

Unit - I

Number Systems

1) Binary Number System

2) Octal Number System ($0, 1, 2, 3, 4, 5, 6, 7$)

3) Decimal Number System ($0, 1, 2, 3, 4, 5, 6, 7, 8, 9$)

4) Hexadecimal Number system ($0, 1, 2, \dots, 9, A, B, \dots, F$)

Ques-1

$$23.47_{10} = (23.47)_{10} = 1101.11_2$$

$$\begin{array}{r} 2 | 23 \rightarrow 1 \\ 2 | 11 \rightarrow 1 \\ 2 | 5 \rightarrow 1 \\ 2 | 2 \rightarrow 0 \\ \hline & 1 \end{array} \quad (23)_{10} \rightarrow (10111)_2$$

Ans

$$0.47 \times 2 = 0.94 \quad 0.94 \times 2 = 1.88$$

$$0.88 \times 2 = 1.76 \quad 1.76 \times 2 = 3.52$$

$$0.52 \times 2 = 1.04 \quad 1.04 \times 2 = 0.08$$

$$0.08 \times 2 = 0.16 \quad 0.16 \times 2 = 0.32$$

$$0.32 \times 2 = 0.64 \quad 0.64 \times 2 = 1.28$$

$$0.128 \times 2 = 0.256 \quad 0.256 \times 2 = 0.512$$

$$0.512 \times 2 = 1.024 \quad 1.024 \times 2 = 2.048$$

$$2.048 \times 2 = 4.096 \quad 4.096 \times 2 = 8.192$$

$$8.192 \times 2 = 16.384 \quad 16.384 \times 2 = 32.768$$

$$32.768 \times 2 = 65.536 \quad 65.536 \times 2 = 131.072$$

$$131.072 \times 2 = 262.144 \quad 262.144 \times 2 = 524.288$$

$$524.288 \times 2 = 1048.576 \quad 1048.576 \times 2 = 2097.152$$

Ques-2

$$(10111.011110)_2 = (\quad)_{10}$$

$$\begin{aligned}
 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + \\
 &\quad 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + \\
 &\quad 1 \times 2^{-5} + 0 \times 2^{-6} \\
 &= 16 + 4 + 2 + 1 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \\
 &= 23 + 0.25 + 0.125 + 0.0625 + 0.03125 \\
 &= 23.46875
 \end{aligned}$$

Ques-3

$$(23.178)_{10} \rightarrow (\quad)_8$$

$$\begin{array}{r}
 8 | 23 \rightarrow 7 \\
 8 | 2 \rightarrow 2 \\
 0 \\
 \hline
 1.424 = 5 \times 28.0
 \end{array}$$

$$0.178 \times 8 = 1.424 = 10.0$$

$$0.356 \times 8 = 2.848 = 2$$

$$0.848 \times 8 = 6.784 = 0.1(10.0)$$

$$0.424 \times 8 = 3.392 = 3$$

$$0.392 \times 8 = 3.136 = 0.1(3)$$

$$0.136 \times 8 = 1.088 = 1$$

$$= (0.1331)_8$$

$$= (23.1331)_8$$

Ques-4

$$(27.1331)_8 = (\quad)_{10}$$

$$\begin{aligned}
 &= 2 \times 8^4 + 1 \times 8^3 + 3 \times 8^2 + 3 \times 8^1 + 1 \times 8^0 + \\
 &\quad 1 \times 8^{-1} + 1 \times 8^{-2} + 1 \times 8^{-3} + 1 \times 8^{-4} + \\
 &\quad 1 \times 8^{-5} + 0 \times 8^{-6} \\
 &= 128 + 512 + 192 + 24 + 1 + \frac{1}{8} + \frac{1}{64} + \frac{1}{512} + \frac{1}{4096} + \\
 &\quad 15360 = 15177618 \\
 &= 23 + 0.125 + 0.046875 + 0.005514 + \\
 &\quad 0.000229
 \end{aligned}$$

Ques-5

$$(92.27)_{10} = (\quad)_{16}$$

$$\begin{array}{r}
 16 | 92 \rightarrow 12 \rightarrow 0010 \\
 16 | 5 \rightarrow 5 \\
 0 \\
 \hline
 0110 = (56)_{16}
 \end{array}$$

$$0.27 \times 16 = 4.32 = 4$$

$$0.32 \times 16 = 5.12 = 5$$

$$0.12 \times 16 = 1.92 = 1$$

$$0.92 \times 16 = 14.72 = 14 \rightarrow E$$

$$(0.27)_{10} = (0.451E)_{16}$$

$$= (56.451E)_{16}$$

Ques - 16

$$(5C.451E)_{16} \rightarrow (\quad)_{10}$$

$$\begin{aligned} & 5 \times 16^1 + C \times 16^0 + 4 \times 16^{-1} + 5 \times 16^{-2} + 1 \times 16^{-3} + E \times 16^{-4} \\ = & 80 + 12 + \frac{4}{16} + \frac{5}{256} + \frac{1}{4096} + \frac{14}{65536} \\ = & 92 + 0.25 + 0.01953 + 0.000244 + 0.000213 \\ = & (92.269987)_{10} \end{aligned}$$

Octal Digit

Binary Representation

0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Hexadecimal

Binary Representation

0	()	= 010000_2
1	()	= 0001_2
2	()	= 0010_2
3	()	= 0011_2
4	()	= 0100_2
5	()	= 0101_2
6	()	= 0110_2
7	()	= 0111_2
8	()	= 1000_2
9	()	= 1001_2
A	()	= 1010_2
B	()	= 1011_2
C	()	= 1100_2
D	()	= 1101_2
E	()	= 1110_2
F	()	= 1111_2

Ques - 7

$$\begin{aligned} (23.47)_8 &= (\quad)_2 \\ &= (010011 \cdot 100111)_2 \end{aligned}$$

Ques - 8

$$\begin{aligned} (01011101 \cdot 11001011)_2 &= (\quad)_8 \\ &= (135 \cdot 626)_8 \end{aligned}$$

Ques - 9

$$(4AE \cdot D7F)_{16} = ()_2$$

1000

$$(01001010\ 1110 \cdot 11010111\ 1111)_2$$

Ques - 10

$$(16101110111 \cdot 101011011)_2 = ()_{16}$$

$$= (577 \cdot D6C)_{16}$$

0101
1101
0011
1011
0110
1111

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1011
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0110
1111

Codes

(Binary Decimal Code) ^{BCD}
~~(Binary Decimal Code) BCD~~

Decimal	Binary	BCD Code	Notes
0	0000	0000	0
1	0001	0100	0001
2	0010	0010	0010
3	0011	0001	0011
4	0100	0000	0100
5	0101	0010	0100
6	0110	0100	0001
7	0111	0101	0101
8	1000	0000	0101
9	1001	1001	1100
10	1010	0100	N.E (Not Equal)
11	1011	1011	N.E (Not Equal)
12	1100	1100	N.E = 01(F2) N.S.
13	1101	1101	N.S.
14	1110	1110	N.S.
15	1111	1111	N.S.

