :

We know that $\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \frac{S}{N}$

$$\therefore \quad 30 = 10 \log_{10} \frac{S}{N}$$

$$\therefore \quad \frac{S}{N} = 1000$$

Channel coding theorem states that information can be received without error if,

$$R \leq C$$

We have $R = 2.56 \times 10^8$ and $C = B \log_2 \left(1+\frac{S}{N}\right)$. Hence above equation becomes,

$$2.56 \times 10^8 \leq B \log_2 \left(1+\frac{S}{N}\right)$$

i.e. $$2.56 \times 10^8 \leq B \log_2 (1+1000)$$

or $$B \geq \frac{2.56 \times 10^8}{\log_2 (1001)} \quad \text{i.e. } 25.68 \text{ MHz}$$

Thus the transmission channel must have a bandwidth of 25.68 MHz to transmit the resultant video signal.

Ex.2.14.8 *A voice grade telephone channel has a bandwidth of 3400 Hz. If the signal to noise ratio (SNR) on the channel is 30 dB, determine the capacity of the channel. If the above channel is to be used to transmit 4.8 kbps of data determine the minimum SNR required on the channel* **[Dec-2002, 6 Marks]**

Sol. : (i) Given data

Channel bandwidth, B = 3400 Hz

$$\left(\frac{S}{N}\right)_{dB} = 30 \text{ dB}$$

We know that $\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \frac{S}{N}$

$$\therefore \quad 30 = 10 \log_{10} \frac{S}{N}$$

$$\therefore \quad \frac{S}{N} = 1000$$

(ii) To calculate capacity of the channel :

Capacity of the channel is given as,

$$C = B \log_2 \left(1+\frac{S}{N}\right)$$

$$= 3400 \log_2 (1+1000)$$

$$= 33.888 \text{ k bits/sec}$$

(iii) To obtain minimum $\left(\frac{S}{N}\right)$ for 4.8 kbps data :

Here the data rate is 4.8 kbps. From channel coding theorem,

$$R \leq C$$

Here R = 4.8 kbps and $C = B\log_2\left(1+\frac{S}{N}\right)$. Hence above equation becomes,

$$4.8 \text{ kbps} \leq B \log_2 \left(1+\frac{S}{N}\right)$$

i.e. $$4800 \leq 3400 \log_2 \left(1+\frac{S}{N}\right)$$

i.e. $$\log_2 \left(1+\frac{S}{N}\right) \geq 1.41176$$

$$\frac{\log_{10}\left(1+\frac{S}{N}\right)}{\log_{10} 2} \geq 1.41176$$

$$\therefore \quad \frac{S}{N} \geq 1.66$$

This means $\left(\frac{S}{N}\right)_{min} = 1.66$ to transmit data at the rate of 4.8 kbps.

Ex.2.14.9 *For an AWGN channel with 4.0 kHz bandwidth, the noise spectral density $\eta / 2$ is 1.0 pico watts/Hz and the signal power at the receiver is 0.1 mW. Determine the maximum capacity, as also the actual capacity for the above AWGN channel.*
[Dec-2003, 5 marks]

10^{-12}

Sol. : (i) To obtain actual capacity :

This part is identical to Ex. 2.14.5. The capacity calculated in this example as,

$$C = 54.44 \text{ k bits/sec}$$

(ii) To obtain maximum capacity :

Signal power is given as, $S = 0.1 \times 10^{-3}$ W

Maximum capacity of gaussian channel is given as,

$$C_\infty = 1.44 \frac{S}{N_0}$$

Here $\frac{N_0}{2} = 1 \times 10^{-12}$ watts/Hz. Hence above equation becomes,

$$C_\infty = 1.44 \frac{0.1 \times 10^{-3}}{2 \times 10^{-12}}$$

$$= 72 \times 10^6 \text{ Hz or } 72 \text{ MHz.}$$

Review Questions

1. *State and explain Shannon's theorem on channel capacity.*
2. *State and explain Shannon-Hartley theorem.*

Unsolved Example

1. *A 2 kHz channel has signal to noise ratio of 24 dB.*

   a) Calculate maximum capacity of this channel.

   b) Assuming constant transmitting power calculate maximum capacity when channel bandwidth is i) halved and ii) reduced to quarter of its original value.

Ans. :

$$C = 15.95 \times 10^3 \text{ bits/sec}$$

$$C_{1/2} = 8.97 \times 10^3 \text{ bits/sec}$$

$$C_{1/4} = 4.99 \times 10^3 \text{ bits/sec}$$

University Questions

1. *In a facsimilie transmission of a picture, there are about $\left[2.25 \times 10^6\right]$ picture elements per frame. For good reproduction, twelve brightness levels are necessary. Assuming all these, levels to be equiprobable, calculate the channel bandwidth required to transmit one picture in every three minutes for a single to noise power ratio of 30 dB. If SNR requirement increases to 40 dB, calculate the new bandwidth. Explain the trade-off between bandwidth and SNR, by comparing the two results.* **[Ans. : Please refer Ex. 2.14.6] [May-2000, 8 Marks]**
2. *Show that the channel capacity of an ideal AWGN channel with infinite bandwidth is given by*

   $$C_\infty = \frac{1}{ln 2} \frac{s}{\eta}$$

   where 's' is the average signal power and $\eta / 2$ is the power spectral density of white gaussian noise.
   [Ans. : Please refer Ex. 2.14.3] [Dec-2000, 8 Marks]