

Lab .1)  $E[f(x)] = \int_{-\infty}^{\infty} dx f(x) P_X(x)$

a)  $E[f(w) + g(v)]$

$$= \int_{-\infty}^{\infty} dx (f(w) + g(v)) P_X(x)$$

$$= \int_{-\infty}^{\infty} dx f(w) P_X(x) + \int_{-\infty}^{\infty} dx g(v) P_X(x)$$

$$= E[f(w)] + E[g(v)]$$

b)  $E[X + a] = \int_{-\infty}^{\infty} dx (x + a) P_X(x)$

$$= \int_{-\infty}^{\infty} dx x P_X(x) + a \int_{-\infty}^{\infty} dx P_X(x)$$

$$= E[X]$$

(Since the sum of all probabilities is 1.)

$$c) E[aX] = \int_{-\infty}^{\infty} dx (aX) P_X(x) = a \int_{-\infty}^{\infty} dx x P_X(x)$$

$$= aE[X]$$

$$d) \text{Var}(aX) = E[(aX - E[aX])^2]$$

$$= E[(aX - aE[X])^2] \quad (\text{Using c})$$

$$= E[a^2(X - E[X])^2]$$

$$= a^2 E[(X - E[X])^2] \quad (\text{Again, using c})$$

$$= a^2 \text{Var}(X)$$

$$2) \text{ Linearity : } E[af(x) + bg(y)]$$

$$= aE[f(x)] + bE[g(y)]$$

$$\text{Let } Z = X + Y$$

$$\text{Var}(Z) = E[(Z - E[Z])^2]$$

$$= E[Z^2 - 2ZE[Z] + (E[Z])^2]$$

$$= E[(X+Y)^2 - 2(X+Y)E[X+Y]$$

$$+ (E[X+Y])^2]$$

$$\begin{aligned}
&= E[\underline{x}^2 + 2\underline{xy} + \underline{y}^2 - 2(\underline{x} + \underline{y})(E[x] \\
&\quad + E[y]) + (E[x] + E[y])^2] \\
&= E[(x^2 - 2x E[x] + E[x]^2) + (y^2 \\
&\quad - 2y E[y] + E[y]^2) + 2xy - 2xE[y] \\
&\quad - 2yE[x] + 2E[x]E[y]] \\
&= E[(x - E[x])^2] + E[(y - E[y])^2] \\
&\quad + 2E[xy] - 2E[x]E[y] - \cancel{2E[y]E[x]} \\
&\quad + \cancel{2E[x]E[y]} \\
&= \text{Var}(x) + \text{Var}(y) + 2C_{xy}
\end{aligned}$$

where  $C_{xy} = E[xy] - E[x]E[y]$