Ques - 11

$$1)(7)_{10}$$

$$= (0111)_{BCD}$$

$$= (111)_2$$

$$2)(19)_{10} = (10011)_2 = (00011001)_{BCD}$$

Excess-3 code

Digit BCD Excess-3 code

Digit	BCD	Excess-3 code
0	0000	0011 → 3
1	0001	0100 → 4
2	0010	0101 → 5
3	0011	00110 → 6
4	0100	0111 → 7
5	0101	1000 → 8
6	0110	1001 → 9
7	0111	01010 → 10 → A
8	1000	11001 → 11 → B
9	1001	00100 → 12 → C
P	100	P

Gray Code

$$① (27)_{10} = (1011)_2 = (0011) \text{ gray code}$$

$$\begin{array}{r} 2 | 27 \rightarrow 11 \\ 2 | 13 \rightarrow 11 \\ 2 | 6 \rightarrow 0 \\ 2 | 3 \rightarrow 1 \\ \hline \end{array} \quad (27)_{10} = (11011)_2$$

$$(10011000) = (11001) = (P)(S)$$

$$(11011)_2 \rightarrow (1010110) \text{ gray code}$$

$$= (10110) \text{ gray code}$$

$$② (38)_{10}$$

$$\begin{array}{r} 2 | 38 \rightarrow 0 \\ 2 | 19 \rightarrow 1 \\ 2 | 9 \rightarrow 0 \\ 2 | 4 \rightarrow 0 \\ 2 | 2 \rightarrow 0 \\ \hline \end{array} = (100110)_2$$

$$(100110)_2$$

$$= (110101) \text{ gray code}$$

$$(100110)_2$$

BCD Addition

$$(23)_{BCD} = 0010001$$

$$+ (47)_{BCD} = 0100011$$

$$\begin{array}{r} 00000110 \\ + 01110000 \\ \hline 01110000 \end{array}$$

$$\begin{array}{r} 01000011 \\ 01110010 \\ \hline 10110101 \end{array}$$

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$$\begin{array}{r} (43)_{BCD} = 01000011 \quad (11011) \\ (72)_{BCD} = 01110010 \\ \hline 10110101 \quad (01101) \\ + 00000110 \\ \hline 10111011 \quad (88) \text{ (c)} \\ \text{Add 6} \end{array}$$

$$\begin{array}{r} 90110101 \\ 00100000 \\ \hline 000100010101 \\ (115)_{BCD} \end{array}$$

$$\begin{array}{r} Ques. \quad (11001) \\ (49)_{BCD} = 01001001 \quad (101011) \\ (38)_{BCD} = 00111000 \\ \hline 10000001 \quad (011001) \\ + 00000111 \\ \hline 10000111 \rightarrow (81)_{BCD} \end{array}$$

Ques. 1) Add 6 in lower nibble

① its value greater than 9

② addition of D₃ bit carry is generated

ii) Add 6 in upper nibble

① its value greater than 9

② addition of D₇ bit carry is generated

Ques. 2) Add 6 in lower nibble

$$\begin{array}{r} (94)_{BCD} = 10010100 \\ (83)_{BCD} = 10000011 \quad (\text{PSA}) + (\text{SA}) \text{ (i)} \\ \hline 00010111 \quad (177)_{BCD} \\ 01100000 \\ \hline 000101110111 \\ 1 \quad 7 \quad 7 \quad (177) \end{array}$$

1) Binary Addition

$$(27)_{10} = (11011)_2 = 00011011$$

$$(38)_{10} = (100110)_2 = 00100110$$

$$\begin{array}{r} 01000001 \quad (01000001) \\ + 00010001 \quad (00010001) \\ \hline 01000001 \quad (01000001) \end{array}$$

(65)₁₀

2) Octal Addition

$$\begin{array}{r} i) (76)_8 + (43)_8 = (1100100)_2 = 10001100 \\ (76)_8 = (1100100)_2 \\ (43)_8 = (10101)_2 \\ \hline (141)_8 \end{array}$$

$$\begin{array}{r} ii) (47)_{10} \\ (56)_8 \\ \hline 01250_8 \end{array}$$

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3) Hexadecimal Addition

Hexadecimal Addition	00	10
i) $(1AB)_{16} + (A24)_{16}$	01	11
	02	12
	03	13
	04	14
	05	15
	06	16
	07	17
	08	18
	09	19
	0A	1A
	0B	1B
	0C	1C
	0D	1D
	0E	1E
	0F	1F

$$\text{ii) } \begin{array}{l} (A \text{ F D})_{16} \\ (B \text{ 9 A})_{16} \\ \hline \end{array} = \begin{array}{l} 1100100 \\ 0110010 \\ \hline 1010010010 \end{array}$$

Subtraction

① Subtraction using 1's complement

$$(38)_{10} - (27)_{10}$$

$$(100110)_2 - (11011)_2$$

21100 20100 20200 20300 20400 20500 20600

5 8-81 1011 11 8 (EP)

$$38 + (-27)$$

$$(-27)_{10} = \text{0000011001}$$

$$1^{\text{st}} \text{ compl.} = 11100100$$

Handwritten diagram illustrating a binary addition problem:

$$\begin{array}{r}
 \textcircled{1} \quad 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \\
 + 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 0 \ 0 \\
 \hline
 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0
 \end{array}$$

The diagram shows the addition of two binary numbers. The top number is 00001010 and the bottom number is 101100100. The result is 11010010. A circled '1' at the start of the carry chain indicates a leading zero. Blue arrows point from the carry chain to the sum bits. One arrow points to the fourth column from the left, labeled "Add carry here". Another arrow points to the fifth column from the left, labeled "Add carry here". A third arrow points to the second column from the left, labeled "discard carry".

$$\textcircled{1} \quad -\sqrt{(27)_{10}} = -(38)_{10} \mid 0100111$$

$$27 = (000\ 11011)_2$$
~~27 = (00011011)_2~~

$$38 = (00100110)_2$$

$$(-38)_{10} = (11011001)_2 \equiv 0189$$

$271 \times 11 = 2981$

$$-38 = 11011001$$

$$\overline{(11110100)_2}$$

$$(\overline{00001011})_{1's\ complement} = (-11)_{10}$$

When carry does not come, take its complement of answer

$$s((1+\delta N)000) \Rightarrow -s(1+\delta T))_+ = s_1(Ts)$$

$$f(0,1100100) = \varphi(0,11001) = \varphi(88)$$

$$+ \omega_1(8\%) + \omega_1(FS) = \omega_1(8\%) - \omega_1(FS)$$

01100100 11 - 38

~~-mbox 211 ← 10011011~~

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Subtraction using 2's complement

a) $(38)_{10} - (27)_{10}$

$$(38)_{10} = (100110)_2 = (00100110)_2$$

$$(27)_{10} = (11011)_2 = (00011011)_2$$

$$(38)_{10} - (27)_{10} = (38)_{10} + (-27)_{10}$$

$$(27)_{10} = (00011011)_2$$

$\begin{array}{r} 11100100 \\ \hline \end{array} \rightarrow 1\text{'s complement}$

$$\begin{array}{r} + 1 \\ \hline 11100101 \end{array} \rightarrow 2\text{'s complement}$$

$$(38)_{10} = 00100110 \rightarrow 01 (88)$$

$$(-27)_{10} = 11100101$$

$$\textcircled{1} \quad \begin{array}{r} 00001011 \\ \hline = (11)_{10} \end{array}$$

discard last digit
00001011

$$= (11)_{10}$$

b) $(27)_{10} - (38)_{10}$ now for sub. plus with

$$(27)_{10} = (11011)_2 = (00011011)_2$$

$$(38)_{10} = (100110)_2 = (00100110)_2$$

$$(27)_{10} - (38)_{10} = (27)_{10} + (-38)_{10}$$

$$38_{10} = 00100110$$

$\begin{array}{r} 11011001 \\ \hline \end{array} \rightarrow 1\text{'s complement}$

$$\begin{array}{r} + 1 \\ \hline 11011010 \end{array} \rightarrow 2\text{'s complement}$$

$$\begin{array}{r} (27)_{10} = 000100110 \\ (-38)_{10} = 11011010 \\ \hline = 11110101 \end{array}$$

no carry \therefore no is negative

2's complement

$$\begin{array}{r} 1's \text{ comp} \rightarrow 00001010 \\ + 1 \\ \hline 00001011 \end{array} \rightarrow (11)_{10}$$

Subtraction using 7's complement

a) $(47)_8 - (36)_8$

$$(47)_8 + (-36)_8$$

= 77

$$\begin{array}{r} -36 \\ 41 \\ \hline \end{array} \rightarrow 7\text{'s complement}$$

