



- N.B. : (1) Questions no. 1 is compulsory.  
(2) Attempt any three questions from Q. 2 to Q. 6  
(3) Use of statistical table permitted.  
(4) Figures to the right indicate full marks.

1. (a) Evaluate  $\int_C (z - z^2) dz$ , where C is the upper half of the circle  $|z|=1$ . 5
  - (b) If  $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ , then find the eigen values of  $6A^{-1} - A^2 + 2I$  5
  - (c) State whether the following statement is true or false with reasoning: "The regression coefficients between 2x and 2y are the same as those between x and y." 5
  - (d) Construct the dual of the following L.P.P. 5
- Maximise  $Z = 3x_1 + 17x_2 + 9x_3$   
Subject to  $x_1, x_2, x_3 \geq 0$   
 $-3x_1 + 2x_2 \leq 1$   
 $2x_1 + x_2 - 3x_3 = 1$   
 $x_1, x_2, x_3 \geq 0$
2. (a) Evaluate  $\int_C \frac{e^{2z}}{(z+1)^4} dz$ , where C is the circle  $|Z-1|=3$  6
  - (b) Show that the matrix  $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$  is derogatory. 6
  - (c) A manufacturer knows from his experience that the resistance of resistors he produces is normal with  $\mu=100$  ohms and standard deviation  $\sigma=2$  ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms? 8
  3. (a) A discrete random variable has the probability distribution given below: 6
- |      |     |    |     |    |     |    |
|------|-----|----|-----|----|-----|----|
| x    | -2  | -1 | 0   | 1  | 2   | 3  |
| p(x) | 0.2 | k  | 0.1 | 2k | 0.1 | 2k |
- Find k, the mean and variance

[ TURN OVER ]

- (b) Solve the following L.P.P. by simplex method 6
- Maximise  $Z = 3x_1 + 2x_2$   
Subject to  $x_1 + x_2 \leq 4$   
 $x_1, x_2 \geq 0$
- (c) Expand  $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$  around  $z=0$ , indicating region of convergence. 8
  4. (a) Find the first two moments about the origin of Poisson distribution and hence find mean and variance. 6
  - (b) Calculate R and r from the following data: 6
- |   |     |     |     |     |     |
|---|-----|-----|-----|-----|-----|
| x | 12  | 17  | 22  | 27  | 32  |
| y | 113 | 119 | 117 | 115 | 121 |
- (R - the rank correlation coefficient, r - correlation coefficient)
- (c) Show that the matrix  $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$  is diagonalisable. 8
- Find the transforming matrix and the diagonal matrix.
5. (a) A tyre company claims that the lives of tyres have mean 42,000 kms with S.D of 4000 kms. A change in the production process is believed to result in better product. A test sample of 81 new tyres has a mean life of 42,500 kms. Test at 5% level of significance that the new product is significantly better than the old one. 6
  - (b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5 + 3 \sin \theta}$  using Cauchy's residue theorem. 6
  - (c) Using the Kuhn-Tucker conditions solve the following N.L.P.P. 8
- Minimise  $Z = 7x_1 + 5x_2 - 6x_3$   
Subject to  $x_1 + 2x_2 \leq 10$   
 $x_1 + 3x_2 \leq 9$   
 $x_1, x_2 \geq 0$

[ TURN OVER ]

6. (a) 300 digits were chosen at random from a table of random numbers. The frequency of digits was as follows. 6

Digit	0	1	2	3	4	5	6	7	8	9	Total
Frequency	28	29	33	31	26	35	32	30	31	25	300

Using  $\chi^2$  test examine the hypothesis that the digits were distributed in equal numbers in the table.

- (b) Use the dual simplex method to solve the following L.P.P. 6
- Minimise  $Z = 6x_1 + x_2$   
Subject to  $2x_1 + x_2 \geq 3$   
 $x_1, x_2 \geq 0$
- (c) (i) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches. 4
  - (ii) A random variable X has the following probability distribution 4
- |      |               |               |               |               |
|------|---------------|---------------|---------------|---------------|
| x    | 0             | 1             | 2             | 3             |
| p(x) | $\frac{1}{6}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{1}{6}$ |
- Find M.G.F about the origin and hence first four raw moments.

