Enhancing Education Policy Estimation: A Novel Ridge Fuzzy Regression Approach for Handling Multicollinearity with Fuzzy Input Data

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Abstract

Multicollinearity often complicates regression analysis, both in classical and fuzzy input setup. This research introduces a new approach that combines ridge regression with fuzzy regression to tackle correlated covariate's impact, with a specific focus on improving education policy systems. Our method utilizes the α-level estimation algorithm and a dataset where Grade Point Average (GPA) serves as a fuzzy input, while input variables remain crisp. We assess our estimator's performance using RMSE and MAPE. This applied research showcases the potential of our method in enhancing education policies through more accurate data-driven decision-making.

Keywords: Education Policy, Multicollinearity, Ridge Regression Model, Ridge Fuzzy Regression Model, Multiple Fuzzy Linear Regression

1 Introduction

Fuzziness often emerges in the domain of education policy formulation and curriculum design within universities due to the multifaceted nature of educational systems. Education policies must contend with diverse student populations, varying learning styles, and evolving societal needs, all of which introduce inherent uncertainty. Curriculum settings, likewise, need to accommodate a wide range of disciplines, teaching methods, and evolving educational paradigms. These complexities make it challenging to precisely quantify the impact of policy changes or curriculum modifications, giving rise to fuzzy elements in decision-making processes. Consequently, incorporating fuzzy logic and fuzzy regression models can provide a more nuanced and adaptive approach to address the intricacies of education policy and curriculum planning in the ever-evolving landscape of higher education.

Among some commonly applied statistical techniques, Linear and Ridge regression models are well-established for handling precise datasets effectively. Nevertheless, real-world data often deviates from precision, presenting us with non-precise data, including linguistic or imprecise data. Such data structures fall outside the scope of classical statistical theory. To address this challenge, researchers have introduced the fuzzy regression model.

The inception of the fuzzy regression model is attributed to Tanaka et al.(1982) [1], and since the beginning, it has found utility in the analysis of unconventional datasets. Analogous to traditional statistical theory's multiple regression models, fuzzy multiple regression models also contend with the challenge of multicollinearity. To address this concern, we introduce the ridge fuzzy regression model, a cutting-edge method that combines ridge regression and the fuzzy regression model.. This fusion's main goal is to lessen multicollinearity's negative effects when they occur. We propose the α-level estimation approach, which is based on the α-level ridge loss function, to build the ridge fuzzy regression model.

The format of this paper goes as, Section [2], begins by presenting the methodology encompassing the fuzzy ridge regression model, also fuzzy numbers and the alpha-level estimate process are also explained. Additionally, we outline the criteria for evaluating performance. Moving on to Section [3], We explore the use of the fuzzy multiple regression model in practice, applying the α-level estimation method to a dataset pertaining to student performance. Here, we discuss the fuzzy coefficients alongside their respective alpha-levels, concluding with a performance comparison against a simpler fuzzy regression model. Section [4] marks the culmination of our work, where we emphasize the significance of these estimation techniques within the realm of education. Lastly, in Section [5], we explore potential avenues for future research in this domain.

2 Methodology

The primary model of multiple linear regressoin is defined as,

$$Y = X\beta + \epsilon \tag{1}$$

with the least square MSE for the regression coefficient is

$$\hat{\beta_{OLS}} = (X'X)^{-1} X'Y \tag{2}$$

in the presence of multicollinearity or when $p > n$, model experiences non-orthogonality hence makes the estimators biased. Ridge regssion model in handles these situations well.

2.1 Ridge Regression Model

Hoerl and Kennard (1976) [4] introduced the ridge regression model to resolve the issue of non-orthongonality by incorportaing a positive constant with $X'X$ and got the estimate of β as,

$$\hat{\beta}_{RIDGE} = (X'X + kI)^{-1} X'Y \tag{3}$$

, where $k(> 0)$ is the postive tuning parameter.
The formula for the MSE of the Ridge Estimator is given as,

$$MSE(\hat{\beta}_{RIDGE}) = \sigma^2 \left((X'X + kI)^{-1} X'X(X'X + kI)^{-1}\right) + (k^2\beta'(X'X + kI)^{-2}\beta) \tag{4}$$

The bias-variance trade-off idea serves as another justification for using the ridge regression model. As the regularization parameter (k) increases, it forces the regression coefficients closer to zero, effectively shrinking the penalty term. This process reduces the model's flexibility, leading to decreased variance but increased bias. Consequently, by substantially curbing variance while accepting a moderate increase in bias, ridge regression can yield improved outcomes compared to the least squares estimation method.

