Recess From ___ am/pm till ___ am/pm / Lunch From ___ am/pm till ___ am/pm		1	2	3	4	5	6	7	8	9	10
		From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm	From ___ am/pm Till ___ am/pm
Monday	Subject										
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Tuesday	Subject										
	Teacher										
Wednesday	Subject										
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Thursday	Subject										
	Teacher										
Friday	Subject										
	Teacher										
Saturday	Subject										
	Teacher										

INDEX

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Serial No	Date	Title	Page No	Teacher's Signature/Remarks

July 16, 2019

LINEAR PROGRAMMING PROBLEM

Linear Programming Problem is used for optimization i.e. find max. or min values. It generally consist of three parts: objective, constraints, non-negative restriction

Graphical Method to solve LPP:
Procedure

1.) Convert inequalities to equations
2.) Draw the equations (lines) on graph
3) Find the feasible region.
4.) Find the corner points of the feasible region.
5.) Find the values of objective function on corner point and select the appropriate ans.

Ques1 A firm manufactures headache pills in two sizes A and B. Pill A contains two grains of aspirin, 5 grains of bicarbonate and 1 grain of ~~prot~~ codline. Size B contains 1 grain of aspirin, 6 grains of codline and 8 of bicarbonate. It has been found by users it requires 12 grains of aspirin, 74 grains of bicarbonate & 24 of codline for providing immediate effect. Determine graphically least number of pills.

	A	B	
aspirin	2	1	12
bicarbonate	5	8	74
codline	1	6	24

min $z = x + y$

$$2x + y \geq 12$$
$$5x + 8y \geq 74$$
$$x + 6y \geq 24$$
$$x, y \geq 0$$

$2x + y = 12$ $\qquad \frac{x}{6} + \frac{y}{12} = 1$

$5x + 8y = 74$ $\qquad \frac{x}{74/5} + \frac{y}{74/8} = 1 \Rightarrow \frac{x}{14.8} + \frac{y}{9.25} = 1$

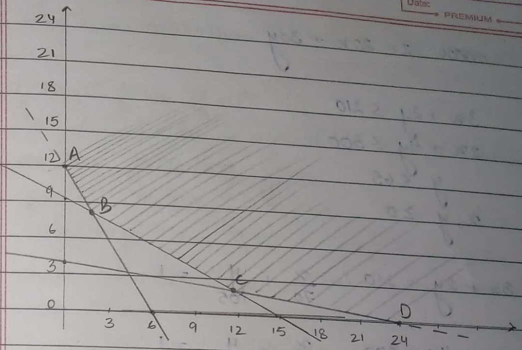
$x + 6y = 24$ $\qquad \frac{x}{24} + \frac{y}{4} = 1$

A $(0, 12)$ $\qquad z = 12$

B $(2, 8)$ $\qquad z = 10$ $\qquad$ min.

C $(11.45, 2.09)$ $\qquad z = 13$

D $(24, 0)$ $\qquad z = 24$



Ques A firm manufactures two type of electric item A and B and makes a profit of ₹20/unit A and Rs 30 per unit of B. Each unit of A requires 3 motors & 2 transformers. Each unit of B requires 2 motors and 4 transformers. The total supply of these is restricted to 210 motor & 300 transformers. Type B is an export model which requires a stabilizer which has supply restricted to 65 units per month. Formulate LPP for maximizing profit & solve graphically.

	A	B	
motor	3	2	210
transformer	2	4	300
stabilizer	0	1	65

Page No. 004.

max $z = 20x + 30y$

$3x + 2y \le 210$

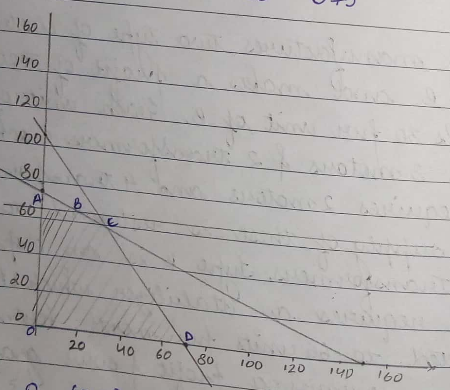
$2x + 4y \le 300$

$y \le 65$

$x, y \ge 0$

$3x + 2y = 210 \qquad \frac{x}{70} + \frac{y}{105} = 1$

$2x + 4y = 300 \qquad \frac{x}{150} + \frac{y}{75} = 1$



O	(0,0)	$z = 0$
A	(0,65)	$z = 1950$
B	(20,65)	$z = 400 + 1950 = 2350$
C	(30,60)	$z = 600 + 1800 = 2400$
D	(70,0)	$z = 1400$

Page No. 005.

So, z is maximum at $x = 30$, $y = 60$.

July 17, 2019

General form of LPP

An LPP with n variables can be written in the following form

find $x_1, x_2 \dots x_n$

optimize $z = c_1x_1 + c_2x_2 + \dots c_nx_n$

s.t. $a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n$ = or <= or >= b_1

$\vdots$

$a_{n1}x_1 + a_{n2}x_2 + \dots a_{nn}x_n$ = or <= or >= b_n

and $x_1, x_2 \dots x_n \ge 0$

Simplex Method

Ques Solve the LPP by simplex method:

maximize $z = 5x_1 + 3x_2$

s.t. $3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$ & $x_1, x_2 \ge 0$

We take x_3 and x_4 as slack variables, so, LPP becomes

max $z = 5x_1 + 3x_2 + 0x_3 + 0x_4$

s.t. $3x_1 + 5x_2 + x_3 + 0x_4 = 15$

$5x_1 + 2x_2 + 0x_3 + x_4 = 10$

$x_1, x_2 \ge 0$

			C_j	5	3	0	0
C_B	B	x_B	b	y_1	y_2	y_3	y_4
0	α_3	x_3	15	3	5	1	0
0	α_4	x_4	10	$\boxed{5}$	2	0	1
	$Z_j - C_j$			-5 ↑	-3	0	0

$15 - 6 + 5$ $\quad 3 - 5$ $\quad 5 - 6/5$ $\quad 1 - 0$ $\quad 0 - 3/5$

key element $\min\left[15/3 , 10/5\right]$

entering vector x_1

departing vector x_4

			C_j	5	3	0	0
C_B	B	x_B	b	y_1	y_2	y_3	y_4
0	α_3	x_3	9	0	$\boxed{19/5}$	1	$-3/5$
5	α_1	α_1	2	1	$2/5$	0	$1/5$
	$Z_j - C_j$			0	-1 ↑	0	1

key element $\min\left[2/2/5 , 9/19/5\right]$

entering vector x_2

departing vector x_3

			C_j	5	3	0	0
C_B	B	x_B	b	y_1	y_2	y_3	y_4
3	α_2	x_2	$45/19$	0	1	$5/19$	$-3/19$
5	α_1	x_1	$20/19$	1	0	$-2/19$	$19/95$
	$Z_j - C_j$			0	0	$5/19$	$10/19$

$2 - 18/19$ $\quad 1 - 0$ $\quad 2/5 - 2/5$ $\quad 0 - 2/19$ $\quad 1/5 - -6/195$

$x_1 = 20/19 \qquad x_2 = 45/19$

$z = 5(20/19) + 3(45/19)$

July 18, 2019

Ques Solve the given LPP by simplex method

min $z = x_1 - 3x_2 + 2x_3$

s.t. $3x_1 - x_2 + 3x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

& $x_1, x_2, x_3 \ge 0$

To convert the problem into maximization, we take $z^* = -z$ and also we take 3 slack variables: x_4, x_5, x_6. Hence LPP become

max $z^* = -x_1 + 3x_2 - 2x_3 + 0x_4 + 0x_5 + 0x_6$

$3x_1 - x_2 + 3x_3 + x_4 + 0.x_5 + 0.x_6 = 7$

$-2x_1 + 4x_2 + 0x_3 + 0x_4 + x_5 + 0.x_6 = 12$

$-4x_1 + 3x_2 + 8x_3 + 0x_4 + 0x_5 + x_6 = 10$

& $x_1, x_2, x_3 \ge 0$

			C_j	-1	3	-2	0	0	0
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_6
0	α_4	x_4	7	3	-1	3	1	0	0
0	α_5	x_5	12	-2	$\boxed{4}$	0	0	1	0
0	α_6	x_6	10	-4	3	8	0	0	1
	$Z_j - C_j$			1	-3 ↑	2	0	0	0

key element: min [3, 3.33]
departing vector x_5
entering vector x_2

C_B	B	x_B	c_j / b	y_1 (−1)	y_2 (3)	y_3 (−2)	y_4 (0)	y_5 (0)	y_6 (0)
0	α_4	x_4	10	$\boxed{5/2}$	0	3	1	1/4	0
3	α_2	x_2	3	−1/2	1	0	0	1/4	0
0	α_6	x_6	1	−5/2	0	8	0	−3/4	1
$Z_j - C_j$				−1/2 ↑	0	2	0	3/4	0

key element 5/2
entering vector x_1
departing vector x_4

C_B	B	x_B	c_j / b	y_1 (−1)	y_2 (3)	y_3 (−2)	y_4 (0)	y_5 (0)	y_6 (0)
−1	α_1	x_1	4	1	0	6/5	2/5	1/10	0
3	α_2	x_2	5	0	1	6/10	2/10	3/10	0
0	α_6	x_3	11	0	0	1.1	1	−1/2	1
$Z_j - C_j$				0	0	13/5	1/5	4/5	0

$x_1 = 4$ $x_2 = 5$ $x_3 = 0$

$z^* = -x_1 + 3x_2 - 2x_3 = 11$

$z = -z^* = -11$

July 19, 2019

Procedure for simplex method

1) convert the problem into maximization if it is of minimization by taking $-z = z^*$.

2) convert all the b_i's into positive if they are negative

3) converts the constraints into equation by taking slack, surplus or artificial variables.

4) construct initial simplex table.

5) ~~calculate ke~~ select key element, entering vector and departing vector

6) construct new simplex table if $Z_j - C_j < 0$ for this make key element 1 & all other element above & below key element 0 (zero)

7) repeat step 5 and 6 till all $Z_j - C_j > 0$

Big M method

Ques Solve the LPP by Big M method

min $z = x_1 + x_2$

s.t. $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

& $x_1, x_2 \geq 0$

To convert problem into maximization, we take $z^* = -z$ and also we take surplus and artificial variables x_3, x_4, x_5 and x_6. Now, the LPP becomes

max $z^* = -x_1 - x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$

$2x_1 + x_2 - x_3 + 0.x_4 + x_5 + 0.x_6 = 4$

$x_1 + 7x_2 + 0.x_3 - x_4 + 0.x_5 + x_6 = 7$

$x_1, x_2 \geq 0$

			c_j	-1	-1	0	0	-M	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_6
-M	α_5	x_5	4	2	1	-1	0	1	0
-M	α_6	x_6	7	1	$\boxed{7}$	0	-1	0	1
	$z_j - c_j$			-3M+1	-8M+1 ↑	M	M	0	0 ↓

key element 7

departing vector y_6 x_6

entering vector y_2 x_2

			c_j	-1	-1	0	0	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5
-M	α_5	x_5	3	$\boxed{13/7}$	0	-1	1/7	1
-1	α_2	x_2	1	1/7	1	0	-1/7	0
	$z_j - c_j$			$\frac{-13M-1}{7}+1$ ↑	0	M	$\frac{-M+1}{7}$	0 ↓

key element 13/7

departing vector x_5

entering vector x_1

			c_j	-1	-1	0	0
C_B	B	x_B	b	y_1	y_2	y_3	y_4
-1	α_1	x_1	21/13	1	0	-7/13	1/13
-1	α_2	x_2	10/13	0	1	-1/13	-2/13
	$z_j - c_j$			0	0	8/13	11/13

$x_1 = 21/13$ $x_2 = 10/13$

$z^* = -x_1 - x_2 = -31/13$

$z = -z^* = 31/13$

July 23, 2019

Ques Solve the LPP:

min $z = 5x_1 + 2x_2$

$3x_1 + x_2 = 4$

$2x_1 + x_2 \geq 3$

$x_1 + 2x_2 \leq 3$

To convert problem into maximization we take $z^* = -z$ and also we take slack surplus and artificial variable. x_3, x_4, x_5 & x_6
Now, the LPP becomes

max $z^* = -z$

$$= -5x_1 - 2x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$$

$$3x_1 + x_2 + 0.x_3 + 0.x_4 + x_5 + 0.x_6 = 4$$
$$2x_1 + x_2 - x_3 + 0.x_4 + 0.x_5 + x_6 = 3$$
$$x_1 + 2x_2 + 0.x_3 + x_4 + 0.x_5 + 0.x_6 = 3$$

			c_j	-5	-2	0	0	-M	-M
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5	y_6
-M	α_5	x_5	4	$\boxed{3}$	1	0	0	1	0
-M	α_6	x_6	3	2	1	-1	0	0	1
0	α_4	x_4	3	1	2	0	1	0	0
	$z_j - c_j$			-5M+5 ↑	-2M+2	M	0	0	0

key element 3
departing element x_5
entering element x_1

			c_j	-5	-2	0	0	-M
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_6
-5	α_1	x_1	4/3	1	1/3	0	0	0
-M	α_6	x_6	1/3	0	$\boxed{1/3}$	-1	0	1
0	α_4	x_4	5/3	0	5/3	0	1	0
	$z_j - c_j$			0	$\frac{-5-M}{3}+2$	M	0	0

key element min $\left[\frac{4/3}{1/3}, \frac{1/3}{1/3}, \frac{5/3}{5/3}\right]$
departing vector x_6
entering vector x_2

			c_j	-5	-2	0	0
C_B	B	x_b	b	y_1	y_2	y_3	y_4
-5	α_1	x_1	1	1	0	1	0
-2	α_2	x_2	1	0	1	-3	0
0	α_4	x_4	0	0	0	5	1
	$z_j - c_j$			0	0	1	0

$x_1 = 1 \quad x_2 = 1$

$z^* = -5x_1 - 2x_2 = -7$

$z = -z^* = 7$

Inconsistency & Redundancy in LPP
Now, we observe that the basis matrix B

To convert problem into maximization we take $z^* = -z$ and also we take slack surplus and artificial variable x_3, x_4, x_5 & x_6. Now, the LPP becomes

$$\max \quad z^* = -z$$
$$= -5x_1 - 2x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$$

$$3x_1 + x_2 + 0.x_3 + 0.x_4 + x_5 + 0.x_6 = 4$$
$$2x_1 + x_2 - x_3 + 0.x_4 + 0.x_5 + x_6 = 3$$
$$x_1 + 2x_2 + 0.x_3 + x_4 + 0.x_5 + 0.x_6 = 3$$

			c_j	-5	-2	0	0	-M	-M
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5	y_6
-M	α_5	x_5	4	$\boxed{3}$	1	0	0	1	0
-M	α_6	x_6	3	2	1	-1	0	0	1
0	α_4	x_4	3	1	2	0	1	0	0
	$z_j - c_j$			-5M+5 ↑	-2M+2	M	0	0	0

key element 3
departing element x_5
entering element x_1

			c_j	-5	-2	0	0	-M
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_6
-5	α_1	x_1	4/3	1	1/3	0	0	0
-M	α_6	x_6	1/3	0	$\boxed{1/3}$	-1	0	1
0	α_4	x_4	5/3	0	5/3	0	1	0
	$z_j - c_j$			0	$\frac{-5-M}{3}+2$	M	0	0

key element $\min\left[\frac{4/3}{1/3} \quad \frac{1/3}{1/3} \quad \frac{5/3}{5/3}\right]$
departing vector x_6
entering vector x_2

			c_j	-5	-2	0	0
C_B	B	x_b	b	y_1	y_2	y_3	y_4
-5	α_1	x_1	1	1	0	1	0
-2	α_2	x_2	1	0	1	-3	0
0	α_4	x_4	0	0	0	5	1
	$z_j - c_j$			0	0	1	0

$x_1 = 1 \qquad x_2 = 1$
$z^* = -5x_1 - 2x_2 = -7$
$z = -z^* = 7$

Inconsistency & Redundancy in LPP
Now, we observe that the basis matrix B

gives the solution in one of the following form:

<u>case 1</u> B has no artificial vector. In this case the solution is optimal and feasible

<u>case 2</u> B has artificial vector & value is zero. In this case redundancy exist and solution will be optimal

<u>case 3</u> B has artificial vector whose value is nonzero. In this case there is no feasible solution.

Solve the LPP

$\max. \ z = 3x_1 + 2x_2$

s.t. $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

We take slack, surplus and artificial variables x_3, x_4 and x_5

Now, the LPP becomes

$\max \ z = 3x_1 + 2x_2 + 0x_3 + 0x_4 - Mx_5$

$2x_1 + x_2 + x_3 \cancel{+x_6} + 0x_4 + 0x_5 = 2$

$3x_1 + 4x_2 + 0.x_3 - x_4 + x_5 = 12$

		C_j		3	2	0	0	-M	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_6
-M	α_6	x_6	2	$\boxed{2}$	1	1	0	0	1
-M	α_5	x_5	12	3	4	0	-1	1	0
	$Z_j - C_j$			-5M+3	-5M+2	-M	M	0	0
				↑					

key element min $[2/2, 12/3]$

departing vector x_6

entering vector x_1

		C_j		3	2	0	0	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5
3	α_1	x_1	1	1	1/2	1/2	0	0
-M	α_5	x_5						

		C_j		3	2	0	0	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5
0	α_3	x_3	2	2	$\boxed{1}$	1	0	0
-M	α_5	x_5	12	3	4	0	-1	1
	$Z_j - C_j$			-3M-3	-4M+2	0	M	0

key element

departing vector x_3

entering vector x_2

gives the solution in one of the following form :

case 1 B has no artificial vector. In this case the solution is optimal and feasible

case 2 B has artificial vector & value is zero. In this case redundancy exist and solution will be optimal

case 3 B has artificial vector whose value is nonzero. In this case there is no feasible solution.

Ques Solve the LPP

max. $z = 3x_1 + 2x_2$

s.t. $2x_1 + x_2 \le 2$

$3x_1 + 4x_2 \ge 12$

$x_1, x_2 \ge 0$

We take slack, surplus and artificial variables x_3, x_4 and x_5

Now, the LPP becomes

max $z = 3x_1 + 2x_2 + 0x_3 + 0x_4 - Mx_5$

$2x_1 + x_2 + x_3 \cancel{+ x_6} + 0x_4 + 0x_5 = 2$

$3x_1 + 4x_2 + 0.x_3 - x_4 + x_5 = 12$

		c_j		3	2	0	0	-M	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_6
-M	α_6	x_6	2	$\boxed{2}$	1	1	0	0	1
-M	α_5	x_5	12	3	4	0	-1	1	0
	$z_j - c_j$			-5M+3	-5M+2	-M	M	0	0
				↑					

key element min $[2/2 ; 12/3]$

departing vector x_6

entering vector x_1

		c_j		3	2	0	0	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5
3	α_1	x_1	1	1	1/2	1/2	0	0
-M	α_5	x_5						

		c_j		3	2	0	0	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5
0	α_3	x_3	2	2	$\boxed{1}$	1	0	0
-M	α_5	x_5	12	3	4	0	-1	1
	$z_j - c_j$			-3M-3	-4M+2	0	M	0

key element

departing vector x_3

entering vector x_2

			c_j	3	2	0	0	$-M$
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5
2	α_2	x_2	2	2	1	1	0	0
$-M$	α_5	x_5	4	-5	0	-4	-1	1
$z_j - c_j$				$5M+4-3$	0	$4M+2$	M	0

~~B has artificial vector whose value is non-zero.~~

We see that all $z_j - c_j$ are positive and basis matrix B consist of artificial vector x_5 whose value is 4 (non-zero). Hence there is no feasible solution.

Ques. Solve the LPP

max. $z = 3x_1 + 2x_2 + x_3$

s.t. $-3x_1 + 2x_2 + 2x_3 = 8$

$-3x_1 + 4x_2 + x_3 = 7$

$x_1, x_2, x_3 \geq 0$

We take artificial variable x_4 & x_5. Now, the LPP becomes

max. $z = 3x_1 + 2x_2 + x_3 - Mx_4 - Mx_5$

$-3x_1 + 2x_2 + 2x_3 + x_4 + 0 \cdot x_5 = 8$

$-3x_1 + 4x_2 + x_3 + 0 \cdot x_4 + x_5 = 7$

			c_j	3	2	1	$-M$	$-M$
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5
$-M$	α_4	x_4	8	-3	2	2	1	0
$-M$	α_5	x_5	7	-3	$\boxed{4}$	1	0	1
$z_j - c_j$				$6M-3$	$-6M-2$ ↑	$-3M-1$	0	0

key element min $[8/2, 7/4]$

departing vector x_5

entering vector x_2

			c_j	3	2	1	$-M$
C_B	B	x_b	b	y_1	y_2	y_3	y_4
$-M$	α_4	x_4	$9/2$	$-3/2$	0	$\boxed{3/2}$	1
2	α_2	x_2	$7/4$	$-3/4$	1	$1/4$	0
$z_j - c_j$				$\frac{3}{2}M - \frac{9}{2}$	0	$-\frac{3}{2}M - \frac{1}{2}$ ↑	0

key element min $\left[\frac{9/2}{3/2}, \frac{7/4}{1/4}\right]$

departing vector x_4

entering vector x_3

			c_j	3	2	1
C_B	B	x_b	b	y_1	y_2	y_3
1	α_3	x_3	3	-1	0	1
2	α_2	x_2	1	$-1/2$	1	0
$z_j - c_j$				-5	0	0

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~~Key element~~ Here $Z_j - C_j$ is ~~less~~ most
~~departing ele~~ negative but we cannot calculate key element as y_{11} & y_{12} are bothe negative.

July 25, 2019

Two Phase Method

Ques

$$\min z = 2x_1 + 9x_2 + x_3$$

s.t.

$$x_1 + 4x_2 + 2x_3 \geq 5$$

$$3x_1 + x_2 + 2x_3 \geq 4$$

$$\& \; x_1, x_2, x_3 \geq 0$$

We convert the problem into maximization we take $z^* = -z$. We also take surplus and artificial variable x_4, x_5, x_6 & x_7 now, the dpp becomes

$$z^* = -z$$

$$= -2x_1 - 4x_2 - x_3 + 0 \cdot x_4 + 0 \cdot x_5 - x_6 - x_7$$

such that

$$x_1 + 4x_2 + 2x_3 + x_4 + 0 \cdot x_5 + x_6 - 0 \cdot x_7 = 5$$

$$3x_1 + x_2 + 2x_3 + 0x_4 - x_5 + 0 \cdot x_6 + x_7 = 4.$$

		c_j		0	0	0	0	0	-1	-1
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5	y_6	y_7
-1	α_6	x_6	5	1	$\boxed{4}$	2	-1	0	1	0
-1	α_7	x_7	4	3	1	2	0	-1	0	1
	$Z_j - C_j$			-4	-5 ↑	-4	1	1	0	0

Key element $\min[5/4 \quad 4/1]$
departing vector x_6
entering vector x_2

			c_j	0	0	0	0	0	-1
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5	y_7
0	α_2	x_2	5/4	1/4	1	1/2	-1/4	0	0
-1	α_7	x_7	11/4	$\boxed{11/4}$	0	3/2	1/4	-1	1
		$Z_j - C_j$		-11/4 ↑	0	-3/2	-1/4	2	0 ↓

key element $\min[5/4 / 1/4 \quad 11/4 / 11/4]$
departing vector x_7
entering vector x_1.

			c_j	0	0	0	0	0
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5
0	α_2	x_2	1	0	1	4/11	-3/11	1/11
0	α_1	x_1	1	1	0	6/11	1/11	-4/11
	$Z_j - C_j$			0	0	0	0	0

Phase II

			c_j	-2	-9	-1	0	0
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5
-9	α_2	x_2	1	0	1	4/11	-3/11	1/11
-2	α_1	x_1	1	1	0	$\boxed{6/11}$	1/11	-4/11
	$Z_j - C_j$			0	0	-37/11 ↑	+25/11	-1/11

~~key element~~ Here $Z_i - C_1$ is ~~less~~ most ~~departing ele~~ negative but we cannot calculate key element as y_{11} & y_{12} are bothe negative.

July 25, 2019

Two Phase Method

Ques min $z = 2x_1 + 9x_2 + x_3$

s.t. $x_1 + 4x_2 + 2x_3 \geq 5$

$3x_1 + x_2 + 2x_3 \geq 4$

& $x_1, x_2, x_3 \geq 0$

We convert the problem into maximization we take $z^* = -z$. We also take surplus and artificial variable x_4, x_5, x_6 & x_7 now, the LPP becomes

$$z^* = -z$$

$$= -2x_1 - 4x_2 - x_3 + 0 \cdot x_4 + 0 \cdot x_5 - x_6 - x_7$$

such that

$$x_1 + 4x_2 + 2x_3 + x_4 + 0 \cdot x_5 + x_6 - 0 \cdot x_7 = 5$$

$$3x_1 + x_2 + 2x_3 + 0x_4 - x_5 + 0 \cdot x_6 + x_7 = 4.$$

			c_j	0	0	0	0	0	-1	-1
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_6	y_7
-1	α_6	x_6	5	1	$\boxed{4}$	2	-1	0	1	0
-1	α_7	x_7	4	3	1	2	0	-1	0	1
	$z_j - c_j$			-4	-5	-4	1	1	0	0

↑

key element min[5/4 4/1]

departing vector x_6

entering vector x_2

			c_j	0	0	0	0	0	-1
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_7
0	α_2	x_2	5/4	1/4	1	1/2	-1/4	0	0
-1	α_7	x_7	11/4	$\boxed{11/4}$	0	3/2	1/4	-1	1
	$z_j - c_j$			-11/4	0	-3/2	-1/4	2	0

↑ ↓

key element min[5/4/1/4 11/4/11/4]

departing vector x_7

entering vector x_1.

			c_j	0	0	0	0	0
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5
0	α_2	x_2	1	0	1	4/11	-3/11	1/11
0	α_1	x_1	1	1	0	6/11	1/11	-4/11
	$z_j - c_j$			0	0	0	0	0

Phase II

			c_j	-2	-9	-1	0	0
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5
-9	α_2	x_2	1	0	1	4/11	-3/11	1/11
-2	α_1	x_1	1	1	0	$\boxed{6/11}$	1/11	-4/11
	$z_j - c_j$			0	0	-37/11	+25/11	-1/11

↑

key element min [11/4 11/6]
departing vector x_1
entering vector x_3

C_B	B	x_b	c_j / b	y_1 (−2)	y_2 (−9)	y_3 (−1)	y_4 (0)	y_5 (0)
−9	α_2	x_2	1/3	−2/3	1	0	−1/3	$\boxed{1/3}$
−1	α_3	x_3	11/6	11/6	0	1	1/6	−2/3
	$z_j - c_j$			37/6	0	0	17/6	−7/3 ↑

key element 1/3
departing vector x_2
entering vector. x_5

C_B	B	x_b	c_j / b	y_1 (−2)	y_2 (−9)	y_3 (−1)	y_4 (0)	y_5 (0)
0	α_5	x_5	1	−2	3	0	−1	1
−1	α_3	x_3	5/2	1/2	2/3	1	−1/2	0
	$z_j - c_j$			3/3	7	0	1/2	0

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 5/2$

$z^* = -5/2$

$z = -z^* = 5/2$

July 26, 2019

Duality:

Every LPP is associated with another LPP called its dual. The original problem is termed as primal problem. The optimal solution of both the problems gives the information concerning the optimal solution of the other i.e. if optimal solution of one is known then optimal solution of the other can be searched out easily

In particular when the no of variables in LPP is less than no of constraints then it is useful to solve its dual problem.

General form of LPP is given by

$z = c_1x_1 + c_2x_2 + c_3x_3 \dots$

s.t. $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \dots a_{1n}x_n \geq / \leq / = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \dots a_{2n}x_n \geq / \leq / = b_2$

$\vdots$

$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 \dots a_{nn}x_n \geq / \leq / = b_n$

$x_1, x_2 \dots x_n \geq 0$

Page No. 022.

In matrix form, it can be written as

optimize $Z = CX$

$AX \geq$ or $\leq$ or $= B$.

$X \geq 0$.

General Procedure for finding the dual.

1) Conversion of LPP into standard form

(a) if problem is of minimization then, all the constrains must consist of '$\geq$' symbol

(b) if problem is of maximization then, all the constraints must contain $\leq$ symbol

(c) if constraint is an equation, then we convert it into two inequalities.

(d) if variable is unrestricted in the sign then we convert it into restricted variable

2) Transpose the coefficient matrix A

3) Interchange the role of B and C

4) Reverse the sign of inequalities.

Page No. 023.

5) Convert the problem to maximization if primal is of minimization & vice-versa.

Que Find the dual of given LPP

$\min Z = 2x_1 + 2x_2 + 4x_3$

such that $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

$\min Z = 2x_1 + 2x_2 + 4x_3$

$2x_1 + 3x_2 + 5x_3 \geq 2$

$-3x_1 - x_2 - 7x_3 \geq -3$

$-x_1 - 4x_2 - 6x_3 \geq -5$

$x_1, x_2, x_3 \geq 0$

$C = [2, 2, 4]$ $\quad C^T = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$B = \begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix}$ $\quad B^T = [2 \ \ -3 \ \ -5]$

$$A = \begin{bmatrix} 2 & 3 & 5 \\ -3 & -1 & -7 \\ -1 & -4 & -6 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & -3 & -1 \\ 3 & -1 & -4 \\ 5 & -7 & -6 \end{bmatrix}$$

Given problem is in the form:

min $Z = CX$

$AX \geq B$

$X \geq 0$

Hence, dual will be given by:

max $Z_D = B^T W$

$A^T W \leq C^T$

$W \geq 0$

when $W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

Therefore, dual will be:

max $Z_D = 2w_1 - 3w_2 - 5w_3$

$2w_1 - 3w_2 - 4w_3 \leq 2$

$3w_1 - w_2 - 4w_3 \leq 2$

$5w_1 - 7w_2 - 6w_3 \leq 4$

$w_1, w_2, w_3 \geq 0$

July 29, 2019

Ques Find the dual of given LPP:

min $Z = x_1 + x_2 + x_3$

s.t. $x_1 - 3x_2 + 4x_3 = 5$

$2x_1 - 2x_2 \leq -3$

$2x_2 - x_3 \geq 5$

& $x_1, x_2 \geq 0$

x_3 is unrestricted in sign.

For converting LPP into standard form we take $x_3 = x_3' - x_3''$

where $x_3', x_3'' \geq 0$

also, we replace equation $x_1 - 3x_2 + 4x_3 = 5$ by

$x_1 - 3x_2 + 4x_3 \leq 5 \Rightarrow -x_1 + 3x_2 - 4x_3 \geq -5$

and $x_1 - 3x_2 + 4x_3 \geq 5$

∴ Standard form is given by:

min $z = x_1 + x_2 + x_3' - x_3''$

$x_1 - 3x_2 + 4(x_3' - x_3'') \geq 5$

$-x_1 + 3x_2 - 4(x_3' - x_3'') \geq -5$

$-2x_1 + 2x_2 \geq 3$

$2x_2 - (x_3' - x_3'') \geq 5$

hence we obtain:

Page No. 026

$$A = \begin{bmatrix} 1 & -3 & 4 & -4 \\ -1 & 3 & -4 & 4 \\ -2 & 2 & 0 & 0 \\ 0 & 2 & -1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ -5 \\ 3 \\ 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 1 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -1 & -2 & 0 \\ -3 & 3 & 2 & 2 \\ 4 & -4 & 0 & -1 \\ -4 & 4 & 0 & 1 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \qquad W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 5 & -5 & 3 & 5 \end{bmatrix}$$

Dual is given by:

max $z_D = 5w_1 - 5w_2 + 3w_3 + 5w_4$

$w_1 - w_2 - 2w_3 \;\cancel{\geq}\; \leq 1$

$-3w_1 + 3w_2 + 2w_3 + 2w_4 \leq 1$

$4w_1 - 4w_2 - w_4 \leq 1$

$-4w_1 + 4w_2 + w_4 \leq -1$

where $w_1, w_2, w_3, w_4 \geq 0$

If we take $w_1 - w_2 = w_1'$ (unrestricted sign) then dual becomes

Page No. 027

max $Z_D = 5w_1' + 3w_3 + 5w_4$

$w_1' - 2w_3 \leq 1$

$-3w_1' + 2w_3 + 2w_4 \leq 1$

$\left.\begin{array}{l} 4w_1 - w_4 \leq 1 \\ -4w_1 + w_4 \leq -1 \end{array}\right\} \; 4w_1 - w_4 = 1$

and $w_3, w_4 \geq 0$

$w_1' = w_1 - w_2$ is unrestricted in sign.

Ques min $z = x_1 - 3x_2 - 2x_3$

$3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 - 4x_2 \geq 12$

$-4x_1 + 3x_2 + 3x_3 = 10$

$x_1, x_2 \geq 0$; x_3 is unrestricted.