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Question Nos. 2 to 6.

1. (a) Evaluate  $\int_C (z - z^2) dz$  where C is the upper half of the circle  $|z| = 1$ . What is the value of the integral for the lower half of the same circle? 5
- (b) If  $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ . Find the eigen values of  $A^3 + 5A + 8I$ . 5
- (c) The regression lines of a sample are  $x + 6y = 6$  and  $3x + 2y = 10$ . Find (1) mean of  $x$  and  $y$  and (2) coefficient of correlation between  $x$  and  $y$ . 5
- (d) A machine is claimed to produce nails of mean length 5 cm. and standard deviation of 0.45 cm. A random sample of 100 nails gave 5.1 cm. as average length. Does the performance of the machine justify the claim? Mention the level of significance you apply. 5
2. (a) Show that the matrix  $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$  is derogatory. 6
- (b) Evaluate  $\int \frac{z+3}{z^2+2z+5} dz$ , where C is the circle (i)  $|z| = 1$ . (ii)  $|z+1-i| = 2$ . 6
- (c) The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 cm and the standard deviation is 0.005 cm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 0.496 to 0.508 cm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed. 8
3. (a) A continuous random variable  $X$  has the following probability law  $f(x) = kx^2 e^{-x}$ ,  $x \geq 0$ . Find  $k$ , mean and variance. 6
- (b) Solve the following LPP by Simplex method :-  

$$\begin{aligned} \text{Max } z &= x_1 + 4x_2 \\ \text{Subject to } 2x_1 + x_2 &\leq 3 \\ 3x_1 + 5x_2 &\leq 9 \\ x_1 + 3x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$
 6
- (c) Find Laurent's series which represents the function  $f(z) = \frac{2}{(z-1)(z-2)}$  when  
 (i)  $|z| < 1$  (ii)  $1 < |z| < 2$  (iii)  $|z| > 2$ . 8

[ TURN OVER

GN-Con.:6855-14.

QP Code : 12413

2

4. (a) The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sums of the squares of the deviation from the means are 26.94 and 18.73 respectively. Can the samples be considered to have been drawn from the same population? 6
- (b) Calculate the correlation coefficient from the following data :  

$$\begin{array}{cccccccccccc} X : & 23 & 27 & 28 & 29 & 30 & 31 & 33 & 35 & 36 & 39 \\ Y : & 18 & 22 & 23 & 24 & 25 & 26 & 28 & 29 & 30 & 32 \end{array}$$
 6
- (c) Show that the following matrix is Diagonalizable. Find the transforming matrix and the Diagonal matrix.  

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$
 8
5. (a) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6
- (b) Evaluate the following integral by contour integration  

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$$
 6
- (c) Use Kuhn Tucker method to solve the NLPP :-  

$$\begin{aligned} \text{Max } Z &= -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2 \\ \text{St } x_1 + x_2 &\leq 2 \\ 2x_1 + 3x_2 &\leq 12 \\ x_1, x_2 &\geq 0. \end{aligned}$$
 8
6. (a) For special security in a certain protected area, it was decided to put three lighting bulbs on each pole. If each bulb has a probability  $p$  of burning out in the first 100 hours of service, calculate the probability that at least one of them is still good after 100 hours.  
 If  $p = 0.3$ , how many bulbs would be needed on each pole to ensure 99% safety that atleast one is good after 100 hours? 6
- (b) Use Duality to solve the following LPP :  

$$\begin{aligned} \text{Max } Z &= 2x_1 + x_2 \\ \text{Subject to } 2x_1 - x_2 &\leq 2 \\ x_1 + x_2 &\leq 4 \\ x_1 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$
 6
- (c) The number of car accidents in a metropolitan city was found to be 20, 17, 12, 6, 7, 15, 8, 5, 16 and 14 per month respectively. Use  $\chi^2$  test to check whether these frequencies are in agreement with the belief that occurrence of accidents was the same during 10 months period. Test at 5% level of Significance. 8

GN-Con.:6855-14.