47

+ 41

110

carry

$$= (10)_8$$

$$+ 1$$

$$\hline (11)_8$$

$$8 (11)_8$$

$$110$$

$$110$$

b) $(36)_8 - (47)_8 = (36)_8 + (-47)_8$

$$\begin{array}{r} 77 \\ - 47 \\ \hline 30 \end{array} \rightarrow 7\text{ comp}$$

$$\begin{array}{r} 36 \\ + 30 \\ \hline 66 \end{array} \leftarrow \text{PP}$$

$$\begin{array}{r} 77 \\ - 66 \\ \hline 11 \end{array} \rightarrow 7\text{ comp.}$$

$$(36)_8 - (47)_8$$

Subtraction using 8's complement (FH)

a) $(47)_8 - (36)_8 = (47)_8 + (-36)_8$

$$\begin{array}{r} 77 \\ - 36 \\ \hline 41 \end{array} \rightarrow 7\text{'s compn}$$

$$\begin{array}{r} 77 \\ - 36 \\ \hline 41 \\ + 1 \\ \hline 42 \end{array} \rightarrow 8\text{'s compn}$$

$$\begin{array}{r} 47 \\ + 42 \\ \hline 111 \\ \text{answ} \\ \text{carry} \\ \text{true} \end{array} \rightarrow (11)_8$$

b) $(36)_8 - (47)_8$

$$= (36)_8 + (-47)_8$$

$$\begin{array}{r} 77 \\ - 47 \\ \hline 30 \\ + 1 \\ \hline 31 \end{array} \rightarrow 8\text{'s compn}$$

$$\begin{array}{r} 36 \\ 31 \\ \hline 47 \\ + 31 \\ \hline 67 \end{array}$$

$$\begin{array}{r} 77 \\ - 67 \\ \hline 10 \\ + 1 \\ \hline 11 \end{array} \rightarrow 8\text{ w}$$

$$= (-11)_8 \text{ 2.01 parity error condition}$$

Subtraction using 9's complement (8)

a) $(88)_{10} - (47)_{10}$

$$(88)_{10} + (47)_{10} = 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$$

$$= 99$$

$$- 47$$

$$\begin{array}{r} 52 \\ \hline (52)_{10} \rightarrow 9\text{'s comp} \end{array}$$

$$\begin{array}{r} 88 \\ 52 \\ \hline 40 \\ + 1 \\ \hline 41 \end{array} \rightarrow (41)_{10}$$

$$\text{b) } (47)_{10} - (88)_{10} = (47)_{10} + (-88)_{10}$$

$$\begin{array}{r} 99 \\ - 88 \\ \hline 11 \end{array} \rightarrow 9\text{'s comp.}$$

$$\begin{array}{r} 88 \\ + 11 \\ \hline 99 \end{array} \quad \begin{array}{r} 38 \\ 18 \\ \hline 56 \end{array} \quad \begin{array}{r} 49 \\ 19 \\ \hline 68 \end{array} \quad \begin{array}{r} 58 \\ 18 \\ \hline 76 \end{array}$$

$$\begin{array}{r} 99 \\ - 58 \\ \hline 41 \end{array} = (-41)_{10}$$

Subtraction using 10's comp.

$$\text{a) } (88)_{10} - (47)_{10} = (88)_{10} + (-47)_{10}$$

$$\begin{array}{r} 99 \\ - 47 \\ \hline 52 \end{array} \rightarrow 9\text{'s comp.} + (38)$$

$$\begin{array}{r} + 1 \\ \hline 53 \end{array} \rightarrow 10\text{'s comp.}$$

$$\begin{array}{r} 88 \\ 53 \\ \hline 141 \end{array} \quad \begin{array}{r} 78 \\ 13 \\ \hline 91 \end{array} \quad \begin{array}{r} 68 \\ 22 \\ \hline 90 \end{array} \quad \begin{array}{r} 18 \\ 18 \\ \hline 36 \end{array}$$

$$\text{b) } (47)_{10} - (88)_{10} = (47)_{10} + (-88)_{10}$$

$$\begin{array}{r} 99 \\ - 88 \\ \hline 11 \end{array} \rightarrow 9\text{'s comp.}$$

$$\begin{array}{r} 01 \\ 10 \\ \hline 11 \end{array} \rightarrow 10\text{'s comp.}$$

$$\begin{array}{r} 47 \\ + 12 \\ \hline 59 \end{array} \quad \begin{array}{r} 12 \\ 12 \\ \hline 24 \end{array} \quad \begin{array}{r} 47 \\ + 12 \\ \hline 59 \end{array}$$

$$\begin{array}{r} 99 \\ - 59 \\ \hline 40 \end{array} \rightarrow 9\text{'s comp.}$$

$$\begin{array}{r} 40 \\ + 1 \\ \hline 41 \end{array} \rightarrow 10\text{'s comp.}$$

$$= (-41)_{10}$$

$$(3854) - (2519)$$

$$(3954) - (3519)$$

$$1744$$

$$2854$$

$$P3D8$$

$$7772$$

$$7813 - 7772 = 41$$

$$4118$$

$$(10171 -) = \text{problem}$$

Subtraction using 15's and 16's complement (d)

$$1) (A2B6)_{16} - (2476)_{16}$$

$$(A2B6)_{16} + (-2476)_{16} \text{ using 15's comp.}$$

$$\begin{array}{r} \text{FFFF} \\ - 2476 \\ \hline (\text{DB89})_{15\text{comp}} \end{array}$$

$$\begin{array}{r} \text{A}^{\oplus} \text{2B6} \\ + \text{DB89} \\ \hline \text{① 7E3F} \end{array}$$

$$\begin{array}{r} = (7E3F)_{16} \\ + 1 \\ \hline (\text{7E40})_{16} \end{array}$$

00	10
01	11
02	12
03	13
04	14
05	15
06	16
07	17
08	18
09	19
0A	1A
0B	1B
0C	1C
0D	1D
0E	1E
0F	1F

$$2) (2476)_{16} - (A2B6)_{16}$$

$$(2476)_{16} + (-A2B6)_{16}$$

$$\begin{array}{r} \text{FFFF} \\ - A2B6 \\ \hline \text{5D59} \end{array}$$

$$\begin{array}{r} \text{① 2476} \\ + 5D59 \\ \hline 81BF \end{array}$$

$$\begin{array}{r} \text{FFFF} \\ - 81BF \\ \hline \text{① 7E40} \end{array}$$

$$\text{No carry} = (-7E40)_{16}$$

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Q2 X1

$$1) \text{ Q2.Xa) } (A2B6)_{16} - (2476)_{16} \text{ using 16's complement.}$$

$$= (A2B6)_{16} + (-2476)_{16}$$

FFFF

- 2476

$\begin{array}{r} \text{DB89} \\ + 1 \\ \hline \text{DB8A} \end{array}$ → 15's comp.

$\begin{array}{r} \text{DB8A} \\ + 1 \\ \hline \text{DB8B} \end{array}$ → 16's comp.

A[⊕]2B6

+ DB8A

① 7E40

Digit

$$16 - (7E40)$$

$$b) (2476)_{16} - (A2B6)_{16}$$

$$(2476)_{16} + (-A2B6)_{16}$$

FFFF

- A2B6

$\begin{array}{r} \text{5D49} \\ + 1 \\ \hline \text{5D4A} \end{array}$ → 15's comp

+ 1

$\begin{array}{r} \text{5D4A} \\ + 1 \\ \hline \text{5D4B} \end{array}$ → 16's comp

① 2476

+ 5D4A

$\begin{array}{r} \text{81CO} \\ + 1 \\ \hline \text{81CB} \end{array}$

- 81CO

$\begin{array}{r} \text{7E3F} \\ + 1 \\ \hline \text{7E40} \end{array}$

$$= (-7E40)_{16}$$

$$1 \times 0 = 0$$

Binary Multiplication

$$1 \times 1 = 1$$

$$0 \times 1 = 0$$

$$1) (38)_{10} \times (27)_{10}$$

$$7717$$

$$(38)_{10} = (100110)_2$$

$$(27)_{10} = (\underline{\underline{100}}11)_2$$

$$\begin{array}{r} 100110 \\ \times 111 \\ \hline \end{array}$$

$$\begin{array}{r} 000110 \\ \times 10 \\ \hline 000110 \\ \times x \\ \hline \end{array}$$

$$(100001010)_2$$

$$2^8 + 2^3 + 2^1 = (286)_{10}$$

Boolean Multiplication

$$1 \times 1 = 1$$

$$0 \times 1 = 0$$

$$\begin{array}{r} 38 \rightarrow 0 \\ 2 | 19 \rightarrow 1 \\ 2 | 9 \rightarrow 1 \\ 2 | 4 \rightarrow 0 \\ 2 | 2 \rightarrow 0 \\ \hline & 11 \end{array}$$