For converting LPP to standard form we take $x_3 = x_3' - x_3''$

where $x_3', x_3'' \geq 0$

also we replace equation $-4x_1 + 3x_2 + 3x_3 = 10$ by $-4x_1 + 3x_2 + 3x_3 \leq 10 \Rightarrow 4x_1 - 3x_2 - 3x_3 \geq -10$

and $-4x_1 + 3x_2 + 3x_3 \geq 10$

∴ Standard form is given by

min $z = x_1 - 3x_2 - 2(x_3' - x_3'')$

$-3x_1 + x_2 - 2(x_3' - x_3'') \geq -7$

$2x_1 - 4x_2 \geq 12$

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$4x_1 - 3x_2 - 3(x_3' - x_3'') \geq -10$
$-4x_1 + 3x_2 + 3(x_3' - x_3'') \geq 10$

hence we obtain:

$$A = \begin{bmatrix} -3 & 1 & -2 & 2 \\ 2 & -4 & 0 & 0 \\ 4 & -3 & -8 & 8 \\ -4 & 3 & 8 & -8 \end{bmatrix} \quad A^T = \begin{bmatrix} -3 & 2 & 4 & -4 \\ 1 & -4 & -3 & 3 \\ -2 & 0 & -8 & 8 \\ 2 & 0 & 8 & -8 \end{bmatrix}$$

$$B = \begin{bmatrix} -7 \\ 12 \\ -10 \\ 10 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -3 & -2 & 2 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 \\ -3 \\ -2 \\ 2 \end{bmatrix}$$

$B^T = [-7 \quad 12 \quad -10 \quad 10]$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3' \\ x_3'' \end{bmatrix} \quad W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

Dual is given by:
max $z_0 = -7w_1 + 12w_2 - 10w_3 + 10w_4$
s.t. $-3w_1 + 2w_2 + 4w_3 - 4w_4 \leq 1$

$w_1 - 4w_2 - 3w_3 + 3w_4 \leq -3$
$-2w_1 - 3w_3 + 8w_4 \leq -2$
$2w_1 + 3w_3 - 8w_4 \leq 2$
where $w_1, w_2, w_3, w_4 \geq 0$

If we take $w_3 - w_4 = w_3'$ (unrestricted sign) then dual becomes:
min $z_0 = -7w_1 + 12w_2 \mp 10\, w_3'$
s.t. $-3w_1 + 2w_2 + 4w_3' \leq 1$
$w_1 - 4w_2 - 3w_3' \leq -3$
$\left.\begin{array}{l} -2w_1 - 8w_3' \leq -2 \\ 2w_1 + 8w_3' \leq 2 \end{array}\right\}$ $2w_1 + 8w_3' = 2$
and $w_1, w_2 \geq 0$
$w_3' = w_3 - w_4$ is unrestricted in sign

July 30, 2019

Ques Use duality to solve the following problem
min $z = 3x_1 + x_2$
$x_1 + x_2 \geq 1$
$2x_1 + 3x_2 \geq 2$
& $x_1, x_2 \geq 0$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad C = [3 \quad 1]$$

$A^T = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $B^T = [1 \quad 2]$ $C^T = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

min $Z = CA$

$AX \geq B$

$X \geq 0$

then dual be

max $Z_D = B^T A^T$

$A^T W \geq C^T$

$W \geq 0$

max $Z_D = w_1 + 2w_2$

$w_1 + 2w_2 \leq 3$

$w_1 + 3w_2 \leq 1$

$w_1, w_2 \geq 0$

We take slack variables w_3 & w_4

max $Z_D = w_1 + 2w_2 + 0.w_3 + 0.w_4$

$w_1 + 2w_2 + w_3 + 0.w_4 \leq 3$

$w_1 + 3w_2 + 0.w_3 + w_4 = 1$

$w_1, w_2 \geq 0$

			C_j	1	2	0	0
C_B	B	W_b	b	y_1	y_2	y_3	y_4
0	α_3	w_3	3	1	2	1	0
0	α_4	w_4	1	1	$\boxed{3}$	0	1
	$Z_j - C_j$			-1	-2 ↑	0	0

key element min[3/2 , 1/3]

departing vector w_4

entering vector w_2

			C_j	1	2	0	0
C_B	B	W_b	b	y_1	y_2	y_3	y_4
0	α_3	w_3	7/3	1/3	0	1	-2/3
2	α_2	w_2	1/3	$\boxed{1/3}$	1	0	1/3
	$Z_j - C_j$			-1/3 ↑	0	0	2/3

key element : min [$\frac{7/3}{1/3}$ $\frac{1/3}{1/3}$]

departing vector w_2

entering vector w_1

			C_j	1	2	0	0
C_B	B	W_B	b	y_1	y_2	y_3	y_4
0	α_3	w_3	2	0	-1	1	-1
1	α_1	w_1	1	1	3	0	1
	$Z_j - C_j$			0	1	0	1

$w_1 = 1$ $w_2 = 0$

$Z_D = 1$

$x_1 = Z_3 - C_3 = 0$

$x_2 = Z_4 - C_4 = 1$

$Z = 1$

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Ques Use duality to solve the following LPP

min $z = 2x_1 + 9x_2 + x_3$

s.t. $x_1 + 4x_2 + 2x_3 \geq 5$

$3x_1 + x_2 + 2x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

$A = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 2 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $\quad B^T = [5 \quad 4]$

$C = [2 \quad 9 \quad 1]$ $\quad C^T = \begin{bmatrix} 2 \\ 9 \\ 1 \end{bmatrix}$

$W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

dual be

max $z_D = 5w_1 + 4w_2$

s.t. $w_1 + 3w_2 \leq 2$

$4w_1 + w_2 \leq 9$

$2w_1 + 2w_2 \leq 1$

We take slack variables w_3, w_4, w_5

Now, the LPP becomes

max $z_D = 5w_1 + 4w_2 + 0.w_3 + 0.w_4 + 0.w_5$

s.t. $w_1 + 3w_2 + w_3 + 0.w_4 + 0.w_5 = 2$

$4w_1 + w_2 + 0.w_3 + w_4 + 0.w_5 = 9$

$2w_1 + 2w_2 + 0.w_3 + 0.w_4 + w_5 = 1$

$w_1, w_2, w_3, w_4, w_5 \geq 0$

			c_j	5	4	0	0	0
C_B	B	W_B	b	y_1	y_2	y_3	y_4	y_5
0	α_3	w_3	2	1	3	1	0	0
0	α_4	w_4	9	4	1	0	1	0
0	α_5	w_5	1	$\boxed{2}$	2	0	0	1
	$z_j - c_j$			-5 ↑	-4	0	0	0

key element $\quad$ min $[2/1, 9/4, 1/2]$

departing vector $\quad w_5$

entering vector $\quad w_1$

			c_j	5	4	0	0	0
C_B	B	W_B	b	y_1	y_2	y_3	y_4	y_5
0	α_3	w_3	3/2	0	2	1	0	-1/2
0	α_4	w_4	7	0	-3	0	1	-2
5	α_1	w_1	1/2	1	1	0	0	1/2
	$z_j - c_j$			0	1	0	0	5/2

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$w_1 = 1/2$ $\quad w_2 = 0$

$z_D = 5/2$

$x_1 = z_3 - c_3 = 0$

$x_2 = z_4 - c_4 = 0$

$x_3 = z_5 - c_5 = 5/2$ $\quad z = 5/2$

Ques Apply duality to solve the following by simplex method.

$\max \; z = x_1 + x_2$

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

& $x_1, x_2 \geq 0$

Given LPP in standard form

$\max \; z = x_1 + x_2$

$-2x_1 - x_2 \leq -4$

$-x_1 - 7x_2 \leq -7$

$x_1, x_2 \geq 0$

Then dual be

$\min \; z_D = -4x_1 - 7x_2$

$-2w_1 - w_2 \geq 1$

$-w_1 - 7w_2 \geq 1$

$w_1, w_2 \geq 0$

To convert the problem to maximization, we take $z_D^* = -z_D$. Also we take surplus and artificial variables

Now, the LPP becomes

$\max \; z_D^* = 4w_1 + 7w_2 + 0.w_3 + 0w_4 - Mw_5 - Mw_6$

$-2w_1 - w_2 - w_3 + w_5 = 1$

$-w_1 - 7w_2 - w_4 + w_6 = 1$

			c_j	4	7	0	0	-M	-M
C_B	B	W_B	C_j	y_1	y_2	y_3	y_4	y_5	y_6
-M	α_5	w_5	1	-2	-1	-1	0	1	0
-M	α_6	w_6	1	-1	-7	0	-1	0	1
$z_j - c_j$				3M-4	8M-7	M	M	0	0

Since all $z_j - c_j$ are positive and B has ~~an~~ non zero artificial vector. So, feasible solution of primal as well as dual doesn't exist.

July 31, 2019

Assignment Problem:

It is a particular type of LPP in which object is to find the optimum allocation of number of jobs to no of persons/machines.

Suppose n workers are available to perform n jobs then assignment problem is written in the form of cost matrix as follows:

Persons	Jobs 1	2	. . .	n
1	C_{11}	C_{12}		C_{1n}
2	C_{21}	C_{22}		C_{2n}
3	⋮	⋮		⋮
n	C_{n1}	C_{n2}		C_{nn}

where C_{ij} is the cost of performing j^{th} job by i^{th} machine/person. then we have to allocate jobs to the person such that cost is minimum.

Sums A department has four subordinates and four task the estimate of the time each subordinate would take to perform each task is given in the cost matrix below.

Tasks	Subordinats I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should the task be allotted so as to minimize the total time taken.

Row Reduced matrix

	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

1) We obtain row reduced matrix by ~~Column &~~ subtracting minimum of each row from all other elements of corresponding row

2) Obtain column reduced matrix. Subtract minimum of each column from all other elements of corresponding column

Column Reduced Matrix

	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

3) Zero assignment:

In matrix so obtained we start with

Scanned by CamScanner

first row and check if it consist of single zero. If found assign that zero and cross all the zeroes in corresponding column.

Next we examine the second row and repeat the process.

We all rows are examined then we examine the column. Then repeat the process till all zeroes are assigned or crossed.

If each row & column consist of assigned zero then solution is optimal

	I	II	III	IV
A	$\boxed{0}$	14	9	3
B	9	20	$\boxed{0}$	22
C	23	$\boxed{0}$	3	$\cancel{0}$
D	9	12	14	$\boxed{0}$

A → I 8
B → III 4
C → II 19
D → IV 10

41 hrs.

Ques There are five jobs to be assigned one each to five machines. The cost matrix is given below.
Solve the assignment problem.

	I	II	III	IV	V
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

Row Reduced Matrix

	I	II	III	IV	V
A	3	9	0	8	12
B	3	1	6	0	9
C	1	4	3	0	4
D	4	7	0	11	9
E	4	0	2	1	5

Column Reduced Matrix

	I	II	III	IV	V
A	2	9	$\boxed{0}$	8	8
B	2	1	6	$\boxed{0}$	5
C	$\boxed{0}$	4	3	$\cancel{0}$	$\cancel{0}$
D	3	7	$\cancel{0}$	11	5
E	3	$\boxed{0}$	2	1	1

	I	II	III	IV	V
A	2	9	$\boxed{0}$	8	8
B	~~2~~	~~1~~	~~6~~	$\boxed{0}$	5
C	$\boxed{0}$	~~4~~	~~8~~3	~~0~~	~~0~~
D	3	7	~~0~~	11	5
E	~~3~~	$\boxed{0}$	~~2~~	~~1~~	~~1~~

	I	II	III	IV	V
A	$\boxed{0}$	7	0	6	6
B	2	1	8	$\boxed{0}$	5
C	~~0~~	4	5	~~0~~	$\boxed{0}$
D	1	5	$\boxed{0}$	9	3
E	3	$\boxed{0}$	4	1	1

August 2, 2019

Ques Apply the principle of duality to solve the LPP: max $z = 3x_1 + 4x_2$

s.t. $x_1 - x_2 \le 1$

$x_1 + x_2 \ge 4$

$x_1 - 3x_2 \le 3$ & $x_1, x_2 \ge 0$

Given LPP in standard form

max $z = 3x_1 + 4x_2$

$x_1 - x_2 \le 1$

$-x_1 - x_2 \le -4$

$x_1 - 3x_2 \le 3$ s.t. $x_1, x_2 \ge 0$

$A = \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ 1 & -3 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & -3 \end{bmatrix}$

$B = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ $\quad B^T = [1 \ -4 \ 3]$

$C^T = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$C = [3 \ 4]$

Then its dual be

min $z_D = w_1 - 4w_2 + 3w_3$

s.t. $w_1 - w_2 + w_3 \ge 3$

$-w_1 - w_2 - 3w_3 \ge 4$ s.t. $w_1, w_2, w_3 \ge 0$

where $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

To convert the problem to maximization, we take $z_D^* = -z_D$. Also we take surplus and artificial variable w_4, w_5, w_6, w_7

Now, the LPP becomes:

~~max $z_D^* = 4w_2$~~

max $z_0^* = -w_1 + 4w_2 - 3w_3 + 0w_4 + 0w_5 - Mw_6 - Mw_7$

s.t. $w_1 - w_2 + w_3 - w_4 + 0.w_5 + w_6 + 0.w_7 = 3$

$-w_1 - w_2 - 3w_3 + 0.w_4 - w_5 + 0.w_6 + w_7 = 4$

$w_1, w_2, w_3, w_4, w_5, w_6, w_7 \geq 0$

			c_j	-1	4	-3	0	0	-M	-M
C_B	B	W_B	b	y_1	y_2	y_3	y_4	y_5	y_6	y_7
-M	α_6	w_6	3	1	-1	1	-1	0	1	0
-M	α_7	w_7	4	-1	-1	-3	0	-1	0	1
$z_j - c_j$				1	2M-4	2M+3	M	M	0	0

Since all $z_j - c_j$ are positive and B has non zero artificial variables. So there is no feasible solution of dual. Hence no feasible solution of primal as well.

August 6, 2019

Process for Drawing lines:

If solution is not optimal, we find the optimal solution by drawing minimum number of horizontal and vertical lines so as to cover all the zeroes. We follow the following process:

mark the rows in which there is no zero assignment.

mark the columns ~~in tick~~ which consist of zero in marked rows.

mark the rows which have assignment in marked columns

repeat step 2 and 3 until the chain of marking ends.

draw minimum no of lines through unmarked rows and marked columns.

select the smallest element from uncovered elements then subtract it from all uncovered element and add it to every element that lies on the intersection of the lines and leave the remaining elements unchanged

now we do the zero assignment.

Ques A national truck rental service has a surplus of one truck in each of the cities 1, 2, 3, 4, 5 and 6 and it is deficit of one truck in each of the cities

7, 8, 9, 10, 11 and 12 The distance in kilometres between the cities is displayed below. How should the truck be dispersed so as to minimize the total distance.

	7	8	9	10	11	12
1	31	62	29	42	15	41
2	12	19	39	55	71	40
3	17	29	50	41	22	22
4	35	40	38	42	27	33
5	19	30	29	16	20	23
6	72	30	30	50	41	20

Row Reduced matrix:

	7	8	9	10	11	12
1	16	47	14	27	0	26
2	0	7	27	43	59	28
3	0	12	33	24	5	5
4	8	13	11	15	0	6
5	3	14	13	0	4	7
6	52	10	10	30	21	0

Column Reduced Matrix

	7	8	9	10	11	12
1	16	40	4	27	$\boxed{0}$	26
2	$\cancel{0}$	$\boxed{0}$	17	43	59	28
3	$\boxed{0}$	5	23	24	5	5
4	8	6	1	15	$\cancel{0}$	6
5	3	7	3	$\boxed{0}$	4	7
6	52	3	$\boxed{0}$	31	21	$\cancel{0}$

Zero matrix

	7	8	9	10	11	12
1	16	40	4	27	$\boxed{0}$	26 ✓
2	$\cancel{0}$	$\boxed{0}$	17	43	59	28
3	$\boxed{0}$	5	23	24	5	5
4	8	6	1	15	$\cancel{0}$	6 ✓
5	3	7	3	$\boxed{0}$	4	7
6	52	3	$\boxed{0}$	31	21	$\cancel{0}$

	7	8	9	10	11	12
1	15	39	3	26	$\boxed{0}$	25
2	$\cancel{0}$	$\boxed{0}$	17	43	60	20
3	$\boxed{0}$	5	23	24	6	5
4	7	5	$\boxed{0}$	14	$\cancel{0}$	5
5	3	7	3	$\boxed{0}$	5	7
6	52	3	$\cancel{0}$	30	22	$\boxed{0}$

1 → 11	15	
2 → 8	19	
3 → 7	17	
4 → 9	28	
5 → 10	16	
6 → 12	20	

~~Ques A company has five jobs to be done the matrix below shows the re.~~

Ques A car hire company has five depot A, b, c, d, e. A customer requires cars in each town namely A, B, C, D, E. Distance between depots (origin) and town (destination) are given in the following matrix

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

how should the cars be assigned to customers so as to minimize the distance travelled.

Row Reduced matrix

	a	b	c	d	e
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Column Reduced matrix

	a	b	c	d	e
A	30	$\boxed{0}$	35	30	15
B	15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$
C	30	$\cancel{0}$	35	30	20
D	$\boxed{0}$	$\cancel{0}$	20	$\cancel{0}$	5
E	20	$\cancel{0}$	25	15	15

Zero matrix

	a	b	c	d	e	
A	30	$\boxed{0}$	35	30	15	✓
B	15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$	
C	30	$\cancel{0}$	35	30	20	✓
D	$\boxed{0}$	$\cancel{0}$	20	$\cancel{0}$	5	
E	20	$\cancel{0}$	35	15	15	✓

	a	b	c	d	e
A	15	$\cancel{0}$	20	15	$\boxed{0}$
B	15	15	$\boxed{0}$	10	$\cancel{0}$
C	15	$\boxed{0}$	20	15	5
D	$\boxed{0}$	15	20	$\cancel{0}$	5
E	5	$\cancel{0}$	10	$\boxed{0}$	$\cancel{0}$

A → e 200
B → c 130
C → b 110
D → a 50
E → d 80

August 7, 2019

Q. Solve the minimal assignment problem given below:

	A	B	C	D	E
1	10	20	4	24	8
2	8	20	6	28	12
3	12	26	8	32	12
4	14	28	8	36	18
5	16	28	12	36	24

Row reduced matrix

	A	B	C	D	E
1	6	16	0	20	4
2	2	14	0	22	6
3	4	18	0	24	4
4	6	20	0	28	10
5	4	16	0	24	12

Column reduced matrix

	A	B	C	D	E	
1	4	2	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	
2	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2	2	
3	2	4	$\cancel{0}$	4	$\boxed{0}$	
4	4	6	$\boxed{0}$	8	6	✓
5	2	2	$\cancel{0}$	4	8	✓
6			✓			

Zero Assignment:

	A	B	C	D	E
1	4	2	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$
2	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2	2
3	2	4	$\cancel{0}$	4	$\boxed{0}$
4	4	6	$\boxed{\cancel{0}}$	8	6
5	2	2	$\cancel{0}$	4	8

col 4

	A	B	C	D	E
1	4	2	2	$\boxed{0}$	$\cancel{0}$
2	$\boxed{0}$	$\cancel{0}$	2	2	2
3	2	4	2	4	$\boxed{0}$
4	2	4	$\boxed{0}$	6	4
5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	2	6

4	2	2	$\boxed{0}$	$\cancel{0}$
$\cancel{0}$	$\boxed{0}$	2	2	2
2	4	2	4	$\boxed{0}$
2	4	$\boxed{0}$	6	4
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2	6

$1 \rightarrow D$
$2 \rightarrow A$
$3 \rightarrow E$
$4 \rightarrow C$
$5 \rightarrow B$

$1 \rightarrow D$
$2 \rightarrow B$
$3 \rightarrow E$
$4 \rightarrow C$
$5 \rightarrow A$

Unbalanced Assignment Problem

If the number of rows is not equal to no of columns in assignment problem then we add a dummy row or column with zero (0) cost

Maximal Assignment Problem

The maximization assignment problem can be converted to minimization problem by creating a lost opportunity matrix by using one of the following method.

1. The first method is by putting a -ve sign before the cost in the matrix and then solve by usual method.

2. In second method we locate the largest value in matrix and subtract each element in the matrix from this value and then solve it by usual method.

Ques Solve the following assignment problem for ~~ma~~ minimization.

	J1	J2	J3	J4
W1	7	5	8	4
W2	5	6	7	4
W3	8	7	9	8

As the problem is unbalanced hence, we add a dummy row with cost 0 Hence the problem becomes

	J1	J2	J3	J4
W1	7	5	8	4
W2	5	6	7	4
W3	8	7	9	8
W4	0	0	0	0

Row reduced matrix

	J1	J2	J3	J4
W1	3	1	4	0
W2	1	2	3	0
W3	1	0	2	1
W4	0	0	0	0

Column Reduced matrix

	J1	J2	J3	J4
W1	3	1	4	$\boxed{0}$
W2	1	2	3	$\cancel{0}$
W3	1	$\boxed{0}$	2	1
W4	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$

	J1	J2	J3	J4
W1	3	1	4	$\boxed{0}$
W2	1	2	3	$\cancel{0}$
W3	1	$\boxed{0}$	2	1
W4	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$

	J1	J2	J3	J4
W1	2	$\cancel{0}$	3	$\boxed{0}$
W2	$\boxed{0}$	1	2	$\cancel{0}$
W3	1	$\boxed{0}$	2	2
W4	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1

Ans A company has a team of four salesperson and there are four districts where the company wants to start its business. The company estimates the profit per day in Rs. for each salesperson for each district in the table given below. After taking into the capabilities of the salesperson & nature of district, assign a district to each salesperson.

	1	2	3	4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

	1	2	3	4
A	0	6	2	5
B	2	5	1	1
C	1	1	3	4
D	3	4	2	1

Row reduced matrix

	1	2	3	4
A	0	6	2	5
B	1	4	0	0
C	0	0	2	3
D	2	3	1	0

Column reduced matrix

	1	2	3	4
A	$\boxed{0}$	6	2	5
B	1	4	$\boxed{0}$	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$	2	3
D	2	3	1	$\boxed{0}$

Ques Solve the following assignment problem for minimization

	a	b	c	d
A	310	620	290	420
B	120	190	390	550
C	170	290	500	410
D	350	400	380	420

Row reduced matrix

	a	b	c	d
A	20	330	0	130
B	0	70	270	430
C	0	120	330	240
D	0	50	30	170

Column reduced matrix

20	280	$\boxed{0}$	60
$\boxed{0}$	20	270	360
$\cancel{0}$	70	330	170
$\cancel{0}$	$\boxed{0}$	30	$\cancel{0}$

	a	b	c	d
A	20	280	$\boxed{0}$	60
B	$\boxed{0}$	20	270	360
C	$\cancel{0}$	70	330	170
D	$\cancel{0}$	$\boxed{0}$	30	$\cancel{0}$

	a	b	c	d
A	40	280	$\boxed{0}$	60
B	$\cancel{0}$	$\boxed{0}$	250	340
C	$\boxed{0}$	50	310	150
D	20	$\cancel{0}$	30	$\boxed{0}$

290 + 190 + 170 + 420

1070

August 8, 2019

Ques

Delhi-Jaipur			Jaipur-Delhi		
flight no	depot	arrival	no	depot	arrival
1	7:00AM	8:00AM	101	8:00 AM	9:15 AM
2	8:00AM	9:00AM	102	8:30AM	9:45 AM
3	1:30 PM	2:30 PM	103	12:00 Noon	1:15 PM
4	6:30 PM	7:30PM	104	5:30PM	6:45 PM
5					

An airline operates 7 days a week as shown below:

A crew must have minimum lay over time of 5 hour between flights obtain the

Column reduced matrix

	1	2	3	4
A	$\boxed{0}$	6	2	5
B	1	4	$\boxed{0}$	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$	2	3
D	2	3	1	$\boxed{0}$

Ques Solve the following assignment problem for minimization

	a	b	c	d
A	310	620	290	420
B	120	190	390	550
C	170	290	500	410
D	350	400	380	420

Row reduced matrix

	a	b	c	d
A	20	330	0	130
B	0	70	270	430
C	0	120	330	240
D	0	50	30	170

Column reduced matrix

20	280	$\boxed{0}$	60
$\boxed{0}$	20	270	360
$\cancel{0}$	70	330	170
$\cancel{0}$	$\boxed{0}$	30	$\cancel{0}$

	a	b	c	d
A	20	280	$\boxed{0}$	60
B	$\boxed{0}$	20	270	360
C	$\cancel{0}$	70	330	170
D	$\cancel{0}$	$\boxed{0}$	30	$\cancel{0}$

	a	b	c	d
A	40	280	$\boxed{0}$	60
B	$\cancel{0}$	$\boxed{0}$	250	340
C	$\boxed{0}$	50	310	150
D	20	$\cancel{0}$	30	$\boxed{0}$

290 + 190 + 170 + 420

1070

August 8, 2019

Ques

Delhi-Jaipur			Jaipur-Delhi		
flight no	depot	arrival	no	depot	arrival
1	7:00AM	8:00AM	101	8:00 AM	9:15 AM
2	8:00AM	9:00AM	102	8:30AM	9:45 AM
3	1:30 PM	2:30 PM	103	12:00 Noon	1:15 PM
4	6:30 PM	7:30 PM	104	5:30PM	6:45 PM
5					

An airline operates 7 days a week as shown below:

A crew must have minimum lay over time of 5 hour between flights obtain the pairing of flight that minimize lay over time away from home for any given pair The crew will be based at the city that results in the smaller lay over. For each pair also mention the town where the crew should be based

We take 1 hr = 4 unit

Layover at Jaipur

	101	102	103	104
1	96	98	112	38
2	92	94	108	34
3	70	72	86	108
4	50	52	66	88

Layover at Delhi

	101	102	103	104
1	87	85	71	49
2	91	89	75	53
3	113	111	97	75
4	37	35	21	95

Minimum Layover Time

	101	102	103	104
1	87*	85*	71*	38
2	91*	89*	75*	34
3	70	72	86	75*
4	37*	35*	21*	88

Row Reduced matrix

	101	102	103	104
1	49	47	33	0
2	57	55	41	0
3	0	2	16	5
4	16	14	0	67

Column Reduced Matrix

	101	102	103	104
1	49	45	33	$\boxed{0}$
2	57	53	41	~~0~~
3	$\boxed{0}$	~~0~~	16	5
4	16	12	$\boxed{0}$	67

	101	102	103	104
1	16	12	~~0~~	~~0~~
2	24	20	8	$\boxed{0}$
3	$\boxed{0}$	~~0~~	16	38
4	16	12	$\boxed{0}$	100

	101	102	103	104
1	4	$\boxed{0}$	~~0~~	~~0~~
2	12	8	8	$\boxed{0}$
3	$\boxed{0}$	~~0~~	28	50
4	4	~~0~~	$\boxed{0}$	100

101	102	103	104
4	~~0~~	$\boxed{0}$	~~0~~
12	8	8	$\boxed{0}$
$\boxed{0}$	~~0~~	28	50
4	$\boxed{0}$	~~0~~	100

1 → 102 = 85*
2 → 104 = 34
3 → 101 = 70
4 → 103 = 21*

1 → 103 = 71*
2 → 104 = 34
3 → 101 = 70
4 → 102 = 35*

August 9, 2019

Transportation Problem:

In big manufacturing companies, generally a single commodity is manufactured in different units situated at different places. So that it can be distributed everywhere with minimum cost and time such type of problem of distribution are known as transportation problem.

Methods to solve.

- Northwest Corner Rule
- Lowest cost entry method
- Vogel's approximation method

Ques Solve the transportation problem

From	D_1	D_2	D_3	Available
01	2	7	4	5
02	3	3	1	8
03	5	4	7	7
04	1	6	2	14
Demand	7	9	18	

From	D_1	D_2	D_3	Available
01	2(5)	7	4	~~5~~ 0
02	3(2)	3(6)	1	~~8~~ ~~6~~ 0
03	5	4(3)	7(4)	~~7~~ ~~4~~ 0
04	1	6	2(14)	~~14~~ 0
Demand	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~18~~ ~~14~~ 0	

$2\times5 + 3\times2 + 3\times6 + 4\times3 + 7\times4 + 2\times14$

$= 102$

Northwest Corner Rule

In this method we select the north west cell and allocate minimum of demand and supply and accordingly we change the demand and supply matrix. In this process one row or one column will be removed. Then again we select the new northwest cell and repeat the process till both demand and supply exhaust.

Ques Solve the transportation problem

From	1	2	3	Supply
A	16	19	12	14
B	22	13	19	16
C	14	28	8	12
Demand	10	15	17	

$1 \to 102 = 85^*$ $\quad$ $1 \to 103 = 71^*$

$2 \to 104 = 34$ $\quad$ $2 \to 104 = 34$

$3 \to 101 = 70$ $\quad$ $3 \to 101 = 70$

$4 \to 103 = 21^*$ $\quad$ $4 \to 102 = 35^*$

August 9, 2019

Transportation Problem:

In big manufacturing companies, generally a single commodity is manufactured in different units situated at different places. So that it can be distributed everywhere with minimum cost and time. Such type of problem of distribution are known as transportation problem.

Methods to solve.

- Northwest Corner Rule
- Lowest cost entry method
- Vogel's approximation method

Ques Solve the transportation problem

From	D_1	D_2	D_3	Available
01	2	7	4	5
02	3	3	1	8
03	5	4	7	7
04	1	6	2	14
Demand	7	9	18	

From	D_1	D_2	D_3	Available
01	2(5)	7	4	~~5~~ 0
02	3(2)	3(6)	1	~~8~~ ~~6~~ 0
03	5	4(3)	7(4)	~~7~~ ~~4~~ 0
04	1	6	2(14)	~~14~~ 0
Demand	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~18~~ ~~14~~ 0	

$2\times5 + 3\times2 + 3\times6 + 4\times3 + 7\times4 + 2\times14$

$= 102$

Northwest Corner Rule

In this method we select the north west cell and allocate minimum of demand and supply and accordingly we change the demand and supply matrix. In this process one row or one column will be removed. Then again we select the new northwest cell and repeat the process till both demand and supply exhaust.

Ques Solve the transportation problem

From	1	2	3	Supply
A	16	19	12	14
B	22	13	19	16
C	14	28	8	12
Demand	10	15	17	

From	1	2	3	Supply
A	16 (10)	19 (4)	12	~~14~~ ~~4~~ 0
B	22	13 (11)	19 (5)	~~16~~ ~~5~~ 0
C	14	28	8 (12)	~~12~~ 0
Demand	~~10~~ 0	~~15~~ ~~11~~ 0	~~17~~ ~~12~~ 0	

16×10 + 19×4 + 13×11 + 19×5 + 8×12

160 + 76 + 143 + 95 + 96

570

Lowest Cost entry method

Ques Solve the transportation problem

To

From	D_1	D_2	D_3	Available
01	2	7	4	5
02	3	3	1	8
03	5	4	7	7
04	1	6	2	14
Demand	7	9	18	

From	D_1	D_2	D_3	Available
01	2	7(2)	4 (3)	~~5~~ 2
02	3	3	1(8)	~~8~~ 0
03	5	4(7)	7	~~7~~ 0
04	1(7)	6	2 (7)	~~14~~ ~~7~~ 0
Demand	~~7~~ 0	~~9~~ 2	~~18~~ ~~10~~ ~~3~~ 0	

1×7 + 7×2 + 4×7 + 4×3 + 1×8 + 2×7

= 83

In this process we select the lowest cost cell and allocate the minimum of demand and supply If there is more than one position of lowest cell then we can select any one of them Then we change the demand & supply matrix accordingly Then repeat the process of selection of lowest cost cell & allocation till demand & supply exhaust.

Ques Solve the transportation problem

To

From	1	2	3	Available
A	16 (9)	19	12 (5)	~~14~~ ~~9~~ 0
B	22 (1)	13 (15)	19	~~16~~ ~~1~~ 0
C	14	28	8 (12)	~~12~~ 0
Demand	~~10~~ ~~1~~ 0	~~15~~ 0	~~17~~ ~~5~~ 0	

16×9 + 22×1 + 13×15 + 12×5 + 8×12

~~493~~

August 13, 2019

Vogel's Approximation Method:

Ques Solve the following transportation problem by Vogel's Approximation method

	D_1	D_2	D_3		Penalty			
01	2(5)	7	4	~~5~~0	2 ←			
02	3	3	1(8)	~~8~~0	2	2 ←		
03	5	4(7)	7	7	1	1	1	1
04	1(2)	6(2)	2(10)	~~14~~4	1	1	1	5
	~~7~~2	9	~~18~~~~10~~0					
Penalty	1	1	1					
	2	1	1					
	4	2	5					
	4	2	↑					

Step Wise Solution

2(5)	7	4	~~5~~0	2 ←
3	3	1	8	2
5	4	7	7	1
1	6	2	14	1
~~7~~2	9	18		
1	1	1		

3	3	1(8)	~~8~~0	2 ←
5	4	7	7	1
1	6	2	14	1
2	9	~~18~~10		
2	1	1		

5	4	7	7	1
1	6	2(10)	~~14~~4	1
2	9	~~10~~0		
4	2	5		
		↑		

5	4	7	1
1(2)	6	~~4~~2	5
~~2~~0	9		
4	2		

4(7)	~~7~~0
6(2)	2
~~9~~2	

Vogel's Approximation Method:

1. we calculate row & column penalty by finding the difference between smallest and next smallest cost.

2. we select the highest penalty, if there are equal penalties we select any one of them.

3. from selected penalty (maybe row or column) we find the minimum cost and allocate minimum of demand and supply.