2.2 Fuzzy Regression Model

However, in practical scenarios, we frequently encounter linguistic or imprecise data, exemplified by terms like 'young,' 'tall,' or 'high.' To address such data types, Zadeh's introduction of fuzzy sets [5] has proven invaluable. Fuzzy regression models, harnessing the power of fuzzy data represented through fuzzy sets, have garnered attention and demonstrated their utility across various domains. Analogous to traditional multiple regression techniques, multicollinearity issues often arise within fuzzy multiple regression models. This creates a requirement for the ridge fuzzy regression model, which smoothly combines the ridge regression principles with fuzzy data in an effort to overcome the difficulties presented by multicollinearity.

2.3 Fuzzy Numbers

A fuzzy set is an ordered pair set $A = \{(x, \mu_A(x)) : x\epsilon X\}$, where $\mu_A(X) : X \to [0,1]$ represents the membership function of x in a set A.

A fuzzy set's support is determined by $H(A) = \{x\epsilon R | \mu_A(x) > 0\}$. A set $A_\alpha = \{x\epsilon R^1 : \mu_A(x) \geq \alpha\}$ contains all the elements in X for an α-level set, where A is crisp set with range of α as $[0,1]$, while A_α membership value in A is greater than or equal to α. An α-level set of fuzzy set A can be represented as,

$$A(\alpha) = [l_A(\alpha), r_A(\alpha)] \tag{5}$$

where A is normal, i.e. $\mu_A(x_0) = 1$ for some $x_0\epsilon\mathbb{R}$ and $\mu_A(x)$ is convex.

A fuzzy number A is considered to be an LR-fuzzy number as a special case if the following membership function describes it:

$$\mu_A(x) = \begin{cases} \mathcal{L}_A\left(\frac{n-x}{l_n}\right) & \text{for } 0 \leq n - x \leq l_n \\ \mathcal{R}_A\left(\frac{x-n}{r_n}\right) & \text{for } 0 \leq x - n \leq r_n \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where, $'n'$ is the mode, and $'l'_n$, $'r'_n$ are called the left and right spreads respectively and $'r'$ and $'l' > 0$. These spreads, which can be symmetric or asymmetric, illustrate how fuzzy a number is. $\mathcal{L}_A$ and $\mathcal{R}_A$ are shape functions of A and They are constant, left-continuous, and non-increasing functions that confirm the characteristics of the class of fuzzy sets in order to ensure that:

$$\mathcal{L}_A(0) = \mathcal{R}_A(0) = 1 \tag{7}$$

and,

$$\mathcal{L}_A(x) = \mathcal{R}_A(x) = 0, \quad x \in \mathbb{R}[0,1) \tag{8}$$

In particular, if

$$\mathcal{L}_A(x) = \mathcal{R}_A(x) = 1 - x \tag{9}$$

Afterwards, A is known as a triangular fuzzy number and is represented by,

$$A = (n, l_n, r_n) \tag{10}$$

A fuzzy set can be represented by either its membership function or its α-level sets, according to Zadeh's (1975) resolution identity theorem [7]. Assume that A is a fuzzy number with a membership function of $\mu_A(x)$ and a set of $A(\alpha)$ levels. This theorem states that it can be written as follows:

$$\mu_A(x) = \sup_{\alpha \in [0,1]} \{\alpha \cdot I_A(\alpha)(x)\} \tag{11}$$

where $I_A(\alpha)(x)$ displays the defining characteristic function for the α-level set of A.

2.4 Fuzzy Regression Model

Consider the multiple fuzzy linear regression model:

$$Y_i = A_0 + A_1 X_{i1} + \cdots + A_p X_{ip} + E_i, i = 1, \ldots, n \tag{12}$$

The classical regression model is a specific case of the fuzzy regression model, just as crisp data is a special example of fuzzy data.