B

Boolean Algebra

1) Commutative Property

$$a) x+y = y+x \quad x = y + x \rightarrow \text{OR logic}$$

$$b) x \cdot y = y \cdot x \quad x \cdot y = y \cdot x \rightarrow \text{AND logic}$$

OR GATE

AND GATE

$$x \ y \ z \ 1 + xz \ x \ y \ z \text{ (iii)}$$

$$0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \text{ found}$$

$$1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0$$

$$1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \text{ (i) } x + y + z$$

2) Associative Property

$$a) (x+y)+z = x+(y+z)$$

$$b) s(x \cdot y) \cdot z = s \cdot x \cdot (y \cdot z) = s \cdot x \cdot y \cdot z$$

Q)

3) De Morgan's Theorem

$$a) (x+y)' = x'y' + xy'$$

$$b) (\overline{x+y})' = \overline{x} \cdot \overline{y} = x' + y'$$

$$c) (xy)' = x' + y'$$

$$d) (\overline{x} \cdot \overline{y}) = \overline{x} + \overline{y}$$

Postulates

$$i) x + 0 = x$$

$$ii) x + x' = 1$$

$$iii) x + x = x$$

$$iv) x + 1 = x$$

$$v) x \cdot x = x$$

$$vi) x \cdot x' = 0$$

$$vii) x \cdot 1 = x$$

$$viii) x \cdot 0 = 0$$

Proof or Minimize the expression using Boolean algebra

$$1. x + xy = x$$

$$x(1+y)$$

$$x \cdot 1 = x$$

$$R.H.S$$

$$2. xy + x'z + yz = xy + x'z$$

L.H.S

$$xy + x'z + yz(x+x')$$

$$= xy + x'z + xyz + x'y'z$$

$$= xy(1+z) + x'z(1+y)$$

$$= xy \cdot 1 + x'z \cdot 1$$

$$= xy + x'z$$

R.H.S

$$3. xy + xz + x\bar{y}z(xy + z) = 1$$

L.H.S

$$xy + xz + x\bar{y}z \cdot xy + x\bar{y}zz$$

$$= xy + xz + x\bar{y}z$$

$$= x(y + \bar{y}z) + x\bar{z}$$

$$= x(y + z) + x\bar{z}$$

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$$= xy + xz + \underline{xz} + \underline{yz} = (x+y) \cdot (x+z) \cdot (y+z) = 1$$

$$= (x+y)(\bar{x}\bar{z}+z)(\bar{y}+xz) = \bar{xy}$$

$$L.H.S$$

$$(x+y)(\bar{x}\bar{z}+z)(\bar{y} \cdot \underline{\bar{x}\bar{z}})yz + xy + yz =$$

$$= (x+y)(\bar{x}\bar{z}+z)(y \cdot \bar{x}\bar{z}) + xy + yz =$$

$$= (x\bar{x}\bar{z} + xz + \bar{x}\bar{z}y + yz)(y \cdot \bar{x}\bar{z})$$

$$= (0 + xz + \bar{x}\bar{z}y + yz)(y + \bar{x} + z)$$

$$= (y(z + \bar{x}\bar{z}) + xz)(y + (\bar{x} + z))$$

$$= (y(z + \bar{x}) + xz)(\bar{x}y + y\bar{z})$$

$$= (yz + y\bar{x} + xz)(\bar{x}y + y\bar{z})$$

$$= \bar{x}yyz + \bar{y}yyz\bar{z} + \bar{x}\bar{x}yz + xyz\bar{z}$$

$$+ x\bar{x}yz + xyz\bar{z} + \bar{x}yz\bar{z} + xyz\bar{z}$$

$$= \bar{x}yz + 0 + \bar{x}y + xyz\bar{z} + 0 + 0$$

$$= \bar{x}y(1+z) + \bar{x}y\bar{z} + \bar{x}yz$$

$$= \bar{x}\bar{y} + \bar{x}y(1 + \bar{z}) + (\bar{x} + \bar{y})\bar{z}$$

$$= \bar{x}\bar{y} + (\bar{x}y + \bar{y}\bar{z} + \bar{x}\bar{y}\bar{z})\bar{z}$$

$$= \bar{x}\bar{y} + (\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z})\bar{z}$$

$$5. A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C = C + B + \bar{C}\bar{D} =$$

$$L.H.S$$

$$A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C$$

$$= A\bar{B}C + B(1 + \bar{D}) + AB\bar{D} + \bar{A}C$$

$$= A\bar{B}C + B + AB\bar{D} + \bar{A}C$$

$$= C(\bar{A} + A\bar{B}) + B + AB\bar{D}$$

$$= C(\bar{A} + \bar{B}) + B + AB\bar{D}$$

$$= \bar{A}C + \bar{B}C + B + AB\bar{D}$$

$$= \bar{A}C + B + C + AB\bar{D}$$

$$= C(1 + \bar{A}) + B(1 + A\bar{D}) = C + B = B + C$$

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$$6) (x+y)(y+z)(x+z) = xy + xz + yz + x^2y + y^2z + x^2z =$$

$$\begin{aligned} & (xy + xz + yz + y^2z)(x + z) \\ &= x^2y + xz^2 + xy + xyz + xyz + xz^2 + yz^2 \\ &= xy + xz + yz + xyz + xz^2 + yz^2 \\ &= xy + xz + yz + xyz + xyz(x + z) \\ &= xy + xz + yz(x + z) \\ &= xy + xz + yz \end{aligned}$$

$$7) If x\bar{y} + \bar{x}y = z / (\bar{x}\bar{y} + (\bar{x} + y))$$

Then show that $\bar{x}z + \bar{z}x = y + \bar{y}$

$$So \bar{x} + \bar{y} + \bar{x}\bar{y} + \bar{x}\bar{z} + \bar{z}\bar{x} + \bar{z}\bar{y}$$

$$x\bar{z} + \bar{x}z$$

Put the values of z

$$\begin{aligned} &= x(\bar{x}\bar{y} + \bar{x}y) + \bar{x}(x\bar{y} + \bar{x}y) \\ &= x(\bar{x}\bar{y} \cdot \bar{x}y) + \bar{x}x\bar{y} + \bar{x}\bar{x}y \\ &= x((\bar{x} + \bar{y}) \cdot (\bar{x} + y)) + 0 + \bar{x}y \\ &= x((\bar{x} + y) \cdot (x + \bar{y})) + \bar{x}y \\ &= x(x\bar{y} + \bar{x}\bar{y} + xy + y\bar{y}) + \bar{x}y \\ &= x(\bar{x}\bar{y} + xy) + \bar{x}y \\ &= x\bar{x}\bar{y} + xxy + \bar{x}y \\ &= xy + \bar{x}y \\ &= y(x + \bar{x}) \end{aligned}$$

$$= y \cdot 1 = y + (\bar{y} + 1)y =$$

$$\bar{A}B\bar{A} + \bar{B} + (\bar{A}\bar{B} + \bar{A}) =$$

$$\bar{A}B\bar{A} + \bar{B} + (\bar{A} + \bar{A}) =$$

$$\bar{A}B\bar{A} + \bar{B} + \bar{A} =$$

$$\bar{A}B\bar{A} + \bar{B} + \bar{C} + \bar{A} =$$

$$D + D^2 = D + D = (DA + 1)D + (\bar{A} + 1)D =$$

Basic Gates

1) NOT GATE

symbol :