4. we remove row or column whichever is zero

5. we again calculate row & column penalty and repeat the above process till demand and supply exhaust

Optimum Solution:
The solution so obtained is optimum if number of allocations is $m+n-1$ and if d_{ij} are positive

If above conditions are not satisfied, solution is not optimal then, we use modi method to make it optimal

Que Solve the following transportation problem

	D1	D2	D3	D4	
O1	1	2	1	4	30
O2	3	3	2	1	50
O3	4	2	5	9	20
	20	40	30	10	

	D1	D2	D3	D4		Penalty				
O1	1(20)	2	1(10)	4	~~30~~ ~~10~~ 0	0	0	0	1 ←	
O2	3	3(20)	2(20)	1(10)	~~50~~ 40	1	1	1	1	
O3	4	2(20)	5	9	~~20~~ 0	2	2 ←			
	~~20~~ 0	~~40~~ 20	~~30~~ 20	~~10~~ 0						
Penalty	2	0	1	3 ↑						
	2	0	1							
	2	1	1							
	↑	1	1							

	D1	D2	D3	D4
O1	1(20)	2	1(10)	4
O2	3	3(20)	2(20)	1(10)
O3	4	2(20)	5	9

no of allocations $= 3+4-1 = 6$

hence first condition is satisfied

we calculate $d_{ij} = C_{ij} - (u_i + v_j)$ for unallocated cell.

		1(10)	4	$u_1 = -1$
1(20)	2	2(20)	1(10)	$u_2 = 0$
3	3(20)	5 ~~2(20)~~	9	$u_3 = -1$
4	2(20)			
$v_1 = 2$	$v_2 = 3$	$v_3 = 2$	$v_4 = 1$	

$$\begin{bmatrix} & 2 & & 4 \\ 3 & & & \\ 4 & & 5 & 9 \end{bmatrix} - \begin{bmatrix} & 2 & & 0 \\ 2 & & & \\ 1 & & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} & 0 & & 4 \\ 1 & & & \\ 3 & & 4 & 9 \end{bmatrix}$$

all are positive
∴ solution is optimal.

Rough work

$u_2 = 0$ for allocated cell:

$C_{11} = u_1 + v_1 \Rightarrow 1 = -1 + v_1 \Rightarrow v_1 = 2$

$C_{13} = u_1 + v_3 \Rightarrow 1 = u_1 + 2 \Rightarrow u_1 = -1$

$C_{22} = u_2 + v_2 \Rightarrow 3 = v_2$

$C_{23} = u_2 + v_3 \Rightarrow 2 = v_3$

$C_{24} = u_2 + v_4 \Rightarrow 1 = v_4$

$C_{32} = u_3 + v_2 \Rightarrow u_3 = -1$

August 14, 2019

Ques A company is spending Rs 1000 on transportation of its unit to four warehouse from three factories. What can be maximum saving by optimum scheduling. Solve the following transportation problem.

factory	Warehouse W1	W2	W3	W4	Capacity
~~F1~~					
~~F2~~ F1	19	30	50	10	7
~~F3~~ F2	70	30	40	60	9
~~requirement~~	40	8	70	20	18
requirement	5	8	7	14	

	W1	W2	W3	W4	Cap.					
F1	19(5)	30	50	10(2)	~~7~~ ~~2~~ 0	9	9	40	40	
F2	70	30	40(7)	60(2)	9	10	20	20	20	
F3	40	8(8)	70	20(10)	18(10) 0	12	20	50 ←		
req	~~5~~ 0	~~8~~ 0	7	~~14~~ ~~4~~ 2						
	21	22	10	10						
	21	↑	10	10						
	↑		10	10						
			10	50						
				↑						

Test for optimality

here $n = 4$ $m = 3$

∴ no of allocations $= 6 = 3 + 4 - 1$

we calculate $d_{ij} = c_{ij} - (u_i + v_j)$ [

19(5)	30	50	10(2)	10
70	30	40(7)	60(2)	60
40	8(8)	70	20(10)	20
9	-12	-20	0	

$d_{ij} = c_{ij} - (u_i + v_j)$ [for unallocated cell

$$d_{ij} = \begin{bmatrix} - & 30 & 50 & - \\ 70 & 30 & - & - \\ 40 & - & 70 & - \end{bmatrix} - \begin{bmatrix} 10 & -2 & -10 & - \\ 189 & 48 & 0 & - \\ 29 & - & 40 & - \end{bmatrix} \begin{bmatrix} - & -2 & -10 \\ 69 & 48 & - \\ 29 & - & 0 \end{bmatrix}$$

$$= \begin{bmatrix} - & 32 & 60 & - \\ 1 & -18 & - & - \\ 11 & - & 70 & - \end{bmatrix}$$

here we see that in the cell C_{22} the entry is -18 which is less than zero. So, solution is not optimal

Therefore we allocate θ to the corresponding cell.

19(5)	30	50	10(2)
70	30(θ)	40(7)	60(2-θ)
40	8(8-θ)	70	20(10+θ)

$\{\theta = 2\}$

19(5)	30	50	10(2)	$u_1 = -32$
70	30(2)	40(7)	60	$0 = u_2$
40	8(6)	70	20(12)	$u_3 = -22$
$v_1 = 51$	$v_2 = 30$	$v_3 = 40$	$v_4 = 42$	

$d_{ij} = c_{ij} - (u_i + v_j)$ [for unallocated cell]

$$= \begin{bmatrix} - & 30 & 50 & - \\ 70 & - & - & 60 \\ 40 & - & 70 & - \end{bmatrix} - \begin{bmatrix} - & -2 & 8 & - \\ 51 & - & - & 42 \\ 29 & - & 18 & - \end{bmatrix}$$

$$= \begin{bmatrix} - & 32 & 42 & - \\ 19 & - & - & 18 \\ 11 & - & 52 & - \end{bmatrix}$$

Therefore solution is optimal:

$19 \times 5 + 30 \times 2 + 40 \times 7 + 10 \times 2 + 8 \times 6 + 20 \times 12$

743

Saving is $1000 - 743 = 257$

August 19, 2019

Unbalanced Transportation Problem

~~When number of rows and columns~~ If demand is not equal to supply then the transportation problem is unbalanced. In order to solve such problem it is converted to balanced problem by introducing some dummy where either zero cost or given cost is taken and then problem is solved by usual method.

Ques In the following unbalanced transportation problem there is not enough supply. Suppose there are penalty cost for every unsupplied unit which are given by 5, 3, 2 for destination 1, 2, 3 resp. Find the optimal solution.

destination

	1	2	3	supply
1	5	1	7	10
2	6	4	6	80
3	3	2	5	15
demand	75	20	50	~~145~~ 105

	1	~~2~~ 2	3		Penalty		
1	5	1(10)	7	~~10~~	4 ←		
2	6(60)	4(10)	6(10)	~~80~~ ~~70~~ 60	2	2	2
3	3(15)	2	5	15	1	1	1
4	5	3	2(40)	~~40~~	1	1	2
	~~75~~ 60	~~20~~ 10	~~50~~ 10				
	2	1	3				
	2	1	3				
	3	1	↑				
	↑						

Now, no of allocations $= m+n-1 = 6$.

we calculate $d_{ij} = C_{ij} - (u_i + v_j)$

	1	2	3	
1	5	1(10)	7	-3
2	6(60)	4(10)	6(10)	0
3	3(15)	2	5	-3
4	5	3	2(40)	-4
	6	4	6	

$$d_{ij} = \begin{bmatrix} 5 & - & 7 \\ - & - & - \\ - & 2 & 5 \\ 5 & 3 & - \end{bmatrix} - \begin{bmatrix} 3 & - & 3 \\ - & - & - \\ - & 1 & 3 \\ 2 & 0 & - \end{bmatrix} = \begin{bmatrix} 2 & - & 4 \\ - & - & - \\ - & 1 & 2 \\ 3 & 3 & - \end{bmatrix}$$

all are positive

∴ the solution is optimal.

$= 1\times10 + 6\times60 + 4\times10 + 6\times10 + 3\times15$

$= 515$

Degeneracy in Transportation:

Whenever number of allocations is less than $m+n-1$ then we say that there is degeneracy in the problem.

Resolution of Degeneracy:

We allocate an extremely small amount (very close to zero) to lowest cost cell so that no of allocated cells is equal to $m+n-1$. We allocate ~~lowest am~~ small amount such that unique values of u_i and v_j can be determined.

Ques Solve the given transportation problem:

	1	2	3	4	supply
1	8	10	7	6	50
2	12	9	4	7	40
3	9	11	10	8	30
demand	25	32	40	23	

	1	2	3	4		Penalty		
1	8(25)	10(2)	7	6(23)	50 27 2	1	2 ←	2 ←
2	12	9	4(40)	7	40 0	3 ←		
3	9	11(30)	10	8	30	1	1	2
	25 0	32	40 0	23 0				
	1	1	3	1				
	1	1		2				
	1	1						

Now the matrix be:

	1	2	3	4
1	8(25)	10(2)	7	6(23)
2	12	9	4(40)	7
3	9	11(30)	10	8

Now, no of allocations $= 5 < m+n-1 = 6$.

Number of allocation is $5 < m+n-1 = 6$ hence solution is degenerate. Now we allocate Δ to lowest cost cell which is at the position (1,3) f(2,4)

	1	2	3	4	
1	8(25)	10(2)	7(Δ)	6(23)	$0 = u_1$
2	12	9	4(40)	7	$u_2 = -3$
3	9	11(30)	10	8	$u_3 = 1$
	8	10	7	6	

$d_{ij} = C_{ij} - (u_i + v_j)$

$$d_{ij} = \begin{bmatrix} - & - & - & - \\ 12 & 9 & - & 7 \\ 9 & - & 10 & 8 \end{bmatrix} - \begin{bmatrix} - & - & - & - \\ 5 & 7 & - & 3 \\ 9 & - & 8 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} - & - & - & - \\ 7 & 2 & - & 4 \\ 0 & - & 2 & 1 \end{bmatrix}$$

all d_{ij} are positive
∴ solution is optimal.

$= 8 \times 25 + 10 \times 2 + 6 \times 23 + 4 \times 40 + 11 \times 30$

Ques Find the optimal solution for following transportation problem

	I	II	III	IV	supply	Penalty			
A	5(6)	1(25)	3(3)	3	~~34~~/~~9~~/3	2 ←	0	2	2
~~B~~	~~3(15)~~	~~3~~	~~5~~	~~4~~	15/0	0	1	2	
C	6	4	4(12)	3	12	1	1	2	2
D	4	1	4(2)	2(17)	19/2	1	2 ←	0	
demand	21/6/0	25/0	17	17/0					
	1	0	1	1					
	1		1	1					
	1		1						
	1		1						

no of assignments $= m+n-1 = 4+4-1$
$= 7$ ~~6~~

	I	II	III	IV	
A	$5(6-\theta)$	1(25)	$3(3+\theta)$	$3(\theta)$	$u_1 = 0$
B	3(15)	3	5	4	$u_2 = -2$
C	6	4	4(12)	3	$u_3 = 1$
D	$4(\theta)$	1	$4(2-\theta)$	$2(17-\theta)$	u_4 1
	5	1	3	1	

we calculate $d_{ij} = C_{ij} - (u_i + v_j)$ for unallocated cells.

$$\begin{bmatrix} - & - & - & 3 \\ - & 3 & 5 & 4 \\ 6 & 4 & - & 3 \\ 4 & 1 & - & - \end{bmatrix} - \begin{bmatrix} - & - & - & 1 \\ - & -1 & 1 & -1 \\ 6 & 2 & - & 2 \\ 6 & 2 & - & - \end{bmatrix}$$

$$\begin{bmatrix} - & - & - & 2 \\ - & 4 & 4 & 5 \\ 0 & 2 & - & 1 \\ -2 & -1 & - & - \end{bmatrix}$$

here we see that C_{41} entry is -2 which is less than zero
So solution is not optimal

Therefore we allocate θ to corresponding cell.

$5(6-\theta)$	$1(25)$	$3(3+\theta)$	$3(\cancel{\theta})$
$3(15)$	3	5	4
6	4	$4(12)$	3
$4(\theta)$	1	$4(27-\theta)$	$2(17-\cancel{\theta})$

$\theta = \min((6-\theta), (2-\theta)) = 0$

$\theta = \cancel{6}\,2$

5	$1(25)$	$3(3)$	$3(6)$	$u_1 = 0$
$3(15)$	3	5	4	u_2
6	4	$4(12)$	3	u_3
$4(6)$	1	$4(2)$	$2(11)$	$u_4 = -1$
4	1	34	3	

$5(4)$	$1(25)$	$3(5)$	3	u_1	$= 0$
$3(15)$	3	5	4	u_2	-2
6	4	$4(12)$	3	u_3	1
$4(2)$	1	$\cancel{4}\,4$	$2(17)$	u_4	-1
5	1	3	3		

Ques Solve the following transportation problem

	D	E	F	G	
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
	80	90	110	160	

	D	E	F	G	H							
A	42	48	38(10)	37(150)	0	160 150	37	1	1	1	1	1
B	40(80)	49	52	51(10)	0(60)	150 90 10	40	9	11 ←	1		
C	39	38(90)	40(100)	43	0	190 100	38	1	1	3 ←		
	80 0	90	110 10	160	60							
	1	10	2	6	0							
	1	10	2	6								
	1		2	6								
			2	6								
			14	14								
			1									

August 21, 2019

Optimization:

Optimizing multivariable without constraint

Hessian Matrix

If f is a function of variable $x_1, x_2 \dots x_n$ then matrix of the form

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \dots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \dots & \dots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \dots & & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \dots & & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

then

1. if $H(x)$ is positive definite then there exist minima.
   $H(x)$ is positive definite if principle leading minors are ~~alternate~~ positive ~~negative~~

2. if $H(x)$ is negative definite then there exist maxima
   It is negative definite if all principle minors are negative positive alternate

3. if sign of minors do not meet above two conditions then, there is saddle point

Ques Find the point x_1, x_2, x_3 at which the function $z = f(x_1, x_2, x_3) = -x_1^2 - x_2^2 - x_3^2 + x_2 x_3 + x_1 + 2x_3$ has optimal values.

$\frac{\partial f}{\partial x_1} = -2x_1 + 1 = 0 \Rightarrow x_1 = 1/2$

$\frac{\partial f}{\partial x_2} = -2x_2 + x_3 = 0 \Rightarrow x_3 = 2x_2$

$\frac{\partial f}{\partial x_3} = -2x_3 + x_2 + 2 = 0 \Rightarrow x_2 = 2/3 \quad x_3 = 4/3$

$\therefore$ point $(1/2, 2/3, 4/3)$

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

$\frac{\partial^2 f}{\partial x_1^2} = -2 \qquad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0 \qquad \frac{\partial^2 f}{\partial x_1 \partial x_3} = 0$

$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0 \qquad \frac{\partial^2 f}{\partial x_2^2} = -2 \qquad \frac{\partial^2 f}{\partial x_2 \partial x_3} = 1$

$$\frac{\partial^2 f}{\partial x_3 \partial x_1} = 0 \qquad \frac{\partial^2 f}{\partial x_3 \partial x_2} = 1 \qquad \frac{\partial^2 f}{\partial x_3^2} = -2$$

$$H(x) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$M_1 = -2$

$$M_2 = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4$$

$$M_3 = \begin{vmatrix} -2 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{vmatrix} = -2(4-1) = 6$$

$\therefore H(x)$ is positive definite
hence there exist minima

Ques Find the extreme point of the function

$$f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$$

$$\frac{\partial f}{\partial x_1} = 3x_1^2 + 4x_1 = 0 \Rightarrow x_1(3x_1 + 4) = 0$$

$$\frac{\partial f}{\partial x_2} = 3x_2^2 + 8x_2 = 0 \Rightarrow x_2(3x_2 + 8) = 0$$

$x_1 = 0$ or $x_1 = -4/3$

$x_2 = 0$ or $x_2 = -8/3$

Therefore points are:
$(0,0)$, $(0,-8/3)$, $(-4/3, 0)$, $(-4/3, -8/3)$

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x_1^2} = 6x_1 + 4 \qquad \frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = 0 \qquad \frac{\partial^2 f}{\partial x_2^2} = 6x_2 + 8$$

at $(0,0)$

$$H(x) = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$$

$M_1 = 4$ $\quad M_2 = 32$ ~~minima~~ ~~saddle pt.~~

at $(0, -8/3)$

$$H(x) = \begin{bmatrix} 4 & 0 \\ 0 & -8 \end{bmatrix}$$

$M_1 = 4$ $\quad M_2 = -32$ saddle pt

at $(-4/3, 0)$

$$H(x) = \begin{bmatrix} -4 & 0 \\ 0 & 8 \end{bmatrix}$$

$M_1 = -4$ $M_2 = -32$ ~~maxima~~ saddle point

at $(-4/3, -8/3)$

$$H(x) = \begin{bmatrix} -4 & 0 \\ 0 & -8 \end{bmatrix}$$ +ve definite

$M_1 = -4$ $M_2 = 32$ minima

Ques $f = x^2 + 4y^2 + 4z^2 + 4xy + 4xz + 16yz$
Find its extreme point

$\frac{\partial f}{\partial x} = 2x + 4y + 4z = 0 \Rightarrow x + 2y + 2z = 0$

$\frac{\partial f}{\partial y} = 8y + 4x + 16z = 0 \Rightarrow x + 2y + 4z = 0$

$\frac{\partial f}{\partial z} = 8z + 4x + 16y = 0 \Rightarrow x + 4y + 2z = 0$

$(x, y, z) = (0, 0, 0)$

$$H(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{bmatrix}$$

$\frac{\partial^2 f}{\partial x^2} = 2$ $\frac{\partial^2 f}{\partial x \partial y} = 4$ $\frac{\partial^2 f}{\partial x \partial z} = 4$

$\frac{\partial^2 f}{\partial y \partial x} = 4$ $\frac{\partial^2 f}{\partial y^2} = 8$ $\frac{\partial^2 f}{\partial y \partial z} = 16$

$\frac{\partial^2 f}{\partial z \partial x} = 4$ $\frac{\partial^2 f}{\partial z \partial y} = 16$ $\frac{\partial^2 f}{\partial z^2} = 8$

$$H(x) = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 8 & 16 \\ 4 & 16 & 8 \end{bmatrix}$$

$= -128$

$M_1 = 2$ $M_2 = 0$ $M_3 = 2(64 - 256) = -384$

August 22, 2019

hence, saddle point at $(0,0,0)$

Optimization of Multivariable function with constraints

method of substitution
Lagrange's method of multipliers.

Method of Substitution

Ques Optimize the function $z = x_1^2 + (x_2+1)^2 + (x_3-1)^2$
s.t. $x_1 + 5x_2 - 3x_3 = 6$

$z = x_1^2 + (x_2+1)^2 + (x_3-1)^2$ — (1)

$x_1 + 5x_2 - 3x_3 = 6$ — (2)

from ②

$$x_3 = \frac{x_1 + 5x_2 - 6}{3}$$

∴ ① becomes

$$z = x_1^2 + (x_2+1)^2 + (x_3-1)^2$$

$$= x_1^2 + (x_2+1)^2 + \left(\frac{x_1 + 5x_2 - 6}{3} - 1\right)^2$$

$$= x_1^2 + (x_2+1)^2 + \frac{1}{9}(x_1 + 5x_2 - 9)^2$$

∴ z is function of two variable x_1 & x_2

$$\frac{\partial z}{\partial x_1} = 2x_1 + \frac{2}{9}(x_1 + 5x_2 - 9)$$

$\Rightarrow \quad 18x_1 + 2x_1 + 10x_2 - 18 = 0$

$\Rightarrow \quad 20x_1 + 10x_2 = 18$

$\Rightarrow \quad 2x_1 + x_2 = 18/10$ ——— ③

$$\frac{\partial z}{\partial x_2} = 2(x_2+1) + \frac{10}{9}(x_1 + 5x_2 - 9)$$

$= \cancel{2}\,18x_2 + 18 + 10x_1 + 50x_2 - 90 = 0$

$= 10x_1 + 68x_2 - 72 = 0$ ——— ④

solving ③ & ④ we get

$x_1 = 2/5 \qquad x_2 = 1$

$$x_3 = \frac{x_1 + 5x_2 - 6}{3} = \frac{-1}{5}$$

∴ point is $(2/5, 1, -1/5)$

$$H(x) = \begin{bmatrix} \frac{\partial^2 z}{\partial x_1^2} & \frac{\partial^2 z}{\partial x_1 \partial x_2} \\ \frac{\partial^2 z}{\partial x_2 \partial x_1} & \frac{\partial^2 z}{\partial x_2^2} \end{bmatrix}$$

$$\frac{\partial^2 z}{\partial x_1^2} = 2 + \frac{2}{9} = \frac{20}{9} \qquad \frac{\partial^2 z}{\partial x_1 \partial x_2} = \frac{10}{9}$$

$$\frac{\partial^2 z}{\partial x_2 \partial x_1} = \frac{10}{9} \qquad \frac{\partial^2 z}{\partial x_2^2} = 2 + \frac{50}{9} = \frac{68}{9}$$

$$H(x) = \begin{bmatrix} 20/9 & 10/9 \\ 10/9 & 68/9 \end{bmatrix}$$

$M_1 = 20/9 \qquad M_2 = \frac{20 \times 68 - 100}{9} > 0$

H(x) is positive definite;
hence minima at $(2/5, 1, -1/5)$

Lagranges Method of Multipliers:

If the problem is in the form

$z = f(x_1, x_2 \ldots x_n)$

s.t. $g_1(x_1, x_2 \ldots x_n) = 0$

$g_2(x_1, x_2 \ldots \quad) = 0$

$g_3(x_1, x_2 \ldots \quad) = 0$ then,

in this method we construct lagranges function given by

$$\mathcal{L}(x_1, x_2, x_3 \ldots x_n, \lambda_1, \lambda_2, \lambda_3) = f(x_1, x_2 \ldots x_n) + \lambda g_1 + \lambda_2 g_2 + \lambda_3 g_3$$

where $\lambda_1, \lambda_2, \lambda_3$ are lagranges multipliers

For maxima & minima we know that first derivatives are zero. Hence we obtain

$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial x_2} = \frac{\partial L}{\partial x_3} \ldots \frac{\partial L}{\partial x_n} = \frac{\partial L}{\partial \lambda_1} = \frac{\partial L}{\partial \lambda_2} = \frac{\partial L}{\partial \lambda_3} = 0$$

Solving above equations we obtain the values of $x_1, x_2, x_3 \ldots x_n, \lambda_1, \lambda_2, \lambda_3$

Then we construct the matrix.

$$\begin{vmatrix} L_{11}-z & L_{12} & L_{13} & \ldots & L_{1n} & g_{11} & g_{21} & g_{31} \\ L_{21}-z & L_{22}-z & L_{23} & \ldots & L_{2n} & g_{12} & g_{22} & g_{32} \\ L_{31} & L_{32} & L_{33}-z & \ldots & L_{3n} & g_{13} & g_{23} & g_{33} \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ L_{n1} & L_{n2} & L_{n3} & \ldots & L_{nn}-z & g_{1n} & g_{2n} & g_{3n} \\ g_{11} & g_{12} & g_{13} & \ldots & 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} & \ldots & 0 & 0 & 0 \\ g_{31} & g_{32} & g_{33} & \ldots & 0 & 0 & 0 \end{vmatrix} = 0$$

August 23, 2019

Ques Find the dimension of a box of largest volume ~~ste~~ inscribed in a sphere of radius a

Let the length breadth and height of the box be $2x$, $2y$ and $2z$. Hence the volume

$V = 2x \cdot 2y \cdot 2z$

$= 8xyz$

which is optimized subject to the condition

$x^2 + y^2 + z^2 = a^2$

$\Rightarrow z^2 = a^2 - y^2 - x^2$

$z = \sqrt{a^2 - y^2 - x^2}$

So,

$V = 8xy\sqrt{a^2 - y^2 - x^2}$

$$\frac{\partial V}{\partial x} = 8y\left[1 \cdot \sqrt{a^2 - y^2 - x^2} + \frac{x \times -\cancel{2}x}{\cancel{2}\sqrt{a^2 - x^2 - y^2}}\right]$$

$$= 8y\left[\frac{a^2 - y^2 - x^2 - x^2}{\sqrt{a^2 - y^2 - x^2}}\right]$$

$$\frac{\partial V}{\partial x} = 8y\left[\frac{a^2-y^2-2x^2}{\sqrt{a^2-y^2-x^2}}\right]$$

$$\frac{\partial V}{\partial y} = 8x\left[\frac{a^2-2y^2-x^2}{\sqrt{a^2-y^2-x^2}}\right]$$

For max & min

$$\frac{\partial V}{\partial x} = 0 = \frac{\partial V}{\partial y}$$

$$a^2-2x^2-y^2=0 \quad \text{—— A}$$

$$a^2-2y^2-x^2=0 \quad \text{—— B}$$

~~A×2~~ B×2 − A we get

$$2a^2-4y^2-2x^2-a^2+2x^2+y^2=0$$

$$a^2-3y^2=0$$

$$y^2 = \frac{a^2}{3}$$

$$y = \frac{a}{\sqrt{3}}$$

from (A)

$$a^2-2x^2-\left(\frac{a}{\sqrt{3}}\right)^2=0$$

$$x = \frac{a}{\sqrt{3}}$$

from (1)

$$z = \sqrt{a^2-\left(\frac{a}{\sqrt{3}}\right)^2-\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{a}{\sqrt{3}}$$

∴ point is $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

Now, $$\frac{\partial^2 V}{\partial x^2} = 8y\left[\frac{-4x}{\sqrt{a^2-x^2-y^2}} + (a^2-2x^2-y^2)\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{a^2-x^2-y^2}}\right)\right]$$

at $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

$$\frac{\partial^2 V}{\partial x^2} = \frac{-32\,(a/\sqrt{3})^2}{\sqrt{a^2-(a/\sqrt{3})^2-(a/\sqrt{3})^2}} + 0$$

as at $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

$$a^2-2x^2-y^2=0$$

$$= -32\left(\frac{a}{\sqrt{3}}\right)^2 \Big/ \left(\frac{a}{\sqrt{3}}\right) = \frac{-32a}{\sqrt{3}}$$

Similarly, $$\frac{\partial^2 V}{\partial y^2} = \frac{-32a}{\sqrt{3}}$$

Now, $$\frac{\partial^2 V}{\partial y\,\partial x} = 8\cdot\left[\left(1\cdot\frac{a^2-2x^2-y^2}{\sqrt{a^2-x^2-y^2}}\right) \frac{y}{\sqrt{a^2-x^2-y^2}}\ x - 2y + y\,(a^2-2x^2-y^2)\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{a^2-x^2-y^2}}\right)\right]$$

at $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

$$\frac{\partial V}{\partial x} = 8y\left[\frac{a^2-y^2-2x^2}{\sqrt{a^2-y^2-x^2}}\right]$$

$$\frac{\partial V}{\partial y} = 8x\left[\frac{a^2-2y^2-x^2}{\sqrt{a^2-y^2-x^2}}\right]$$

For max & min

$$\frac{\partial V}{\partial x} = 0 = \frac{\partial V}{\partial y}$$

$a^2-2x^2-y^2=0$ ——— A

$a^2-2y^2-x^2=0$ ——— B

~~A×2~~ B×2 − A we get

$2a^2-4y^2-2x^2-a^2+2x^2+y^2=0$

$a^2-3y^2=0$

$$y^2 = \frac{a^2}{3}$$

$$y = \frac{a}{\sqrt{3}}$$

from (A)

$$a^2-2x^2-\left(\frac{a}{\sqrt{3}}\right)^2=0$$

$$x = \frac{a}{\sqrt{3}}$$

from (1)

$$z = \sqrt{a^2-\left(\frac{a}{\sqrt{3}}\right)^2-\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{a}{\sqrt{3}}$$

∴ point is $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

Now, $$\frac{\partial^2 V}{\partial x^2} = 8y\left[\frac{-4x}{\sqrt{a^2-x^2-y^2}} + (a^2-2x^2-y^2)\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{a^2-x^2-y^2}}\right)\right]$$

at $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

$$\frac{\partial^2 V}{\partial x^2} = \frac{-32\,(a/\sqrt{3})^2}{\sqrt{a^2-(a/\sqrt{3})^2-(a/\sqrt{3})^2}} + 0$$

as at $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

$a^2-2x^2-y^2=0$

$$= -32\left(\frac{a}{\sqrt{3}}\right)^2 \Big/ \left(\frac{a}{\sqrt{3}}\right) = \frac{-32a}{\sqrt{3}}$$

Similarly, $$\frac{\partial^2 V}{\partial y^2} = \frac{-32a}{\sqrt{3}}$$

Now, $$\frac{\partial^2 V}{\partial x\,\partial y} = 8.\left[\left(1.\ \frac{a^2-2x^2-y^2}{\sqrt{a^2-x^2-y^2}}\right)\frac{y}{\sqrt{a^2-x^2-y^2}}\ x-2y + y\,(a^2-2x^2-y^2)\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{a^2-x^2-y^2}}\right)\right]$$

at $(a/\sqrt{3},\ a/\sqrt{3},\ a/\sqrt{3})$

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Page No. 090.

$$\frac{\partial^2 V}{\partial y \partial x} = 8\left[\frac{-2(a/\sqrt{3})^2}{a^2 - a^2/3 - a^2/3}\right]$$

$$= \frac{-16a}{\sqrt{3}}$$

Now, $H(x) = \begin{bmatrix} \frac{\partial^2 V}{\partial x^2} & \frac{\partial^2 V}{\partial x \partial y} \\ \frac{\partial^2 V}{\partial y \partial x} & \frac{\partial^2 V}{\partial y^2} \end{bmatrix}$

$$= \begin{vmatrix} -32a/\sqrt{3} & -16a/\sqrt{3} \\ -16a/\sqrt{3} & -32a/\sqrt{3} \end{vmatrix}$$

$M_1 = -32a/\sqrt{3}$

$$= -16a/\sqrt{3} \times -16a/\sqrt{3} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$M_2 = 4a^2 > 0$

$\because$ $H(x)$ is negative definite

maxima at $(a/\sqrt{3}, a/\sqrt{3}, a/\sqrt{3})$

Ques Find the stationary points of the function

$f(x_1, x_2, x_3) = 2x_1x_2x_3 - 4x_1x_3 + x_1^2 + x_2^2 + x_3^2 - 2x_1 - 4x_2 + 4x_3$

Page No. 091.

$$\frac{\partial f}{\partial x_1} = 2x_2x_3 - 4x_3 + 2x_1 - 2$$

$$\frac{\partial f}{\partial x_2} = 2x_1x_3 + 2x_2 - 4$$

$$\frac{\partial f}{\partial x_3} = 2x_1x_2 - 4x_1 + 2x_3 + 4$$

August 27, 2019

Ques Find the point on the plane $x+y+3z=1$ which is nearest to the point $(-1,0,1)$

Let the point be x, y, z then by distance formula we have

$$f = (x+1)^2 + (y-0)^2 + (z-1)^2$$
$$= (x+1)^2 + y^2 + (z-1)^2$$

Now, constraint is that point must lie on plane $g(x,y,z) = x + 2y + 3z - 1$

Now, Lagranges function is

$$L = f + \lambda g$$
$$L = (x+1)^2 + y^2 + (z+1)^2 + \lambda(x+2y+3z-1)$$

Now for maxima and minima

$$\frac{\partial L}{\partial x} = 0 = \frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} = \frac{\partial L}{\partial \lambda}$$

$$L_1 = \frac{\partial L}{\partial x} = 2(x+1) + \lambda$$

$$\Rightarrow x = -\frac{\lambda}{2} - 1 = \frac{-\lambda-2}{2}$$

$$L_2 = \frac{\partial L}{\partial y} = 2y + 2\lambda \Rightarrow y = -\lambda$$

$$L_3 = \frac{\partial L}{\partial z} = 2(z-1) + 3\lambda$$

$$\Rightarrow z = \frac{-3\lambda+2}{2}$$

$$L_4 = \frac{\partial L}{\partial \lambda} = x + 2y + 3z - 1$$

from above

$$\frac{-\lambda-2}{2} + 2(-\lambda) + 3\left(\frac{-3\lambda+2}{2}\right) - 1 = 0$$

$$-\lambda - 2 - 4\lambda - 9\lambda + 6 - 2 = 0$$

$$-14\lambda + 2 = 0$$

$$\lambda = 1/7$$

$$\therefore x = \frac{-1/7 - 2}{2} = \frac{-15}{14}$$

$$y = -\lambda = -1/7$$

$$z = 11/14$$

$\therefore$ point is $(-15/14, -1/7, 11/14)$

Now for maxima and minima at this point we find

$$\begin{vmatrix} L_{11}-z & L_{12} & L_{13} & g_{11} \\ L_{21} & L_{22}-z & L_{23} & g_{12} \\ L_{31} & L_{32} & L_{33}-z & g_{13} \\ g_{11} & g_{12} & g_{13} & 0 \end{vmatrix} = 0$$

Now $L_{11} = 2$ $\quad L_{12} = 0$ $\quad L_{13} = 0$

$L_{21} = 0$ $\quad L_{22} = 2$ $\quad L_{23} = 0$

$L_{31} = 0$ $\quad L_{32} = 0$ $\quad L_{33} = 2$

$g_{11} = 1$ $\quad g_{12} = 2$ $\quad g_{13} = 3$

$$\therefore \begin{vmatrix} 2-z & 0 & 0 & 1 \\ 0 & 2-z & 0 & 2 \\ 0 & 0 & 2-z & 3 \\ 1 & 2 & 3 & 0 \end{vmatrix} = 0$$

$$(2-z)\begin{vmatrix} 2-z & 0 & 2 \\ 0 & 2-z & 3 \\ 2 & 3 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 2-z & 0 \\ 0 & 0 & 2-z \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$(2-z)\left[(2-z)(-9) + 2(-2(2-z))\right] + (2-z)(2-z) = 0$$

$$(2-z)(2-z)\left[-9-4+1\right] = 0$$

$$(2-z)(2-z) = 0$$

$z = 2, 2$ positive values

$\therefore$ minima at $\left(-15/14, -1/7, 11/14\right)$

Ques Optimize the function $f(x) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$ such that $x_1 - x_2 = 0$ and $x_1 + x_2 + x_3 = 1$

Lagranges function is

$L = f(x) + \lambda g_1 + \lambda_2 g_2$

$g_1 = x_1 - x_2$

$g_2 = x_1 + x_2 + x_3 - 1$

$\therefore L = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) + \lambda_1(x_1 - x_2) + \lambda_2(x_1 + x_2 + x_3 \ldots$

$\frac{\partial L}{\partial x_1} = \frac{1}{\cancel{2}}\cancel{2}x_1 + \lambda_1 + \lambda_2 \Rightarrow x_1 = -\lambda_1 - \lambda_2$

$\frac{\partial L}{\partial x_2} = x_2 - \lambda_1 + \lambda_2 \Rightarrow x_2 = \lambda_1 - \lambda_2$

$\frac{\partial L}{\partial x_3} = x_3 + \lambda_2 \Rightarrow x_3 = -\lambda_2$

$\frac{\partial L}{\partial \lambda_1} = x_1 - x_2 \Rightarrow x_1 = x_2$

$\frac{\partial L}{\partial \lambda_2} = x_1 + x_2 + x_3 - 1 \Rightarrow 2x_1 + x_3 - 1 = 0$

$2x_1 - \lambda_3 - 1 = 0$

$x_1 = \frac{\lambda_3 + 1}{2}$

$\lambda_1 = 0$

$\therefore x_1 = 1/3 \quad x_2 = 1/3 \quad x_3 = 1/3$

$L_{11} = 1 \quad L_{12} = 0 \quad L_{13} = 0$

$L_{21} = 0 \quad L_{22} = 1 \quad L_{23} = 0$

$L_{31} = 0 \quad L_{32} = 0 \quad L_{33} = 1$

$g_{11} = 1 \quad g_{12} = -1 \quad g_{13} = 0$

$g_{21} = 1 \quad g_{22} = 1 \quad g_{23} = 1$

$$\begin{vmatrix} L_{11}-z & L_{12} & L_{13} & g_{11} & g_{21} \\ L_{21} & L_{22}-z & L_{23} & g_{12} & g_{22} \\ L_{31} & L_{32} & L_{33}-z & g_{13} & g_{23} \\ g_{11} & g_{12} & g_{13} & 0 & 0 \\ g_{21} & g_{22} & g_{23} & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1-z & 0 & 0 & 1 & 1 \\ 0 & 1-z & 0 & -1 & 1 \\ 0 & 0 & 1-z & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{vmatrix} = 0$$

$C_4 \to C_4 - C_5$

$$\begin{vmatrix} 1-z & 0 & 0 & 0 & 1 \\ 0 & 1-z & 0 & -2 & 1 \\ 0 & 0 & 1-z & -1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{vmatrix} = 0$$

$$(1-z)\begin{vmatrix} 1-z & 0 & -2 & 1 \\ 0 & 1-z & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 0 & 1-z & 0 & -2 \\ 0 & 0 & 1-z & -1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$R_3 \to R_3 + R_4$ $\qquad$ $R_3 \to R_3 - R_4$

$$(1-z)\begin{vmatrix} 1-z & 0 & -2 & 1 \\ 0 & 1-z & -1 & 1 \\ -2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1-z & 0 & -2 \\ 0 & 0 & 1-z & -1 \\ 0 & -2 & -1 & 0 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$(1-z)\left[(1-z)\begin{vmatrix} 1-z & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix} + 1\begin{vmatrix} 0 & -2 & 1 \\ 1-z & -1 & 1 \\ 1 & 0 & 0 \end{vmatrix}\right] + \begin{vmatrix} 1-z & 0 & -2 \\ 0 & 1-z & -1 \\ -2 & -1 & 0 \end{vmatrix}$$

$$(1-z)\left[(1-z)(0) + 1(+2(1) + 1(-1))\right] + (1-z)(-1) + (-2)(2(1-z)) = 0$$

$$(1-z) = 0$$

$$z = 1$$

positive, hence minima.