2.5 α-level estimation algorithm

To estimate the parameters of our ridge fuzzy regression model, we use the α-level ridge loss function. When used with the traditional ridge regression model, it serves to reduce the conventional ridge loss function. The following is a description of the α-level estimation [2] algorithm, which relies on the α-level ridge loss function:

Step 1: Make the dependent variable Y's $\alpha - level$. The set of a "alpha-level" is given by for any positive integer j. $A = \{\alpha_l : \alpha_l \epsilon (0,1), l = 1, 2, ..., s\} \cup \{0, 1\}$.

Step 2: Find the estimator $\hat{l_{A_j}}(1)$ using the ridge estimation method. $\hat{r_{A_j}}(1)$ of $l_{A_j}(1)$ and $r_{A_j}(1)$ by reducing the modified ridge loss with the α-level functions:

$$\sum_{i=1}^{n} \left(l_{Y_i}(1) - \sum_{j=1}^{p} l_{A_j}(1) l_{X_{ij}}(1) \right)^2 + k \sum_{j=1}^{p} l_{A_j}(1)^2 \tag{13}$$

and

$$\sum_{i=1}^{n} \left(r_{Y_i}(1) - \sum_{j=1}^{p} r_{A_j}(1) r_{X_{ij}}(1) \right)^2 + k \sum_{j=1}^{p} r_{A_j}(1)^2 \tag{14}$$

Step 3: In Step 1, we make the assumption that the set A contains the elements $\alpha^* = \max_{\alpha_j} A$. The intermediate estimators $\bar{l_{A_j}}(\alpha^*)$ and $\bar{r_{A_j}}(\alpha^*)$ are then discovered by minimizing the following:

$$\sum_{i=1}^{n} \left(l_{Y_i}(\alpha^*) - \sum_{j=1}^{p} l_{A_j}(\alpha^*) l_{X_{ij}}(\alpha^*) \right)^2 + k \sum_{j=1}^{p} l_{A_j}(\alpha^*)^2 \tag{15}$$

and

$$\sum_{i=1}^{n}\left(r_{Y_i}(\alpha^*)-\sum_{j=1}^{p} r_{A_j}(\alpha^*) r_{X_{ij}}(\alpha^*)\right)^2+k\sum_{j=1}^{p} r_{A_j}(\alpha^*)^2 \tag{16}$$

respectively.
Thus, we obtain the estimators $\hat{l_{A_j}}(\alpha^*)$ and $\hat{r_{A_j}}(\alpha^*)$ through altering the intermediate estimators $\bar{l_{A_j}}(\alpha^*)$ and $\bar{r_{A_j}}(\alpha^*)$ so that The estimated coefficients obtained from Step 3 are used to create a membership function with a predetermined shape. For the same, we employ the following min and max operators, which are:

$$\hat{l_{A_j}}(\alpha^*)=\min\left\{\bar{l_{A_j}}(\alpha^*), \hat{l_{A_j}}(1)\right\} \tag{17}$$

and

$$\hat{r_{A_j}}(\alpha^*)=\max\left\{\bar{r_{A_j}}(\alpha^*), \hat{r_{A_j}}(1)\right\} \tag{18}$$

Step 4: Moving forward, for any $\alpha\epsilon(0,1)$, we obtain the intermediate estimators $\bar{l_{A_j}}(\alpha_l)$ and $\bar{r_{A_j}}(\alpha_l)$ of $l_{A_j}(\alpha_l)$ and $r_{A_j}(\alpha_l)$, employing the altered ridge loss functions created in Step 3. We also discover the estimators. $\hat{l_{A_j}}(\alpha_l)$ and $\hat{r_{A_j}}(\alpha_l)$ of $l_{A_j}(\alpha_l)$ and $r_{A_j}(\alpha_l)$ by altering the intermediate estimators so that they adopt the form of the pre-defined form's membership function.