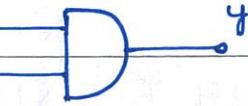
$$y = \bar{x}$$

Truth Table

x	y
0	1
1	0

2) AND GATE

symbol :



$$y = a \cdot b$$

Truth Table

a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

3) OR GATE

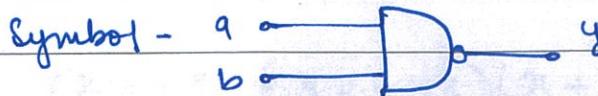
symbol :



Truth Table

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

iv) NAND Gate

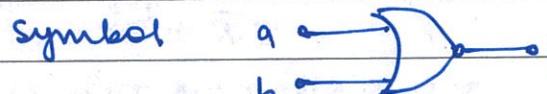


Truth Table

$$y = \overline{a \cdot b}$$

a	b	y
0	0	1
0	1	0
1	0	0
1	1	0

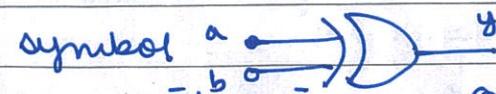
v) NOR Gate



$$y = \overline{a + b}$$

a	b	y
0	0	1
0	1	0
1	0	0
1	1	0

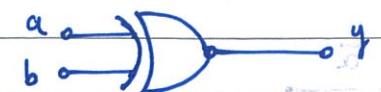
vi) XOR Gate



$$y = \overline{ab} + ab = a \oplus b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

vii) XNOR Gate



$$y = \overline{a \bar{b}} + ab = a \oplus b$$

T.T

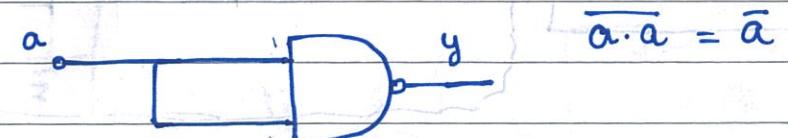
a	b	y
0	0	1
0	1	0
1	0	0
1	1	1

Universal Gates

i) NAND Gates

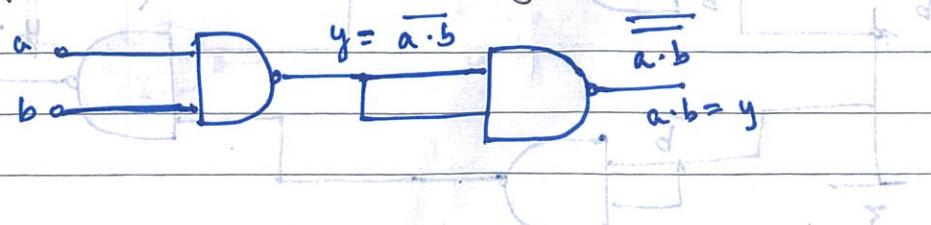
ii) NOR Gates

1) Implement NOT Gate using NAND Gate

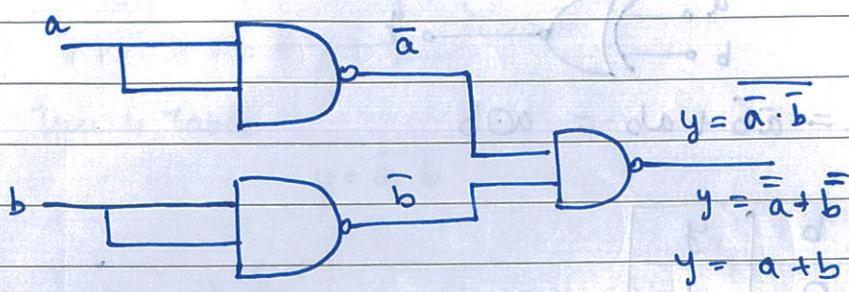


a	y
0	1
1	0

2) Implement AND Gate using NAND Gate

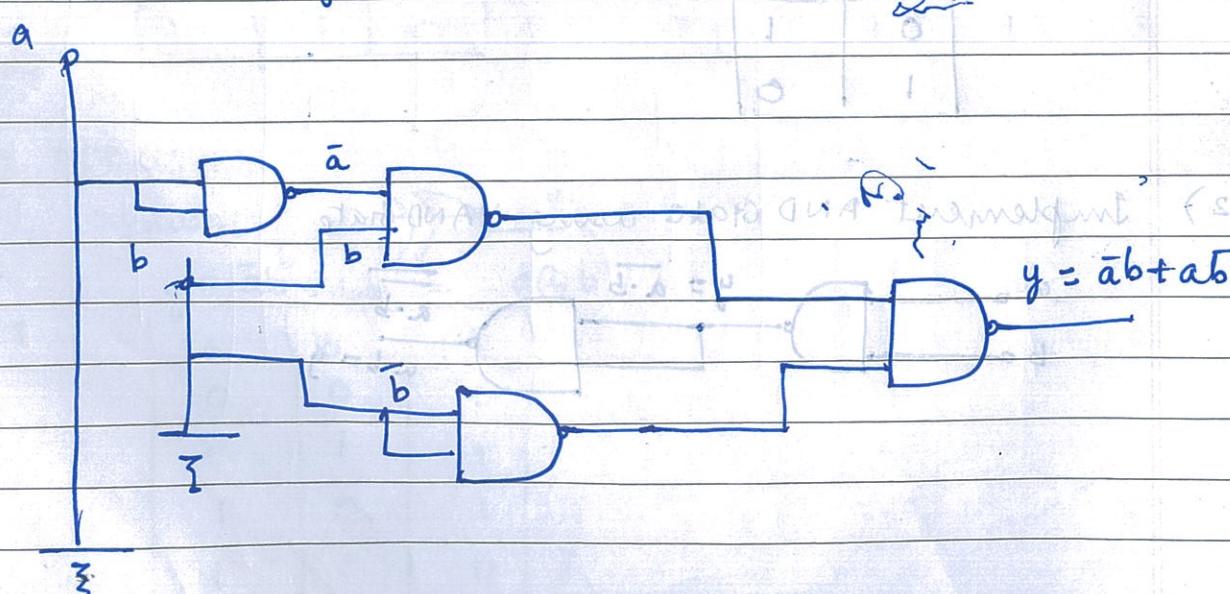
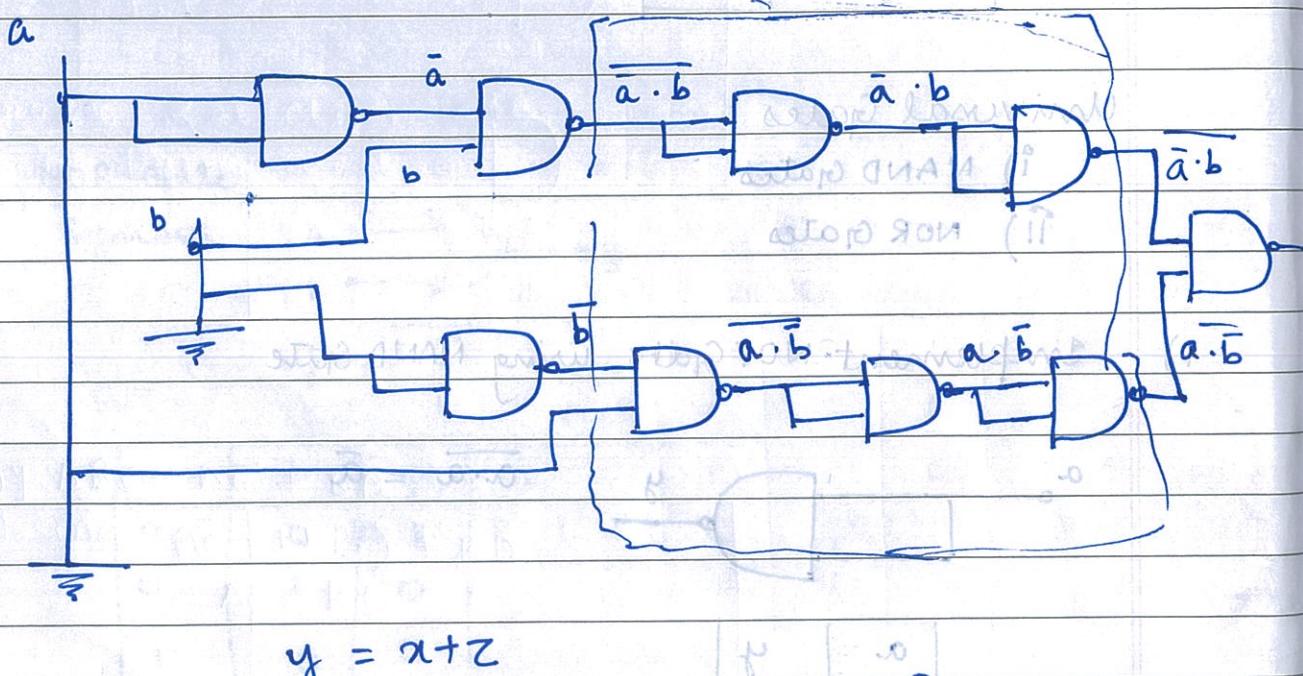


