

QUESTION BANK - II (TEST - I)
SE - IT (SEM - IV) APPLIED MATHEMATICS - IV

Q1. The prob distribution of a random variable X is given by

X	-3	-2	-1	0	1	2
$P(X=x)$	0.01	0.1	0.2	$2k$	0.2	k

Find (i) k (ii) $P(X < 0)$ (iii) $V(X)$

Q2. A random variable X has the prob distribution
 $P(X=0) = P(X=2) = p$, $P(X=1) = 1-2p$, $0 \leq p \leq 2/3$.
 For what value of p is the $\text{Var}(X)$ maximum?

Q3. Find k in each of the following cases so that the following can be regarded prob distribution:
 Also find cumulative distribution, expectation, and variance of X .

a)

$X=x$	-3	2	50
$P(X=x)$	$(k-4)/5$	$2/k$	$(11-k)/5k$

b)

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 1 \\ (2-x)^2 & 1 \leq x \leq 2 \end{cases}$$

c)

$$f(x) = \begin{cases} kx^2 e^{-x} & 0 \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

Q4. Find the prob distribution function given the cumulative distribution function as

a)	x	1	2	3	4
	F(x)	15/61	25/61	55/61	1

b)
$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

Q5. Calculate first four raw moments about origin and first four central moments for X which denotes the number of tails in three tosses of an unbiased coin.

Q6. Compute the first three moments about 37 from the following data and then find them about the mean.

x_i	31	34	37	40	43	46
f_i	2	4	26	47	15	6

Q7. Find the m.g.f of the following, hence find mean and variance

a)
$$f(x) = \begin{cases} x & 0 \leq x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

b) X	1	3	4	5
P(X=x)	0.4	0.1	0.2	0.3

Q8. Calculate the coefficient of correlation from the following data :

i)

X	30	33	25	10	33	75	40	85	90	95
Y	68	65	80	85	70	30	55	18	15	10

ii) $N=10$, $\sum x = 100$, $\sum y = 150$, $\sum (x-10)^2 = 180$
 $\sum (y-15)^2 = 215$, $\sum (x-10)(y-15) = 60$

Q9. Calculate Spearman's Rank correlation coefficient.

(i) Distribution of marks in Economics and Mathematics of 10 students in a certain test are given below:

Eco. :	25	28	32	36	38	40	39	42	41	45
Maths :	70	80	85	75	57	65	48	50	54	66

(ii)

X :	104	104	104	101	100	99	98	96	93	92
Y :	101	103	100	100	98	95	96	104	92	92

Binomial, Poisson, Normal Distribution

- 1) A die is thrown repeatedly 36 times in all. Find $E(X)$ and $V(X)$ where X is the number of sixes obtained.
- 2) The probability that a car travelling along a certain road will have a tyre burst is 0.05. Find the probability that among 17 cars:
 - (a) exactly one has a burst tyre,
 - (b) at most three have a burst tyre,
 - (c) two or more have burst tyres.
- 3) A Transmission channel transmits zeros and ones in strings of length 8. (Call these words). possible distortion may change a one to a zero or vice versa; assume this distortion occurs with probability .01 for each digit, independently. An error-correcting code is employed in the construction of the word such that the receiver can deduce the word correctly if at most one digit is in error. What is the probability the word is decoded incorrectly?
- 4) An examination consists of 10 multi-choice questions, in each of which a candidate has to deduce which one of five suggested answers is correct. A completely unprepared student may be assumed to guess each answer completely randomly. What is the probability that this student gets 8 or more questions correct? Draw the appropriate moral !

- 5) Fit a Binomial distribution to the following data:

X	0	1	2	3	4	5	6
Frequency	5	18	28	12	7	6	4

- 6) Find the moment generating function for Binomial distribution and hence the mean and the variance .
- 7) Find the recurrence relation for the Binomial distribution.
- 8) A radioactive source emits 4 particles on average during a five-second period.
 - a) Calculate the probability that it emits 3 particles during a 5-second period.
 - b) Calculate the probability that it emits at least one particle during a 5-second period.
 - c) During a ten-second period, what is the probability that 6 particles are emitted
- 9) The number of typing mistakes made by a secretary has a Poisson distribution. The mistakes are made independently at an average rate of 1.65 per page. Find the probability that a three-page letter contains no mistakes.
- 10) A 5-litre bucket of water is taken from a swamp. The water contains 75 mosquito larvae. A 200mL flask of water is taken from the bucket for further analysis. What is
 - a) the expected number of larvae in the flask?
 - b) the probability that the flask contains at least one mosquito larva?