August 28, 2019.

Ques Find the dimension of cylindrical tin with top and bottom made up of metal sheet to maximize its volume such that total surface area is equal to $A_0 = 24\pi$

Let the radius of the base of cylindrical tin be r, height be h then

volume $= \pi r^2 h$

s.t.

$$2\pi r^2 h + 2\pi r h = A_0 = 24\pi$$

$$\mathcal{L} = \pi r^2 h + \lambda(2\pi r^2 + 2\pi r h - 24\pi)$$

max V. $= \pi r^2 h$

Then Lagranges function be:

$$L = V + \lambda g$$

$$L = \pi r^2 h + \lambda(2\pi r^2 + 2\pi r h - 24\pi)$$

$$\frac{\partial L}{\partial r} = 2\pi r h + \lambda(4\pi r + 2\pi h)$$

$$\frac{\partial L}{\partial h} = \pi r^2 + \lambda(\cancel{2\pi r} + 2\pi r)$$

$$\frac{\partial L}{\partial \lambda} = 2\pi r^2 + 2\pi r h - 24\pi$$

Thus equating with zero

$\lambda = -1 \quad \mu = 2 \quad h = 4$

$$\begin{vmatrix} L_{11}-z & L_{12} & g_{11} \\ L_{21} & L_{22}-z & g_{12} \\ g_{11} & g_{12} & 0 \end{vmatrix} = 0$$

$L_{11} = 2\pi h + 4\pi\lambda = 8\pi - 4\pi = 4\pi$

$L_{12} = 2\pi r + 2\pi\lambda = 4\pi - 2\pi = 2\pi$

$L_{21} = 2\pi r + 2\pi\lambda = 2\pi$

$L_{22} = 0$

$g_{11} = 4\pi r + 2\pi h = 16\pi$

$g_{12} = 2\pi r = 4\pi$

$$\begin{vmatrix} 4\pi - z & 2\pi & 16\pi \\ 2\pi & -z & 4\pi \\ 16\pi & 4\pi & 0 \end{vmatrix} = 0$$

$$4\pi \begin{vmatrix} 4\pi - z & 2\pi & 4 \\ 2\pi & -z & 1 \\ 4 & 1 & 0 \end{vmatrix}$$

$$4 \begin{vmatrix} 2\pi & -z \\ 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4\pi - z & 2\pi \\ 4 & 1 \end{vmatrix} = 0$$

$4(2\pi - 4z) - 1(4\pi - z - 8\pi) = 0$

$z = \frac{-12\pi}{17} = -ve$

hence maxima.

Ques Find the ~~solution~~ volume of greatest parallelopiped inscribed in ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Ques Find the solution of the following problem using Lagranges method of multipliers.

Maximize $f(x,y) = Kx^{-1}y^{-2}$

$g(x,y) = x^2 + y^2 - a^2$

Then Lagranges function be

$L = f(x,y) + \lambda g(x,y)$

$L = Kx^{-1}y^{-2} + \lambda(x^2 + y^2 - a^2)$

Differentiating w.r.t x, y & z.

$\frac{\partial L}{\partial x} = -Ky^{-2}x^{-2} + 2\lambda x$

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$$\frac{\partial L}{\partial y} = -2Kx^{-1}y^{-3} + 2\lambda y$$

$$\frac{\partial L}{\partial \lambda} = x^2 + y^2 - a^2$$

Equating to zero. the above equations:

$$2\lambda x - \frac{K}{x^2y^2} = 0 \Rightarrow 2\lambda x^3y^2 - K = 0$$

$$\Rightarrow 2\lambda x^3y^2 = K$$

$$2\lambda y - \frac{2K}{xy^3} = 0 \Rightarrow \lambda xy^4 - K = 0$$

$$\lambda xy^4 = K \qquad x = \frac{K}{\lambda y^4}$$

$$x^2 + y^2 - a^2 = 0 \Rightarrow x^2 + y^2 = a^2$$

$$\cancel{2\lambda x^3y^2 = \lambda xy^4}$$

$$y = \sqrt{2}x \Rightarrow x^2 + 2x^2 = a^2$$

$$y = \frac{\sqrt{2}a}{\sqrt{3}} \qquad 3x^2 = a^2$$

$$\sqrt{3}x = a \qquad x = \frac{a}{\sqrt{3}}$$

$$\lambda xy^4 - K = 0$$

$$\lambda = \frac{K}{xy^4} \Rightarrow \lambda = \frac{K}{(a/\sqrt{3})(\sqrt{2}/\sqrt{3}\,a)^4}$$

$$\lambda = \frac{K}{(a/\sqrt{3})(4/9\,a^4)}$$

$$\lambda = \frac{K\,9\sqrt{3}}{4a^5}$$

$$\begin{vmatrix} L_{11}-z & L_{12} & g_{11} \\ L_{21} & L_{22}-z & g_{12} \\ g_{11} & g_{12} & 0 \end{vmatrix} = 0$$

$$L_{11} = \frac{\partial}{\partial x}(-Ky^{-2}x^{-2} + 2\lambda x) = 2Ky^{-2}x^{-3} + 2\lambda$$

$$L_{12} = \frac{\partial}{\partial y}(-Ky^{-2}x^{-2} + 2\lambda x) = 2Ky^{-3}x^{-2}$$

$$L_{21} = \frac{\partial}{\partial x}(-2Kx^{-1}y^{-3} + 2\lambda y) = +2Ky^{-3}x^{-2}$$

$$L_{22} = \frac{\partial}{\partial y}(-2Kx^{-1}y^{-3} + 2\lambda y) = 6Kx^{-1}y^{-4} + 2\lambda$$

$$L_{11} = 2\cdot K\left(\frac{\sqrt{2}a}{\sqrt{3}}\right)^{-2}\left(\frac{a}{\sqrt{3}}\right)^{-3} + 2\,\frac{K\,9\sqrt{3}}{4a^5}$$

$$\frac{\partial L}{\partial y} = -2Kx^{-1}y^{-3} + 2\lambda y$$

$$\frac{\partial L}{\partial \lambda} = x^2 + y^2 - a^2$$

Equating to zero. the above equations:

$$2\lambda x - \frac{K}{x^2y^2} = 0 \Rightarrow 2\lambda x^3 y^2 - K = 0$$

$$\Rightarrow 2\lambda x^3 y^2 = K$$

$$2\lambda y - \frac{2K}{xy^3} = 0 \Rightarrow \lambda x y^4 - K = 0$$

$$\lambda x y^4 = K \qquad x = \frac{K}{\lambda y^4}$$

$$x^2 + y^2 - a^2 = 0 \Rightarrow x^2 + y^2 = a^2$$

$$\cancel{2\lambda x^3 y^2 = \lambda x y^4}$$

$$y = \sqrt{2}x \Rightarrow x^2 + 2x^2 = a^2$$

$$y = \frac{\sqrt{2}a}{\sqrt{3}} \qquad 3x^2 = a^2$$

$$\sqrt{3}x = a \qquad x = \frac{a}{\sqrt{3}}$$

$$\lambda x y^4 - K = 0$$

$$\lambda = \frac{K}{xy^4} \Rightarrow \lambda = \frac{K}{(a/\sqrt{3})\left(\frac{\sqrt{2}}{\sqrt{3}}a\right)^4}$$

$$\lambda = \frac{K}{(a/\sqrt{3})(4/9\, a^4)}$$

$$\lambda = \frac{K\, 9\sqrt{3}}{4a^5}$$

$$\begin{vmatrix} L_{11}-z & L_{12} & g_{11} \\ L_{21} & L_{22}-z & g_{12} \\ g_{11} & g_{12} & 0 \end{vmatrix} = 0$$

$$L_{11} = \frac{\partial}{\partial x}\left(-Ky^{-2}x^{-2} + 2\lambda x\right) = 2Ky^{-2}x^{-3} + 2\lambda$$

$$L_{12} = \frac{\partial}{\partial y}\left(-Ky^{-2}x^{-2} + 2\lambda x\right) = 2Ky^{-3}x^{-2}$$

$$L_{21} = \frac{\partial}{\partial x}\left(-2Kx^{-1}y^{-3} + 2\lambda y\right) = +2Ky^{-3}x^{-2}$$

$$L_{22} = \frac{\partial}{\partial y}\left(-2Kx^{-1}y^{-3} + 2\lambda y\right) = 6Kx^{-1}y^{-4} + 2\lambda$$

$$L_{11} = 2K\left(\frac{\sqrt{2}a}{\sqrt{3}}\right)^{-2}\left(\frac{a}{\sqrt{3}}\right)^{-3} + 2\,\frac{K\,9\sqrt{3}}{2^4 a^5}$$

August 29, 2019.

Optimization of problem with inequality constraint

Kuhn Tucker's Conditions (K-T)

If problem is in the form

$\min f(x_i)$ $[i = 1, 2, 3 \dots n]$

such that $g_j(x_i) \le 0$ $[j = 1, 2 \dots m]$

then, we construct Lagranges function:

$$L = f(x_i) + \lambda_1 g_1 + \lambda_2 g_2 + \dots \lambda_m g_m$$

then Kuhn Tucker's condition are given by

1. $\frac{\partial L}{\partial x_i} = 0$ $[i = 1, 2, 3 \dots n]$
2. $\lambda_j g_j = 0$ $[j = 1, 2, 3 \dots m]$
3. $g_j \le 0$
4. $\lambda_j \ge 0$

Now, we have following

Constraint	max	min
$\le$	$\lambda_j \le 0$	$\lambda_j \ge 0$
$\ge$	$\lambda_j \ge 0$	$\lambda_j \le 0$

Ques Solve the following problem minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

s.t. $x_1 + x_2 + x_3 \ge 5$

$x_2 x_3 - 2 \ge 0$

$x_1 \ge 0$, $x_2 \ge 0$ $x_3 \ge 2$

determine whether KT conditions are satisfied at following points?

$$x_1 = \begin{bmatrix} 3/2 \\ 3/2 \\ 2 \end{bmatrix} \quad x_2 = \begin{bmatrix} 4/3 \\ 2/3 \\ 3 \end{bmatrix} \quad x_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

Lagranges function is given by

$$L = x_1^2 + x_2^2 + x_3^2 + \lambda_1(x_1 + x_2 + x_3 - 5) + \lambda_2(x_2 x_3 - 2) + \lambda_3(x_1) + \lambda_4(x_2) + \lambda_5(x_3 - 2)$$

Kuhn Tucker's conditions are given by:

1. $\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial x_2} = \frac{\partial L}{\partial x_3} = 0$
2. $\lambda_1 g_1 = \lambda_2 g_2 = \lambda_3 g_3 = \lambda_4 g_4 = \lambda_5 g_5 = 0$
3. $g_1, g_2, g_3, g_4, g_5 \ge 0$
4. $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \le 0$

∴ here we have

$2x_1 + \lambda_1 + \lambda_3 + \lambda_5 = 0$ ——— ①

$2x_2 + \lambda_1 + x_3\lambda_2 + \lambda_4 = 0$ ——— ②

$2x_3 + \lambda_1 + \lambda_2 x_2 + \lambda_5 = 0$ ——— ③

$\lambda_1(x_1 + x_2 + x_3 - 5) = 0$ ——— ④

$\lambda_2(x_2 x_3 - 2) = 0$ ——— ⑤

$\lambda_3 x_1 = 0$ ——— ⑥

$\lambda_4 x_2 = 0$ ——— ⑦

$\lambda_5(x_3 - 2) = 0$ ——— ⑧

$x_1 + x_2 + x_3 - 5 \geq 0$

$x_2 x_3 - 2 \geq 0$

$x_1 \geq 0$

$x_2 \geq 0$

$x_3 - 2 \geq 0$

& $\lambda_j \leq 0$ $[j = 1, 2, \dots, 5]$

1.) at point $(3/2, 3/2, 2)$

④ ⇒ $\lambda_1\left(\frac{3}{2} + \frac{3}{2} + 2 - 5\right) = 0$ ⇒ $\lambda_1 \neq 0$

⑤ ⇒ $\lambda_2\left(\frac{3}{2} \times 2 - 2\right) = 0$ ⇒ $\lambda_2 = 0$

⑥ ⇒ $\lambda_3\left(\frac{3}{2}\right) = 0$ ⇒ $\lambda_3 = 0$

⑦ ⇒ $\lambda_4\left(\frac{3}{2}\right) = 0$ ⇒ $\lambda_4 = 0$

⑧ ⇒ $\lambda_5(2 - 2) = 0$ ⇒ $\lambda_5 \neq 0$

Now from ①

$2 \cdot 3/2 + \lambda_1 + 0 + 0 = 0$

$\lambda_1 = -3 \leq 0$

from ③ $2 \cdot 2 + -3 + 0 + \lambda_5 = 0$

$\lambda_5 = -1 \leq 0$

∴ $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \leq 0$

⇒ All KJ conditions are fulfilled.

2.) At point $(4/3, 2/3, 3)$

④ ⇒ $\lambda_1(4/3 + 2/3 + 3 - 5) = 0$ ⇒ $\lambda_1 \neq 0$

⑤ ⇒ $\lambda_2(2/3 \cdot 3 - 2) = 0$ ⇒ $\lambda_2 \neq 0$

⑥ ⇒ $\lambda_3(4/3) = 0$ ⇒ $\lambda_3 = 0$

⑦ ⇒ $\lambda_4(2/3) = 0$ ⇒ $\lambda_4 = 0$

⑧ ⇒ $\lambda_5(3 - 2) = 0$ ⇒ $\lambda_5 = 0$

Now, from ①

$2(4/3) + \lambda_1 + \cancel{\lambda_2} + 0 = 0$

$\lambda_1 + \cancel{\lambda_2} = -8/3$ $\cancel{\lambda_1 = -\lambda_2 \; 8/3}$

from ② $2(2/3) + -\cancel{8/3} + 3\lambda_2 + 0 = 0$

$3\lambda_2 = 4/3$

$\lambda_2 > 0$

∴ KT conditions are not satisfied.

$\therefore$ here we have

$2x_1 + \lambda_1 + \lambda_3 + \lambda_5 = 0$ —— (1)

$2x_2 + \lambda_1 + x_3\lambda_2 + \lambda_4 = 0$ —— (2)

$2x_3 + \lambda_1 + \lambda_2 x_2 + \lambda_5 = 0$ —— (3)

$\lambda_1(x_1 + x_2 + x_3 - 5) = 0$ —— (4)

$\lambda_2(x_2 x_3 - 2) = 0$ —— (5)

$\lambda_3 x_1 = 0$ —— (6)

$\lambda_4 x_2 = 0$ —— (7)

$\lambda_5(x_3 - 2) = 0$ —— (8)

$x_1 + x_2 + x_3 - 5 \geq 0$

$x_2 x_3 - 2 \geq 0$

$x_1 \geq 0$

$x_2 \geq 0$

$x_3 - 2 \geq 0$

& $\lambda_j \leq 0$ $\quad [j = [1, 2, \dots 5]]$

1.) at point $(3/2, 3/2, 2)$

(4) $\Rightarrow \lambda_1\left(\frac{3}{2} + \frac{3}{2} + 2 - 5\right) = 0 \quad \Rightarrow \lambda_1 \neq 0$

(5) $\Rightarrow \lambda_2\left(\frac{3}{2} \times 2 - 2\right) = 0 \quad \Rightarrow \lambda_2 = 0$

(6) $\Rightarrow \lambda_3\left(\frac{3}{2}\right) = 0 \quad \Rightarrow \lambda_3 = 0$

(7) $\Rightarrow \lambda_4\left(\frac{3}{2}\right) = 0 \quad \Rightarrow \lambda_4 = 0$

(8) $\Rightarrow \lambda_5(2 - 2) = 0 \quad \Rightarrow \lambda_5 \neq 0$

Now from (1)

$2 \cdot 3/2 + \lambda_1 + 0 + 0 = 0$

$\lambda_1 = -3 \leq 0$

from (3) $\quad 2 \cdot 2 + -3 + 0 + \lambda_5 = 0$

$\lambda_5 = -1 \leq 0$

$\therefore \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \leq 0$

$\Rightarrow$ All KT conditions are fulfilled.:

2.) At point $(4/3, 2/3, 3)$

(4) $\Rightarrow \lambda_1(4/3 + 2/3 + 3 - 5) = 0 \quad \Rightarrow \lambda_1 \neq 0$

(5) $\Rightarrow \lambda_2(2/3 \cdot 3 - 2) = 0 \quad \Rightarrow \lambda_2 \neq 0$

(6) $\Rightarrow \lambda_3(4/3) = 0 \quad \Rightarrow \lambda_3 = 0$

(7) $\Rightarrow \lambda_4(2/3) = 0 \quad \Rightarrow \lambda_4 = 0$

(8) $\Rightarrow \lambda_5(3 - 2) = 0 \quad \Rightarrow \lambda_5 = 0$

now, from (1)

$2(4/3) + \lambda_1 + \cancel{\lambda_2} + 0 = 0$

$\lambda_1 + \cancel{\lambda_2} = -8/3 \quad \cancel{\lambda_1 = -\lambda_2 - 8/3}$

from (2) $\quad 2(2/3) + -8/3 + 3\lambda_2 + 0 = 0$

$3\lambda_2 = 4/3$

$\lambda_2 > 0$

$\therefore$ KT conditions are not satisfied.

3.) at point (2, 1, 2)

④ ⇒ $\lambda_1(2+1+2-5)=0$ $\qquad \lambda_1 \neq 0$

⑤ ⇒ $\lambda_2(1.2-2)=0$ $\qquad \lambda_2 \neq 0$

⑥ ⇒ $\lambda_3(2)=0$ $\qquad \lambda_3 = 0$

⑦ ⇒ $\lambda_4(1)=0$ $\qquad \lambda_4 = 0.$

⑧ ⇒ $\lambda_5(2-2)=0$ $\qquad \lambda_5 \neq 0$

now from ① $\qquad$ from ②

$2.2+\lambda_1+0=0$ $\qquad 2.1-4+2\lambda_2+0=0$

$\lambda_1=-4$ $\qquad \lambda_2 = 1$

from ③ $\quad 2.2+(-4)+1.1+\lambda_5=0$

$\lambda_5=-1$

Since $\lambda_2 = 1 > 0$

∴ KT conditions are not satisfied.

August 30, 2019

Ques Use KT conditions to

min $z = f(x,y,z) = x^2+y^2+z^2+20x+10y$

s.t. $\quad x \geq 40$

$x+y \geq 80$

$x+y+z \geq 120$

Lagranges function is given by:

$L = z + \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3$

$g_1 = x-40 \geq 0$

$g_2 = x+y-80 \geq 0$

$g_3 = x+y+z-120 \geq 0$

$L = x^2+y^2+z^2+20x+10y+\lambda_1(x-40)+\lambda_2(x+y-80)$
$+\lambda_3(x+y+z-120)$

Kuhn Tucker's condition

1. $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} = 0$

2. $\lambda_1 g_1 = \lambda_2 g_2 = \lambda_3 g_3 = 0$

3. $g_1, g_2, g_3 \geq 0$

4. $\lambda_j \leq 0$

$\frac{\partial L}{\partial x} = 2x+20+\lambda_1+\lambda_2+\lambda_3=0$ —— ①

$\frac{\partial L}{\partial y} = 2y+10+\lambda_2+\lambda_3=0$ —— ②

$\frac{\partial L}{\partial z} = 2z+\lambda_3=0$ —— ③

$\lambda_1(x-40)=0$ —— ④

$\lambda_2(x+y-80)=0$ —— ⑤

$\lambda_3(x+y+z-120)=0$ —— ⑥

$x - 40 \geq 0$

$x + y - 80 \geq 0$

$x + y + z - 120 \geq 0$

Now ~~that~~ let $\lambda_1, \lambda_2, \lambda_3 \neq 0$

$\therefore$ (4) $\Rightarrow$ $x - 40 = 0$ $\quad x = 40$

(5) $\Rightarrow$ $x + y - 80 = 0$ $\quad y - 80 + 40 = 0$

$y = 40$

(6) $\Rightarrow$ $x + y + z - 120 = 0$ $\quad 40 + 40 + z - 120 = 0$

$z = 40$

$\therefore x = y = z = 40$

By (3) $\quad 2z + \lambda_3 = 0$

$\lambda_3 = -2z = -80 \leq 0$

By (2) $\quad 2y + 10 + \lambda_2 + \lambda_3 = 0$

$90 - 80 + \lambda_3 = 0 \quad \lambda_3 = -10 \leq 0$

By (1) $\quad 2x + 20 + \lambda_1 + \lambda_2 + \lambda_3 = 0$

$80 + 20 + \lambda_1 - 80 - 10 = 0$

$\lambda_1 = -10 \leq 0$

$\therefore$ All KT conditions are satisfied at $x = y = z = 40$

$\therefore Z_{min} = (40)^2 + (40)^2 + (40)^2 + 20 \times 40 + 10 \times 40$

$= 1600 \times 3 + 800 + 400$

$= 6000$

Ques Solve using KT conditions minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

s.t. $g_1(x) = 2x_1 + x_2 - 5 \leq 0$

$g_2(x) = x_1 + x_3 - 2 \leq 0$

$g_3(x) = 1 - x_1 \leq 0$

$g_4(x) = 2 - x_2 \leq 0$

$g_5(x) = -x_3 \leq 0$

Lagranges function is given by:

$$L = f(x) + \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3 + \lambda_4 g_4 + \lambda_5 g_5$$

$$L = x_1^2 + x_2^2 + x_3^2 + \lambda_1(2x_1 + x_2 - 5) + \lambda_2(x_1 + x_3 - 2) + \lambda_3(1 - x_1) + \lambda_4(2 - x_2) + \lambda_5(-x_3)$$

Kuhn Tucker's conditions:

1. $\frac{\partial L}{\partial x_1} = 0 = \frac{\partial L}{\partial x_2} = \frac{\partial L}{\partial x_3}$

2. $\lambda_1 g_1 = \lambda_2 g_2 = \lambda_3 g_3 = \lambda_4 g_4 = \lambda_5 g_5 = 0$

3. $g_1, g_2, g_3, g_4, g_5 \leq 0$

4. $\lambda_j g_j \geq 0 \quad [j = 1, 2 \ldots 5]$

$$\frac{\partial L}{\partial x_1} = 2x_1 + 2\lambda_1 \cancel{x_1} + \lambda_2 \cancel{x_1} - \lambda_3 = 0 \quad ①$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + \lambda_1 - \lambda_4 = 0 \quad ②$$

$$\frac{\partial L}{\partial x_3} = 2x_3 + \lambda_2 - \lambda_5 = 0 \quad ③$$

$\lambda_1(2x_1 + x_2 - 5) = 0$ ④

$\lambda_2(x_1 + x_3 - 2) = 0$ ⑤

$\lambda_3(1 - x_1) = 0$ ⑥

$\lambda_4(2 - x_2) = 0$ ⑦

$\lambda_5(-x_3) = 0$ ⑧

$2x_1 + x_2 - 5 \le 0$ ⑨

$x_1 + x_3 - 2 \le 0$ ⑩

$1 - x_1 \le 0$ ⑪

$2 - x_2 \le 0$ ⑫

$-x_3 \le 0$ ⑬

by ⑥ $\lambda_3(1 - x_1) = 0$

let $\lambda_3 \ne 0 \Rightarrow 1 - x_1 = 0$

$\Rightarrow x_1 = 1$

⑦ $\Rightarrow \lambda_4(2 - x_2) = 0$

let $\lambda_4 \ne 0 \Rightarrow 2 - x_2 = 0$

$x_2 = 2$

⑧ $\Rightarrow \lambda_5(-x_3) = 0$

let $\lambda_5 \ne 0 \Rightarrow x_3 = 0$

④ $\Rightarrow \lambda_1(2x_1 + x_2 - 5) = 0$

$\lambda_1(2 + 2 - 5) = 0 \Rightarrow \lambda_1 = 0$

⑤ $\Rightarrow \lambda_2(x_1 + x_3 - 2) = 0$

$\lambda_2(1 + 0 - 2) = 0 \Rightarrow \lambda_2 = 0$

① $\Rightarrow 2x_1 + 2\lambda_1 + \lambda_2 - \lambda_3 = 0$

$2 + 0 + 0 - \lambda_3 = 0$

$\lambda_3 = +2 \ge 0$

② $\Rightarrow 2x_2 + \lambda_1 - \lambda_4 = 0$

$4 + 0 - \lambda_4 = 0$

$\lambda_4 = 4 \ge 0$

③ $\Rightarrow 2x_3 + \lambda_2 - \lambda_5 = 0$

$0 + 0 - \lambda_5 = 0 \Rightarrow \lambda_5 = 0 \ge 0$

$\therefore x_1 = 1,\ x_2 = 2,\ x_3 = 0$

$\lambda_1 = \lambda_2 = \lambda_5 = 0$

$\lambda_3 = 2 \ge 0 \qquad \lambda_4 = 4 \ge 0$

min $= 1^2 + 2^2 + 0^2 = 5$

Ques max $Z = 2x_1 - x_1^2 + x_2$

s.t. $2x_1 + 3x_2 \le 6$

$2x_1 + x_2 \le 4$

$x_1, x_2 \ge 0 \Rightarrow -x_1, -x_2 \le 0$

Lagranges function is given by:

$L = z + \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3 + \lambda_4 g_4$

$L = 2x_1 - x_1^2 + x_2 + \lambda_1(2x_1 + 3x_2 - 6)$
$+ \lambda_2(2x_1 + x_2 - 4) + \lambda_3(-x_1) + \lambda_4(-x_2)$

Kuhn Tucker's Conditions

1. $\frac{\partial L}{\partial x_1} = 0 = \frac{\partial L}{\partial x_2}$

2. $\lambda_1 g_1 = \lambda_2 g_2 = \lambda_3 g_3 = \lambda_4 g_4 = 0$

3. $g_1, g_2, g_3, g_4 \le 0$

4. $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \le 0$

$\frac{\partial L}{\partial x_1} = 2 - 2x_1 + 2\lambda_1 + 2\lambda_2 - \lambda_3 = 0$ ——— (1)

$\frac{\partial L}{\partial x_2} =$

September 2, 2019

Ques A firm producing small water heater contracted to supply 50 water heaters at the end of first month, 50 at the end of second and 50 at the third month. The cost of producing x water heater in any month is given by $x^2 + 1000$. To carry a water heater from 1 month to next cost ₹20 per unit. Assuming that there is no initial inventory determine the no of water heaters to be produced in each month to minimize the total cost.