iii) Implement OR Gate using NAND gate



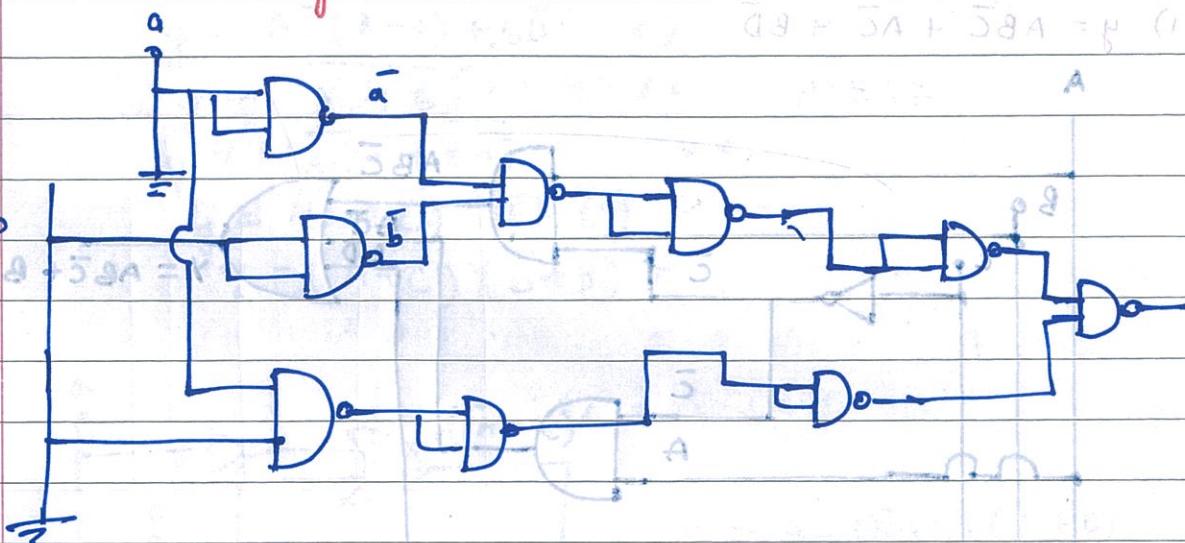
iv) Implement XOR Gate using NAND gate

$$y = \bar{a}b + \bar{b}a$$



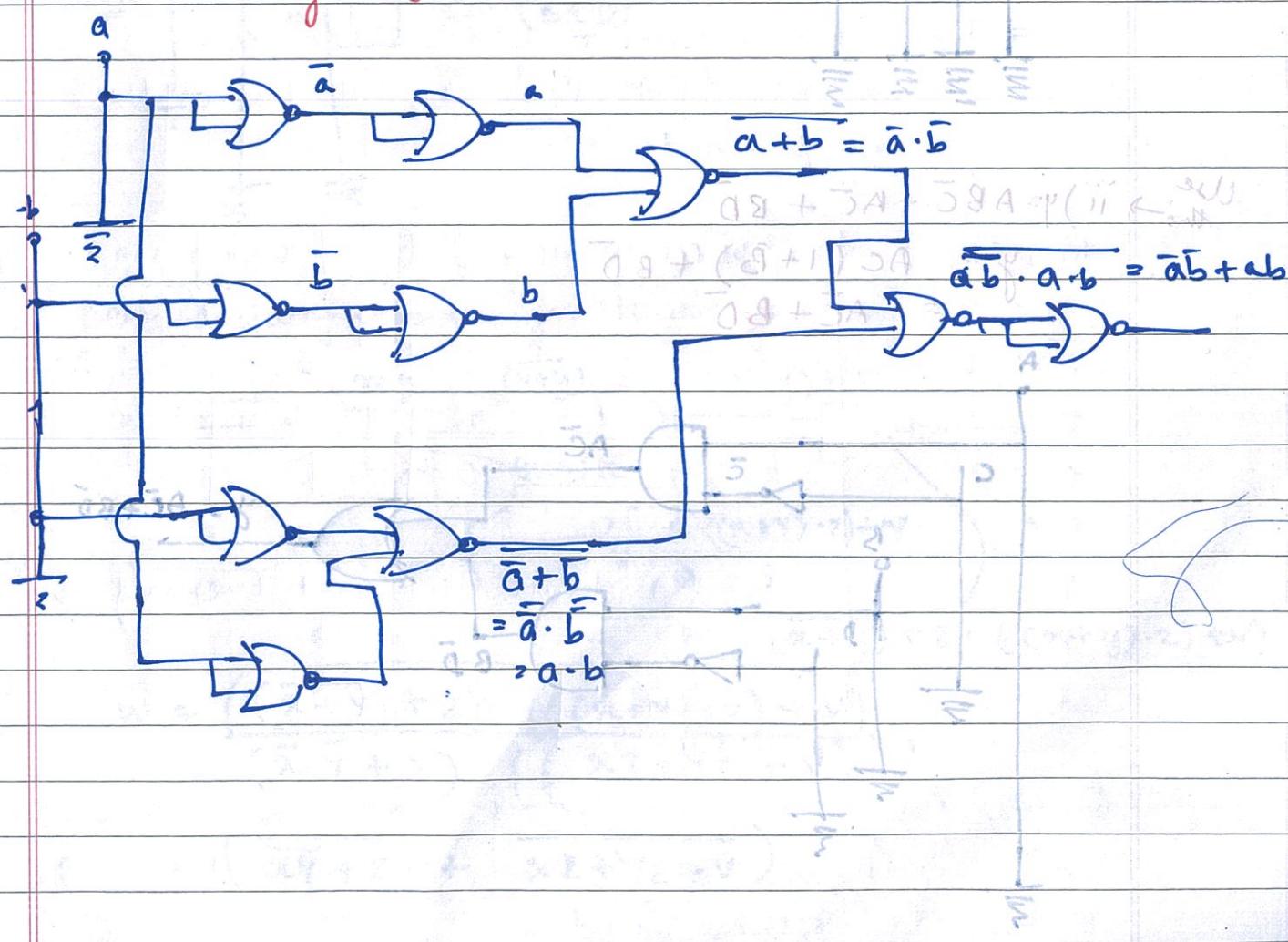
v) Implement XNOR Gate using NAND gate

$$y = \bar{a}\bar{b} + ab$$



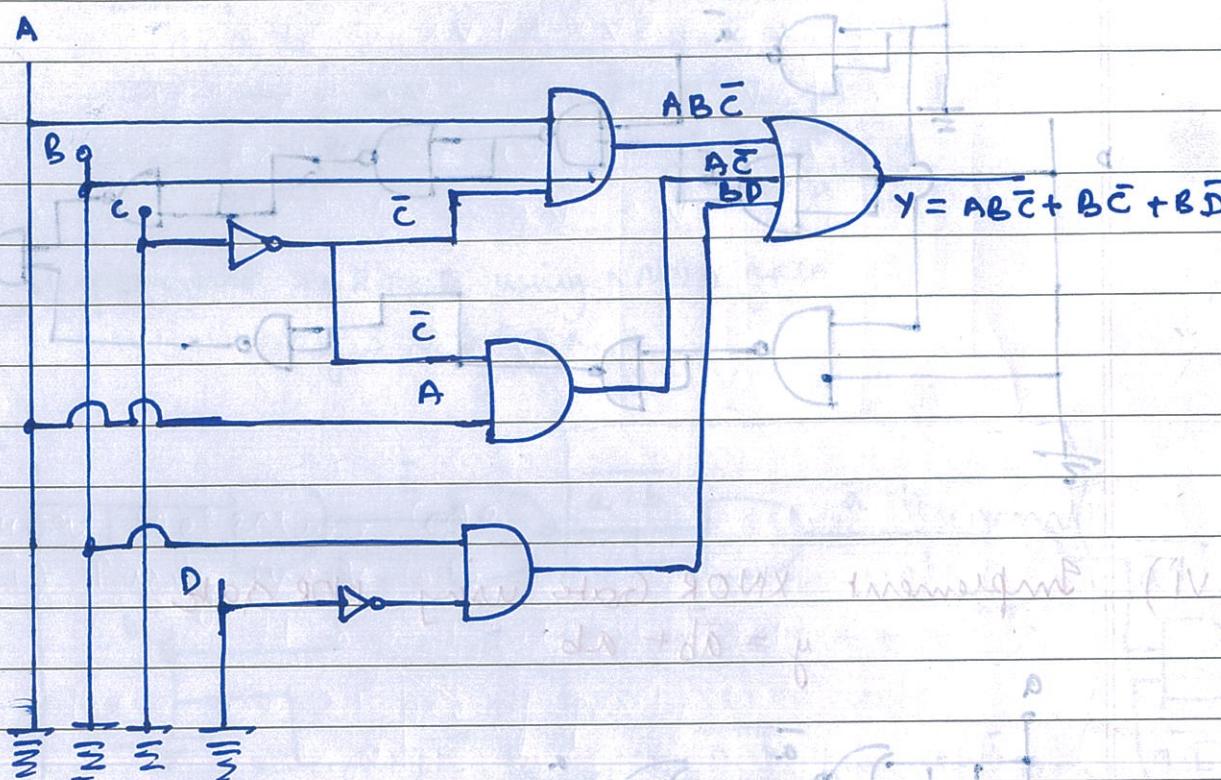
vi) Implement XNOR Gate using NOR gate

$$y = \bar{a}\bar{b} + ab$$



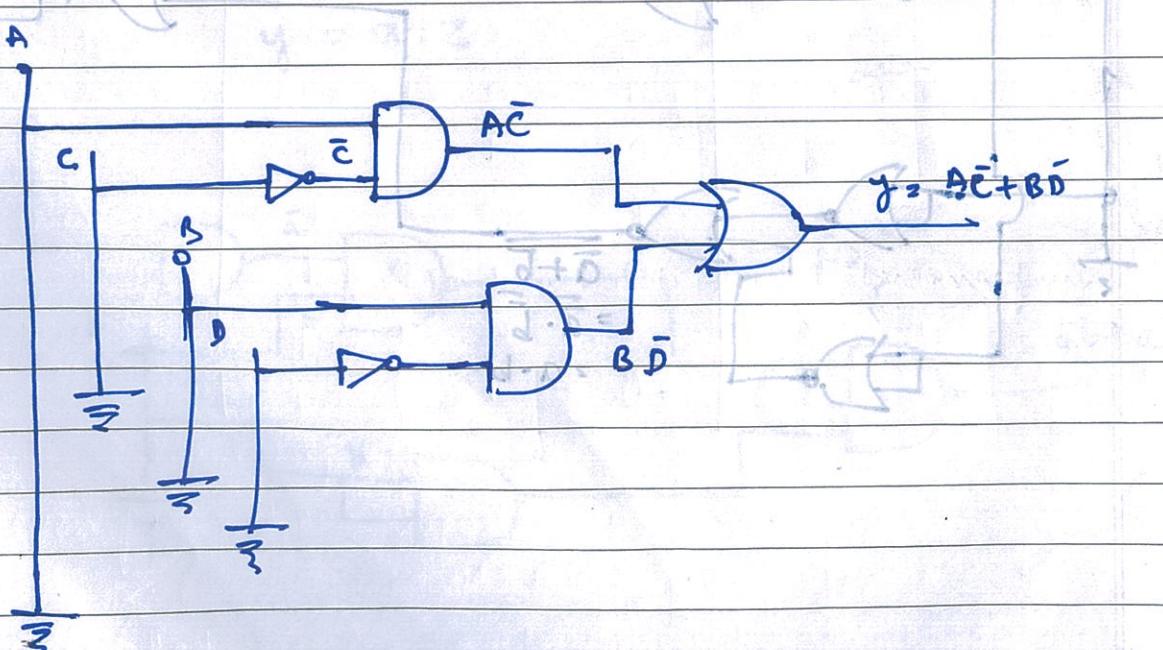
Logic Diagram from Boolean Expression

$$i) y = AB\bar{C} + A\bar{C} + B\bar{D}$$



$$\text{Use this } ii) y = AB\bar{C} + A\bar{C} + B\bar{D}$$

$$y = A\bar{C}(1+B) + B\bar{D}$$