Binomial, Poisson, Normal Distribution

11) Find the moment generating function for Poisson distribution and hence the mean and the variance.

12) Find the recurrence relation for the Poisson distribution.

13) Fit a Poisson distribution to the following data:

X	0	1	2	3	4	5	6	7	8
Frequency	70	137	135	89	44	17	6	2	0

14) Potassium blood levels in healthy humans are normally distributed with a mean of 17.0 mg/100 ml, and standard deviation of 1.0 mg/100 ml. Elevated levels of potassium indicate an electrolyte balance problem, such as may be caused by Addison's disease. However, a test for potassium level should not cause too many "false positives". What level of potassium should we use so that only 2.5 % of healthy individuals are classified as "abnormally high"?

15) For a particular type of wool the number of 'crimps per 10cm' follows a normal distribution with mean 15.1 and standard deviation 4.79.

(a) What proportion of wool would have a 'crimp per 10 cm' measurement of 6 or less?

(b) If more than 7% of the wool has a 'crimp per 10 cm' measurement of 6 or less, then the wool is unsatisfactory for a particular processing. Is the wool satisfactory for this processing?

16) The finish times for marathon runners during a race are normally distributed with a mean of 195 minutes and a standard deviation of 25 minutes.

a) What is the probability that a runner will complete the marathon within 3 hours?

b) Calculate to the nearest minute, the time by which the first 8% runners have completed the marathon.

c) What proportion of the runners will complete the marathon between 3 hours and 4 hours?

17) The download time of a resource web page is normally distributed with a mean of 6.5 seconds and a standard deviation of 2.3 seconds.

a) What proportion of page downloads take less than 5 seconds?

b) What is the probability that the download time will be between 4 and 10 seconds?

c) How many seconds will it take for 35% of the downloads to be completed?

18) The marks obtained by students in an examination follow a normal distribution. If 30% of the students got below 35 marks and 10% got above 60 marks, find the % of students who got marks between 40 and 50.

19) If X and Y are two independent normal variates with means 3, 8 and standard deviation 4, 5 respectively, find the probability that a point will lie between the lines $5X+3Y=8$ and $5X+3Y=15$.

Binomial, Poisson, Normal Distribution

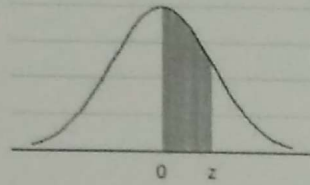
- 20) The mean and standard deviation (S.D) of marks obtained by students in Mathematics and Physics are given below:

	Mean	S.D
Mathematics	50	10
Physics	55	12

Assuming the marks in the two subjects to be independent normal variates, obtain the probability that a student scores marks between 100 and 130 marks in the two subjects taken together.

- 21) The time when a city-bus arrives at a certain bus stop is distributed normally with a mean of 8.25a.m. and standard deviation of 4 minutes. What is the least time one should arrive at this stop that probability of catching the bus is 0.99?
- 22) A coin is tossed 500 times. Find the probability of getting 240 to 280 heads if the probability of a single head is 0.4. Use Binomial approximation to Poisson distribution.
- 23) At a telephone exchange, on an average 16 calls are received per hour. Find the chance that during a certain hour, the number of calls received lies between 8 and 20 (including both), using normal approximation to Poisson distribution.
- 24) At a lawyer's office on an average 9 clients arrive per hour. On a certain day the lawyer is late by an hour. Find the chance that during that hour's time before lawyer arrives at least 15 clients have arrived, using normal approximation to Poisson distribution.

Table of Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Note:

Middle 50% area = ± 0.6745
 Middle 90% area = ± 1.6449
 Middle 95% area = ± 1.9600
 Middle 99% area = ± 2.5758

For any Normal Variate (with Mean = μ and Std. Deviation = σ):
 $\mu \pm 1\sigma = 0.6827$
 $\mu \pm 2\sigma = 0.9545$
 $\mu \pm 3\sigma = 0.9973$