Let x_1, x_2, x_3 be heaters produced in first, second and third month resp.
Then, the total cost will be given by:

$\min C = (x_1^2 + 1000) + (x_2^2 + 1000) + (x_3^2 + 1000)$
$+ 20(x_1 - 50) + 20(x_1 + x_2 - 100)$

$\min C = x_1^2 + x_2^2 + x_3^2 + 40x_1 + 20x_2$

The constraint of the above problem are given by:

$g_1 = x_1 - 50 \ge 0$

$g_2 = x_1 + x_2 - 100 \ge 0$

$g_3 = x_1 + x_2 + x_3 - 150 \ge 0$

$\mathcal{L} = C + \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3$

Now, Kuhn Tucker conditions be

1. $\frac{\partial L}{\partial x_1} = 0 = \frac{\partial L}{\partial x_2} = \frac{\partial L}{\partial x_3}$

2. $\lambda_1 g_1 = \lambda_2 g_2 = \lambda_3 g_3 = 0$

3. $g_1, g_2, g_3 \geq 0$

4. $\lambda_1, \lambda_2, \lambda_3 \leq 0$

$\therefore$ we have

$\mathcal{L} = x_1^2 + x_2^2 + x_3^2 + 20x_2 + 40x_1 + \lambda_1(x_1 - 50) + \lambda_2(x_1 + x_2 - 100) + \lambda_3(x_1 + x_2 + x_3 - 150)$

$\frac{\partial L}{\partial x_1} = 2x_1 + 40 + \lambda_1 + \lambda_2 + \lambda_3 = 0$ —— (1)

$\frac{\partial L}{\partial x_2} = 2x_2 + 20 + \lambda_2 + \lambda_3 = 0$ —— (2)

$\frac{\partial L}{\partial x_3} = 2x_3 + \lambda_3 = 0$ —— (3)

$\lambda_1(x_1 - 50) = 0$ —— (4)

$\lambda_2(x_1 + x_2 - 100) = 0$ —— (5)

$\lambda_3(x_1 + x_2 + x_3 - 150) = 0$ —— (6)

$g_1, g_2, g_3 \geq 0$
$\lambda_1, \lambda_2, \lambda_3 \leq 0$

Now, by (4) we have
$\lambda_1 = 0$ or $x_1 - 50 = 0$

Case I : $\lambda_1 = 0$

$x_3 = -\lambda_3/2$ $\quad x_2 = \frac{-20 - \lambda_2 - \lambda_3}{2}$ // By (1), (2) & (3)

$x_1 = \frac{-40 - \lambda_2 - \lambda_3}{2}$

$\therefore$ (5) $\Rightarrow$

$\lambda_2 \left[\frac{-40 - \lambda_2 - \lambda_3}{2} + \frac{-20 - \lambda_2 - \lambda_3}{2} - 100 \right] = 0$

$\lambda_2 \left[\frac{-2\lambda_2 - 2\lambda_3 - 260}{2} \right] = 0$

$\lambda_2 [\lambda_2 + \lambda_3 + 130] = 0$

(6) $\Rightarrow$

$\lambda_3 \left[\frac{-40 - \lambda_2 - \lambda_3}{2} + \frac{-20 - \lambda_2 - \lambda_3}{2} - \frac{\lambda_3}{2} - 150 \right] = 0$

$\lambda_3 \left[\frac{-2\lambda_2 - 3\lambda_3 - 360}{2} \right] = 0$

$\lambda_3 \left[\lambda_2 + \frac{3\lambda_3}{2} + 180 \right] = 0$

$\therefore$ we have

$\lambda_2(\lambda_2 + \lambda_3 + 130) = 0$

$\lambda_3(\lambda_2 + \frac{3\lambda_3}{2} + 180) = 0$

Now from above we have following cases:

1.) $\lambda_2 = 0 \quad \lambda_3 \neq 0$

$\lambda_2 + \frac{3\lambda_3}{2} + 180 = 0 \qquad 3\lambda_3 = -360$

$\lambda_3 = -120$

$\Rightarrow x_3 = 60, \; x_2 = 50, \; x_1 = 40$

does not satisfy $g_1 \geq 0$

2.) $\lambda_2 \neq 0, \quad \lambda_3 = 0$

$\lambda_2 + \lambda_3 + 130 = 0$

$\lambda_2 = -130$

$x_3 = 0 \quad x_2 = 55 \quad x_1 = 45$

again $g_1 \geq 0$ not satisfied.

3.) $\lambda_2 \neq 0$ and $\lambda_3 \neq 0$

$\left.\begin{aligned} \lambda_2 + \lambda_3 + 130 = 0 \\ \lambda_2 + \frac{3\lambda_3}{2} + 180 = 0 \end{aligned}\right\} \quad \lambda_2 = -30 \quad \lambda_3 = -100$

$x_3 = 50 \quad x_2 = 55 \quad x_1 = 45$

$g_1 \geq 0$ not satisfied

4.) $\lambda_2 = 0 \quad \lambda_3 = 0$

$x_3 = 0 \quad x_2 = -10 \quad x_1 = 20$

g_1, g_2, g_3 are not satisfied.

case II: Let $x_1 = 50$

$\therefore \lambda_3 = -2x_3$

$\lambda_2 = -2x_2 - 20 - \lambda_3$

$\lambda_2 = -2x_2 - 20 + 2x_3$

$\lambda_1 = -2x_1 - 40 - \lambda_2 - \lambda_3$

$\lambda_1 = -x_1 - 40 + 2x_2 + 20 - \cancel{2x_3} + \cancel{2x_3}$

$\lambda_1 = -2x_1 + 2x_2 - 20$

By (5)

$(-2x_2 - 20 + 2x_3)(x_1 + x_2 - 100) = 0$

By (6)

$(-2x_3)(x_1 + x_2 + x_3 - 150) = 0$

$\therefore$ we prove

(1) $-2x_2 - 20 + 2x_3 = 0$, ~~or~~ $-2x_3 = 0$

(2) $-2x_2 - 20 + 2x_3 = 0 \qquad x_1 + x_2 + x_3 - 150 = 0$

(3) $x_1 + x_2 - 100 = 0 \qquad -2x_3 = 0$

Page No. 120.

(4) $x_1+x_2-100=0 \quad x_1+x_2+x_3-150=0$

1.) $-2x_2-20+2x_3=0 \quad , \quad -2x_3=0$

$\Rightarrow x_3=0$

$\Rightarrow -2x_2=20 \quad \Rightarrow x_2=-10$

$g_1, g_2, g_3 \geq 0$ not satisfied.

2.) $-2x_2-20+2x_3=0 \quad x_1+x_2+x_3-150=0$

$x_2+x_3-100=0$

$2x_2+2x_3-200=0$

$4x_3-220=0 \quad x_3=\frac{220}{4}=55$

$x_2+x_3=100 \quad \Rightarrow x_2=45$

$g_2 \geq 0$ not satisfied.

3.) $x_3=0$

$x_1+x_2-100=0 \quad x_2=50$

$g_3 \geq 0$ not satisfied.

4.) $x_1+x_2-100 \quad x_1+x_2+x_3-150=0$

$x_2=50 \quad x_3=50$

$x_1=x_2=x_3=50$

all $g_i \geq 0$ $[i=1,2,3]$ satisfied.

Page No. 121.

September 4, 2019

Ques Consider the following optimization problem

max $f=-x_1-x_2$

s.t. $x_1^2+x_2 \geq 2$

$x_1+3x_2 \geq 4$

$x_1+x_2^4 \leq 30$

Find whether the design vector $x=(1,1)$ satisfies KT conditions and if satisfies KT conditions find the value of Lagranges multipliers

Lagranges function is given by

$\mathcal{L}=f+\lambda_1 g_1+\lambda_2 g_2+\lambda_3 g_3$

$g_1 = x_1^2+x_2-2 \geq 0$

$g_2 = x_1+3x_2-4 \geq 0$

$g_3 = -x_1-x_2^4+30 \geq 0$

$\therefore \mathcal{L}=-x_1-x_2+\lambda_1(x_1^2+x_2-2)+\lambda_2(x_1+3x_2-4)$
$+\lambda_3(-x_1-x_2^4+30)$

Kuhn Tucker's conditions are given by:

$\frac{\partial L}{\partial x_1}=\frac{\partial L}{\partial x_2}=0$

$\lambda_1 g_1=\lambda_2 g_2=\lambda_3 g_3=0$

$g_1, g_2, g_3 \geq 0$

$\lambda_1, \lambda_2, \lambda_3 \geq 0$

$$\frac{\partial L}{\partial x_1} = -1 + 2\lambda_1 x_1 + \lambda_2 - \lambda_3 = 0 \quad \text{--- (1)}$$

$$\frac{\partial L}{\partial x_2} = -1 + \lambda_1 + 3\lambda_2 - 4\lambda_3 x_2^3 = 0 \quad \text{--- (2)}$$

$$\lambda_1 (x_1^2 + x_2 - 2) = 0 \quad \text{--- (3)}$$

$$\lambda_2 (x_1 + 3x_2 - 4) = 0 \quad \text{--- (4)}$$

$$\lambda_3 (-x_1 - x_2^4 + 30) = 0 \quad \text{--- (5)}$$

$$x_1^2 + x_2 - 2 \geq 0 \quad \text{--- (6)}$$

$$x_1 + 3x_2 - 4 \geq 0 \quad \text{--- (7)}$$

$$-x_1 - x_2^4 + 30 \geq 0 \quad \text{--- (8)}$$

at point (1, 1)

$\lambda_1 (1^2 + 1 - 2) = 0$ $\qquad \lambda_1 \neq 0$

$\lambda_2 (1 + 3 - 4) = 0$ $\qquad \lambda_2 \neq 0$

$\lambda_3 (-1 - 1 + 30) \geq 0$ $\qquad \lambda_3 = 0$

put in (1) & (2)

$-1 + 2\lambda_1 + \lambda_2 = 0$

$-1 + \lambda_1 + 3\lambda_2 = 0 \quad \times 2$

$$\begin{array}{rl} & -1 + 2\lambda_1 + \lambda_2 = 0 \\ - & -2 + 2\lambda_1 + 6\lambda_2 = 0 \\ & + \quad - \quad\quad - \\ \hline & 1 \qquad -5\lambda_2 = 0 \end{array}$$

$\lambda_2 = 1/5$

Substituting in (1)

$-1 + 2\lambda_1 + 1/5 = 0$

$2\lambda_1 = 4/5$

$\lambda_1 = 2/5$

$\therefore$ All KT conditions are fulfilled.

Page No. 122.

$$\frac{\partial L}{\partial x_1} = -1 + 2\lambda_1 x_1 + \lambda_2 - \lambda_3 = 0 \quad \text{—— (1)}$$

$$\frac{\partial L}{\partial x_2} = -1 + \lambda_1 + 3\lambda_2 - 4\lambda_3 x_2^3 = 0 \quad \text{—— (2)}$$

$\lambda_1 (x_1^2 + x_2 - 2) = 0$ —— (3)

$\lambda_2 (x_1 + 3x_2 - 4) = 0$ —— (4)

$\lambda_3 (-x_1 - x_2^4 + 30) = 0$ —— (5)

$x_1^2 + x_2 - 2 \geq 0$ —— (6)

$x_1 + 3x_2 - 4 \geq 0$ —— (7)

$-x_1 - x_2^4 + 30 \geq 0$ —— (8)

at point (1,1)

$\lambda_1 (1^2 + 1 - 2) = 0$ $\quad \lambda_1 \neq 0$

$\lambda_2 (\cancel{2}1 + 3 - 4) = 0$ $\quad \lambda_2 \neq 0$

$\lambda_3 (-1 - 1 + 30) \geq 0$ $\quad \lambda_3 = 0$

put in (1) & (2)

$-1 + 2\lambda_1 + \lambda_2 = 0$

$-1 + \lambda_1 + 3\lambda_2 = 0 \quad \times 2$

$-1 + 2\lambda_1 + \lambda_2 = 0$

$- \quad -2 + 2\lambda_1 + 6\lambda_2 = 0$

$+ \quad - \quad -$

$1 \quad -5\lambda_2 = 0$

Page No. 123.

$\lambda_2 = 1/5$

Substituting in (1)

$-1 + 2\lambda_1 + 1/5 = 0$

$2\lambda_1 = 4/5$

$\lambda_1 = 2/5$

$\therefore$ All KT conditions are fulfilled.

UNIT-3

1. What is optimization techniques?
   Gokhale Article 1.2

2. History of optimization technique
   pdf (not in Gokhale)

3. Application of optimization technique
   Article 1.5 Chapter 1.

4. Phases of operation research. pdf.

5. Role of operation research. pdf

6. Scope of operation research pdf

7. Classification of operation research problem.
   (i). Classification based on constraint

   Optimization problem can be classified as constrained and unconstrained based on presence or absence of constraint in the problem.

   ~~conts~~ unconstrained max $z = x_1^2 + x_2^2$

constrained max $z = x_1^2 + x_2^2$

s.t. $x_1 + x_2 \leq 2$

$x_2 \leq 1$

$x_1, x_2 \geq 0$

(ii) Classification based on nature of design variable

It can be classified into following two forms: Static optimization problem.

If the variable involved are static in nature then it is static optimization problem.

Dynamic optimization problem

If design parameters are varying in nature with time then they are dynamic programming problem.

(iii) Classification based on nature of equation involved.

It can be categorized into following four forms:
- Linear Programming Problem
- Non linear Programming Problem
- Geometric Programming
- Quadratic Programming

Linear Programming
If the objective function and constraint involved are linear function then the problem is linear programming problem.

Non Linear
If any of one function ~~involved~~ among objective function and constraint is non linear then the problem is called non linear programming problem.

example

LPP: $\max z = x_1 + 2x_2 + 3x_3$

s.t. $2x_1 + x_2 + x_3 \le 2$

$x_1 + x_2 \ge 1$

$x_1 + x_3 + 2x_2 \le 5$

$x_1, x_2, x_3 \ge 0$

NLPP: $\max z = x_1 + x_2 + x_3$

s.t. $x_1 + 2x_2 + x_3 \le 2$

$x_1^2 + x_2^2 \ge 1$

$x_1, x_2, x_3 \ge 0$

Geometric
A problem in which objective function and constraint are expressed as posinomials is GPP.
A function is called posinomial if it can be expressed as sum of terms of each of the form

$$c_1 x_1^{a_{11}} x_2^{a_{12}} x_3^{a_{13}} \dots x_n^{a_{1n}}$$

i.e. $h(x) = c_1 x_1^{a_{11}} x_2^{a_{12}} \dots x_n^{a_{1n}} + c_2 x^{a_{21}} x_2^{a_{22}} x_3^{a_{23}} \dots x_n^{a_{2n}} + c_n x_1^{a_{n1}} x_2^{a_{n2}} \dots x_n^{a_{nn}}$

Quadratic Programming problem
A non linear programming problem with quadratic objective function and linear constraint is called QPP.

ex $\max z = x_1^2 + x_2^2 + x_3^2$

s.t $x_1 + x_2 + x_3 \ge 3$

$x_1 + 2x_2 \ge 1$

and $x_1, x_2, x_3 \ge 0$

(iv) Classification based on permissible value of design variable
These are classified into following two

forms.

Integer valued programming problem
If some or all variable of the problem are restricted to take only integer values then problem is called integer valued

Real valued programming problem
If ~~all or~~ some variables take any real values then it is called real valued programming problem.

(v) Classification based on deterministic nature of variable.
These are further classified into following two forms

Stochastic
If variable involved are probabilistic in nature then it is called stochastic programming problem.

deterministic
If variable involved is deterministic in nature.

(vi) Classification based on number of objective function.

No Objective Function
These are feasibility problems.
for example: design of integrated circuits

Single Objective Function
If there is only one objective function

Multiple Objective Function
$\max f(x_1), f(x_2) \dots f(x_n)$
s.t. $g_j(x) \geq 0$

(vii) Classification based on Physical Structure.
Article 1.6.3

Optimal and non optimal.

September 9, 2019

PROBABILITY

Probability mathematical (Classical) def:
The probability of happening of event A is denoted by $P(A) = m/n$ where the odds in favour of A is $\left(\frac{m}{n-m}\right)$ and odd against A ~~are~~ is given by $\frac{n-m}{m}$ where there are n exhaustive mutually exclusive and equally likely cases and m of them are favourable to an event A.

Axiomatic definition of probability:
In an any random experiment let S be the sample space and A be any event happening in this sample space then it satisfies following 3 axioms:

1. $0 \le P(A) \le 1$
2. $P(S) = 1$
3. if A and B are two mutually exclusive events happening in sample space S then $P(A \cup B) = P(A) + P(B)$

Imp ~~Find the~~
probability of happening of event A $P(A)$

probability of not happening of event A $P(\bar{A}) = 1 - P(A)$

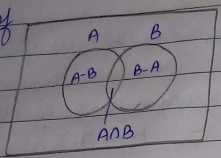
probability of happening of either A or B $P(A \cup B)$

probability of happening of both A and B $P(A \cap B)$

Conditional probability
probability of happening of A when B has already occurred $P(A/B)$

Imp Addition theorem of probability
If A and B are two events happening in sample space S then probability of occurrence of A or B i.e. atleast one is given by
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof



now $A \cup B = (A-B) \cup (A \cap B) \cup (B-A)$

$$\therefore P(A \cup B) = P[(A-B) \cup (A \cap B) \cup (B-A)]$$
$$= P(A-B) + P(A \cap B) + P(B-A)$$

$$= P(A-B) + P(A \cap B) + P(B-A) + P(A \cap B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$\left[\begin{array}{l} \text{as } P(A-B) + P(A \cap B) = P(A) \\ P(B-A) + P(A \cap B) = P(B) \end{array} \right]$$

now, if A, B, C are three events then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$P(A \cup B \cup C) = P(S \cup C)$ as $S = A \cup B$

$$= P(S) + P(C) - P(S \cap C)$$
$$= P(A \cup B) + P(C) - P((A \cup B) \cap C)$$
$$= P(A) + P(B) - P(A \cap B) + P(C) - P[(A \cap C) \cup (A \cap B)]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - [P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Independent events

Two events A and B are said to be independent if and only if
$$P(A \cap B) = P(A) \cdot P(B)$$

Conditional probability:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Ques A problem in maths is given to 3 students A, B, C whose chance of solving are $1/2$, $1/3$ and $1/4$ resp. Find the probability that problem will be solved.

$P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) + P(A \cap B \cap C)$
$- P(A \cap B) - P(A \cap C) - P(B \cap C)$

Since A, B, C are independent

$P(A \cap B) = P(A) \cdot P(B) = 1/6$
$P(B \cap C) = P(B) \cdot P(C) = 1/12$
$P(A \cap C) = P(A) \cdot P(C) = 1/8$
$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = 1/24$

$P(A \cup B \cup C) = 1/2 + 1/3 + 1/4 + 1/24$
$- 1/6 - 1/12 - 1/8$

$= 3/4$

or

$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C})$
$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$
$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})$
$= 1 - 1/2 \cdot 2/3 \cdot 3/4$
$= 1 - 1/4$
$= 3/4$

Ques A candidate is selected for interview of management trainee for 3 companies. For the first company there are 12 candidates for the second there are 15 and for the third 10. What are the chances of getting in atleast one of the companies

probability for being in 1, 2, 3 companies are $1/12$, $1/15$ and $1/10$

P(selected in atleast one)
$= 1 - P$(not being selected in any)
$= 1 - 11/12 \cdot 14/15 \cdot 9/10$
$=$

Ques each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing a dice. Find the probability that equation will have real roots

The roots of the equation will be real if
$b^2 - 4ac \geq 0$
$b^2 \geq 4ac$

each coefficient can take value from 1 to 6
$\therefore$ total no of possible cases can be
$6 \times 6 \times 6 = 216$

Also ac can take value from 1 to 9

$\therefore$ following are number of favourable cases

ac	a	c	$4ac$	b	Possible cases
1	1	1	4	2,3,4,5,6	$1\times5 = 5$
2	1	2	8	3,4,5,6	$2\times4 = 8$
	2	1			
3	3	1	12	4,5,6	$2\times3 = 6$
	1	3			
4	2	2	16	4,5,6	$3\times3 = 9$
	4	1			
	1	4			
5	1	5	20	5,6	$2\times2 = 4$
	5	1			
6	2	3	24	5,6	$4\times2 = 8$
	3	2	2		
	6	1			
	1	6			
7	not possible				
8	2	4	32	6	$2\times1 = 2$
	4	2			
9	3	3	36	6	$1\times1 = 1$

$\therefore$ favourable no of cases are $5+8+6+9+4+8+2+1$

43

$\therefore$ possibility that roots will be real $\frac{43}{216}$

Ques A speaks truth in 75% cases and B in 80% cases. Find the probability that they are likely to contradict each other in stating the same fact

probability that

A speaks truth $P(A)$ $75/100$

$P(\bar{A})$ $25/100$

B speaks truth $P(B)$ $80/100$

$P(\bar{B})$ $20/100$

Probability that A and B will contradict

$P(A\bar{B}) + P(\bar{A}B)$

$P(A).P(\bar{B}) + P(\bar{A}).P(B)$

$75/100 . 20/100 + 25/100 . 80/100$

$3/4 . 1/5 + 1/4 . 4/5$

$7/20$

Ques The odds that a book will be favourably reviewed by 3 independent critics are 3:2, 4:3 and 2:3 respectively. What is the probability that majority will be favourable.

$P(A) = 3/5$ $P(B) = 4/7$ $P(C) = 2/5$

$P(\bar{A}) = 2/5$ $P(\bar{B}) = 3/7$ $P(\bar{C}) = 3/5$

Probability that majority are in favour

$= P(AB\bar{C}) + P(\bar{A}BC) + P(A\bar{B}C) + P(ABC)$

$= P(A)P(B)P(\bar{C}) + P(\bar{A})P(B)P(C) + P(A)P(\bar{B})P(C) + P(A)P(B)P(C)$

$= \frac{3}{5}\cdot\frac{4}{7}\cdot\frac{3}{5} + \frac{2}{5}\cdot\frac{4}{7}\cdot\frac{2}{5} + \frac{3}{5}\cdot\frac{3}{7}\cdot\frac{2}{5} + \frac{3}{5}\cdot\frac{4}{7}\cdot\frac{2}{5}$

$= 94/175$

Ques A and B take turns in throwing two dice and the person who throws 9 first is to be awarded a prize. Show that ~~A~~ if A has the first turn their chances of winning the prize are in the ratio 9:8

The following are favourable cases of getting 9 on a dice

(3,6), (6,3), (4,5), (5,4)

∴ favourable no of cases are 4.

total no of cases 36.

probability of throwing nine ~~winning~~ $4/36$ $1/9$

probability of not winning $8/9$

A will win if a throw 9 in first chance or 3rd or 5th chance or ...

Therefore chances of winning of A is given by

$\frac{1}{9} + \left(\frac{8}{9}\right)^2\left(\frac{1}{9}\right) + \left(\frac{8}{9}\right)^4\left(\frac{1}{9}\right) + \dots$

$a = 1/9$ $r = (8/9)^2$ $S = \frac{a}{1-r}$

$S = \frac{1/9}{1-(8/9)^2} = \frac{1/9}{\frac{81-64}{9^2}}$

$P(A) = 9/17$

now probability of B throwing 9 (winning)

$1 - P(A)$

$1 - 9/17 = 8/17$

∴ ratio of winning $\frac{9/17}{8/17} = 9:8$

Ques A student takes his examination in four subjects $\alpha, \beta, \gamma, \delta$. He estimated his chances of passing in $\alpha = 4/5$, β is $3/4$, γ is $5/6$ and δ $2/3$. To qualify the exam he must pass in α and atleast two other subjects. What is the probability that he qualifies.

$P(\alpha\beta\gamma\bar{\delta}) + P(\alpha\beta\bar{\gamma}\delta) + P(\alpha\bar{\beta}\gamma\delta) + P(\alpha\beta\gamma\delta)$

$= P(\alpha)P(\beta)P(\gamma)P(\bar{\delta}) + P(\alpha)P(\beta)P(\bar{\gamma})P(\delta)$
$+ P(\alpha)P(\bar{\beta})P(\gamma)P(\delta) + P(\alpha)P(\beta)P(\gamma)P(\delta)$

$P(\alpha) = 4/5 \quad P(\beta) = 3/4 \quad P(\gamma) = 5/6 \quad P(\delta) = 2/3$

September 16, 2019

Ques A random variable x has the following probability distribution

X	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

a) find K
b) evaluate $P(x<6)$ $P(x\geq 6)$ $P(0<x<5)$
c) if $P(x\leq c) > 1/2$ find c
d) find $P\left(\frac{1.5<x<4.5}{x>2}\right)$
e) determine distribution function of X

$\sum_{i=0}^{7} P(x_i) = 1$

$0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$10K^2 + 9K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$K = -1 \quad 1/10$

Thus $K = 1/10$

(ii) $P(X<6) = 1 - P(X\geq 6)$
$= 1 - [P(6) + P(7)]$
$= 1 - (2/100 + 17/100)$
$= 81/100$

$P(X\geq 6) = (2/100 + 17/100)$
$19/100$

$P(0<X<5) = P(1) + P(2) + P(3) + P(4)$
$= K + 2K + 3K + 2K$
$= 8K$
$= 8/10$

(iii) $P(X\leq c) > 1/2$

$P(X\leq 2) = K + 2K = 3K = 3/10$ X

$P(X\leq 3) = K+2K+2K = 5K = 5/10$ X

$P(X\leq 4) = K+2K+2K+3K = 8K = 8/10 > 1/2$

$\therefore C = 4$

(iv) $P\left(\frac{1.5<X<4.5}{x>2}\right)$

$P(A/B) = \frac{P(A\cap B)}{P(B)}$

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$$\frac{P((1.5 < X < 4.5) \cap (X > 2))}{P(X > 2)}$$

$$\frac{P(2 < X < 4.5)}{P(X>2)} = \frac{P(3) + P(4)}{1 - P(X \leq 2)}$$

$$= \frac{5K}{1-3K} = \frac{5/100}{1 - 3/100}$$

$$= \frac{5}{97}$$

(v) Distribution function

X	f(x)	
0	0	0
1	1/10	K
2	3/10	K+2K
3	5/10	5K
4	8/10	8K
5	81/100	$8K + K^2$
6	83/100	$8K + 3K^2$
7	100/100 = 1	$9K + 10K^2$

September 12, 2019.

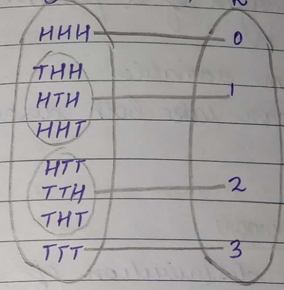
RANDOM VARIABLES

A random variable X is a function defined from sample space S to R (set of real numbers) i.e. domain is sample space S and range is R

For example:

If we toss a coin thrice, then if X is a random variable of no of tails then we have

X : S ⟶ R



1/8

3/8

3/8

1/8

There are three type of random variable

discrete random variable

It has either finite or countably infinite number of values which can take only integral values.

for example:

the number shown when dice is thrown

no of complaints received in a call centre

continuous random variable.
A random variable which can take infinite no of values in an interval i.e it can take fractional or decimal values also
for example
price of a house
height or weight of a person.

mixed random variable.
These variable can take both discrete and continuous values.

PROBABILITY DISTRIBUTION
The probability distribution of discrete random variable list all possible values that the random variable can assume and their corresponding probabilities
for example
if we throw a dice then distribution function is given by

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

Probability mass function
Let X be a discrete random variable such that $P(X=x_i) = P_i \;\forall i$ the P_i is said to be PMF if it satisfies following two properties
(i) $0 \le P_i \le 1$
(ii) $\Sigma P_i = 1$

Ques Check whether following are PMF or not.
(i) $P(X=x_i) = \frac{x-2}{2}$ $\quad x_i = 1,2,3,4$

X	1	2	3	4
P(X)	-1/2	0	1/2	1

here $P(X=1) = -1/2 < 0$
$\therefore$ P is not PMF

(ii) $P(X=x_i) = x^2/25$ $\quad x_i = 1,2,3,4$

X	1	2	3	4
P(X)	1/25	4/25	9/25	16/25

$\Sigma P_i = 1/25 + 4/25 + 9/25 + 16/25$
$= 30/25 \neq 1$
$\therefore$ P is not PMF

Cumulative Distribution Function/ Discrete Distribution Function:

If X is a discrete random variable then CDF is given by

$$f_x(x) = P(X=x) = \begin{cases} 0 ; & -\infty \le x < x_1 \\ 0+P_1 & x_1 \le x < x_2 \\ 0+P_1+P_2 & x_2 \le x \le x_3 \\ \vdots & \\ 0+P_1+P_2 \dots P_n = 1 & x_n \le x < \infty \end{cases}$$

for example:
If we throw a dice then CDF is

$$f_x(x) = P(X=x) = \begin{cases} 0 & -\infty \le x < 1 \\ 1/6 & 1 \le x < 2 \\ 1/6 + 1/6 = 2/6 & 2 \le x < 3 \\ 2/6 + 1/6 = 3/6 & 3 \le x < 4 \\ 3/6 + 1/6 = 4/6 & 4 \le x < 5 \\ 4/6 + 1/6 = 5/6 & 5 \le x < 6 \\ 5/6 + 1/6 = 6/6 = 1 & 6 \le x < \infty \end{cases}$$

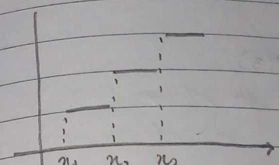
$\therefore$ we see that CDF satisfies following properties

(i) $0 \le f(x) \le 1$

(ii) $f_x(x)$ is monotonic increasing function

(iii) $f_x(-\infty) = 0$. $f_x(\infty) = 1$

graph is given by



September 16, 2019.

Ques A random variable X has following probability distribution

X	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	3K

1) find K

2) evaluate $P(X<2)$ $P(X \ge 2)$ $P(-2 < X < 2)$

3) find the minimum value of $*$ s.t. $P(x \le 1) > 0.36$

4) determine distribution function of x

1) $\sum_{i=-2}^{3} P(x_i) = 1$

$0.1 + K + 0.2 + 2K + 0.3 + 3K = 1$

$6K = 1 - 0.6 \quad K = \frac{0.4}{6} = \frac{0.2}{3} = 0.06 = \frac{1}{15}$

2.) $P(x<2)$

$1 - P(x \geq 2) = 1 - [P(2) + P(3)]$

$= 1 - [0.3 + 3K]$

$= 1 - 0.5$

$= 0.5$

$P(x \geq 2) = P(2) + P(3)$

$= 0.5$

$P(-2 < x < 2)$

$= P(-1) + P(0) + P(1)$

$= K + 0.2 + 2K$

$= 0.2 + 3K = 0.4.$

3.) $P(x \leq 1) > 0.36$

$P(x \leq 1) = P(-1) + (-2) + P(0) + P(1)$

$= 0.1 + K + 0.2 + 2K$

$= 0.3 + 3K$

$P(x \leq 1) > 0.36$

$0.3 + 3K > 0.36$

$3K > 0.06$

$K > 0.02$

(4) Distribution function of x

x	
-2	0.1
-1	$0.1 + K =$
0	$0.2 + 0.1 + K$
1	$0.3 + 3K$
2	$0.6 + 2K$
3	$0.6 + 6K = 1$

Ques If the random variable X takes the value 1, 2, 3, 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$ find the probability distribution and distribution function of x.

Let $P(X=3) = 30K$

$2P(X=1) = 30K \Rightarrow P(X=1) = 15$

Similarly $P(X=2) = 10K$, $P(X=4) = 6K$

$\sum_{i=1}^{4} P(x_i) = P(1) + P(2) + P(3) + P(4) = 1$

$\Rightarrow 30K + 10K + 6K + 15K = 1$

$\Rightarrow 61K = 1 \quad \therefore K = 1/61$

Probability distribution

X	1	2	3	4
$P(x)$	$15/61$	$10/61$	$30/61$	$6/61$

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X	F(x) P(
1	$15/61$
2	$25/61$
3	$55/61$
4	$61/61 = 1$

September 17, 2019

CONTINUOUS RANDOM VARIABLE:

Probability density function (PDF)

If X is continuous random variable then f(x) is probability density function if it satisfies following two properties

(i) $f(x) \geq 0 \ \forall \ x \in R$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Cumulative distribution function

If X is a continuous random variable then F(x) is cumulative distribution function defined as $F(x) = P(X \geq x) = \int_{x}^{\infty} f(x)\, dx$

where f(x) is PDF

and it satisfies following property

(i) $F'(x) = f(x)$ is continuous

(ii) $F(-\infty) = 0$

(iii) $F(\infty) = \int_{-\infty}^{\infty} f(x)\, dx = 1$

Ques Find the value of K so that

$$f(x) = \begin{cases} 0 & x \leq 0 \\ Kxe^{-4x^2} & x > 0 \end{cases}$$

may be probability density function of X.

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{0} f(x)\, dx + \int_{0}^{\infty} f(x)\, dx = 1$$

$$0 + \int_{0}^{\infty} Kxe^{-4x^2}\, dx = 1 \qquad 4x^2 = t, \quad 8x\,dx = dt$$

$$K\int_{0}^{\infty} e^{-t}\frac{dt}{8} = 1$$

$$\frac{K}{8}\left[\frac{e^{-t}}{-1}\right]_0^{\infty} = 1$$

$$-\frac{K}{8}\left[\frac{1}{e^{\infty}} - \frac{1}{e^0}\right] = 1$$

$$\underline{K = 8.}$$

Ques a) Find the constant c such that $f(x) = \begin{cases} cx^2 & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$ is a pdf.

b) find $P(1 < x < 2)$

c) find distribution function.

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{0} f(x)\, dx + \int_{0}^{3} f(x)\, dx + \int_{3}^{\infty} f(x)\, dx$$

Page No. 150.

X	F(X) Pl.
	15/61
1	25/61
2	55/61
3	61/61 = 1
4	

September 17, 2019

Ques CONTINUOUS RANDOM VARIABLE:

Probability density function (PDF)

If X is continuous random variable then f(x) is probability density function if it satisfies following two properties

(i) $f(x) \geq 0 \quad \forall x \in R$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Cumulative distribution function

If X is a continuous random variable then F(x) is cumulative distribution function defined as $F(x) = P(X \geq x) = \int_{x}^{\infty} f(x)\,dx$

where f(x) is PDF

and it satisfies following property

(i) $F'(x) = f(x)$ is continuous

(ii) $F(-\infty) = 0$

(iii) $F(\infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1$

Page No. 151.

Ques Find the value of K so that

$f(x) = \begin{cases} 0 & x \leq 0 \\ Kxe^{-4x^2} & x > 0 \end{cases}$ may be probability density function of X.