$$iii) y = \overline{ABC} + \overline{AC} + \overline{BD} = \overline{S}\overline{P}\overline{R} + \overline{S}\overline{P}\overline{R} + \overline{S}\overline{P}\overline{R}$$

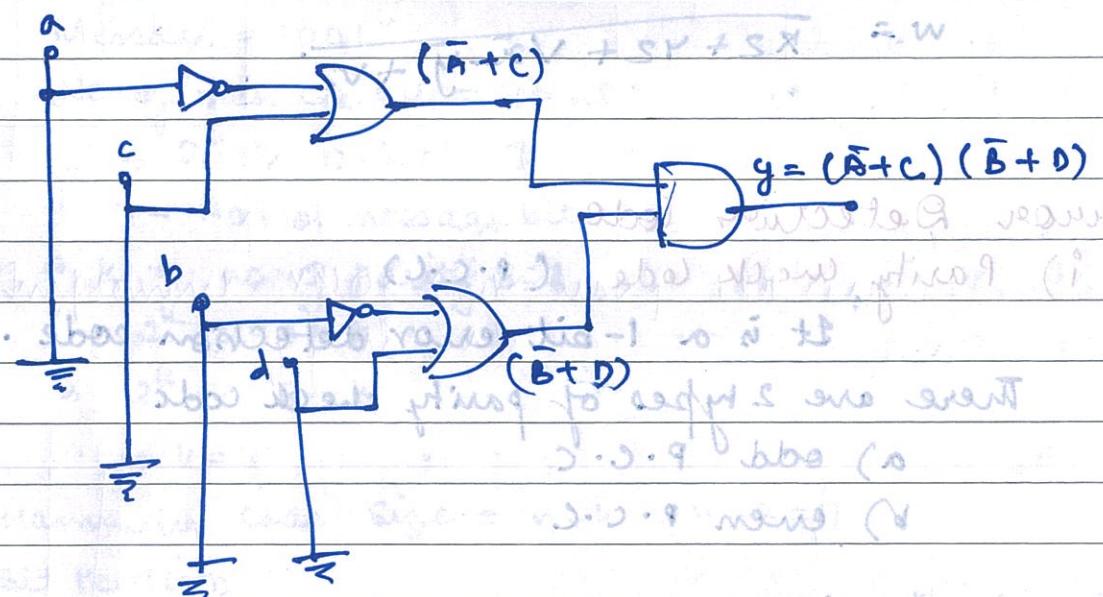
$$y = \overline{A\bar{C}(B+1)} + \overline{B\bar{D}} = \overline{S}\overline{P}\overline{R}$$

$$y = \overline{A\bar{C} + B\bar{D}} = \overline{S}\overline{P}\overline{R}$$

$$y = \overline{\overline{A\bar{C}} \cdot \overline{(B\bar{D})}} = \overline{S}\overline{P}\overline{R}$$

$$y = (\overline{A} + \overline{C})(\overline{B} + \overline{D}) = \overline{S}\overline{P}\overline{R}$$

$$y = (\overline{A} + C)(\overline{B} + D) = \overline{S}\overline{P}\overline{R}$$



