PBSR Assignment 2

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Question 5

- Consider the following model:

$$r_t^{TCS} = \alpha + \beta r_t^{Nifty} + \varepsilon,$$

where $\mathbb{E}(\varepsilon) = 0$ and $\mathbb{V}ar(\varepsilon) = \sigma^2$.

1. Estimate the parameters of the models $\theta = (\alpha, \beta, \sigma)$ using the method of moments type plug-in estimator discussed in the class.

Let $x_i = r_t^{Nifty}$ and $y_i = r_t^{TCS}$

Then we are given $y_i = \alpha + \beta x_i + \epsilon$

where $\mathbb{E}(\epsilon) = 0$ and $Var(\epsilon) = \sigma^2$

$\mathbb{E}[\epsilon] = 0 \implies \mathbb{E}[y_i] = \alpha + \beta\,\mathbb{E}[x_i]$

$Var[\epsilon] = \sigma^2$ and independence of x_i and $\epsilon \implies Var(y_i) = \beta^2 Var(x_i) + \sigma^2$

x_i and ϵ are independent and $E[\epsilon] = 0 \implies \mathbb{E}[\epsilon x_i] = 0 \implies \mathbb{E}[x_i y_i] = \alpha\,\mathbb{E}[x_i] + \beta\,\mathbb{E}[x_i^2]$

now we can estimate $\mathbb{E}[x_i]$, $\mathbb{E}[y_i]$, $Var(x_i)$ and $Var(y_i)$ from the sample,

using $\mathbb{E}[x_i y_i] = Cov(x_i, y_i) + \mathbb{E}[x_i]\,\mathbb{E}[y_i]$ and $E[x_i^2] = Var(x_i) + E[x_i]^2$ and estimating $Cov(x_i, y_i)$ from the sample, we solve 3 equation in 3 variables to get the estimates for α, β and σ

solving these 3 equation gives $\alpha = \mathbb{E}[y_i] - \mathbb{E}[x_i]\beta$, $\beta = \frac{Cov(x_i,y_i)}{Var(x_i)}$ and $\sigma = \sqrt{Var(y_i) - Cov(x_i, y_i)\beta}$

```
library(quantmod)

getSymbols('TCS.NS')
```

```
## [1] "TCS.NS"
```

```
getSymbols('^NSEI')
```

```
## [1] "^NSEI"
```

```
TCS_rt = diff(log(TCS.NS$TCS.NS.Adjusted))
Nifty_rt = diff(log(NSEI$NSEI.Adjusted))
retrn = cbind.xts(TCS_rt,Nifty_rt)
retrn = na.omit(data.frame(retrn))
```

```r
beta = cov(retrn$NSEI.Adjusted,retrn$TCS.NS.Adjusted)/var(retrn$NSEI.Adjusted)

alpha = mean(retrn$TCS.NS.Adjusted) - mean(retrn$NSEI.Adjusted)*beta

sigma = sqrt(var(retrn$TCS.NS.Adjusted) - cov(retrn$NSEI.Adjusted,retrn$TCS.NS.Adjusted)*beta)
```

$\alpha = 4.557948 \times 10^{-4}$ $\beta = 0.743205$ $\sigma = 0.0161853$

2. Estimate the parameters using the `lm` built-in function of `R`. Note that `lm` using the OLS method.

```r
linear_model = lm(TCS.NS.Adjusted ~ NSEI.Adjusted, data=retrn)
summary(linear_model)
```

```
##
## Call:
## lm(formula = TCS.NS.Adjusted ~ NSEI.Adjusted, data = retrn)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.115331 -0.008764 -0.000082  0.008543  0.120660
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.0004558  0.0002669   1.708   0.0878 .
## NSEI.Adjusted 0.7432050  0.0191692  38.771   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01619 on 3678 degrees of freedom
## Multiple R-squared:  0.2901, Adjusted R-squared:  0.2899
## F-statistic:  1503 on 1 and 3678 DF,  p-value: < 2.2e-16
```

$\alpha = 0.0004558$ $\beta = 0.7432052$ $\sigma = 0.01619$

3. Fill-up the following table

Parameters	Method of Moments	OLS
α	0.00045579502	0.0004558
β	0.7432057	0.7432052
σ	0.0161853	0.01619

4. If the current value of Nifty is 18000 and it goes up to 18200. The current value of TCS is Rs. 3200/-. How much you can expect TCS price to go up?

We can expect TCS price to go up to $3200 + \beta * 200 =$ Rs. 3348.64104/- up by Rs. 148.64104