(3 Hours) [Total Marks : 80]

N.B. : (1) Question No. one is compulsory.

(2) Answer any three questions from Q.2 to Q.6

(3) Use of statistical Tables permitted.

(4) Figures to the right indicate full marks

1. (a) Evaluate the line integral  $\int_0^{1+i} (x^2 - iy) dz$  along the path  $y = x$  5

(b) State Cayley-Hamilton theorem & verify the same for  $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$  5

(c) The probability density function of a random variable  $x$  is

$x$	-2	-1	0	1	2	3
$P(x)$	0.1	$\frac{1}{e}$	0.2	$\frac{2}{e}$	0.3	$\frac{1}{e}$

Find i)  $E$  ii) mean iii) variance 5

(d) Find all the basic solutions to the following problem

$$\text{Maximize } z = x_1 + 3x_2 + 3x_3$$

$$\text{Subject to } x_1 + 2x_2 + 3x_3 = 4$$

$$2x_1 + 3x_2 + 5x_3 = 7$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

2. (a) Find the Eigen values and the Eigen vectors of the matrix  $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$  6

(b) Evaluate  $\oint_C \frac{dz}{z^2(2+4)}$  where  $C$  is the circle  $|z| = 2$  6

(c) If the heights of 500 students is normally distributed with mean 68 inches and standard deviation of 4 inches, estimate the number of students having heights i) less than 62 inches, ii) between 65 and 71 inches. 8

[TURN OVER]

MD-Con. 8175-15.

2 Q.P. Code : 5316

3. (a) Calculate the coefficient of correlation from the following data

$x$	30	33	25	10	33	75	40	25	90	95	45	55
$y$	68	65	80	85	70	30	55	18	12	10	35	45

(b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 100 such samples, how many would you expect to contain 3 defectives i) using the Binomial distribution ii) Poisson distribution. 6

(c) Show that the matrix  $\begin{bmatrix} -9 & 4 & 4 \\ -3 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalizable. Find the transforming matrix and the diagonal matrix. 8

4. (a) Fit a Poisson distribution to the following data

$x$	0	1	2	3	4	5	6	7	8
$f$	56	156	132	92	37	22	11	0	1

(b) Solve the following LPP using Simplex method

$$\text{Maximize } z = 6x_1 - 2x_2 + 3x_3$$

$$\text{Subject to } 2x_1 - x_2 + 2x_3 \leq 2$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

(c) Expand  $f(z) = \frac{2}{(z-2)(z-1)}$  in the regions

$$i) |z| < 1, ii) 1 < |z| < 2, iii) |z| > 2$$

5. (a) Evaluate using Cauchy's Residue theorem  $\oint_C \frac{1-2z}{z(z-1)(z-2)} dz$  where  $C$  is  $|z| = 1.5$  6

[TURN OVER]

MD-Con. 8175-15.

2 Q.P. Code : 5316

(b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6

(c) Solve the following LPP using the Dual Simplex method

$$\text{Minimize } z = 2x_1 + 2x_2 + 4x_3$$

$$\text{Subject to } 2x_1 + 3x_2 + 5x_3 \geq 2$$

$$3x_1 + x_2 + 7x_3 \leq 3$$

$$x_1 + 4x_2 + 6x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$

6. (a) Solve the following NLP using Kuhn-Tucker conditions

$$\text{Maximize } z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$$

$$\text{Subject to } 2x_1 + x_2 \leq 5; \text{ and } x_1, x_2 \geq 0$$

(b) In an experiment on immunization of cattle from Tuberculosis the following results were obtained

	Affected	Not Affected	Total
Inoculated	267	27	294
Not Inoculated	757	135	912
Total	1024	162	1206

Use  $\chi^2$  Test to determine the efficacy of vaccine in preventing tuberculosis. 6

(c) i) The regression line of  $x$  on  $y$  for a sample are  $x + 6y = 6$  and  $3x + 2y = 10$

find a) sample means  $\bar{x}$  and  $\bar{y}$  b) coefficient of correlation between  $x$  and  $y$  4

ii) If two independent random samples of sizes 15 & 8 have respectively the means and population standard deviations as

$$\bar{x}_1 = 980, \bar{x}_2 = 1012; \sigma_1 = 75, \sigma_2 = 80$$

Test the hypothesis that  $\mu_1 = \mu_2$  at 5% level of significance. 4

MD-Con. 8175-15.