$$\hat{l_{A_j}}(\alpha_l)=\begin{cases}\max\{\hat{l_{A_j}}(\alpha^*), \min\{\bar{l_{A_j}}(\alpha_l), \hat{l_{A_j}}(1)\}\} & \text{if } \alpha_l \leq \alpha^* \\ \min\{\bar{l_{A_j}}(\alpha_l), \hat{l_{A_j}}(\alpha^*)\} & \text{if } \alpha_l > \alpha^*.\end{cases} \tag{19}$$

and

$$\hat{r_{A_j}}(\alpha_l)=\begin{cases}\min\{\hat{r_{A_j}}(\alpha^*), \max\{\bar{r_{A_j}}(\alpha_l), \hat{r_{A_j}}(1)\}\} & \text{if } \alpha_l \leq \alpha^* \\ \max\{\bar{r_{A_j}}(\alpha_l), \hat{r_{A_j}}(\alpha^*)\} & \text{if } \alpha_l > \alpha^*\end{cases} \tag{20}$$

Step 5: Carry out the process repeatedly to discover $\hat{l_{A_j}}(0)$ and $\hat{r_{A_j}}(0)$ of $l_{A_j}(0)$ and $r_{A_j}(0)$.

Step 6: Finally, using linear regression on the estimated $\alpha-level$ sets, find the membership functions $\mu_{A_j}(x)$ for the fuzzy regression coefficients A_j $(j=0,1,\ldots,p)$.

$\hat{A}_j(\alpha_l)=\left[\hat{l_{A_j}}(\alpha_l), \hat{r_{A_j}}(\alpha_l)\right]$ $(l=1,2,\ldots,p)$, $\hat{A}_j(0)$, and $\hat{A}_j(1)$ Adding a constraint such as: $\mu_{\hat{A_j}}\left(\hat{l_{A_j}}(1)\right)=\mu_{\hat{A_j}}\left(\hat{r_{A_j}}(1)\right)=1$ at the top of the predefined memebership function satisfying the condition of α level 1.

3 Empirical Application

In this section the performance the ridge fuzzy estimator is illustrated through an example of fuzzy input and crisp output. Where, the mid-point of the response variable (Y_i) is assumed to be centered, and the covariates standarized.

In particular, the Diamond distance is employed as the distance metric, and the measures of RMSE (Root Mean Square Error) and MAPE (Mean Absolute Percentage Error) are used to evaluate the performance.

$$RMSE_{Fuzzy} = \frac{1}{n}\sum_{i=1}^{n} d^2\left(Y_i, \hat{Y_i}\right)$$
$$= \sqrt{\frac{1}{n}\sum_{i=1}^{n} (y_{il} - \hat{y_{il}})^2 + (y_{im} - \hat{y_{im}})^2 + (y_{ir} - \hat{y_{ir}})^2} \tag{21}$$

and

$$MAPE_{Fuzzy} = \frac{100\%}{n}\sum_{i=1}^{n}\left(\left|\frac{y_{il} - \hat{y_{il}}}{y_{il}}\right| + \left|\frac{y_{im} - \hat{y_{im}}}{y_{im}}\right| + \left|\frac{y_{ir} - \hat{y_{ir}}}{y_{ir}}\right|\right) \tag{22}$$

where, the observed values are as $Y_i = (y_{il}, y_{im}, y_{ir})$ and the fitted values as $\hat{Y_i} = (\hat{y_{il}}, \hat{y_{im}}, \hat{y_{ir}})$.

3.1 Example

A survey was conducted among 25 students from an estmeed University. Consider the response below in a form of a dataset in the Table 3.1 :