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$

$0 + \int_{0}^{\infty} Kxe^{-4x^2}\,dx = 1$ $\qquad 4x^2 = t$, $8x\,dx = dt$

$K\int_{0}^{\infty} e^{-t} \frac{dt}{8} = 1$

$\frac{K}{8}\left[\frac{e^{-t}}{-1}\right]_{0}^{\infty} = 1$

$-\frac{K}{8}\left[\frac{1}{e^{\infty}} - \frac{1}{e^{0}}\right] = 1$

$\underline{\underline{K = 8.}}$

Ques a) Find the constant c such that $f(x) = \begin{cases} cx^2 & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$ is a pdf.

b) find $P(1 < x < 2)$

c) find distribution function.

a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx$

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Page No. 182

$$0 + \int_0^3 cx^2 dx + 0 = 1$$

$$\left(\frac{cx^3}{3}\right)_0^3 = 1 \Rightarrow c\left(\frac{27}{3}\right) = 1$$

$c = 1/9.$

(ii) $P(1 < x < 2) =$

$$\int_1^2 f(x)dx = \int_1^2 \frac{x^2}{9} dx$$

$$\frac{1}{9}\left(\frac{x^3}{3}\right)_1^2 = \frac{1}{27}[8-1] = \frac{7}{27}$$

(iii) when $x<0$ $f(x)=0$ $F(x)=0$

when $0<x<3$ $f(x) = \frac{x^2}{9}$

$$F(x) = \int_0^x \frac{x^2}{9} dx = \frac{1}{9}\left(\frac{x^3}{3}\right)_0^x = \frac{x^3}{27}$$

when $x \geq 3$

$$F(x) = \frac{x^3}{27} = \frac{27}{27} = 1$$

$$\therefore F(x) = \begin{cases} 0 & ; x<0 \\ x^3/27 & ; 0<x<3 \\ 1 & ; x \geq 3 \end{cases}$$

// $e^0 = 1$
// $\frac{1}{e^\infty} = 0$

Notes

Ques The distribution function $F(x) = \begin{cases} 1-e^{-2x} & x \geq 0 \\ 0 & x<0 \end{cases}$

find a) PDF b) $P(x>2)$ c) $P(-3 < x \leq 4)$

(i) $f(x) = \frac{d}{dx}(F(x))$

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x<0 \end{cases}$$

(ii) $P(X>2) = \int_2^\infty f(x)dx = \int_2^\infty 2e^{-2x} dx$

$$\left[-e^{-2x}\right]_2^\infty = \left[\frac{1}{e^\infty} - \frac{1}{e^4}\right] = e^{-4}$$

(iii) $P(-3 < x \leq 4)$

$$\int_{-3}^0 f(x)dx + \int_0^4 f(x)dx$$

$$0 + \int_0^4 2e^{-2x}dx = \left[-e^{-2x}\right]_0^4$$

$$= \frac{-1}{e^8} + \frac{e^0}{\cancel{e^0}} = 1 - e^{-8}$$

Ques Let $f(x) = Ke^{-\alpha x}(1-e^{-\alpha x})$, $x>0$ $\alpha>0$

a) find K b) CDF c) $P(X>1)$

$K = 2\alpha$

$$\int_0^\infty f(x)dx = 1 = \int_0^\infty Ke^{-\alpha x}(1-e^{-\alpha x})dx = 1$$

$$K\left[\frac{e^{-\alpha x}}{-\alpha} - \frac{e^{-2\alpha x}}{-2\alpha}\right]_0^\infty = 1$$

$$\frac{-K}{\alpha}[0-1-0+2] = 1 \qquad K = 2\alpha$$

Notes

Ques Solve the following assignment problem

	C_1	C_2	C_3	C_4	C_5	C_6	
B_1	75.2	77.9	77.4	72.3	79	76.6	72.3
B_2	68.3	67.3	69.9	66.4	69.2	68.2	66.4
B_3	71.9	71.2	72.3	70.5	73.4	71.6	70.5
B_4	74.2	70.5	76.1	73.2	74.3	75.4	70.5

Row reduced matrix

2.9	5.6	5.1	0	6.7	4.3
1.9	0.9	1.5	0	2.8	1.8
1.4	0.7	1.8	0	2.9	1.1
3.7	0	5.6	2.7	3.8	4.9
0	0	0	0	0	0
~~Go~~ 0	0	0	0	0	0

Zero Assignment ✓

2.9	5.6	5.1	$\boxed{0}$	6.7	4.3	✓
1.9	0.9	1.5	$\cancel{0}$	2.8	1.8	✓
1.4	0.7	1.8	$\cancel{0}$	2.9	1.1	✓
3.7	$\boxed{0}$	5.6	2.7	3.8	4.9	
$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	

INSPIRATIONAL QUOTES

- It is during our darkest moments that we must focus to see
  Aristotl
- Today I choose life. Every morning when I wake up I can happiness, negativity, pain... To feel the freedom that c being able to continue to make mistakes and choices - tod to feel life, not to deny my humanity but embrace it.
  Kev
- My mission in life is not merely to survive, but to thrive so with some passion, some compassion, some humor, style.
  Ma
- I believe in pink. I believe that laughing is the best calc believe in kissing, kissing a lot. I believe in being s everything seems to be going wrong. I believe that happy prettiest girls. I believe that tomorrow is another day an miracles.
  Elizabeth Barre
- Change your thoughts and you change your world.
  Norman Vi
- Your work is going to fill a large part of your life, and the truly satisfied is to do what you believe is great work. way to do great work is to love what you do. If you ha yet, keep looking. Don't settle. As with all matters of th know when you find it.

S

September 18, 2019.

Ques 1. If $P(X) = \begin{cases} x e^{-x^2/2} & x \geq 0 \\ 0 & x < 0 \end{cases}$

a) show $P(x)$ is pdf b) find cdf ($F(x)$)

Ques 2. A discrete random variable has following probability distribution

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

find a. $P(X<3)$, cdf

a) $\sum_{i=0}^{8} P(x_i) = 1$

$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1.$

$81a = 1 \Rightarrow a = 1/81.$

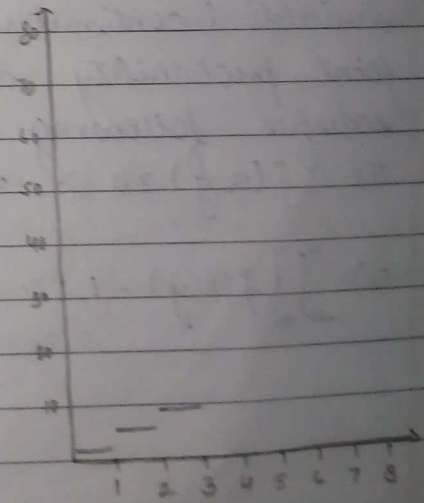
b) $P(X<3)$

$= P(0) + P(1) + P(2)$

$= a + 3a + 5a = 9a = 9/81 = 1/9$

c)

X		F(x)
0	a	1/81
1	4a	4/81
2	9a	9/81
3	16a	16/81
4	25a	25/81
5	36a	36/81
6	49a	49/81
7	64a	64/81
8	81a	81/81 = 1



Ques. $\int_{-\infty}^{\infty} P(x)\,dx = 1$

$\int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} x e^{-x^2/2}\,dx = 1$ $\qquad x^2/2 = t$, $x\,dx = dt$

$0 + \int_{0}^{\infty} e^{-t}\,dt = 1$

$[-e^{-t}]_0^{\infty} = 1$

$-\left[\frac{+1}{e^{\infty}} - \frac{1}{e^{0}}\right] = 1$

$1 = 1$ hence its pdf.

(ii) $F(x) = \begin{cases} -e^{-x^2/2}+1 ; & x \geq 0 \\ 0 ; & x < 0 \end{cases}$

JOINT PROBABILITY DENSITY FUNCTION

If (X, Y) is two dimensional random variable (continuous) then $f(x,y)$ is called joint probability density function if it satisfies following two properties

(i) $f(x,y) \geq 0 \quad \forall\, x,y \in R$

(ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) = 1$

Marginal distribution

(i) Discrete:

If X and Y are two discrete random variable such that $P(X=x, Y=y)$ is joint pmf then the function $P_x = P_{i1} + P_{i2} + P_{i3} \dots P_{in}$ for each x is called marginal distribution of X.

Similarly ~~$P_y = P_1 + P_2$~~ $P_y = P_{1j} + P_{2j} + \dots P_{nj}$ is called marginal distribution of Y.

(ii) Continuous

If x and y are two continuous random variable then the function $g_x = \int_{-\infty}^{\infty} f(x,y)\,dy$ is called marginal distribution function of X where $f(x,y)$ is joint pdf.

Similarly $h_y = \int_{-\infty}^{\infty} f(x,y)\,dx$ is marginal distribution of y.

September 19, 2019

Ques. Determine the value of K for which the function $f(x,y) = Kxy$; $x = 1,2,3$ $y = 1,2,3$ can serve as joint probability distribution $x = 1,2,3$ and $y = 1,2,3$

$f(1,1) = K$ $\quad f(2,1) = 2K$ $\quad f(3,1) = 3K$

$f(1,2) = 2K$ $\quad f(2,2) = 4K$ $\quad f(3,2) = 6K$

$f(1,3) = 3K$ $\quad f(2,3) = 6K$ $\quad f(3,3) = 9K$

now if $f(x,y)$ is joint probability distribution

$$\sum_{i=1}^{3}\sum_{j=1}^{3} f(x_i, y_j) = 1$$

$$K + 2K + 3K + 2K + 4K + 6K + 3K + 6K + 9K = 1$$

$$36K = 1$$

$$K = 1/36$$

$f(x,y)$ is a joint probability distribution if it satisfies following two properties

$$f(x_i, y_j) \geq 0 \quad \forall\, i,j$$

$$\sum_i \sum_j f(x_i, y_j) = 1$$

above properties are satisfied for $k = 36$.

Ques Given the joint probability density function

$$f(x,y) \begin{cases} \frac{3}{5}x(y+x) & 0 < x < 1,\ 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

find $P(x,y) \in A$

where A is the region given by

$\{(x,y);\ 0 < x < 1/2 \quad 1 < y < 2\}$

$$P(x,y) = \int_{y=1}^{2}\int_{x=0}^{1/2} \frac{3}{5} x(y+x)\, dx\, dy$$

$$= \frac{3}{5}\int_1^2 \left(\frac{x^2 y}{2} + \frac{x^3}{3}\right)_0^{1/2} dy$$

$$= \frac{3}{5}\int_1^2 \frac{y}{8} + \frac{1}{8\times 3}\, dy$$

$$= \frac{3}{5}\left(\frac{y^2}{16} + \frac{y}{24}\right)_1^2$$

$$= \frac{3}{5}\left(\frac{4}{16} + \frac{1}{12} - \frac{1}{16} - \frac{1}{24}\right)$$

$$= \frac{3}{5}\left(\frac{3}{16} + \frac{1}{24}\right)$$

$$= \frac{11}{80}$$

Ques For bivariate probability distribution of x, y given below find

x/y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

(i) $P(x \leq 1)$

(ii) $P(y \leq 3)$

(iii) $P(x \leq 1, y \leq 3)$

(iv) $P(x \leq 1 / y \leq 3)$

(v) $P(y \leq 3 | x \leq 1)$

(vi) $P(x + y \leq 4)$

Page No. 006.

(i) $P(x \le 1)$

$P(x=0) + P(x=1)$

$P(x=0, y=1,2\ldots6) + P(x=1, y=1,2,\ldots6)$

$\left(0+0+\frac{1}{32}+\frac{2}{32}+\frac{2}{32}+\frac{3}{32}\right)+\left(\frac{1}{16}+\frac{1}{16}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)$

$\frac{8}{32}+\frac{10}{16}$

$= \frac{7}{8}$

(ii) $P(y \le 3)$

$= P(y=1) + P(y=2) + P(y=3)$

$= P(y=1, x=0,1,2,3) + P(y=2, x=0..3) + P(y=3, x=0,1,2,3)$

$= \left(0+\frac{1}{16}+\frac{1}{32}\right)+\left(0+\frac{1}{16}+\frac{1}{32}\right)+\left(\frac{1}{32}+\frac{1}{8}+\frac{1}{64}\right)$

$= \frac{23}{64}$

(iii) $P(x \le 1, y \le 3)$

$P(x=0, y=1,2,3) + P(x=1, y=1,2,3)$

$= \left(0+0+\frac{1}{32}\right)+\left(\frac{1}{16}+\frac{1}{16}+\frac{1}{8}\right)$

$= \frac{1+2+2+4}{32} = \frac{9}{32}$

(iv) $P(x \le 1 / y \le 3)$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

Page No. 007.

$\frac{P(x \le 1, y \le 3)}{P(y \le 3)}$

$\frac{9/32}{23/64} = \frac{18}{23}$

(v) $P(y \le 3 / x \le 1)$

$= \frac{P(y \le 3, x \le 1)}{P(x \le 1)} = \cancel{\frac{13}{32}}\ \frac{9/32}{7/8}$

(vi) $P(x+y \le 4)$

$= P(x=0, y=1,2,3,4) + P(x=1, y=1,2,3) + P(x=2, y=1,2)$

$= \left(0+0+\frac{1}{32}+\frac{2}{32}\right)+\left(\frac{1}{16}+\frac{1}{16}+\frac{1}{8}\right)+\left(\frac{1}{32}+\frac{1}{32}\right)$

$= \frac{3+4+4+2}{32} = \frac{13}{32}$

Ques The joint pdf. of a two dimensional random variable is given by

$f(x,y) = xy^2 + \frac{x^2}{8} \quad 0 \le x \le 2,\ 0 \le y \le 1$

Compute:

(i) $P(X>1)$

(ii) $P(y<1/2)$ $1/4$

(iii) $P(x>1, y<1/2)$ $5/24$

(iv) $P(x>1 / y<1/2)$ $5/6$

(v) $P(x<y)$

(vi) $P(x+y \le 1)$ $\int_{x=0}^{1}\int_{y=0}^{1-x}$

(i) $P(x>1) = \int_{y=0}^{1}\int_{x=1}^{2}\left(xy^2+\frac{x^2}{8}\right)dx\,dy$

$= \int_{y=0}^{1}\left(\frac{x^2y^2}{2}+\frac{x^3}{24}\right)_1^2 dy$

$$= \int_{y=0}^{1} \left[\left(2y^2 + \frac{1}{3}\right) - \left(\frac{y^2}{2} + \frac{1}{24}\right)\right] dy$$

$$= \int_{y=0}^{1} \left(\frac{3y^2}{2} + \frac{7}{24}\right) dy$$

$$= \left(\frac{y^3}{2} + \frac{7y}{24}\right)_0^1$$

$$= \frac{1}{2} + \frac{7}{24} = \frac{19}{24}$$

(ii) $P(y < 1/2) = \int_{y=0}^{1/2} \int_{x=0}^{2} \left(xy^2 + \frac{x^2}{8}\right) dx\, dy$

$$= \int_{y=0}^{1/2} \left(\frac{x^2y^2}{2} + \frac{x^3}{24}\right)_0^2 dy$$

$$= \int_{y=0}^{1/2} \left(2y^2 + \frac{1}{3}\right) dy$$

$$= \left(\frac{2y^3}{3} + \frac{y}{3}\right)_0^{1/2}$$

$$= \frac{2}{3}\left(\frac{1}{8}\right) + \frac{1}{2}\cdot\frac{1}{3}$$

$$= \frac{1+2}{12} = \frac{1}{4}$$

(iii) $P(x < 1, y < 1/2)$

$$= \int_{y=0}^{1/2} \int_{x=0}^{1} \left(xy^2 + \frac{x^2}{8}\right) dx\, dy$$

$$\int_{y=0}^{1/2} \left(\frac{x^2y^2}{2} + \frac{x^3}{24}\right)_0^1 dy$$

$$\int_{y=0}^{1/2} \left\{\frac{y^2}{2} + \frac{1}{24}\right\} dy$$

$$\left(\frac{y^3}{6} + \frac{y}{24}\right)_0^{1/2}$$

$$\frac{1}{8}\cdot\frac{1}{6} + \frac{1}{2}\cdot\frac{1}{24} = \frac{1}{24}$$

(iv) $P(X > 1 \mid Y < 1/2)$

$$= \frac{P(X > 1, Y < 1/2)}{P(Y < 1/2)}$$

$$\int_{y=0}^{1/2} \int_{x=1}^{2} \left(xy^2 + \frac{x^2}{8}\right) dx\, dy$$

$$\int_0^{1/2} \left(\frac{x^2y^2}{2} + \frac{x^3}{24}\right)_1^2 dy$$

$$\int_0^{1/2} \left(\frac{4y^2}{2} + \frac{8}{24} - \frac{y^2}{2} - \frac{1}{24}\right) dy$$

$$\int_0^{1/2} \left(\frac{3y^2}{2} + \frac{7}{24}\right) dy$$

$$\left(\frac{y^3}{2} + \frac{7y}{24}\right)_0^{1/2}$$

$$\frac{1}{8}\cdot\frac{1}{2} + 7\cdot$$

September

Test.

Ques 1. UP government wants to build 4 bridges over the river Ganga. The government is satisfied with the past construction of 6 contractors and has invited each of them to bid on each of the job. The final bids are as follows

	C_1	C2	C3	C4	C5	C6
B1	75.2	77.9	77.4	72.3	79.0	76.6
B2	68.8	67.3	67.9	66.4	69.2	68.2
B3	71.9	71.2	72.3	70.5	73.4	71.6
B4	74.2	70.5	76.1	73.2	74.3	75.4

Suggest the assignment of the job to the contractors at ideal minimum cost. ~~Solve the~~

Ques 2. Solve the following transportation

	A	B	C	D	E	
X	55	30	40	50	40	40
Y	35	30	100	45	60	20
Z	40	60	95	35	30	40
	25	10	20	30	15	

Find the optimum solution of given transportation problem

Solution 1. We add two dummy rows.

	C1	C2	C3	C4	C5	C6
B1	75.2	77.9	77.4	72.3	79.0	76.6
B2	68.8	67.3	67.9	66.4	69.2	68.2
B3	71.9	71.2	72.3	70.5	73.4	71.6
B4	74.2	70.5	76.1	73.2	74.3	75.4
B5	0	0	0	0	0	0
B6	0	0	0	0	0	0

Row Reduced & Column Reduced Matrix.

	C1	C2	C3	C4	C5	C6	
B1	2.9	5.6	5.1	$\boxed{0}$	6.7	4.3	✓
B2	2.4	0.9	1.5	0	2.8	1.8	✓
B3	1.4	0.7	1.8	0	2.9	1.1	✓
B4	3.7	$\boxed{0}$	5.6	2.7	3.8	4.9	
B5	$\boxed{0}$	0	0	0	0	0	
B6	0	0	$\boxed{0}$	0	0	0	

	C1	C2	C3	C4	C5	C6.	
B1	2.2	4.9	4.4	$\boxed{0}$	6.0	3.6	✓
B2	1.7	0.2	0.8	0	2.1	1.1	✓
B3	0.7	$\boxed{0}$	1.1	0	2.2	0.4	✓
B4	3.7	0	5.6	3.4	3.8	4.9	✓
B5	$\boxed{0}$	0	0	0.7	0	0	
B6	0	0	$\boxed{0}$	0.7	0	0	

	C1	C2	C3	C4	C5	C6.
B1	2.0	4.9	4.2	0		
B2	1.5	0.2	0.6	0		
B3	0.5	0		0		
B4	3.5	0		3.4		
B5	0	0.2	0	0.9	0	0
B6	0	0.2	0	0.9	0	0

	C1	C2	C3	C4	C5	C6.
B1	1.8	4.9	4.0	$\boxed{0}$	5.6	3.2
B2	1.3	0.2	0.4	$\cancel{0}$	1.7	0.7
B3	0.3	$\cancel{0}$	0.7	$\cancel{0}$	1.8	$\boxed{0}$
B4	3.3	$\boxed{0}$	5.2	3.4	3.4	4.5
B5	$\boxed{0}$	0.4	$\cancel{0}$	1.4	$\cancel{0}$	$\cancel{0}$
B6	$\cancel{0}$	0.4	$\boxed{0}$	1.4	0	0

	C1	C2	C3	C4	C5	C6
B1	1.6	4.7	3.8	$\boxed{0}$	5.4	3.0
B2	1.1	$\boxed{0}$	0.2	$\cancel{0}$	1.5	0.5
B3	0.3	$\cancel{0}$	0.7	0.2	1.8	$\boxed{0}$
B4	3.3	$\cancel{0}$	5.2	3.6	3.4	4.5
B5	$\boxed{0}$	0.4	$\cancel{0}$	1.3	$\cancel{0}$	$\cancel{0}$
B6	$\cancel{0}$	0.4	$\boxed{0}$	1.3	$\cancel{0}$	$\cancel{0}$

	C1	C2	C3	C4	C5	C6.
B1	1.4	4.7	3.6	$\boxed{0}$	5.2	2.8
B2	0.9	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1.3	0.3
B3	0.3	0.2	0.7	0.4	1.8	$\boxed{0}$
B4	3.1	$\boxed{0}$	5.0	3.6	3.2	4.3
B5	$\boxed{0}$	0.6	$\cancel{0}$	1.3	$\cancel{0}$	$\cancel{0}$
B6	$\cancel{0}$	0.6	$\cancel{0}$	1.3	$\boxed{0}$	$\cancel{0}$

$B1 \rightarrow C4$ $\quad$ $B2 \rightarrow C3$

$B3 \rightarrow C6$ $\quad$ $B4 \rightarrow C2$

September 24, 2019

Ques The joint density function of bivariant distribution is given by

$$f(x,y) = \begin{cases} \frac{1}{3}(x+y) & 0 \le x \le 2 \quad 0 \le y \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(i) determine marginal distribution of x & y

(ii) show that x & y are stochastically dependent

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$$

$$\int_{x=0}^{2}\int_{y=0}^{1} \frac{1}{3}(x+y)\,dx\,dy$$

$$\frac{1}{3}\int_0^2 \left(xy + \frac{y^2}{2}\right)_0^1 dx$$

$$\frac{1}{3}\left(\frac{x^2}{2} + \frac{x}{2}\right)_0^2 = \frac{1}{3}(2+1) = 1$$

$\therefore f(x,y)$ is a joint distribution function

1. Marginal distribution of x

$$G(x) = \int_0^1 f(x,y)\,dy = \int_0^1 \frac{1}{3}(x+y)\,dy$$

$$= \frac{1}{3}\left(xy + \frac{y^2}{2}\right)_0^1$$

$$= \frac{1}{3}\left(x + \frac{1}{2}\right)$$

2. Marginal distribution of y

$$H(y) = \frac{1}{3}\int_0^2 (x+y)\,dx$$

$$= \frac{1}{3}\left(\frac{x^2}{2} + xy\right) = \frac{2}{3}(1+y)$$

$$G(x)\,H(y) = \frac{1}{3}\left(\frac{2x+1}{2}\right) \cdot \frac{2}{3}(1+y)$$

$$= \frac{1}{9}(2x+1)(1+y) \neq f(x,y)$$

$\Rightarrow x$ & y are stochastically dependent.

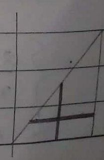
Ques The joint density function of the random variable x y is given by

$$f(x,y) \begin{cases} 8xy & 0 \le x \le 1, \quad 0 \le y \le x \\ 0 & \text{elsewhere} \end{cases}$$

1. determine marginal distribution of x & y
2. conditional probability of x & y.

$$G(x) = \int_{y=0}^{x} f(x,y)\,dy$$

$$= \int_0^x 8xy\,dy$$



$$= \left(8x\frac{y^2}{2}\right)_0^x$$

$$= 4x^3$$

$$H(y) = \int_{x=y}^{1} f(x,y)\,dx$$

$$= \int_{x=y}^{1} 8xy\,dx$$

$$= \left(8y\frac{x^2}{2}\right)_{x=y}^{1}$$

$$= (4y - 4y^3) = 4y(1-y^2)$$

Conditional probability of x

$$\frac{f(x,y)}{H(y)} = \frac{8xy}{4y(1-y^2)}$$

$$= \begin{cases} \dfrac{2x}{1-y^2} & 0 \le x \le 1 \quad 0 \le y \le x \\ 0 & \text{otherwise} \end{cases}$$

Conditional probability of Y

$$\frac{f(x,y)}{G(x)} = \frac{8xy}{4x^3}$$

$$= \begin{cases} \dfrac{2y}{x^2} & 0 \le x \le 1,\ 0 \le y \le x \\ 0 & \text{otherwise} \end{cases}$$

Ques $f(x,y) = \dfrac{9(1+x+y)}{2(1+x)^4(1+y)^4}$ $\quad 0 \le x < \infty \quad 0 \le y \le \infty$

The joint density function of bivariate random variable is given above. Find the conditional probability of Y

$$G(x) = \int_0^\infty f(x,y)\,dy$$

$$= \int_0^\infty \frac{9}{2(1+x)^4} \frac{(1+x+y)}{(1+y)^4}\,dy$$

$$= \frac{9}{2(1+x)^4}\int_0^\infty \left[\frac{1+y}{(1+y)^4} + \frac{x}{(1+y)^4}\right] dy$$

$$= \frac{9}{2(1+x)^4}\left[\frac{1}{-2(1+y)^2} + \frac{x}{-3(1+y)^3}\right]_0^\infty$$

$$= \frac{9}{2(1+x^4)}\left[0 + \frac{1}{2} + 0 + \frac{x}{3}\right]$$

$$= \left(\frac{3+2x}{\cancel{6}_2}\right)\frac{\cancel{9}^3}{2(1+x^4)}$$

Conditional probability of y

$$\frac{f(x,y)}{G(x)} = \frac{\dfrac{9(1+x+y)}{2(1+x)^4(1+y)^4}}{\dfrac{3}{4}\dfrac{(3+2x)}{(1+x)^4}}$$

Ques The joint density function of two continuous random variables is given by

Page No. 018

$$f(x,y) = \begin{cases} cxy & 0<x<4 \quad 1<y<5 \\ 0 & \text{otherwise} \end{cases}$$

(i) determine c

(ii) $P(1<x<2,\ 2<y<3)$

(iii) $P(x\geq 3,\ y\leq 2)$

(iv) marginal distribution of x & y

(i)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) = 1.$$

$$\int_0^4\int_1^5 cxy \, dy \, dx = 1$$

$$\int_0^4 \left(\frac{cxy^2}{2}\right)_1^5 dx = 1$$

$$\int_0^4 cx \times 12 \, dx = 1$$

$$12c\left(\frac{x^2}{2}\right)_0^4 = 1$$

$$6c\times 16 = 1 \qquad c = \frac{1}{96}$$

(ii) $P(1<x<2,\ 2<y<3)$

$$\int_1^2\int_2^3 f(x,y)\, dx\, dy =$$

$$\int_1^2\int_2^3 cxy \, dx\, dy$$

$$\int_1^2 \left(\frac{cxy^2}{2}\right)_2^3 dx = \int_1^2 cx\left(\frac{9-4}{2}\right) dx$$

$$\int_1^2 \frac{5c}{2} x \, dx$$

$$\frac{5c}{2}\left(\frac{x^2}{2}\right)_1^2 = \frac{5c}{4}(4-1)$$

$$= \frac{5}{4}\times 3 \times \frac{1}{96_{32}} = \frac{5}{128}$$

$\frac{7}{128}$

(iii) $P(x\geq 3,\ y\leq 2)$

$$\int_3^4\int_1^2 cxy \, dy\, dx$$

$$\int_3^4 \left(\frac{cxy^2}{2}\right)_1^2 dx = \int_3^4 \left[2cx - \frac{cx}{2}\right] dx$$

$$\left[\frac{2cx^2}{2} - \frac{cx^2}{4}\right]_3^4$$

$$\frac{\cancel{2}c16}{\cancel{2}} - \frac{c16}{4} - \frac{\cancel{2}c9}{\cancel{2}} + \frac{c9}{4}$$

$$c\left[\frac{4\times16 - 16 - 4\times 9 + 9}{4}\right]$$

September 25, 2019

Moments And Expectations

Moments:

For a distribution having the values $x_1, x_2 \ldots x_n$ with frequencies $f_1, f_2 \ldots f_n$ the r^{th} movement about any point 'a' is given by

$$\mu_r' = \frac{1}{N}\sum f_i (x_i - a)^r \quad \text{[discrete R.V]}$$

or

$$\mu_r' = \int_{-\infty}^{\infty} f(x)(x_i - a)^r \, dx \quad \text{[continuous RV]}$$

If we have to find moment about origin then it is given by

$$\mu_r' = \frac{1}{N}\sum f_i x_i^r$$

$$\mu_r' = \int_{-\infty}^{\infty} f(x)\, x^r \, dx$$

Central moments (moments about mean):

$$\mu_r = \frac{1}{N}\sum f_i (x_i - \bar{x})^r \quad \text{[discrete R.V.]}$$

$$\mu_r = \int_{-\infty}^{\infty} f(x)(x - \bar{x})^r \, dx \quad \text{[continuous RV]}$$

Ques Solve the following LPP by big M method

$\min z = 2x_1 + 9x_2 + x_3$

s.t. $x_1 + 4x_2 + 2x_2 \geq 5$

$3x_1 + x_2 + 2x_3 \geq 4$

& $x_1, x_2, x_3 \geq 0$

				-2	-9	-1	0	0	-M	-M	
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5	y_6	y_7	
-M	α_6	y_6	5	1	$\boxed{4}$	2	-1	0	1	0	/4
-M	α_7	y_7	4	3	1	2	0	-1	0	1	
	$z_j - c_j$			-4M+2	-5M+9	-4M+1	M	M	0	0	
					↑				↓		

key element $\min[5/4, 4/1]$

entering vector y_2

departing vector y_6

				-2	-9	-1	0	0	-M	
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5	y_7	
-9	α_2	y_2	5M	1/4	1	1/2	-1/4	0	0	
			~~3~~	~~2~~	~~0~~					
-M	α_7	y_7	11/4	$\boxed{11/4}$	0	3/2	1/4	-1	1	× 4/11
	$z_j - c_j$			$-\frac{(11M+9)}{4}+2$	0	$-\frac{(9+3M)}{2}+1$	$\frac{9}{4}-\frac{M}{4}$	M	0	
				↑						

key element

entering y_1 departing y_7

				-2	-9	-1	0	0
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5
-9	y_2	α_2	1	0	1	5/22	-3/11	1/11
-2	x_1	α_1	1	1	0	$\boxed{6/11}$	1/11	-4/11
	$Z_j - C_j$			0 ↓	0	↑		

key element

entering vector y_3 departing vector y_1

				-2	-9	-1	0	0
C_B	B	x_b	b	y_1	y_2	y_3	y_4	y_5
-9	α_2	x_2	1/3	-2/3	1	0	-1/3	127/66
-1	α_3	x_3	11/6	11/6	0	1	1/6	-2/3

September 26, 2019.

Particular case of the moment of

put $r=0$

$$\mu_0' = \frac{1}{N}\Sigma(x_i-a)^0 f_i \qquad f_i = \frac{\Sigma f_i}{N} = 1$$

$$\mu_0 = \frac{1}{N}\Sigma f_i(x_i-\bar{x})^0 \qquad \frac{\Sigma f_i}{N} = 1$$

$$\therefore \mu_0' = \mu_0 = 1$$

put $r=1$

$$\mu_1' = \frac{1}{N}\Sigma f_i(x_i-a)$$

$$= \frac{1}{N}\Sigma f_i x_i - a\frac{1}{N}\Sigma f_i$$

$$= \bar{x} - a = d \text{ (let)}$$

$$\mu_1 = \frac{1}{N}\Sigma f_i(x_i-\bar{x})$$

$$= \frac{\Sigma f_i x_i}{N} - \bar{x}\frac{\Sigma f_i}{N}$$

$$= \bar{x} - \bar{x} = 0$$

$$\therefore \mu_1' = d \qquad \mu_1 = 0$$

put $r=2$

$$\mu_2 = \frac{1}{N}\Sigma f_i(x_i-\bar{x})^2 = \sigma^2 \text{ (variance)}$$

$\mu_2' = \frac{1}{N} \Sigma f_i (x_i - a)^2$

$= \frac{1}{N} \Sigma f_i (x_i - \bar{x} + \bar{x} - a)^2$

$= \frac{1}{N} \Sigma f_i [(x_i - \bar{x})^2 + (\bar{x} - a)^2 + 2(x_i - \bar{x})(\bar{x} - a)]$

$= \frac{1}{N} \Sigma f_i (x_i - \bar{x})^2 + (\bar{x} - a)^2 \Sigma \frac{f_i}{N} + \frac{2}{N}(\bar{x} - a) \Sigma f_i (x_i - \bar{x})$

$= \sigma^2 + (\bar{x} - a)^2 + 2(\bar{x} - a)\mu_1$

$\mu_2' = \sigma^2 + d^2$

$\mu_2' = \mu_2 + d^2 \qquad \mu_2 = \sigma^2$

Relation between first four moments:
(moments about any point & central moments & vice versa)

moments about any point and central moment

$\mu_r = (\mu_r' - \mu_1')^r$

$\mu_0 = 1 = \mu_0'$

$\mu_1 = (\mu_1' - \mu_1')^1 = 0$

$\mu_2 = (\mu_2' - \mu_1')^2$.