[Total Marks: 80]

N.B. : (1) Question No. one is compulsory.

(2) Answer any three questions from Q.2 to Q.6

(3) Use of statistical Tables permitted.

(4) Figures to the right indicate full marks

1. (a) Find the Eigen values of  $A^2 + 2I$ , where  $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & 0 \\ 3 & 5 & 3 \end{bmatrix}$  and  $I$  is the Identity matrix of order 3. 5

- (b) Evaluate the line integral  $\int_0^{1+i} (x^2 + iy) dz$  along the path  $y = x$  5

- (c) If  $x$  is a continuous random variable with the probability density function given by  $f(x) = \begin{cases} k(x-x^3) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ , Find i) k ii) the mean of the distribution. 5

- (d) Compute Spearman's rank correlation coefficient from the following data

X	18	20	34	52	12
Y	39	23	35	18	46

2. (a) Is the following matrix Derogatory? Justify. 6

$$\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

- (b) Evaluate  $\oint_c \frac{e^{2z}}{(z-1)^2} dz$  where  $c$  is the circle  $|z| = 2$  6

- (c) The marks of 1000 students in an Examination are found to be normally distributed with mean 70 and standard deviation 5, estimate the number of students whose marks will be i) between 60 and 75 ii) more than 75. 8

[Turn over]

2

3. (a) Solve the following non-linear programming problem using Kuhn-Tucker conditions

$$\text{Maximize } z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$$

$$\text{Subject to } 2x_1 + x_2 \leq 5; \text{ and } x_1, x_2 \geq 0 \quad 6$$

- (b) Fit a Binomial distribution to the following data

x	0	1	2	3	4	5	6
F	5	18	28	12	7	6	4

- (c) Is the following matrix diagonalizable? If yes, find the transforming matrix and the diagonal matrix. 8

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

4. (a) Solve the following LPP using Simplex method

$$\text{Maximize } z = 4x_1 + x_2 + 3x_3 + 5x_4$$

$$\text{Subject to } -4x_1 + 6x_2 + 5x_3 + 4x_4 \leq 20$$

$$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$$

$$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$$

$$x_1, x_2, x_3, x_4 \geq 0 \quad 6$$

- (b) If a random variable  $X$  follows the Poisson distribution such that

$$P(X=1) = 2P(X=2), \text{ find the mean, the variance of the distribution and}$$

$$P(X=3) \quad 6$$

- (c) Expand  $f(z) = \frac{1}{z(z-2)(z+1)}$  in the regions

$$i) |z| < 1, ii) 1 < |z| < 2, iii) |z| > 2 \quad 8$$

[Turn over]

3

5. (a) Evaluate using Cauchy's Residue theorem  $\oint_c \frac{2z-1}{z(2z+1)(z+2)} dz$  where  $c$  is

$$|z| = 1. \quad 6$$

- (b) A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:

$$5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6$$

Can it be concluded that the stimulus will increase the blood pressure (at 5% level of significance)? 6

- (c) Solve the following LPP using the Dual Simplex method

$$\text{Maximise } z = -3x_1 - 2x_2$$

$$\text{Subject to } x_1 + x_2 \geq 1$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \geq 10$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0 \quad 8$$

6. (a) Find the equations of lines of regression for the following data

x	5	6	7	8	9	10	11
y	11	14	14	15	12	17	16

- (c) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$  using contour integration. 6

- (b) In an experiments on pea breeding, the following frequencies of seeds were obtained

Round and Yellow	Wrinkled and yellow	Round and green	Wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9:3:3:1.