Q.6 Boolean expression for logic gate

$$i) \begin{array}{c} x+y \\ \downarrow \\ z \\ \downarrow \\ w \end{array} \quad \begin{array}{c} (x+y) \\ \downarrow \\ z \\ \downarrow \\ w \end{array} \quad \begin{array}{c} (x+y)+z \\ \downarrow \\ ((x+y)\cdot z) \\ \downarrow \\ w \end{array}$$

$$w = \frac{((\overline{x}+y)+z) \cdot (((x+y)\cdot z) + v)}{(\overline{x}\cdot \overline{y} + z) \cdot ((x\cdot z + y\cdot z) + v)}$$

$$\text{LHS of part } 2^{\text{nd}} \text{ row } = (\overline{x}\overline{y} + z\overline{w} + x\cdot z + y\cdot z + v)$$

LHS of part 2nd row is given below

$$\begin{aligned}
 &= \bar{x}\bar{y} \cdot \bar{z} + \bar{x}\bar{z} \cdot \bar{y}\bar{z} + \bar{v} \\
 &= \bar{x} + \bar{y} \cdot \bar{z} + (\bar{x} + \bar{z}) \cdot (\bar{y} + \bar{z}) + \bar{v} \\
 &= (x+y) \cdot \bar{z} + (\bar{x} + \bar{z}) \cdot v\bar{y} + v\bar{z} \\
 &= x\bar{z} + y\bar{z} + \bar{x}v\bar{y} + \bar{x}v\bar{z} + \bar{z}v\bar{y} + v\bar{z} \\
 &= x\bar{z} + y\bar{z} + \bar{x}v\bar{y} + \bar{x}v\bar{z} + v\bar{z}[1+v] \\
 &= x\bar{z} + y\bar{z} + \bar{x}v\bar{y} + v\bar{z}[1+\bar{x}] \\
 &= x\bar{z} + y\bar{z} + \bar{x}v\bar{y} + v\bar{z}
 \end{aligned}$$

m

$$w = \overline{x_2 + y_2 + v_x - y} + v_2$$

Error Detection Code

i) Parity Check Code (P.C.C)

It is a 1-bit error detection code.

There are 2 types of parity check code

a) odd P.C.C

b) even P.C.C

MESSAGE	ODD PARITY CHECK	EVEN PARITY CHECK					
x	y	z	p	x	y	z	p
0 0 0	0 0 0	0 0 0	0	0 0 0	0 0 0	0 0 0	0
0 0 1	0 0 1	0 0 1	0	0 0 1	0 0 1	0 0 1	1
0 1 0	0 1 0	0 1 0	0	0 1 0	0 1 0	0 1 0	1
0 1 1	0 1 1	0 1 1	0	0 1 1	0 1 1	0 1 1	0
1 0 0	1 0 0	1 0 0	1	1 0 0	1 0 0	1 0 0	1
1 0 1	1 0 1	1 0 1	0	1 0 1	1 0 1	1 0 1	0
1 1 1	1 1 1	1 1 1	1	1 1 1	1 1 1	1 1 1	1

P → Parity Bit

In even parity check for the no. of one's in the transmitted message is even. In odd P.C.C check for the no. of one's in transmitted message is odd.

Limitations of Parity Check Codes

- 1) In P.C.C we detect only single bit error, but cannot correct the error
- 2) Error detection and correction code

Dinner

a) Hamming Code

b) 1) Hamming Code with Even Parity

2) Hamming Code with Odd Parity

Q1. Determine the single error code for the BCD No. 1001 using even parity.

Sol:-

Message = 1001

No. of message bit = 4 = n = 4

$$\Rightarrow 2^k \geq n+k+1$$

$$\Rightarrow n = \text{No. of message bit}$$

$$\Rightarrow k = \text{no. of parity bits}$$

$$2^k \geq 4+k+1$$

$$\Rightarrow 2^k \geq 5+k$$

$$\Rightarrow k=3$$

$$\text{Hamming Code Size} = n+k = 4+3=7$$

Bit Position

$$\begin{array}{ccccccccc}
 p_1 & p_2 & p_3 & n_1 & n_2 & n_3 & n_4 \\
 \text{(P1 bit check)} & \text{(P2 bit check)} & \text{(P3 bit check)} & \text{(n1 bit check)} & \text{(n2 bit check)} & \text{(n3 bit check)} & \text{(n4 bit check)}
 \end{array}$$

$$(p_1, 1, 3, 5, 7) = (p_1, 1, 0, 1) = (0, 1, 0, 1)$$

$$(p_2, 2, 3, 6, 7) = (p_2, 1, 0, 1) = (0, 1, 0, 1)$$

$$(p_3, 4, 5, 6, 7) = (p_3, 0, 0, 1) = (1, 0, 0, 1)$$

Final Hamming code with even Parity - Q2

$$\begin{array}{ccccccccc}
 p_1 & p_2 & n_1 & p_3 & n_2 & n_3 & n_4 \\
 0 & 0 & 1 & 1 & 0 & 0 & 1
 \end{array}$$

Q2 - Determine the single bit error detection and correction for BCD No - 10110 with odd Parity.

Given Message = 10110 with 5 bits. So $n=5$

No. of message bit = $n=5$

$2^k \geq n+1$ $\Rightarrow k \geq 3$ (as $n=5$)

$2^k \geq 5+1 \Rightarrow k \geq 4$ (as $n=5$)

$2^k \geq n+6 \Rightarrow k \geq 5$ (as $n=5$)

$k=4$ \Rightarrow 4 parity bits are required.

To satisfy the conditions given

D₁ D₂ D₃ D₄ P₁ P₂ P₃ P₄ P₅ P₆ P₇ P₈ P₉

P₁ P₂ 1 P₃ 0 1 1 P₄ 0

$$(P_1, D_3, D_5, D_7, D_9) = (P_1, 1, 0, 1, 0) = (1, 1, 0, 1, 0)$$

$$(P_2, D_3, D_6, D_7) = (P_2, 1, 1, 1) = (0, 1, 1, 1)$$

$$(P_3, D_4, D_5, D_6, D_7) = (P_3, 0, 1, 1) = (1, 0, 1, 1)$$

$$(P_4, D_9) = (P_4, 0) = (1, 0)$$

Final Code

P₁, P₂, n₁, P₃, n₂, n₃, n₄, P₅, n₅

(1, 0, 1, 1, 0, 1, 0, 1, 0) (as $n=5$)

(1, 0, 0, 1, 0) (as $n=5$)

BCD - 1101

Unit - II Minimization Techniques

There are two types of expression :-

i) SOP \rightarrow sum of Product \rightarrow Min Term (m)

ii) POS \rightarrow Product of Sum \rightarrow Max Term (M)

$$y = ABC + CA\bar{B} \quad (\text{Min Term})$$

$$y = (\bar{A} + \bar{B} + C) (A + \bar{B} + C) \quad (\text{Max Term})$$

Digit	Variable	Min Terms SOP (m)		Max Terms (POS) M
		0 0 0	0 0 1	
0	0 0 0	$\bar{A}\bar{B}\bar{C}$	=	
1	0 0 1	$\bar{A}\bar{B}C$	=	
2	0 1 0	$\bar{A}B\bar{C}$	=	
3	0 1 1	$\bar{A}B\bar{C}$	=	
4	1 0 0	$A\bar{B}\bar{C}$	=	
5	1 0 1	$A\bar{B}C$	=	
6	1 1 0	$AB\bar{C}$	=	
7	1 1 1	ABC	=	