Table 3.1: Student Performance Dataset

Slno.	$(\mathbf{y}, \mathbf{s})$	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$
1	3.7 (0.25)	1370	3.6	18	52000
2	3.9 (0.3)	1490	3.8	20	61000
3	3.6 (0.35)	1320	3.5	22	48000
4	3.8 (0.2)	1540	3.9	16	69000
5	3.8 (0.25)	1385	3.7	19	54000
6	3.9 (0.3)	1460	3.8	18	59000
7	3.5 (0.35)	1290	3.4	24	45000
8	4.0 (0.15)	1570	4	15	72000
9	3.4 (0.4)	1250	3.3	25	41000
10	3.9 (0.3)	1505	3.9	17	63000
11	3.7 (0.3)	1400	3.7	21	56000
12	3.9 (0.25)	1510	3.9	18	64000
13	3.6 (0.4)	1310	3.5	23	47000
14	4.0 (0.2)	1560	4	16	71000
15	3.7 (0.25)	1360	3.6	20	51000
16	3.8 (0.3)	1485	3.8	19	60000
17	3.5 (0.35)	1280	3.4	22	44000
18	4.0 (0.15)	1580	4	14	73000
19	3.4 (0.4)	1265	3.3	24	42000
20	3.9 (0.3)	1520	3.9	17	65000
21	3.7 (0.3)	1420	3.7	20	57000
22	3.9 (0.25)	1535	3.9	18	68000
23	3.6 (0.4)	1330	3.5	21	46000
24	4.0 (0.2)	1555	4	15	70000
25	3.8 (0.25)	1390	3.7	19	55000

where, the variables are as follows:

Variable Index	**Variable Name**	**Description**
y	GPA	Student's Current GPA (Fuzzy Input)
s	Spread	The fuzzy spread of the GPA variable
x_1	SAT Score	Student's High School SAT Score
x_2	High School GPA	Student's GPA in high school
x_3	Study Hours Per Week	Hours a student study per week now
x_4	Family Income	Student's current family income (Annual in Rs.)

In the analysis, two diagnostic tools: Variance Inflation Factors (VIF) and the condition number (CN) are used, to thoroughly assess the multicollinearity within our dataset. The VIF values are summarized in Table 3.2, which initially hinted the multicollinearity concerns, a deeper examination via the CN metric revealed a stark reality. The formula for the Condition Number is given as,

$$CN = \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} \tag{23}$$

, where λ_{max} and λ_{min} are respectively the matrix's $(X'X)$ largest and smallest eigen values. With a calculated CN of $1483060 (> 30)$—the dataset exhibited severe multicollinearity. This convergence of evidence from both VIF values and the condition number underscores the unequivocal presence of multicollinearity. Consequently, we deemed Ridge Regression as the appropriate approach to address this challenge, given its capacity to mitigate multicollinearity's adverse effects, enhancing our dataset's predictive accuracy and model stability.

Table 3.2: VIF Values of the Covariates

SAT Score	**High School GPA**	**Study Hours Per Week**	**Family Income**
92.5265	47.7367	8.6340	74.6281

The recently suggested ridge fuzzy regression in this investigation to assess its performance on the response variable, GPA Score. The outcomes, as illustrated in Table 3.3,compare the fitted values obtained by the multiple fuzzy linear model to those produced by the ridge fuzzy regression model. The results unequivocally demonstrate that the ridge fuzzy regression method offers a notably superior description of the original data when comparable to the method of multiple fuzzy linear regression. This result highlights the ridge fuzzy regression model's improved accuracy and effectiveness in capturing the underlying patterns and subtleties within the dataset, highlighting its potential as a useful tool in educational data analysis.