Examine the correspondence between theory and experiment using Chi-square Test

8

QP Code : 541304

(3 Hours) | Total Marks : 80

- N.B. : (1) Question No. one is compulsory.  
 (2) Answer any three questions from Q.2 to Q.6  
 (3) Use of statistical Tables permitted.  
 (4) Figures to the right indicate full marks.  
 (5) Assume suitable data wherever applicable.

1. (a) Find the Eigenvalues and eigenvectors of the matrix.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

5

- (b) Evaluate the line integral  $\int_0^{1+i} (x^2 + iy) dz$  along the path  $y = x$

5

- (c) Find  $k$  and then  $E(x)$  for the p.d.f.

5

$$f(x) = \begin{cases} k(x-x^2), & 0 \leq x \leq 1, k > 0 \\ 0, & \text{otherwise} \end{cases}$$

- (d) Calculate Karl Pearson's coefficient of correlation from the following data.

5

x	100	200	300	400	500
y	30	40	50	60	70

2. (a) Show that the matrix  $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$  is non-derogatory.

6

- (b) Evaluate  $\int \frac{e^{z^2}}{(z+1)^4} dz$  where  $C$  is the circle  $|z-1|=3$

6

- (c) If  $x$  is a normal variate with mean 10 and standard deviation 4 find  
 (i)  $P(x-14 < 1)$  (ii)  $P(5 \leq x \leq 18)$  (iii)  $P(x \leq 12)$

8

QP Code : 541304

3. (a) Find the relative maximum or minimum (if any) of the function

6

$$Z = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 100$$

- (b) If  $x$  is Binomial distributed with  $E(x) = 2$  and  $V(x) = 4/3$ , find the probability distribution of  $x$ .

6

- (c) If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ , find  $A^{10}$ .

8

4. (a) Solve the following L.P.P. by simplex method

6

$$\begin{aligned} &\text{Minimize } z = 3x_1 + 2x_2 \\ &\text{Subject to } 3x_1 + 2x_2 \leq 18 \\ &\quad 0 \leq x_1 \leq 4 \\ &\quad 0 \leq x_2 \leq 6 \\ &\quad x_1, x_2 \geq 0. \end{aligned}$$

- (b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls.

6

- (c) Find Laurent's series which represents the function  $f(z) = \frac{2}{(z-1)(z-2)}$

8

When (i)  $|z| < 1$ , (ii)  $1 < |z| < 2$  (iii)  $|z| > 2$

5. (a) Evaluate  $\int_C \frac{z^2}{(z-1)^2(z+1)} dz$  where  $C$  is  $|z|=2$  using residue theorem

6

- (b) The regression lines of a sample are  $x+6y=6$  and  $3x+2y=10$  Find

6

(i) Sample means  $\bar{x}$  and  $\bar{y}$

(ii) Correlation coefficient between  $x$  and  $y$ . Also estimate  $y$  when  $x = 12$

- (c) A die was thrown 132 times and the following frequencies were observed

8

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Using  $\chi^2$ -test examine the hypothesis that the die is unbiased.

QP Code : 541304

3

6. (a) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9} dx$  using contour integration.

6

- (b) If a random variable  $x$  follows Poisson distribution such that  $P(x=1) = 2P(x=2)$  Find the mean and the variance of the distribution. Also find  $P(x=3)$ .

6

- (c) Use Penalty method to solve the following L.P.P.

8

$$\begin{aligned} &\text{Minimize } z = 2x_1 + 3x_2 \\ &\text{Subject to } x_1 + x_2 \geq 5 \\ &\quad x_1 + 2x_2 \geq 6 \\ &\quad x_1, x_2 \geq 0. \end{aligned}$$

(3 Hours)

[Total Marks : 80]

- N.B.: (1) Question No.1 is compulsory.  
(2) Attempt any three questions from Question No. 2 to 6.  
(3) Use of statistical Tables permitted.  
(4) Figures to the right indicate full marks.