$\mu_2 = \mu_2' - {}^2C_1 \mu_1' (\mu_1')^1 + {}^2C_2 \mu_0' (\mu_1')^2$

$= \mu_2' - 2(\mu_1')^2 + (\mu_1')^2$

$\boxed{\mu_2 = \mu_2' - \mu_1'^2}$

$\mu_3 = (\mu_3' - \mu_1')^3$

$= \mu_3' - {}^3C_1 \mu_2' (\mu_1') + {}^3C_2 (\mu_1')(\mu_1')^2 - {}^3C_3 \mu_0' (\mu_1')^3$

$= \mu_3' - 3\mu_2'\mu_1' + 3(\mu_1')^3 - (\mu_1')^3$

$= \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$\mu_4 = (\mu_4' - \mu_1')^4$

$= \mu_4' - {}^4C_1 \mu_3' \mu_1' + {}^4C_2 \mu_2' (\mu_1')^2 - {}^4C_3 \mu_1 (\mu_1')^3 + {}^4C_4 \mu_0' (\mu_1')^4$

$= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 4(\mu_1')^4 + \mu_1'^4$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$\mu_r' = (\mu_r + d)^r$

$\mu_0 = \mu_0' = 1$

$\mu_1' = (\mu_1 + d)^1 = 0 + d = d$

$\mu_2' = (\mu_2 + d)^2$

$= \mu_2 + {}^2C_1 \mu_1 d + {}^2C_2 \mu_0 d^2$

$= \mu_2 + 0 + d^2$

$\boxed{\mu_2' = \mu_2 + d^2}$

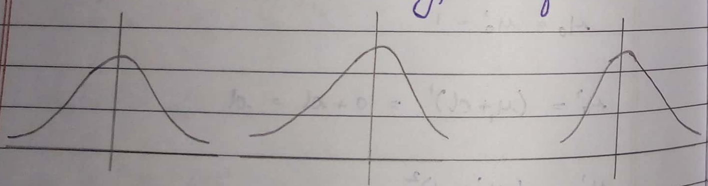
$$\mu_3' = (\mu_3 + d)^3$$
$$= \mu_3 + {}^3C_1 \mu_2 d + {}^3C_2 \mu_1 d^2 + {}^3C_3 \mu_0 d^3$$
$$= \mu_3 + 3\mu_2 d + 3\mu_1 d^2 + \mu_0 d^3$$

$$\boxed{\mu_3' = \mu_3 + 3\mu_2 d + d^3}$$

$$\mu_4' = (\mu_4 + d)^4$$
$$= \mu_4 + {}^4C_1 \mu_3 d + {}^4C_2 \mu_2 d^2 + {}^4C_3 \mu_1 d^3 + {}^4C_4 \mu_0 d^4$$
$$= \mu_4 + 4\mu_3 d + 6\mu_2 d^2 + 4\mu_1 d^3 + \mu_0 d^4$$
$$\mu_4' = \mu_4 + 4\mu_3 d + 6\mu_2 d^2 + d^4$$

Skewness:

One of the important use of the moment is to find the skewness of distribution. Skewness of distribution gives an idea about the lack of symmetry in the data. There are three types of curve.



Curve having negative values of skewness have long tail on left and the

one with positive values of skewness have long tail on the right.

Moment coefficient of skewness is given by
$$\gamma_1 = \sqrt{\beta_1} \quad \text{and} \quad \beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

September 27, 2019

Ques Find the first four moments about the mean for the following distribution.

x	$f(f_i)$	d_i	d_i^2	d_i^3	d_i^4	f_id_i	$f_id_i^2$	$f_id_i^3$	$f_id_i^4$
6	3	-3	9	-27	81	-9	27	-81	243
7	6	-2	4	-8	16	-12	24	-48	96
8	9	-1	1	-1	1	-9	9	-9	9
9	13	0	0	0	0	0	0	0	0
10	8	1	1	1	1	8	8	8	8
11	5	2	4	8	16	10	20	40	80
12	4	3	9	27	81	12	36	108	324
						0	124	18	460

$$\mu_1 = \frac{1}{N}\Sigma f_i(x_i - \bar{x}) \qquad \mu_3 = \frac{1}{N}\Sigma f_i(x_i - \bar{x})^3$$
$$\mu_2 = \frac{1}{N}\Sigma f_i(x_i - \bar{x})^2 \qquad \mu_4 = \frac{1}{N}\Sigma f_i(x_i - \bar{x})^4$$

let $x_i - \bar{x} = d_i$

$$\mu_1 = \frac{1}{N}\Sigma f_i d_i \qquad \mu_3 = \frac{1}{N}\Sigma f_i d_i^3$$
$$\mu_2 = \frac{1}{N}\Sigma f_i d_i^2 \qquad \mu_4 = \frac{1}{N}\Sigma f_i d_i^4$$

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{6(3) + 7(6) + 8(9) + 9(13) + 10(8) + 11(5) + 12(4)}{3 + 6 + 9 + 13 + 8 + 5 + 4}$

$= \frac{18 + 42 + 72 + \quad + 80 + 55 + 48}{48}$

$= 9$

$\mu_1 = 0 \qquad \mu_2 = \frac{\Sigma f_i d_i^2}{N} = \frac{124}{48}$

$\mu_3 = \frac{\Sigma f_i d_i^3}{N} = \frac{18}{40}$

$\mu_4 = \frac{\Sigma f_i d_i^4}{N} = \frac{760}{48}$

Ques Find the first four moments about the mean for following distribution

x	y	d_i	d_i^2	d_i	d_i^2	d_i^3	d_i^4	$f_i d_i$	$f_i d_i^2$	$f_i d_i^3$	$f_i d_i^4$
0	1	-3	9	-4	16	-64	256	-4	16	-64	256
1	8	4	16	-3	9	-27	81	-24	72	-216	648
2	28	24	576	-2	4	-8	16	-56	112	-224	448
3	56	52	2704	-1	1	-1	+1	-56	56	-56	56
4	70	66	4356	0	0	0	0	0	0	0	0
5	56	52	2704	1	1	1	1	56	56	+56	56
6	28	24	576	2	4	8	16	56	112	+224	448
7	8	4	16	3	9	27	81	24	72	216	648
8	1	-3	9	4	16	64	256	4	16	64	256
								0	512	0	2816

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{0(1) + 1(8) + 2(28) + 3(56) + 4(70) + 5(56) + 6(28) + 7(8) + 8}{1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1}$

$= \frac{0 + 8 + 56 + 168 + 280 + 280 + 168 + 56 + 8}{256} = \frac{1024}{256}$

$= 4$

$\mu_1 = 0 \qquad \mu_2 = \frac{\Sigma f_i d_i^2}{N} = \frac{512}{256} = 2.$

$\mu_3 = \frac{\Sigma f_i d_i^3}{N} = \frac{0}{256} = 0$

$\mu_4 = \frac{\Sigma f_i d_i^4}{N} = \frac{2816}{256} = 11$

Ques The first three moments of the distribution about the value 2 are 1, 16, -40

Show that mean = 3 variance = 15 $\mu_3 = -86$

$\mu_1' = 1 \qquad \mu_2' = 16 \qquad \mu_3' = -40$

$\frac{1}{N} \Sigma f_i (x_i - a) = 1$

$\frac{\Sigma f_i x_i}{N} - \frac{a \Sigma f_i}{N} = 1$

$\bar{x} - a = 1$

$\bar{x} - 2 = 1 \qquad \bar{x} = 3$ (mean)

Page No. 030.

$\mu_2 = \sigma^2$ (variance)

$\mu_2 = \mu_2' - \mu_1'^2$

$= 16 - (1)^2 = 15$

also $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^2$

$= -40 - 3(16)(1) + 2(4)^2$

$= -40 + 48 + 2$

$= \cancel{10} - 86$

Ques The first four moments about the value 4 of the variable are -1.5, 17, -30 and 108. Find the first four moments about the mean and also moment about the point a=2

given $\mu_1' = -1.5$

$\frac{1}{N}\sum f_i(x_i - a) = -1.5$

$\bar{x} - a = -1.5$

$\bar{x} = -1.5 + a$

$\bar{x} = 2.5$ (mean)

$\mu_2 = \sigma^2$ (variance)

$= \mu_2' - \mu_1'^2$

$= 17 - (-1.5)^2$

$= 14.75$

Page No. 031.

also $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$ $= \underline{39.75}$

$= -30 - 3(17)(-1.5) + 2(1\cancel{7})^2$

$= -30 + 76.5 + \cancel{576}\ 2\ \cancel{3}\ (2.25)2$

$\cancel{= 624.5}$ $\cancel{= 51}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 108 - 4(-30)(-1.5) + 6(17)$

September 30, 2019 $= 142.3125$

at point $a = 2$

$d = \bar{x} - a$

$= 2.5 - 2 = 0.5$

$\therefore \mu_1' = d = 0.5$

$\mu_2' = \mu_2 + d^2$

$= 14.75 + (0.5)^2 = 15$

$\mu_3' = \mu_3 + 3d\mu_2 + d^3$

$= 39.75 + 3(0.5)(14.75) + (0.5)^3$

$= 62$

$\mu_4' = \mu_4 + 4d\mu_3 + 6d^2\mu_2 + d^4$

$= 142.3125 + 4(0.5)(39.75) + 6(0.5)^2(14.75) + (0.5)^4$

$= 244$

Ques The first four moments of the distribution about the point 5 of the variant at 2, 20, 40, 50 find mean, variance, β_1, β_2 and comment on the nature of distribution

Given at $a = 5$

$\mu_1' = 2$ $\mu_2' = 20$ $\mu_3' = 40$ $\mu_4' = 50$

$\bar{x} - a = \mu_1'$

$\bar{x} - 5 = 2 \Rightarrow \bar{x} = 7$ (mean)

$\mu_2 = \sigma^2$ (variance)

$= \mu_2' - \mu_1'^2 = 20 - (2)^2$

$= 20 - 4 = 16$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= 40 - 3(20)(2) + 2(2)^3$

$= 40 - 120 + 16 = -64$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 50 - 4(40)(2) + 6(20)(2)^2 - 3(2)^4$

$= 50 - 320 + 480 - 48$

$= 162$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(-64)^2}{(16)^3} = 1$

$\beta_2 = \frac{\mu_4}{\mu_2^2} \quad \frac{162}{(16)^2} = 0.63 < 3$

here, β_1 is greater than zero hence the curve is skewed curve. Also $\beta_2 < 3$

therefore curve is platykurtic

Ques Find the skewness and kurtosis of the curve given by the distribution having its first four moments as -1.5, 17, -30 & 108 at the point 4

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3} = \frac{1580.0625}{3209.04688}$

$= 0.492377$ hence skewed

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{142.3125}{217.5625} = 0.654122$

platykurtic

Expectation:

If x is a discrete random variable which takes the value $x_1, x_2 \dots x_n$ with respective probabilities $p_1, p_2 \dots p_n$ then expectation is denoted by $E(x)$ and is given by

$$E(x) = \sum_{i=1}^{n} P_i x_i$$

we know $P_i = \frac{f_i}{N}$

$$E(x) = \sum_{i=1}^{n} \frac{f_i}{N} x_i = \bar{x}$$

Similarly $E(x^2) = \sum_{i=1}^{n} P_i (x_i)^2$

$E(x^2) = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^2$

$= \mu_2'$ (about origin)

If x is a continuous random variable then with probability density function $(f(x))$ then its expectation is given by

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Similarly $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

Properties of expectation:

1. $E(\alpha X \mp Y) = \alpha E(X) \mp E(Y)$
2. $E(XY) = E(X) \cdot E(Y)$ [if X & Y are independent]

If X and Y are two th random variables.

Moment generating function

For finding out various moments without evaluation of integral or sum we use moment generating function (mgf) and it is denoted by $M_X(t)$ and it is equal to expected value of $E(e^{tx})$

Revision

				-4	-2	0	0	0	-M
C_B	B	x_B	b	y_1	y_2	y_3	y_4	y_5	y_7
-4	α_1	x_1	24/5	1	0	-2/5	0	-1/5	0
-M	α_7	x_7	18/5	0	0	1/5	-1	-2/5	1
-2	α_2	x_2	63/5	0	1	1/5	0	-3/5	0
	$z_j - c_j$			0	0				

Duality

Que) Solve the following LPP using duality

min $z = 6x_1 + 12x_2$ // Primal

$x_1 + 4x_2 \geq 7$

$2x_1 + 3x_2 \geq 5$ $\quad x_1, x_2 \geq 0$

$C \begin{bmatrix} 6 & 12 \end{bmatrix}$ $\quad A \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $\quad B \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

$C^T \begin{bmatrix} 6 \\ 12 \end{bmatrix}$ $\quad A^T \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ $\quad B^T \begin{bmatrix} 7 & 5 \end{bmatrix}$

max $z_D = 7w_1 + 5w_2$ // dual

$w_1 + 2w_2 \leq 6$

$4w_1 + 3w_2 \leq 12$

$w_1, w_2 \geq 0$

Page No. 036.

October 16, 2019

Ques ~~A card is~~

October 17, 2019

Ace - 1, 2 = 2, 3 = 3 ... 10 - 10

Jack Queen, king - 10

Probability distribution

X	1	2	3	4	5	6	7	...	10	J	Q	K
P(x)	1/13	1/13	1/13	1/13	1/13	1/13	1/13		1/13	1/13	1/13	1/13

$E(x) = \frac{1}{13} + 2 \times \frac{1}{13} + 3 \times \frac{1}{13} + \dots \left(10 \times \frac{1}{13}\right) \times 4$

$= \frac{1}{13}$

$= 6.5$

Ques From a lot of 25 items which contains five defective a sample of four item is drawn at random

i without replacement

ii with replacement.

Page No. 037.

Find the expected number of defective in the sample

$^nC_r = \frac{n!}{r!(n-r)!}$

x	P(x)	
0	$\frac{^{20}C_4}{^{25}C_4}$	= .3830
1	$\frac{^{20}C_3 \times {^5C_1}}{^{25}C_4}$	= .4506
2	$\frac{^{20}C_2 \times {^5C_2}}{^{25}C_4}$	= .1502
3	$\frac{^{20}C_1 \times {^5C_3}}{^{25}C_4}$	= .0158
4	$\frac{^5C_4}{^{25}C_4}$	= .0004

$E(x) = 0 \times 0.3830 + 1 \times 0.4506 + 2 \times 0.1502 + 3 \times 0.0158 + 4 \times 0.0004$

$= 0 + 0.4506 + 0.3004 + 0.0474 + 0.0016$

$= 0.8$

With replacement

p = probability of non defective item

q = probability of defective item

Page No. 038.

$p = \frac{20}{25} = 0.8$ $\quad q = \frac{5}{25} = 0.2$

x	P(x)		
0	$(0.8)^4$		$= 0.4096$
1	$4(p)^3(q) =$	$4(0.8)^3(0.2)$	$= 0.4096$
2	$6(p)^2(q)^2 =$	$6(0.8)^2(0.2)^2$	$= 0.1536$
3	$4(p)(q)^3 =$	$4(0.8)(0.2)^3$	$= 0.0296$
4	$(q)^4 =$	$(0.2)^3$	$= 0.0016$

$E(x) = 0 \times 0.4096 + 1 \times 0.4096 + 2 \times 0.1536 + 3 \times 0.0296 + 4 \times 0.0016$

Ques A fair coin is tossed 3 times find the mean and variance of number of heads

x	0	1	2	3
P(x)	1/8	3/8	3/8	1/8
x^2	0	1	4	9

mean $= E(x)$

$= 0 \times 1/8 + 1 \times 3/8 + 2 \times 3/8 + 3 \times 1/8$

$= 0 + \frac{3+6+3}{8}$

$= \frac{12}{8} = \frac{3}{2}$

Page No. 039.

Variance $= E(x^2) - [E(x)]^2$

$E(x^2) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 4 \times \frac{3}{8} + 9 \times \frac{1}{8}$

$= \frac{3+12+9}{8} = \frac{24}{8} = 3$

Variance $= 3 - (3/2)^2 = 3/4$

Ques Find the value of $E(x)$, $E(x^2)$ and $E(2x+1)^2$ from the following data

x	-2	-1	0	1
P(x)	1/6	1/2	1/4	1/12
x^2	4	1	0	1
$2x+1$	-3	-1	1	3
$(2x+1)^2$	9	1	1	9

$E(x) = \Sigma P_i x_i$

$= -2 \times 1/6 + -1 \times 1/2 + 0 \times 1/4 + 1 \times 1/12$

$= -1/3 - 1/2 + 1/12$

$= -3/4$

$E(x^2) = \Sigma P_i x_i^2$

$= 1/6 \times 4 + 1/2 \times 1 + 0 \times 1/4 + 1 \times 1/12$

$= 2/3 + 1/2 + 1/12$

$= 5/4$

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$$E(2x+1)^2 = \Sigma P_i (2x+1)^2$$
$$= 9 \times 1/6 + 1 \times 1/2 + 1 \times 1/4 + 9 \times 1/12$$
$$= 3/2 + 1/2 + 1/4 + 3/4$$
$$= 3$$

Ques Let the random variable x be distributed as follows

x	1	2	3	4	5
$P(x)$	$0.1K$	K	$0.2K$	$2K$	$0.3K$

$$\Sigma P_i = 1$$
$$0.1K + K + 0.2K + 2K + 0.3K = 1$$
$$3.6K = 1$$
$$K = 1/3.6$$
$$K = \frac{10}{36} = \frac{5}{18}$$

~~X 1 2 3~~ 4

$P(x)$

$$\Sigma P_i x_i = E(x)$$
$$1 \times 0.1K + 2 \times K + 3 \times 0.2K + 4 \times 2K + 5 \times 0.3K$$
$$0.1K + 2K + 0.6K + 8K + 1.5 \cancel{3} K$$
$$\cancel{16K} \quad 12.2K$$
$$\frac{3\ \cancel{16} \times 5}{\cancel{18}\ 9} = \frac{\cancel{40}}{\cancel{9}}$$

$$E(x) = 12.2K$$
$$= 12.2 \times \frac{5}{18} = \underline{\underline{3.0138}}$$

October 18, 2019

A continuous random variable has a pdf given by $f(x) = Kx^2 e^{-x}$; $x \geq 0$. find the variable K, mean and variance

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_0^{\infty} f(x)\, dx = 1 \quad \Rightarrow \int_0^{\infty} Kx^2 e^{-x}\, dx = 1$$

$$K \int_0^{\infty} x^2 e^{-x}\, dx$$

$$K \left[-x^2 e^{-x} + \int 2x e^{-x} \right]_0^{\infty} = 1$$

$$K \left[-x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) \right]_0^{\infty} = 1$$

$$K \left[0 + 2(0+0) + 2 \right] = 1$$
$$K = 1/2$$

$$\text{mean} = E(x)$$
$$= \int_0^{\infty} x\, f(x)\, dx$$
$$= \int_0^{\infty} x\, Kx^2 e^{-x}\, dx$$

$= \frac{1}{2} \int_0^\infty x^3 e^{-x}\, dx$

$= \frac{1}{2}\left[-x^3 e^{-x} + \int 3x^2 e^{-x}\right]_0^\infty$

$= \frac{1}{2}\left[-x^3 e^{-x} + 3\left[-x^2 e^{-x} + \int 2x\, e^{-x}\right]\right]_0^\infty$

$= \frac{1}{2}\left\{-x^3 e^{-x} + 3\left[-x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right)\right]\right\}_0^\infty$

$= 3$

Variance $= E(x^2) - [E(x)]^2$

$= \cancel{12}\ 12 - (3)^2 = 3$

Binomial Distribution:

Physical condition for binomial distribution

1) The number of trials n is finite

2) each trial has two exhaustive and mutually disjoint outcomes called as success and failure.

3) The trials are independent of each other

4) The probability of success is constant for each trial hence, we have the

binomial probability distribution given by

$B(n, r, p) = {}^nC_r\, p^r\, q^{n-r} = P(X=r)$

$p + q = 1$

probability of getting r success in n trials where p is probability of success and q is probability of failure.

Ques ten coins are tossed together. Find the probability of getting atleast 7 heads.

$p = 1/2 \quad q = 1/2 \quad n = 10$

probability of getting atleast 7 heads

$= P(7) + P(8) + P(9) + P(10)$

$\cancel{{}^{10}C_7\, p}$

$= {}^{10}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$

$= \left(\frac{1}{2}\right)^{10}\left[{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$

$= \left(\frac{1}{2}\right)^{10}\left[\frac{10.9.8}{1.2.3} + \frac{10.9}{1.2} + \frac{10}{1} + 1\right]$

$= \frac{1}{1024}\left[45 + 10 + 1 + 120\right]$

$\frac{176}{1024}$

Ques 10% ~~gold~~ bolt produced by a machine are defective. Find the probability of following when they are checked at random by examining a sample of 5.

(i) none is defective
(ii) one is defective
(iii) atmost one is defective

p is probability for being defective

$p = 1/10 \quad q = 9/10$

$p = 0.1 \quad q = 0.9 \quad n = 5$

$P(0) = {}^5C_0 (0.1)^0 (0.9)^5$

$= 1.(0.9)^5 = .5905$

$P(1) = {}^5C_1 (0.1)^1 (0.9)^4$

$= 5 (0.1)(0.9)^4 = .3281$

at most one is defective

$= P(0) + P(1)$

$= (0.9)^4 [0.9 + 0.5]$

$= 0.9186$

Ques 6 dice are thrown 729 times. How many times do you expect at least 3 dice to show 5 or 6.

let getting a 5 or 6 on the die be success

$p = 1/3 \quad q = 2/3 \quad n = 6$

probability of getting atleast 3 success

$= P(3) + P(4) + P(5) + P(6)$

$= {}^6C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + {}^6C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + {}^6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1 + {}^6C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^0$

$\cancel{= {}^6C_3\left(\frac{1}{3}\right)^3 \left[\left(\frac{2}{3}\right)^3 +\right.}$

$= \left(\frac{1}{3}\right)^3 \left[{}^6C_3\left(\frac{2}{3}\right)^3 + {}^6C_4\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2 + {}^6C_5\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 + {}^6C_6\left(\frac{1}{3}\right)^3\right]$

$= \frac{233}{729}$

no of times to get success $\frac{233}{729} \times 729 = 233$

Ques Out of 800 families with four children each. How many families would be expected to have

(i) 2 boys, 2 girls
(ii) at least one boy
(iii) at most 2 girls

Assume equal probabilities for boys & girls

Page No. 046

let getting a boy be success
$p = \frac{1}{2}$ $q = \frac{1}{2}$

2 boys 2 girls
$P(2) = {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$= 300$

at least one boy
$1 - P(0)$
$1 - {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$

at most 2 girls
$P(2) + P(3) + P(4)$
${}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 + {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0$

October 21, 2019

Ques1. The probability that at any moment one telephone line out of 10 will be busy is 0.2
(i) what is the probably that five lines are busy.
(ii) find the expected number of busy lines and also find out the probability of this number
(iii) what is the probability that all lines are busy.

Page No. 047

Ques2. The probability that a bomb dropped from the plane strikes the target is 1/5. If six bombs are dropped, find the probability that at least two will strike the target and exactly two will strike the target.

Ques3. Assuming that half the population is vegetarian so that chance of being vegetarian is 1/2. Assuming that 100 investigators can take sample of 10 individuals to see whether they are vegetarian, how many investigators would you expect to report that 3 people or less were vegetarians

Answers
1. (i) 0.02642 (ii) 2 = expected no. $P(2) = 0.3019$
(iii) 0.0000001
2. (i) 0.3446 (ii) 0.2457
3. 17

Mean and variance of binomial distribution

mean $= \sum_{r=0}^{n} r\, p(r)$

$= \sum_{r=0}^{n} r\, {}^nC_r p^r q^{n-r}$

$= {}^nC_1 p^1 q^{n-1} + {}^nC_2 p^2 q^{n-2} + {}^nC_3 p^3 q^{n-3} + \dots$

$= npq^{n-1} + \frac{2n(n-1)}{1\cdot 2} p^2 q^{n-2} + \frac{3n(n-1)(n-2)}{1\cdot 2\cdot 3} p^3 q^{n-3} \dots$

$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{1\cdot 2} p^2 q^{n-3} + \frac{(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3} p^3 q^{n-4} + \dots\right]$

$= np\,(p+q)^{n-1}$

$= np \qquad [p+q=1]$

variance $= E(x^2) - [E(x)]^2$

$= \sum_{r=0}^{n} r^2 p(r) - (np)^2$

$= \sum_{r=0}^{n} (r + (r-1)r)\, p(r) - n^2p^2$

$= \sum_{r=0}^{n} r\, p(r) + \left[\sum_{r=0}^{n} r(r-1)\, {}^nC_r p^r q^{n-r}\right] - n^2p^2$

$= np + \left[2(1)\, {}^nC_2 p^2 q^{n-2} + 3(2)\, {}^nC_3 p^3 q^{n-3} + \right.$

$\left. 4(3)\, {}^nC_4 p^4 q^{n-4} + \dots \right] - n^2p^2$

$= np + \left[2\cdot \frac{n(n-1)}{1\cdot 2} p^2 q^{n-2} + 3\cdot \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} p^3 q^{n-3} + \dots\right] - n^2p^2$

$= np + n(n-1)p^2\left[q^{n-2} + \frac{(n-2)}{1\cdot 2} pq^{n-3} + \dots\right] - n^2p^2$

$= np + n(n-1)p^2 (q+p)^{n-2} - n^2p^2$

$= np + n(n-1)p^2 - n^2p^2$

$= np - np^2$

$= np(1-p) = npq$

Moment generating function of binomial distribution

mgf $= E(e^{tx})$

$= \sum e^{tr} p(r)$

$= \sum e^{tr}\, {}^nC_r p^r q^{n-r}$

$= \sum {}^nC_r (pe^t)^r q^{n-r}$

$= (pe^t + q)^n$

Poisson's Distribution

Poisson's distribution is discrete probability distribution It is a limiting case of binomial distribution under following conditions

i. the number of trials (n) is indefinitely large $n \to \infty$

p the probability of success for each trial is indefinitely small $p \to 0$

$n \times p = \lambda$ is finite positive real number.

i.e. a random variable is said to follow Poisson distribution if it assumes only non negative values and probability mass function is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

October 22, 2019 Mean and Variance

Mean $= E(x) = \mu_1'$

$$= \sum_{x=0}^{\infty} x\, P(x)$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-1)!}$$

$$= e^{-\lambda}\left[\frac{\lambda}{0!} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \dots\right]$$

$$= \lambda e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \dots\right]$$

$$= \lambda e^{-\lambda} e^{\lambda} \left[\text{as } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right]$$

$$= \lambda$$

$$\mu_2' = E(x^2)$$

$$= \sum_{x=0}^{\infty} x^2 P(x)$$

$$= \sum_{x=0}^{\infty} (x + (x+1)x) \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} \frac{x(x-1)}{x!} e^{-\lambda}\lambda^x$$

$$= \lambda + \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-2)!}$$

$$= \lambda + e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!}$$

$$= \lambda + e^{-\lambda}\left[\frac{\lambda^2}{0!} + \frac{\lambda^3}{1!} + \frac{\lambda^4}{2!} + \dots\right]$$

$$= \lambda + \lambda^2 e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \dots\right]$$

$$= \lambda + \lambda^2 e^{-\lambda} e^{\lambda}$$

$$= \lambda + \lambda^2$$

distribution It is a limiting case of binomial distribution under following conditions

i. the number of trials (n) is indefinitely large $n \to \infty$.

p the probability of success for each trial is indefinitely small $p \to 0$

$n \times p = \lambda$ is finite positive real number.

i.e. a random variable is said to follow Poisson distribution if it assumes only non negative values and probability mass function is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

October 22, 2019 Mean and Variance

Mean $= E(x) = \mu_1'$

$$= \sum_{x=0}^{\infty} x\, P(x)$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-1)!}$$

$$= e^{-\lambda}\left[\frac{\lambda}{0!} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \cdots\right]$$

$$= \lambda e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= \lambda e^{-\lambda} e^{\lambda} \quad \left[\text{as } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right]$$

$$= \lambda$$

$$\mu_2' = E(x^2)$$

$$= \sum_{x=0}^{\infty} x^2 P(x)$$

$$= \sum_{x=0}^{\infty} (x + (x+1)x) \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} \frac{x(x-1)}{x!} e^{-\lambda}\lambda^x$$

$$= \lambda + \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-2)!}$$

$$= \lambda + e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!}$$

$$= \lambda + e^{-\lambda}\left[\frac{\lambda^2}{0!} + \frac{\lambda^3}{1!} + \frac{\lambda^4}{2!} + \cdots\right]$$

$$= \lambda + \lambda^2 e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= \lambda + \lambda^2 e^{-\lambda} e^{\lambda}$$

$$= \lambda + \lambda^2$$

variance $= \mu_2' - (\mu_1')^2$

$= E(x^2) - [E(x)]^2$

$= \lambda + \lambda^2 - \lambda^2$

$= \lambda$

$\therefore$ mean $=$ variance $= \lambda$

$\mu_3' = \sum_{x=0}^{\infty} x^3 P(x)$

$= \lambda^3 + 3\lambda^2 + \lambda$

$\mu_4' = \sum_{x=0}^{\infty} x^4 P(x)$

$= \lambda^4 + 8\lambda^3 + 7\lambda^2 + \lambda$

Central Moments:

$\mu_1 = 0 \qquad \mu_2 =$ variance $= \lambda$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= (\lambda^3 + 3\lambda^2 + \lambda) - 3(\lambda^2 + \lambda)(\lambda) + 2(\lambda)^3$

$= \lambda$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1' - 3\mu_1'^4$

$= 3\lambda^2 + \lambda$

moment generating function (mgf)

$M_x(t) \doteq E(e^{tx})$

$= \sum_{x=0}^{\infty} e^{tx} P(x)$

$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^x}{x!}$

$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t\lambda)^x}{x!}$

$= e^{-\lambda}\left[1 + \frac{e^t\lambda}{1!} + \frac{(e^t\lambda)^2}{2!} + \dots\right]$

$= e^{-\lambda} e^{e^t\lambda}$

$= e^{\lambda(e^t - 1)}$

Ques Suppose on an average one house in thousand in certain district has a fire during a year. If there are 2000 houses in that district what is the probability that exactly five houses will have a fire during a year. given that $e^{-2} = 0.1353$

Let success be the house has fire

$p =$ probability of success 0.001

$n = 2000$

$\lambda = np = 2$

$P(X=5) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$= \frac{e^{-2}(2)^5}{5!}$

$= \frac{0.1353 \times 32}{120} = 0.036$

Ques A car hire firm has two cars which it hires day to day. The number of demands for a car in each day is distributed as Poisson distribution with mean = 1.5. Calculate the probabilities of days on which neither car is used and probability of the day on which some demand is refused given that $e^{-1.5} = 0.2231$

given mean = 1.5 = np = λ

$P(0) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-1.5}(+1.5)^0}{0!}$

$= e^{-1.5}$

$= 0.2231$

$P(X \geq 2) = 1 - P(X \leq 2)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{e^{-\lambda}\lambda^{0}}{0!} + \frac{e^{-\lambda}\lambda^{1}}{1!} + \frac{e^{-\lambda}\lambda^{2}}{2!}\right]$

$= 1 - e^{-\lambda}[1 + (1.5) + 1.125]$

$= 1 - 0.2231(3.625)$

$= 0.1912625$

Ques A telephone exchange on an average receives four calls per minutes. Find the probability on the basis of Poisson distribution of

(i) two or less call per minute 0.2379

(ii) upto four calls per minute 0.6283

(iii) more than four call per minute. 0.3717

given mean = 4 = λ

(i) two or less calls

$P(0) + P(1) + P(2)$

$\frac{e^{-\lambda}\lambda^{0}}{0!} + \frac{e^{-\lambda}\lambda^{1}}{1!} + \frac{e^{-\lambda}\lambda^{2}}{2!}$

$e^{-\lambda}$

Ques In a town ten accidents took place in span of 50 days. Assuming that the number of accidents follow Poisson distribution find the probability that there will be three or more accidents in a day.

average / mean $= {}^{10}/_{50} = {}^{1}/_{5} = 0.2$

Thus $\lambda = 0.2$

$$P(X \geq 3) = 1 - [P(0) + P(1) + P(2)]$$
$$= 1 - e^{\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!}\right]$$

Ques ~~Assuming that~~ Razor blades are supplied by a manufacturing company in packets of ten there is probability of $1/100$ blades to be defective. Using Poisson's distribution, calculate the number of packets containing

(i) one defective blade

(ii) no

(iii) all defective blades in a consignment of 1000 packets. Given that $e^{-0.2} = .8187$

given $n = 10$ $p = 1/100$

$np = \text{mean} = \lambda$

$= 10/100 = 0.1$

October 23, 2019

Rectangular Distribution :

Let x be a continuous random variable then x is said to have a continuous uniform distribution or rectangular distribution in finite interval (a, b) if its probability density function is constant in the interval and it is given by

$$f(x) = \begin{cases} k & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases}$$

where k is constant. and we know

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_a^b k\, dx = 1$$

$$k\, (x)_a^b = 1$$

$$k = \frac{1}{(b-a)}$$

$$\therefore f(x) = \begin{cases} \frac{1}{b-a} & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases}$$

Ques In a town ten accidents took place in span of 50 days. Assuming that the number of accidents follow Poisson distribution find the probability that there will be three or more accidents in a day.

average / mean $= 10/50 = 1/5 = 0.2$

Thus $\lambda = 0.2$

$$P(X \geq 3) = 1 - [P(0) + P(1) + P(2)]$$
$$= 1 - e^{\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!}\right]$$

Ques ~~Assuming that~~ Razor blades are supplied by a manufacturing company in packets of ten there is probability of 1/100 blades to be defective. Using Poisson's distribution, calculate the number of packets containing

(i) one defective blade

(ii) no

(iii) all defective blade in a consignment of 1000 packets. Given that $e^{-0.2}$ #.8187

given $n = 10 \quad p = 1/100$

$np = \text{mean} = \lambda$

$= 10/100 = 0.1$

October 23, 2019

Rectangular Distribution :

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$$k\,(x)_a^b = 1$$

$$k = \frac{1}{(b-a)}$$

$$\therefore f(x) = \begin{cases} \frac{1}{b-a} & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases}$$

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Page No. 058.