Table 3.3: Fitted Values of GPA

Sl no.	**Fitted Values**		
	$\mathbf{Y = (y, s)}$	$\mathbf{\hat{Y}_{OLS}}$	$\mathbf{\hat{Y}_{RIDGE}}$
1	(3.7, 0.25)	(3.690, 2.566)	(3.705, 0.459)
2	(3.9, 0.3)	(3.820, 2.799)	(3.810, 0.490)
3	(3.6, 0.35)	(3.588, 2.557)	(3.591, 0.510)
4	(3.8, 0.2)	(3.878, 2.808)	(3.933, 0.438)
5	(3.8, 0.25)	(3.785, 2.609)	(3.736, 0.473)
6	(3.9, 0.3)	(3.856, 2.714)	(3.822, 0.462)
7	(3.5, 0.35)	(3.485, 2.544)	(3.513, 0.536)
8	(4.0, 0.15)	(3.974, 2.839)	(3.998, 0.426)
9	(3.4, 0.4)	(3.401, 2.497)	(3.442, 0.548)
10	(3.9, 0.3)	(3.940, 2.769)	(3.894, 0.450)
11	(3.7, 0.3)	(3.750, 2.670)	(3.720, 0.500)
12	(3.9, 0.25)	(3.923, 2.795)	(3.885, 0.464)
13	(3.6, 0.4)	(3.593, 2.559)	(3.572, 0.523)
14	(4.0, 0.2)	(3.980, 2.841)	(3.980, 0.439)
15	(3.7, 0.25)	(3.689, 2.586)	(3.673, 0.485)
16	(3.8, 0.3)	(3.837, 2.773)	(3.819, 0.476)
17	(3.5, 0.35)	(3.510, 2.492)	(3.533, 0.509)
18	(4.0, 0.15)	(3.969, 2.837)	(4.017, 0.413)
19	(3.4, 0.4)	(3.395, 2.503)	(3.462, 0.535)
20	(3.9, 0.3)	(3.917, 2.794)	(3.904, 0.451)
21	(3.7, 0.3)	(3.744, 2.684)	(3.742, 0.487)
22	(3.9, 0.25)	(3.877, 2.836)	(3.903, 0.465)
23	(3.6, 0.4)	(3.615, 2.555)	(3.603, 0.497)
24	(4.0, 0.2)	(3.997, 2.815)	(3.989, 0.425)
25	(3.8, 0.25)	(3.774, 2.617)	(3.740, 0.473)

Finally, Table 3.4 shows the peformance metrics of the ridge fuzzy regression against the multiple fuzzy linear regression model.

Table 3.4: Performance Metrics

	RMSE_Fuzzy	MAPE_Fuzzy (%)
Ridge Fuzzy Regression	0.2893	110
Multiple Fuzzy Regression	3.4003	128

When compared to the Multiple Fuzzy Regression Model in the context of GPA analysis, it was shown that the Ridge Fuzzy Regression model performed better. This superiority was evident in the form of lower Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE) values (shown in Table 3.4).

One crucial factor contributing to this performance gap was the presence of multicollinearity within our dataset.The Ridge Fuzzy Regression model outperformed the Multiple Fuzzy Regression Model in terms of predictive accuracy because it successfully addressed and mitigated the negative impacts of multicollinearity.

The ridge fuzzy regression model is illustrated visually in the plot showing observed versus fitted values (see Fig 1). The fitted values from the ridge fuzzy multiple regression are indicated by red triangles in this image, while the observed values are shown by black triangles.

