$\{X_1, X_2, X_3\}$

$E[X_1] = E[X_2] = E[X_3] = 0$

$Cov(X_1, X_2) = 0 \Rightarrow E[X_1 X_2] = 0$

$|||y, \; E[X_1 X_3] = 0 = E[X_2 X_3]$

$\sigma_{X_1}^2 = 1 = \sigma_{X_2}^2 = \sigma_{X_3}^2 = E[X_1^2]$

$= E[X_2^2] = E[X_3^2] \equiv \sigma^2$

$Y_1 = A_{11} X_1 + A_{12} X_2 + A_{13} X_3$

$Y_2 = A_{21} X_1 + A_{22} X_2 + A_{23} X_3$

$Y_3 = A_{31} X_1 + A_{32} X_2 + A_{33} X_3$

or, $\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} & & \\ & \cdot A_{ij} & \\ & & \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$

$$E[Y_i] = \sum_{j=1}^{3} A_{ij} E[\cancel{X_j}]^{\,0} = 0$$

$$C_{Y_1Y_2} = E[Y_1 Y_2] = A_{11}A_{21} + A_{12}A_{22} + A_{13}A_{23}$$

$$\sigma_{Y_1} = \sqrt{E[Y_1^2]} = \sqrt{A_{11}^2 + A_{12}^2 + A_{13}^2}$$

$$\therefore \rho_{12} = \frac{C_{Y_1Y_2}}{\sigma_{Y_1}\sigma_{Y_2}} = \frac{A_{11}A_{21} + A_{12}A_{22} + A_{13}A_{23}}{\sqrt{A_{11}^2 + A_{12}^2 + A_{13}^2}\sqrt{A_{21}^2 + A_{22}^2 + A_{23}^2}}$$

$$\vec{V}_1 \equiv (A_{11}, A_{12}, A_{13}) \qquad = \frac{\vec{V}_1 \cdot \vec{V}_2}{|\vec{V}_1|\,|\vec{V}_2|}$$

$$\vec{V}_2 \equiv (A_{21}, A_{22}, A_{23})$$

Choose $\vec{V}_1 = (\sin\theta_1, \cos\theta_1, 0)$

& $\vec{V}_2 = (0, \sin\theta_2, \cos\theta_2)$

Then, $\rho_{12} = \sin\theta_2 \cos\theta_1 \equiv \rho_1$

& $Y_1 = X_1 \sin\theta_1 + X_2 \cos\theta_2$

$Y_2 = X_2 \sin\theta_2 + X_3 \cos\theta_2$

But $Y_3 = ?$

$$\rho_{23} = \frac{C_{Y_2 Y_3}}{\sigma_{Y_2}\sigma_{Y_3}} = \frac{\vec{V_2} \cdot \vec{V_3}}{|\vec{V_2}|\,|\vec{V_3}|}$$

where $\vec{V_2} = (A_{21}, A_{22}, A_{23})$

& $\vec{V_3} = (A_{31}, A_{32}, A_{33})$

But we already have,

$\vec{V_2} = (0, \sin\theta_2, \cos\theta_2)$

$\therefore$ let $\vec{V_3} = (\sin\theta_3, 0, \cos\theta_3)$

Then, $\rho_{23} = \cos\theta_2 \cos\theta_3 \equiv \rho_2$

$$\& \quad \gamma_{13} = \frac{\vec{V}_1 \cdot \vec{V}_3}{|\vec{V}_1||\vec{V}_3|} = \sin\theta_1 \sin\theta_3 \equiv \gamma_3$$

$$\underline{A} = \begin{pmatrix} \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & \sin\theta_2 & \cos\theta_2 \\ \sin\theta_3 & 0 & \cos\theta_3 \end{pmatrix}$$