Mean, Variance and moments:

mean $= E(x) = \mu_1'$

$= \int_{-\infty}^{\infty} x f(x)\, dx$

$= \int_a^b \frac{x}{b-a}\, dx$

$= \frac{1}{b-a}\left(\frac{x^2}{2}\right)_a^b$

$= \frac{1}{b-a}\frac{(b^2-a^2)}{2} = \frac{1}{2(b-a)}(b+a)(b-a)$

$= \frac{(a+b)}{2}$

$\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

$= \int_a^b \frac{x^2}{b-a}\, dx = \frac{1}{(b-a)}\left(\frac{x^3}{3}\right)_a^b$

$= \frac{b^3-a^3}{3(b-a)}$

$= \frac{b^2+ab+a^2}{3}$

variance $= \mu_2' - (\mu_1')^2$

$= E(x^2) - [E(x)]^2$

$= \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$

Page No. 059.

$= \frac{b^2+ab+a^2}{3} - \frac{a^2+b^2+2ab}{4}$

$= \frac{4b^2+4ab+4a^2-3a^2-3b^2-6ab}{12}$

$= \frac{a^2+b^2-2ab}{12}$

$= \frac{(b-a)^2}{12}$

$\mu_3' = \int_{-\infty}^{\infty} x^3 f(x)\, dx$

$= \int_a^b x^3 \frac{1}{b-a}\, dx$

$= \frac{1}{(b-a)}\left(\frac{x^4}{4}\right)_a^b$

$\mu_4' = \int_{-\infty}^{\infty} x^4 f(x)\, dx$

$= \int_a^b \frac{x^4}{b-a}\, dx = \frac{1}{(b-a)}\left(\frac{x^5}{5}\right)_a^b$

moment generating function (mgf)

$M_x(t) = E(e^{tx})$

$= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$

$= \int_a^b \frac{e^{tx}}{b-a}\, dx$

Page No. 060

$$= \frac{1}{b-a}\left(\frac{e^{tx}}{t}\right)_a^b$$

$$= \frac{1}{t(b-a)}\left(e^{bt} - e^{at}\right) = \text{mgf}$$

Ques If a random variable λ is uniformly distributed over $(0,10)$ then find the probability that roots of the equation $4x^2 + 4\lambda x + (\lambda+2) = 0$ is real given it is distributed uniformly

$$\therefore f(x) \begin{cases} \frac{1}{10-0} & 0 < x < 10 \\ 0 & \text{otherwise} \end{cases}$$

roots of the given equation be real if

$$b^2 - 4ac \geq 0$$

$$(4\lambda)^2 - 4(4)(\lambda+2) \geq 0$$

$$16\lambda^2 - 16\lambda - 32 \geq 0$$

$$\lambda^2 - \lambda - 2 \geq 0$$

$$(\lambda-2)(\lambda+1) \geq 0$$

$\therefore$ required probability

$$P[(\lambda-2) \geq 0 \ \& \ (\lambda+1) \geq 0] \text{ or}$$

$$P[(\lambda-2) \leq 0 \ \& \ (\lambda+1) \leq 0]$$

Page No. 061

$$= P(\lambda \geq 2 \ \& \ \lambda \geq -1) \text{ or } P(\lambda \leq 2 \ \& \ \lambda \leq -1)$$

$$= P(\lambda \geq 2) \text{ or } P(\lambda \leq -1)$$

$$= \int_2^{\infty} f(x)\,dx + \int_{-\infty}^{-1} f(x)\,dx$$

$$= \int_2^{10} \frac{1}{10}\,dx + 0$$

$$= \frac{1}{10}(x)_2^{10} = \frac{8}{10} = \frac{4}{5}$$

Ques Bus arrives at a particular bus stop after every fifteen minutes starting from 6:00 am. If a passenger arrives at the stop at a random time that is uniformly distribute between 9 to 9:30 am then find the probability that he waits for
(i) less than 5 minutes for a bus
(ii) at least 10 minutes for a bus

Probability density function of $f(x)$ is given by:

$$f(x) = \begin{cases} 1/30 & 0 < x < 30 \\ 0 & \text{otherwise} \end{cases}$$

a person will have to wait for less than 5 minutes if he arrives between 9:10 and 9:15 or he arrives between 9:25 to 9:30

$= P(10 < x < 15)$ or $P(25 < x < 30)$

$$= \int_{10}^{15} \frac{1}{30}\, dx + \int_{25}^{30} \frac{1}{30}\, dx$$

$$= \frac{1}{30}\left[(x)_{10}^{15} + (x)_{25}^{30} \right]$$

$$= \frac{1}{30}(5+5)$$

$$= \frac{10}{30} = \frac{1}{3}$$

(ii) $P(0 < x < 5) + P(15 < x < 20)$

$$\int_{0}^{5} \frac{1}{30}\, dx + \int_{15}^{20} \frac{1}{30}\, dx$$

$$= \frac{1}{30}(5+5)$$

$$= \frac{10}{30} = \frac{1}{3}$$

October 31, 2019

Exponential Distribution.

A random variable x is said to have exponential distribution if with parameter λ if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; \ x \geq 0 \\ 0 & ; \ \text{otherwise} \end{cases}$$

and the corresponding cumulative distribution function is given by

$$F(x) = \int_{0}^{\infty} f(x)\, dx$$

i.e. $$F(x) = \int_{0}^{x} \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_{0}^{x} e^{-\lambda x}\, dx$$

$$= \lambda \left[\frac{e^{-\lambda x}}{\lambda} \right]_{0}^{x} = -\left[e^{-\lambda x} - 1 \right]$$

$$= 1 - e^{-\lambda x}$$

Mean and Variance of exponential distribution

mean $= E(x)$

$$= \int_{0}^{\infty} x f(x)\, dx$$

$$= \int_{0}^{\infty} x \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_{0}^{\infty} x e^{-\lambda x}\, dx \qquad = \lambda \int_{0}^{\infty} x^{2-1} e^{-\lambda x}\, dx$$

$$\cancel{= \lambda \left[\frac{x e^{-\lambda x}}{-\lambda} - \int \frac{1 \cdot e^{-\lambda x}}{-\lambda} \right]_{0}^{x}} \qquad = \frac{\lambda \times \sqrt{2}}{\lambda^2}$$

$$\cancel{= \lambda \left[\frac{x e^{-\lambda x}}{-\lambda} + \frac{e^{-\lambda x}}{\lambda^2} \right]_{0}^{x}} \qquad = \frac{1}{\lambda}$$

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$\cancel{\lambda \frac{x e^{-\lambda x}}{\lambda}} + \frac{e^{-\lambda x}}{}$

as $\left[\int_0^\infty e^{-ax} x^{n-1} = \cancel{\frac{\sqrt{n}}{a^n}} \quad \frac{\Gamma n}{a^n}\right.$

variance $= E(x^2) - [E(x)]^2$

$E(x^2) = \int_0^\infty x^2 f(x)\, dx$

$= \lambda \int_0^\infty x^2 e^{-\lambda x}\, dx$

$= \lambda \int_0^\infty x^{3-1} e^{-\lambda x}\, dx$

$= \lambda \left[\frac{\sqrt{3}}{\lambda^3}\right] = \cancel{\frac{2\sqrt{2}}{\lambda^2}} \quad \frac{2\,\Gamma 2}{\lambda^2}$

$= \frac{2}{\lambda^2}$

variance $= E(x^2) - [E(x)]^2$

$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$

$= \frac{1}{\lambda^2}$

Similarly, $\mu_3' = \int_0^\infty x^3 f(x)\, dx$

$= \frac{\lambda\,\Gamma 4}{\lambda^4}$

and $\mu_4' = \int_0^\infty x^4 f(x)\, dx$

$= \frac{\lambda\,\Gamma 5}{\lambda^5}$

Moment generating function :

$M_x(t) = E(e^{tx})$

$= \int_0^\infty e^{tx} f(x)\, dx$

$= \int_0^\infty e^{tx} \lambda e^{-\lambda x}\, dx$

$= \lambda \int_0^\infty e^{(t-\lambda)x}\, dx$

$= \lambda \left[\frac{e^{(t-\lambda)x}}{t-\lambda}\right]_0^\infty$

$= \frac{\lambda}{\lambda - t}$

Ques The mileage which car owner gets with certain kind of radial tyre is a random variable having exponential distribution with mean 40,000 kms. find the probabilities that one of these types will last (i) at least 20,000 kms
(ii) at most 30,000 kms.

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$$f(x) \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

given mean = 40,000

since mean = $\frac{1}{\lambda}$ $\Rightarrow$ $\lambda = \frac{1}{40,000}$

(i) $x \geq 20,000$

$$\int_{20,000}^{\infty} \frac{1}{40,000} e^{-x/40,000} \, dx$$

$$\frac{1}{40,000} \left[\frac{e^{-x/40,000}}{-1/40,000} \right]_{20,000}^{\infty}$$

$$= \frac{\cancel{1}}{\cancel{1600,000,000}} \left[+e^{-1/2} + - e^{-\infty} \right]$$

$$= e^{-1/2}$$

$$= 0.6065$$

(ii) $x \leq 30000$

$$\int_{0}^{30,000} \frac{1}{40,000} e^{-x/40,000} \, dx$$

$$\frac{1}{40,000} \int_{0}^{30,000} e^{-x/40,000} \, dx$$

$$\frac{1}{40,000} \left[\frac{-e^{-x/40,000}}{1/40,000} \right]_{0}^{30,000} dx$$

$$\left[1 - \frac{1}{e^{\frac{30000}{40000}}} \right]$$

$$1 - \frac{1}{e^{3/4}}$$

$$= 0.5276.$$

November 5, 2019

Normal Distribution

A random variable x is said to have normal distribution if its pdf is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where μ is mean and σ^2 is the variance.

Properties of Normal Distribution

The curve of normal ~~of~~ distribution is bell shaped and symmetrical about the line $x = \mu$

mean, mode and median of distribution coincide i.e. mean = mode = median.

The curve is asymptotic to the base line on either sides.

As x increases numerically, $f(x)$ decreases rapidly and the maximum probability occurs at $x = \mu$ and is equal to $\frac{1}{\sigma\sqrt{2\mu}}$

Mean and Variance of Normal Distribution

$$mean = E(x)$$

$$= \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = z$.

$$\Rightarrow dx = \sigma\, dz$$

$$x = (\sigma z + \mu)$$

$$mean = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu)\, e^{-z^2/2}\, \sigma\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \sigma z\, e^{-z^2/2} dz + \int_{-\infty}^{\infty} \mu\, e^{-z^2/2} dz \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + 2\int_{0}^{\infty} \mu\, e^{-z^2/2} dz \right]$$

$$\left[\text{as } \int_{-a}^{a} f(x)\, dx = \begin{cases} 0 & f(x) \text{ is odd} \\ 2\int_{0}^{a} f(x)\, dx & f(x) \text{ is even} \end{cases} \right]$$

Let $z^2/2 = t$

$$z\, dz = dt$$

$$dz = \frac{dt}{z} = \frac{dt}{\sqrt{2t}}$$

$$\therefore mean = \frac{2}{\sqrt{2\pi}}\, \mu \int_{0}^{\infty} e^{-t} \times \frac{dt}{\sqrt{2t}}$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t}\, t^{-1/2}\, dt$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t}\, t^{1/2 - 1}\, dt$$

$$= \frac{\mu}{\sqrt{\pi}} \times \Gamma\tfrac{1}{2} = \mu$$

$$variance = \mu_2 = E[(x - \bar{x})^2]$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $\frac{x-\mu}{\sigma} = z$

$$dx = \sigma\, dz$$

$\therefore$ variance is

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma^2 z^2 e^{-z^2/2} \times \sigma\, dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2} dz$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} z^2 e^{-z^2/2} dz$$

Let $\frac{z^2}{2} = t \Rightarrow dz = \frac{dt}{\sqrt{2t}}$

$$\therefore \text{variance} = \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} 2t\, e^{-t} \frac{dt}{\sqrt{2t}}$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} e^{-t} t^{1/2} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^{\infty} e^{-t} t^{3/2 - 1} dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \times \Gamma\frac{3}{2}$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \times \frac{1}{2} \times \Gamma\frac{1}{2}$$

$$= \sigma^2$$

Ques X is a normal variate with mean 30 and standard deviation 5, then find the following probabilities

1.) $P(26 \leq X \leq 40)$

2.) $P(X \geq 45)$

3.) $P(|X-30| \leq 5)$

(i) $z_1 = \frac{x-\mu}{\sigma} = \frac{26-30}{5} = \frac{-4}{5} = -0.8$

$z_2 = \frac{40-30}{5} = \frac{10}{5} = 2$

$\therefore P(26 \leq X \leq 40) = P(-0.8 \leq z < 2)$

$= P(-0.8 \leq z \leq 0) + P(0 \leq z \leq 2)$

$= P(0 \leq z \leq 0.8) + P(0 \leq z \leq 2)$

$= 0.2881 + 0.4772$

$= 0.7653$

(ii) $P(X \geq 45)$

$z = \frac{x-\mu}{\sigma} = \frac{45-30}{5} = \frac{15}{5} = 3$

$P(z \geq 3) = 0.5 - P(0 \leq z \leq 3)$

$= 0.5 - 0.4987$

$= 0.0023$

(iii) $P(|X-30| \leq 5)$

$= P(-5 \leq X-30 \leq 5)$

$= P(25 \leq X \leq 35)$

$z_1 = \frac{25-30}{5} = 1$

$z_2 = \frac{35-30}{5} = -1$

$P(|x-30| \le 5) \cong P(-1 \le z \le 1)$

$= P(-1 \le z \le 0) + P(0 \le z \le 1)$

$= 2P(0 \le z \le 1)$

$= 2 \times 0.3413$

$= 0.6826$

November 6/2019

Ques The income of group of 10,000 persons was found to be normally distributed with mean Rs 1750 per month and standard deviation of Rs 50. Show that of this 95% had income exceeding Rs 1668 and 5% had income exceeding 1832

(i) $z_1 = \frac{1668 - 1250}{50}$

$= -1.64$

we have to find

$P(x \ge 1668) \cong P(z \ge -1.64)$

$= P(-1.64 \le z \le 0) + 0.5$

$= 0.5 + P(0 \le z \le 1.64)$

$= 0.5 + 0.4495 = 0.9495$

$\therefore$ number of persons having salary greater than 1668 $= 10,000 \times 0.9495$

$= 9495$

$=$ approax 9590.

(ii) $z_2 = \frac{1832 - 1750}{50} = 1.64$

$\therefore$ we have to find

$P(x \ge 1832)$

$= P(z \ge 1.64)$

$= 0.5 - P(0 \le z \le 1.64)$

$= 0.5 - 0.4495$

$= 0.0505$

$\therefore$ number of persons having salary greater than 1832 $= 10000 \times 0.0505$

$= 505$ = approax ~~510~~ 5%

Ques The local authorities in certain city install 10,000 electric lamps If lamps have average life of 1000 burning hours with standard deviation of 200 Then find what number of lamps would you expect to fail in first 800 hours second b/w 1200 & 800 hours

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After what period of burning hours would you expect that
(i) 10 % of lamps would have failed
(ii) 10 % of lamps will be still burning

(i) $P(X < 800)$

$z = \frac{800 - 1000}{200} = -1$

$P(X < 800) = P(z < -1)$

$= 0.5 - P(-1 \le z \le 0)$

$= 0.5 - P(0 \le z \le -1)$

$= 0.5 - 0.3413$

$=$

(ii) $P(800 \le X \le 1200)$

$z_1 = \frac{800 - 1000}{200} = -1$

$z_2 = \frac{1200 - 1000}{200} = 1$

$P(800 \le X \le 1200)$

$= P(-1 \le z \le 1)$

$= 2P(0 \le z \le 1)$

$= 2 \times 0.3413$

$= 0.6826$

(iii) Let 10% lamps fail after x_1 hours

Let $\frac{x_1 - 1000}{200} = -z_1$

$\therefore P(x \le x_1) = 0.1$

$P(x \le x_1) \simeq P(z \le -z_1) = 0.1$

$= P(-z_1 \le z \le 0) = 0.5 - 0.1$

~~= 0.5 − 0.1~~

$= P(0 \le z \le z_1) = 0.4$

$= z_1 = 1.28$

$\frac{x_1 - 1000}{200} = -1.28 \Rightarrow x_1 = 744$

(iv) Let 10% lamps are still burning after x_1 hours.

Let $\frac{x_1 - 1000}{200} = z_1$

$\therefore P$

Curve Fitting

(i) Fitting of Straight line

$y = a + bx$

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

Ques Fit a straight line to the following set of observations

x	y	x^2	xy
1	2	1	2
2	4	4	8
3	6	9	18
4	8	16	32
5	10	25	50
15	30	55	110

now,
let the equation of line be $y = a + bx$

by method of least square we have

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$30 = 5a + 15b$

$110 = 15a + 55b$

$a = 0$ and $b = 2$

$\therefore$ equation be $y = 2x$

Ques Fit a straight line to the following data

$1 + 0.8x$

x	y	x^2	xy
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
10	16.9	30	47.1

now
let the equation of line be $y = a + bx$

by method of least square we have

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$16.9 = 5a + 10b \quad \times 2$

$47.1 = \cancel{16.9}\ 10a + 30b$

$33.8 = 10a + 20b$

$13.3 = 10b \Rightarrow b = 1.33 \Rightarrow a = 0.72$

November 7, 2019

Fitting of second degree curve or parabola.

$y = a + bx + cx^2$

By least square method.

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

Ques Fit the given set of observations to a parabola

x	y	x^2	x^3	x^4	xy	x^2y
2	8.07	4	8	16	16.14	32.28
4	12.85	16	64	256	51.4	205.6
6	31.47	36	216	1296	188.82	1132.92
8	57.38	64	512	4096	459.04	3672.32
10	91.29	100	1000	10000	912.9	9129.
30	201.06	220	1800	15664	1628.3	14172.12

now let the equation of parabola be

$y = a + bx + cx^2$

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$201.06 = 5a + 30b + 220c$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$1628.3 = 30a + 220b + 1800c$

$14172.12 = 220a + 1800b + 15664c$

$\Rightarrow a = 0.34 \quad b = -0.78 \quad c = 0.99$

Ques Fit a curve $y = ax + bx^2$ to the foll. data

x	y	x^2	x^3	xy
1	1.8	1	1	1.8
2	5.1	4	8	10.2
3	8.9	9	27	26.7
4	14.1	16	64	56.4
5	19.8	25	125	99.0
15	49.7	55	225	194.1

By least square method

$\Sigma y = a\Sigma x + b\Sigma x^2$

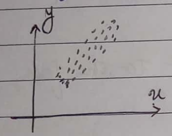
$\Sigma xy = a\Sigma x^2 + b\Sigma x^3$

$49.7 = 15x +$

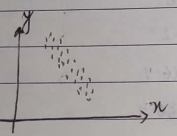
$a = 1.521 \quad b = 0.49$

Corelation :

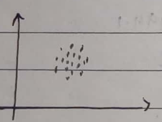
If x and y are two variables and they are plotted on xy plane then graphical representation is known as scatter diagram, which may be in the following forms



$r > 0$
(+ve corelation)



$r < 0$
(-ve corelation)



$r = 0$

If y increases as x increases then there is positive corelation and when y decreases when x increases, there is negative corelation.

when change of one variable does not affect on the second variable then there is no corelation.

Karl Pearson's coefficient of corelation

$$r_{xy} = \frac{\operatorname{cov}(x,y)}{\sqrt{\operatorname{var} x}\sqrt{\operatorname{var} y}}$$

1.) $r_{xy} = \dfrac{\sqrt{\Sigma xy}}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}}$ $\left\{ \begin{array}{l} X = x - \bar{x} \;;\; \bar{x} \text{ is mean} \\ Y = y - \bar{y} \;;\; \bar{y} \text{ is mean} \end{array} \right.$

2.) $r_{xy} = \dfrac{\sqrt{xy - \frac{1}{n}\Sigma x \Sigma y}}{\sqrt{\Sigma X^2 - \frac{1}{n}(\Sigma X)^2}\sqrt{\Sigma Y^2 - \frac{1}{n}(\Sigma Y)^2}}$ $\left\{ \begin{array}{l} x = x - a \;; \\ Y = Y - b \;; \end{array} \right.$

a & b are assumed mean

Ques Calculate the coefficient of corelation r_{xy} between x and y for the following data

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	XY	X^2	Y^2
1	5	-2	1	-2	4	1
2	4	-1	0	0	1	0
3	3	0	-1	0	0	1
4	2	1	-2	-2	1	4
5	6	2	2	4	4	4
				0	10	10

Page No. 082

$\bar{x} = \frac{1+2+3+4+5}{5} = 3$

$\bar{y} = \frac{5+4+3+2+6}{5} = 4$

$r_{xy} = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}} = \frac{0}{\sqrt{10}\sqrt{10}}$

$= 0$

Ques Calculate the coefficient of corelation for following data

x	y	$X = x-\bar{x}$	$Y = y-\bar{y}$	XY	X^2	Y^2
1	9	-4	-3	12	16	9
2	8	-3	-4	12	9	16
3	10	-2	-2	4	4	4
4	12	-1	0	0	1	0
5	11	0	-1	0	0	1
6	13	1	1	1	1	1
7	14	2	2	4	4	4
8	16	3	4	12	9	16
9	15	4	3	12	16	9
45	108			57	60	60

$\bar{x} = \frac{45}{9} = 5 \qquad \bar{y} = \frac{108}{9} = 12$

Page No. 083.

$r_{xy} = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}} = \frac{57}{\sqrt{60}\sqrt{60}}$

$= 0.95 > 0$

positive re corelation

Ques Calculate coefficient of corelation for the following data

x	y	$X = x-a$	$Y = y-b$	XY	X^2	Y^2
1	8	-5	-7	35	25	49
3	12	-3	-3	9	9	9
5	15	-1	0	0	1	0
7	17	1	2	2	1	4
8	18	2	3	6	4	9
10	20	4	5	20	16	25
34	90	-2	0	72	56	96

Let $a = 6$ and $b = 15$ be assumed mean for x & y

$r_{xy} = \frac{\Sigma XY - \frac{1}{n}\Sigma X \Sigma Y}{\sqrt{\Sigma X^2 - \frac{1}{n}(\Sigma X)^2}\sqrt{\Sigma Y^2 - \frac{1}{n}(\Sigma Y)^2}}$

$= \frac{72 - \frac{1}{6}(-2)(0)}{\sqrt{56 - \frac{1}{6}(-2)^2}\sqrt{96 - \frac{1}{6}(0)^2}}$

$= \frac{72}{\sqrt{56 - 2/3}\sqrt{96}} = \frac{72}{7.439 \times 9.797}$

$= 0.987 > 0$

Rank / Corelation of Rank

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

The above formula is not valid when rank is repeated. In that case, the rank is given by

$$r = 1 - \frac{6\Sigma d^2 - \frac{1}{12}P(P^2-1)}{n(n^2-1)}$$

where the factor $\frac{1}{12}P(P^2-1)$ is adjustment factor which is added for each repeated item.

Que Ten competitors in a beauty contest got marks by 3 judges in the following order

1st Judge	1	6	5	10	3	2	4	9	7	8
2nd Judge	6	4	9	8	1	2	3	10	5	7
3rd Judge	3	5	8	4	7	10	2	1	6	9

Use the rank corelation coefficient to discuss which pair of judges have the nearest approach to common taste in beauty

Rank I Judge	x	Rank II Judge	y	Rank III Judge	z	d_1^2 $(x-y)^2$	d_2^2 $(y-z)^2$	d_3^2 $(z-x)^2$
1	10	6	5	3	8	25	9	4
6	5	4	7	5	6	4	1	1
5	6	9	2	8	3	16	1	9
10	1	8	3	4	7	4	16	36
3	8	1	10	7	4	4	36	16
2	9	2	9	10	1	0	64	64
4	7	3	8	2	9	1	1	4
9	2	10	1	1	10	1	81	64
7	4	5	6	6	5	4	1	1
8	3	7	4	9	2	1	4	1
						60	214	200

$$r_{xy} = 1 - \frac{6\Sigma d_1^2}{n(n^2-1)} = 1 - \frac{6\times 60}{10(99)} = 0.636$$

$$r_{yz} = 1 - \frac{6\Sigma d_2^2}{n(n^2-1)} = 1 - \frac{6(214)}{10\times 99} = -0.296$$

$$r_{zx} = 1 - \frac{6\Sigma d_3^2}{n(n^2-1)} = 1 - \frac{6(200)}{10\times 99} = -0.212$$

hence, we conclud that 1st and 2nd judge have nearest approach for marks criteria.

Que The marks obtained by students in mathematics & statistics are given below

Page No. 086

maths	stats	x	y	d^2
10	30	9	9	0
15	42	5	3	4
12	45	8	2	36
17	46	3	1	4
13	33	7	8	1
16	34	4	7	9
24	40	1	4	9
14	35	6	6	0
22	39	2	5	9
				72

$(x-y)^2$

Calculate the rank corelation coefficent
Let rank in maths and statistics be x & y

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6(72)}{9(80)}$$

$$= 1 - 0.6 = 0.4$$

Ques Calculate rank corelation coefficient for the following data

Page No. 087

x	y	rank x	rank y	d $(x-y)^2$
81	10	1	8	49
78	12	2	7	25
73	18	3.5	4.5	2.25
73	18	3.5	4.5	2.25
69	18	5	4.5	0
68	29	6	2	16
62	20	7	3	16
58	24	8	1	49
				~~160~~ 159.5

$$r = 1 - \frac{6\Sigma d^2 + \frac{1}{12}P(P^2-1)}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 159.5 + \frac{1}{12} \times 2(4-1)}{8(64-1)}$$

$$= -0.9$$

Regression

Regression stands for some sort of functional relationship between two or more related variables.

Suppose scatter diagram indicates relation between variables x and y and the dots of

Scanned by CamScanner

Scatter diagram will be concentrated around a curve. This curve is called curve of regression.

Line of regression
When this curve is a line then, it is called line of regression and regression is said to be linear.

There are two types of line of regression.
Line of regression of x on y:
It is given by

$$x-\bar{x} = r\frac{\sigma x}{\sigma y}(y-\bar{y})$$

Line of regression of y on x
It is given by

$$y-\bar{y} = r\frac{\sigma y}{\sigma x}(x-\bar{x})$$

where $\bar{x}$ is mean value of x,
$\bar{y}$ is mean value of y and
r is coefficient of corelation and
σx & σy are standard deviation of x and y respectively.

Ques Find the two line of regression and coefficient of corelation for the following data.

$n = 18$ $\Sigma x = 12$ $\Sigma Y = 18$ $\Sigma x^2 = 60$
$\Sigma Y^2 = \cancel{60}\ 96$ $\Sigma XY = 48$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{12}{18} = 0.67$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{18}{18} = 1$$

$$r = \frac{\Sigma XY \frac{1}{n}\Sigma X \Sigma Y}{\sqrt{\Sigma X^2 - \frac{1}{n}(\Sigma X)^2}\sqrt{\Sigma Y^2 \frac{1}{n}(\Sigma Y)^2}}$$

$$\sigma_x = \sqrt{\Sigma X^2 - \frac{1}{n}(\Sigma X)^2} = \sqrt{60 - \frac{1}{18}(12)^2} = 7.21$$

$$\sigma y = \sqrt{\Sigma Y^2 - \frac{1}{n}(\Sigma Y)^2} = \sqrt{96 - \frac{1}{18}(18)^2} = 8.83$$

$$r = \frac{\Sigma XY - \frac{1}{n}\Sigma X \Sigma Y}{\sigma x\ \sigma y} = \frac{48 - \frac{1}{18} \times 12 \times 18}{7.21 \times 8.83}$$

$$= 0.56$$

Scatter diagram will be concentrated around a curve. This curve is called curve of regression.

Line of regression
When this curve is a line then, it is called line of regression and regression is said to be linear.

There are two types of line of regression.
Line of regression of x on y:
It is given by

$$x - \bar{x} = r \frac{\sigma x}{\sigma y} (y - \bar{y})$$

Line of regression of y on x
It is given by

$$y - \bar{y} = r \frac{\sigma y}{\sigma x} (x - \bar{x})$$

where $\bar{x}$ is mean value of x,
$\bar{y}$ is mean value of y and
r is coefficient of corelation and
σx & σy are standard deviation

of x and y respectively.

Ques Find the two line of regression and coefficent of corelation for the following data.

$n = 18$ $\Sigma x = 12$ $\Sigma Y = 18$ $\Sigma x^2 = 60$
$\Sigma Y^2 = 96$ $\Sigma XY = 48$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{12}{18} = 0.67$$

$$\bar{y} = \frac{\Sigma Y}{n} = \frac{18}{18} = 1$$

$$r = \frac{\Sigma XY - \frac{1}{n} \Sigma X \Sigma Y}{\sqrt{\Sigma X^2 - \frac{1}{n}(\Sigma X)^2} \sqrt{\Sigma Y^2 - \frac{1}{n}(\Sigma Y)^2}}$$

$$\sigma_x = \sqrt{\Sigma X^2 - \frac{1}{n}(\Sigma X)^2} = \sqrt{60 - \frac{1}{18}(12)^2} = 7.21$$

$$\sigma y = \sqrt{\Sigma Y^2 - \frac{1}{n}(\Sigma Y)^2} = \sqrt{96 - \frac{1}{18}(18)^2} = 8.83$$

$$r = \frac{\Sigma XY - \frac{1}{n} \Sigma X \Sigma Y}{\sigma x \, \sigma y} = \frac{48 - \frac{1}{18} \times 12 \times 18}{7.21 \times 8.83}$$

$$= 0.56$$

$\therefore$ Regression line x on y is

$$X - 0.67 = 0.56 \times \frac{7.21}{8.83}(y-1)$$

Regression line of Y on X is

$$Y - 1 = 0.56 \times \frac{8.83}{7.21}(X - 0.67)$$

$$Y - 1 = 0.68\,(X - 0.67)$$

November 11, 2019

Ques In a partially destroyed lab, record of analysis of correlation data, the following results were only legible. The variance of $x = 9$, regression equations

$8x - 10y + 66 = 0$

$40x - 18y = 214$

find (i) mean value of x & y

(ii) variance of y

(iii) coefficient of correlation b/w x & y

$\bar{x}$ and $\bar{y}$ will satisfy regression equation

$8\bar{x} - 10\bar{y} + 66 = 0$ — (1)

$40\bar{x} - 18\bar{y} - 214 = 0$ — (2)

(1) $\times 5$

$40\bar{x} - 50\bar{y} + 330 = 0$ — (3)

(2) − (3) $\quad 32\bar{y} - 116 = 0$

$\bar{y} = \frac{116}{32} = 17$ & $\bar{x} = 13$

iii) variance of y

$8x - 10y + 66 = 0$

$10y = 8x + 66$

$y = \frac{8}{10}x + \frac{66}{10}$

$r\frac{\sigma y}{\sigma x} = \frac{8}{10}$ — (A)

$40x - 18y = 214$

$40x = 18y + 214$

$x = \frac{18}{40}y + \frac{214}{40}$

$\frac{r\sigma x}{\sigma y} = \frac{18}{40}$

$\Rightarrow \frac{\cancel{r}\,\sigma y/\sigma x}{\cancel{r}\,\sigma x/\sigma y} = \frac{8/10}{18/40}$

$\frac{\sigma^2 y}{\sigma x^2} = \frac{8}{10} \times \frac{40}{18}$

$\sigma^2 y = \frac{16}{9} \times \sigma^2_x = 16$

$\sigma y = \pm 4$

Corelation coefficient of x & y
By A and B

Page No. 092.

$$\frac{r\,\sigma y}{\sigma x} \times \frac{r\,\sigma x}{\sigma y} = \frac{8}{10} \times \frac{18}{40}$$

$$r^2 = \frac{9}{25} \Rightarrow r = \pm\frac{3}{5}$$

Ques Two random variables have least square regression lines with equation

$$3x + 2y - 26 = 0$$

$$6x + y - 31 = 0$$

find the mean value and the coefficient of corelation between x and y

ans. $r = -0.5$ $\bar{x} = 4$ $\bar{y} = 7$

Ques Find the line of regression y on x of the following data

x	y
1	9
2	8
3	10
4	12
5	11
6	13
7	14
8	16
9	15
45	

Page No. 093.

also find y at $x = 6.2$

$$y - \bar{y} = \frac{r\,\sigma y}{\sigma x}(x - \bar{x})$$

$\bar{x} = 5$ $\bar{y} = 12$ $r = 0.95$

$$\sigma x = \sqrt{(\Sigma x^2) - \frac{1}{n}(\Sigma x)^2} = \sqrt{60}$$

$$\sigma y = \sqrt{(\Sigma y^2) - \frac{1}{n}(\Sigma y)^2} = \sqrt{60}$$

$$y - 12 = 0.95 \times \frac{\sqrt{60}}{\sqrt{60}}(x - 5)$$

$$y - 12 = 0.95(x - 5)$$