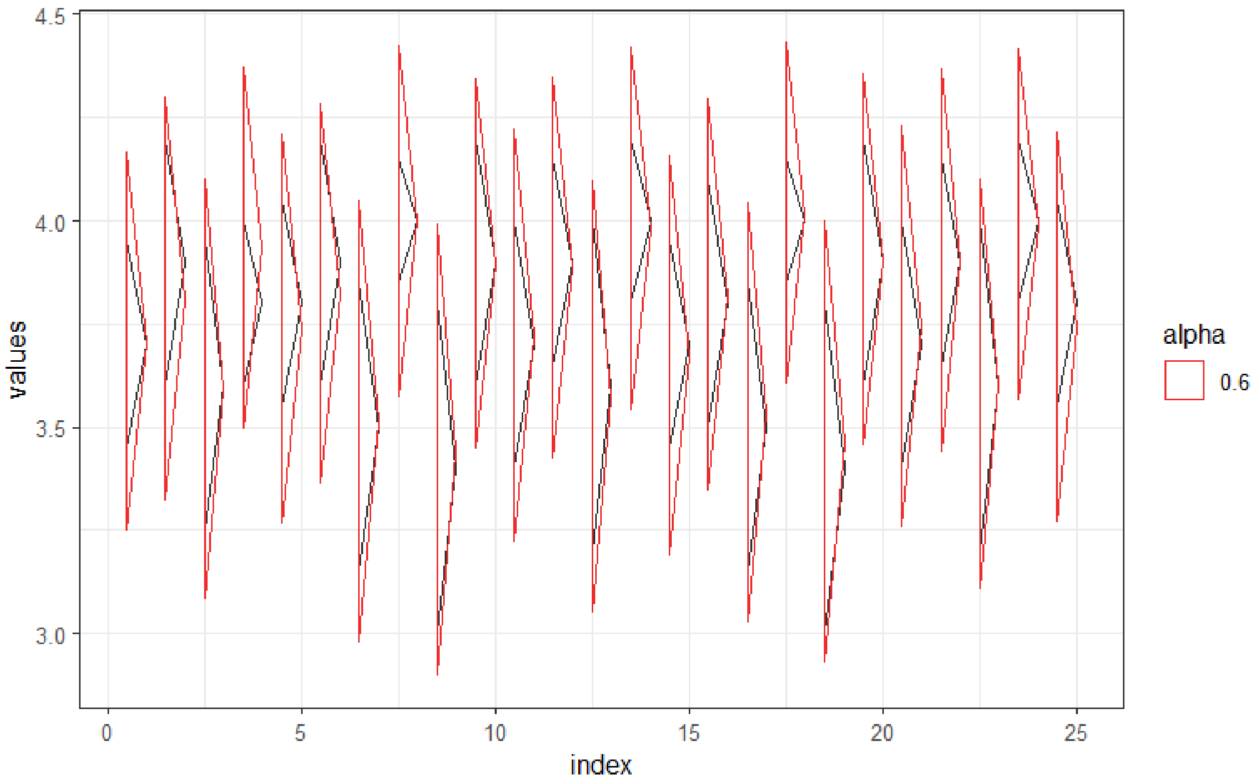

Fig 1: The Observed versus the Estimated values for the Ridge Fuzzy Regression Model

In summary, the findings underscore the robustness of the Ridge Fuzzy Regression model, emphasizing its capability to deliver more accurate predictions when dealing with multicollinear data, making it a valuable tool for modeling complex relationships in such scenarios.

4 Conclusion

In summary, this investigation paves the way for academic institutions, educational governing bodies, ministries, online learning platforms, and diverse institutions to formulate more effective education policies tailored to the needs of students, considering their academic performance and various attributes. Notably, the study demonstrates the superior efficacy of the ridge fuzzy regression model over its classical fuzzy linear regression counterpart in the presence of multicollinearity. This research contributes significantly to the comprehension of handling linguistic data and addressing their inherent fuzziness through key statistical methodologies, elucidating valuable insights attainable through regression analysis techniques.

Furthermore, the practical application of the fuzzy regression model remains largely untapped on a broader scale in real-world scenarios, where human ambiguities prevail across diverse fields. The current research successfully illuminates a noteworthy application within the pivotal realm of education policy, shedding light on the model's potential to navigate linguistic uncertainties effectively. The implications extend beyond the confines of this study, urging a broader exploration of fuzzy regression applications in addressing complex, real-world challenges where ambiguity is a recurrent facet.

In conclusion, this research serves as a significant contribution to understanding and harnessing the potential of fuzzy regression models, particularly in the critical domain of education policy. The findings underscore the relevance of these models in navigating linguistic nuances and offer a promising avenue for future research and practical implementations in diverse sectors encountering inherent uncertainties.

5 Future Scope

The research described here lays the foundation for a wide range of prospective future projects involving the use of the Ridge Fuzzy Regression Model and educational data analysis. Firstly, there is a pressing need to refine and expand the repertoire of fuzzy regression models, exploring hybrid variants and their suitability for complex, multidimensional educational datasets. Secondly, future studies should extend their focus beyond data analysis and delve into the practical implementation and evaluation of education policies informed by the Ridge Fuzzy Regression Model. Investigating the real-world impact of such policies is paramount. Furthermore, the integration of machine learning algorithms with fuzzy regression models holds considerable potential, offering opportunities for more adaptable and robust models. Beyond education, cross-disciplinary applications of the Ridge Fuzzy Regression Model deserve exploration. Lastly, optimizing educational resource allocation based on the model's insights and enhancing interpretability and visualization techniques are vital aspects of future research. These endeavors collectively aim to advance our understanding of educational data, refine modeling techniques, and foster tangible improvements in education policies and practices across diverse contexts.

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