1. (a) Show that  $\int_C \log z \, dz = 2\pi i$ , where  $C$  is the unit circle in the  $z$ -plane. 5
- (b) If  $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$  then find the eigen values of  $4A^{-1} + 3A + 2I$ . 5
- (c) It is given that the means of  $x$  and  $y$  are 5 and 10. If the line of regression of  $y$  on  $x$  is parallel to the line  $20y = 9x + 40$ , estimate the value of  $y$  for  $x = 30$ . 5
- (d) Find the dual of the following L.P.P. 5
 

Maximise  $Z = 2x_1 - x_2 + 3x_3$   
Subject to  $x_1 - 2x_2 + x_3 \geq 4$   
 $2x_1 + x_2 \leq 10$   
 $x_1 + x_2 + 3x_3 = 20$   
 $x_1, x_2 \leq 0$ ,  $x_3$  unrestricted.
2. (a) Evaluate  $\int_C \frac{z+2}{z^2-2z^2} \, dz$ , where  $C$  is the circle  $|Z-2-i| = 2$  6
- (b) Show that  $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$  is derogatory. 6
- (c) In a distribution exactly normal 7% of items are under 35 and 89% of the items are under 63. Find the probability that an item selected at random lies between 45 and 56. 8
3. (a) A continuous random variable has probability density function  $f(x) = 6(x-x)^2$ ,  $0 \leq x \leq 1$ . Find (i) mean (ii) variance. 6
- (b) Solve the following L.P.P. by simplex method 6
 

Maximise  $Z = 4x_1 + 3x_2 + 6x_3$   
Subject to  $2x_1 + 3x_2 + 2x_3 \leq 440$   
 $4x_1 + 3x_3 \leq 470$   
 $2x_1 + 5x_2 \leq 430$   
 $x_1, x_2, x_3 \geq 0$

JP-Con. 9218-15.

[TURN OVER]

Q.P. Code : 3541

2

3. (c) Find all possible Laurent's expansions of the function 8
 

$f(z) = \frac{7z-2}{z(z-2)(z+1)}$  about  $z = -1$
4. (a) Find the moment generating function of Binomial distribution & hence find mean and variance. 6
- (b) Calculate the correlation coefficient from the following data : 6
 

x :	100	200	300	400	500
y :	30	40	50	60	70
- (c) Show that the matrix  $A = \begin{bmatrix} 4 & -4 & 2 \\ 2 & -4 & 1 \end{bmatrix}$  8  
is diagonalisable. Find the transforming matrix and the diagonal matrix.
5. (a) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches. 6
- (b) Evaluate  $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ ,  $a > 0$  using contour integration. 6
- (c) Use Kuhn - Tucker conditions to solve the following N.L.P.P. 8
 

Maximise  $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$   
subject to  $3x_1 + 2x_2 \leq 6$   
 $x_1, x_2 \geq 0$

6. (a) A die was thrown 132 times and the following frequencies were observed. 6
 

No. obtained :	1	2	3	4	5	6	Total
Frequency :	15	20	25	15	29	28	132

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[TURN OVER]

Q.P. Code : 3541

3

- (b) Using duality solve the following L. P. P. 6
 

Maximise  $Z = 5x_1 - 2x_2 + 3x_3$   
Subject to  $2x_1 + 2x_2 - x_3 \geq 2$   
 $3x_1 - 4x_2 \leq 3$   
 $x_1 + 3x_3 \leq 5$   
 $x_1, x_2, x_3 \geq 0$
- (c) (i) A random sample of 50 items gives the mean 6.2 and standard deviation 10.24, can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance? 4
- (ii) Find the M.G.F. of the following distribution. 4
 

X :	-2	3	1
P (X=x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Hence find first four central moments.

Q.P. Code :23022

[Time: Three Hours]

[ Marks:80]

Please check whether you have got the right question paper.

- N.B:
1. Question No 1 is compulsory.
  2. Attempt any three questions from Q.2 to Q.6
  3. Use of statistical table permitted.
  4. Figures to the right indicate full marks.

- Q.1
- a) Evaluate  $\int_C \log z \, dz$  where  $C$  is the unit circle in the  $z$ -plane. 05
  - b) Find the eigen values of the adjoint of  $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$  05
  - c) If the arithmetic mean of regression coefficient is  $p$  and their difference is  $2q$ , find the correlation coefficient. 05
  - d) Write the dual of the following L.P.P.  
 Maximise  $Z = 2x_1 - x_2 + 4x_3$   
 Subject to  $x_1 + 2x_2 - x_3 \leq 5$   
 $2x_1 - x_2 + x_3 \leq 6$   
 $x_1 + x_2 + 3x_3 \leq 10$   
 $4x_1 + x_3 \leq 12$   
 $x_1, x_2, x_3 \geq 0$  05
- Q.2
- a) Evaluate  $\int_C \frac{\cot z}{z} \, dz$  where  $C$  is the ellipse  $9x^2 + 4y^2 = 1$  06
  - b) Show that  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$  is non-derogatory. 06
  - c) If  $X$  is a normal variate with mean 10 and standard deviation 4, find  
 i)  $P(|X - 14| < 1)$ , ii)  $P(5 \leq X \leq 18)$ , iii)  $P(X \leq 12)$  08

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Q.P. Code :23022

- Q.3
- a) Find the expectation of number of failures preceding the first success in an infinite series of independent trials with constant probabilities  $p$  &  $q$  of success and failure respectively. 06
  - b) Using Simplex Method solve the following L.P.P.  
 Maximise  $Z = 10x_1 + x_2 + x_3$   
 Subject to  $x_1 + x_2 - 3x_3 \leq 10$   
 $4x_1 + x_2 + x_3 \leq 20$   
 $x_1, x_2, x_3 \geq 0$  06
  - c) Expand  $f(z) = \frac{1}{z(z+1)(z+2)}$   
 (i) Within the unit circle about the origin.  
 (ii) within the annulus region between the concentric circles about the origin having radii 1 and 2 respectively.  
 (iii) In the exterior of the circle with centre at the origin and radius 2. 08
- Q.4
- a) If  $X$  is Binomial distributed with mean=2 and variance = 4/3, find the probability distribution of  $X$ . 06
  - b) Calculate the value of rank correlation coefficient from the following data regarding score of 6 students in physics & chemistry test.  
 Marks in Physics : 40, 42, 45, 35, 36, 39  
 Marks in Chemistry : 46, 43, 44, 39, 40, 43 06
  - c) Is the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  diagonalisable? If so find the diagonal form and the transforming matrix. 08
- Q.5
- a) A random sample of 50 items gives the mean 6.2 and standard deviation 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance? 06
  - b) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}$ ,  $a>0$  Using Cauchy's residue theorem. 06
  - c) Using Kuhn-Tucker condition to solve the following N.L.P.  
 Maximise  $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$   
 Subject to  $3x_1 + 2x_2 \leq 6$   
 $x_1, x_2 \geq 0$  08

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Q.P. Code :23022

- Q.6
- a) The following table gives the number of accidents in a city during a week. Find whether the accidents are uniformly distributed over a week. 06  

Day:	Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Total.
No. of accidents:	13	15	9	11	12	10	14	84
  - b) If two independent random samples of sizes 15 & 8 have respectively the following means and population standard deviations,  
 $\bar{X}_1 = 980$   $\bar{X}_2 = 1012$   
 $\sigma_1 = 75$   $\sigma_2 = 80$   
 Test the hypothesis that  $\mu_1 = \mu_2$  at 5% level of significance.  
 (Assume the population to be normal) 06
  - c) Using Penalty (Big M) method solve the following L.P.P.  
 Minimise  $Z = 2x_1 + x_2$   
 Subject to  $3x_1 + x_2 \geq 3$   
 $4x_1 + 3x_2 \geq 6$   
 $x_1 + 2x_2 \leq 3$   
 $x_1, x_2 \geq 0$  08